CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

KALMAN FILTER AND ITS USE IN ANTENNA TRACKING SYSTEM FOR SATELLITE TRACKING

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LIST OF SYMBOLS

e_k^-	Estimation Error
F	Linearized State matrix
G	White Noise Sequence Matrix
H	Measurement to State Mapping matrix
K	Kalman Gain Matrix
P	State Estimate Error Covariance Matrix
I	Identity Matrix
P^+	Updated Covariance matrix
P^-	Previous Covariance Matrix
Q	Process Noise Covariance Matrix
V	Satellite Velocity
V_K	Measurement Error Matrix
R	Measurement Error Covariance Matrix
W	Process Noise Vector
X	System State Vector
$\hat{X}$	State Estimate Vector
$\hat{X}^-$	State Estimate before Measurement
$\hat{X}^+$	State Estimate after Measurement
X_0	Horizontal Distance to Satellite
$\tilde{X}$	Linearized Value of the State
Z	Measurement Vector
Z_0	Altitude above the Horizon
Φ	State Transition Matrix
β	Time Correlation Constant
θ_{AZ}	Azimuth Position angle
$\dot{\theta}_{AZ}$	Azimuth Velocity
$\ddot{\theta}_{AZ}$	Azimuth acceleration

θ_{EL}	Elevation Position Angle
$\dot{\theta}_{EL}$	Elevation Velocity
$\ddot{\theta}_{EL}$	Elevation Acceleration
$\Delta\theta_{DEF}$	Deflection Error Signal
σ^2	Variance of the Gauss-Markov Process

ABSTRACT

KALMAN FILTER AND ITS USE IN ANTENNA TRACKING SYSTEM FOR SATELLITE TRACKING

By

Ajay Raja Thamneero

Master of Science in Electrical Engineering

A Kalman Filter used in antenna tracking system is to determine the optimal trajectory of a low earth orbiting satellite. This Kalman filter's function is to estimate the current position of the target from a set of noisy measurements obtained by an antenna tracking system. The target under consideration is an earth orbiting satellite moving in an approximately circular orbit. Tracking is performed by an antenna system which consists of a radio frequency feed at the focus of a parabolic reflector.

After a brief discussion of the antenna system, an in depth description of the discrete Kalman filter is presented. Other forms of Kalman filter are briefly discussed before the paper focuses on the application of extended Kalman filter. The Kalman filter used to make real time estimates is the extended Kalman filter. This form of the Kalman filter results in better estimates than the linearized form because the trajectory is updated with each new measurement.

With the accuracy of the extended Kalman filter comes the possibility of divergence. The most common types of divergence and how to minimize their occurrence is coveredin this report. Since actual data is unavailable for real time calculations, the continuous discrete extended Kalman filter is simulated as a discrete linearized Kalman

filter. The simulation shows that the Kalman filter can provide very accurate estimations of the satellite position angles.

CHAPTER 1
INTRODUCTION

This report studies a Kalman filter for an antenna system used to automatically track low earth orbiting land resource surveillance satellites. The worst case condition occurs when the satellite is at the point of closest approach to the earth. This portion of the satellite trajectory is approximated as a crossover course, with the satellite moving at a constant velocity over flat earth. This is commonly used approximation for the tracking of low earth, circular orbit satellites[1].

Since 1960 when Kalman introduced a method for linear filtering, digital implementations of these recursive algorithms have found widespread use. Kalman filters have been used in missile applications, radar tracking systems and satellite communications. Recently, Kalman filters have been used in applications other than the aerospace industry. They were used by Bell Communications to predict telephone loading capacity. They have been used to position offshore drilling vessels and platforms for the oil industry.

Of the myriad of uses for the Kalman filter, airplane and missile tracking applications, analyzed in coordinate systems other than azimuth elevation, appear to dominate the literature.

Sparseness of satellite tracking information appears to be linked to its origin in airplane and missile tracking. Since satellite tracking models are based on, if not equal to, airplane and missile tracking models, the tendency is to work on the easier problem. After seeing the dynamic equations for tracking a satellite and recording the real data, this practice becomes acceptable.

This topic is not new, so plenty should have been written on it. Alas, it appears that it has not received the attention it warrants.

The discrete and continuous Kalman filters are the two basic forms of kalman filters. The paper discusses about both the forms of Kalman filters and the combination of both filters. Both the discrete and continuous forms require that the system dynamic equations be linear. For nonlinear systems, the options are the linearized Kalman filter or the extended Kalman filter. The linearized form linearizes the equations about a nominal trajectory independent of measurement data. It follows the pre-determined trajectory even if it is incorrect. The extended form linearizes the equations about a trajectory which changes with each new measurement. In the case where the first trajectory was slightly incorrect, this form uses the new measurements to correct itself.

For real time applications, where the system dynamic equations are invariably nonlinear, the extended Kalman filter should be used. The primary reason for using this technique is that it yields better estimates of the system states than the linearized form because of utilization of new measurements. The price for this increased accuracy is an increased probability that the error covariance matrix will lose its symmetry and cease to be positive definite. To avoid this occurrence, simulation models should be run several times to ensure that the error covariance matrix remains bounded, symmetric and positive definite.

Simulations of the results are implemented using a linearized Kalman filter. The nominal trajectory that the linearized filter must follow is generated using a Gaussian random number generator. The extended Kalman filter is not used in the simulation because it requires continuous updates of measurements in order that a new trajectory is estimated. Since continuous measurements are unavailable in a simulation, the closest approximation is to have discrete measurements generated by a random number generator. These measurements are assumed to occur every ten milliseconds.

CHAPTER 2

KALMAN FILTER

The Kalman filter is a mathematical method named after Rudolf E Kalman. Its purpose is to use measurements that are observed overtime that contain noise or random variations and other inaccuracies, and produce values that tend to be closer to the true values of the measurements and their associated calculated values. The Kalman filter has many applications ion technology and is an essential part of the development of space and military technology.

When Kalman introduced his new approach to linear filtering in 1960, the world did not take a holiday. It already had the Wiener filter, which approached and performed well. After Kalman released what amounted to an update to his first paper, someone actually tried to use it. But with computers being slow at that time, these new filters weren't very efficient or cost effective.

A Kalman filter is a simple method for solving a weighted least squares problem. This is accomplished by processing new information recursively instead of in a batch. Recursive computations are more efficient than batch processing because there is no need to store all past information so that current estimates can be computed.

The Kalman filter produces estimates of the true values of measurements and their associated calculated values by predicting a value, estimating the uncertainty of the predicted value, and computing a weighted average of the predicted value and the measured value. The most weight is given to the value with the least uncertainty. The estimates produced by the method tend to be closer to the true values than the original measurements because the weighted average has a better estimated uncertainty than either of the values that went into the weighted average. The basic Kalman filter comes in two forms continuous and discrete Kalman filter.

2.1 CONTINUOUS KALMAN FILTER

The sampling time is assumed to be small enough to approximate continuous state estimate and covariance error propagation. In other words, if measurements are collected every nanosecond and propagation occurs at the same time or more frequently, the process can usually be considered continuous.

The continuous Kalman filter is usually not implemented but it can be used as an aid in better understanding the properties of discrete Kalman filter. For small sampling times the discrete Kalman filter in fact becomes a continuous Kalman filter. Because the continuous Kalman filter does not the derivation of a fundamental matrix for filtering equations, the continuous filter can also be used as a check of the discrete filter. The continuous Kalman filter under any steady state conditions, to derive the transfer functions that exactly represent the filter. These transfer functions cannot only be used to better understand the operation of Kalman filter but can also aid with such mundane issues as helping to choose the correct amount of process noise to use.

Continuous Kalman filtering theory first requires that our model of the real world be described by a set of differential equations. These equations must be cast in matrix or state space form as

$$\dot{x} = Fx + Gu + w \qquad 2.1$$

where x is the column vector with the states of the system, F is the system dynamics matrix, u is the deterministic or control vector and w is the white noise process represented as a column vector. There is a process noise matrix Q that is related to the process noise column vector according to

$$Q = E[ww^T] \qquad 2.2$$

The process noise sometimes has no physical meaning; it is often used as a device for telling the filter that we understand the filter's model of the real world may not be precise, that is larger values of process noise indicate in our model of the real world. The continuous Kalman filter also requires the measurements be linearly related to the states according to

$$z = Hx + v \tag{2.3}$$

where z is the measurement vector, H is the measurement matrix and v is the measurement white noise. The measurement noise matrix R is related to the measurement noise vector v according to

$$R = E[vv^T] \tag{2.4}$$

where the continuous measurement noise matrix consists of spectral densities describing the measurement noise. The resultant continuous Kalman filter is described by the matrix differential equation

$$\dot{\hat{x}} = F\hat{x} + Gu + K[z - H\hat{x}] \tag{2.5}$$

The continuous Kalman filter no longer requires the fundamental matrix for the propagation of the states. State propagation is accomplished by numerically integrating the matrix differential equation involving the system dynamics matrix. The Kalman gain K, required by the preceding Kalman filtering differential equation are now obtained by first integrating the nonlinear matrix differential equation for the covariance matrix

$$\dot{P} = -PH^T R^{-1} HP + PF^T + FP + Q \tag{2.6}$$

And solving the matrix equation for the gain in terms of the covariance matrix or

$$K = PH^T R^{-1}$$

The usefulness of the continuous Kalman filters as an efficient tool has unfortunately gone the way of the analog computer. It has not completely disappeared because control systems courses are taught using continuous dynamic equations and the continuous Kalman filter is useful for conceptualization of system behavior. Apart from this small usage, its sole remaining purpose is to provide fodder for books and research projects.

2.2 DISCRETE KALMAN FILTER

In the discrete Kalman filter, error propagation and state estimation are performed at discrete points in time. This type of filter is the most frequently used version of the two forms for practical real time applications. Complex calculations and faster digital computers sounded the death knell for the wiener filter and made the discrete Kalman filter a popular choice among designers.

Before the discrete Kalman filter can be used, the equations must be in a certain form. Without regard to the means of discretization of the system, the filter address the general problem trying to estimate the state of a discrete time controlled process that is governed by the linear stochastic difference equation

$$X_k = A_{k-1}X_{k-1} + B_k U_{k-1} + W_{k-1} \qquad 2.7$$

where

X_k = system state vector at time t_k

A_k = state transition matrix which is applied to the previous state X_{k-1}

B_k = control input matrix which is applied to control vector U_{k-1}

W_k = process noise matrix

The other half of the system equations is the measurement (observation) equation. Measurements occur at discrete points in time in accordance with the relationship

$$Z_k = H_k X_k + V_k \qquad 2.8$$

where

Z_k = measurement vector at time t_k

H_k = matrix connecting measurements to state

V_k = measurement error matrix

The random variables W_k and V_k represent the process and measurement noise. They are assumed to be independent of each other and with normal probability distributions

$W_k \sim$ N(0,Q)

$V_k \sim$ N(0,R)

Where

Q = process noise covariance

R = measurement noise covariance

Both the process noise and measurement noise are assumed to be uncorrelated zero mean sequences with known covariance. In real problems, it is highly probable that cross correlation terms are non zero and can effect system operation. Without actual experimental data it is difficult to determine the system auto correlation function or the power spectral density or whether the process is Gauss Markov. To simplify calculations, the noise and errors are assumed to be uncorrelated zero mean sequences.

Physically, the process noise is a representation of all random disturbances that perturb the target from a known trajectory. Rather than model these uncertainties as unknown trajectory, they are modeled as noise. Measurement errors arise from biases, imperfect calibrations. The covariance matrices of the process noise and measurement errors are defined by

$$E[W_k W_k^T] = Q_k$$
$$E[V_k V_k^T] = R_k$$
$$E[W_k V_k^T] = 0$$

Before the Kalman filter can begin to work, it needs two sets of initial conditions, one for the states and other for the error covariance matrix. The Kalman filter's ability to utilize this a prori data sets it apart from other filtering techniques. The first estimate of the state is its initial condition. To obtain this information, the filter designer can

1. Search the specification governing the performance of the system.

2. Use a known point of the chosen trajectory.

3. Use data collected from similar systems.

Once the system has been started, the remaining state estimates are calculated using the equation

$$\hat{X}_k^- = A_{k-1}\,\hat{X}_{k-1}^+ \qquad 2.9$$

where

(^) = the value is an estimate

(-) = the time immediately before a measurement

(+) = the time immediately after a measurement.

The other initial condition required is the state estimate error covariance matrix. Procedures for finding the initial conditions for this matrix are similar to those finding the initial conditions of the states. With this initial condition and the condition of the states, the filter begins the recursive process. All other estimates of the covariance are calculated using

$$P_k^- = A_{k-1}P_{k-1}^+A_{k-1}^T + Q_{k-1} \qquad 2.10$$

where

Q_k = process noise covariance matrix

P_k = state estimate error covariance matrix.

The P and Q matrices are an important part of the filter. The P matrix is a measure of the uncertainties in the states, i.e. it shows how close the estimate is to the actual trajectory. The matrix Q is for all unknown parameters. For example, if some states were to be eliminated from the state equations, the effects of these missing states would be modeled as elements of the Q matrix. It is good practice to have a nonzero Q. although this creates a suboptimal filter, it does provide flexibility, which in practical filtering problems is a good feature.

After the initial conditions have been established, the recursive process of Kalman filtering can begin. The first equation to be calculated in the cycle is the Kalman gain. When calculated to minimize the mean square estimation error

$$e_k^- = X_k - \hat{X}_k^-$$

The gain matrix is

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \quad 2.11$$

With

K_k = Kalman gain

R_k = measurement error covariance matrix.

The Kalman gain can interpret as a noise rejection matrix. It extracts the greatest signal content from noisy measurements by reducing the errors in the states. With the gain already calculated, the state estimate is updated by a linear blending of the measurement at time t_k and the prior state estimate using the equation

$$\hat{X}_k^+ = \hat{X}_k^- + K_k[Z_k - H_k \hat{X}_k^-] \quad 2.12$$

where

$\hat{X}_k^+$ = updated estimate

$\hat{X}_k^-$ = previous estimate

The Kalman gain blends the new measurements with the old by applying the right amount of corrections to the estimates. Assigning a larger weight to the most recent measurement give more credence to the newly measured data without completely discarding the old. To complete the set of equations which comprise the filter, one additional equation will be needed

$$P_k^+ = [1 - K_k H_k] P_k^- \qquad 2.13$$

where

P_k^+ = updated covariance matrix

P_k^- = previous covariance matrix

Equations (1) through (7) discussed are for the simplest and safest Kalman filter. They are simple because the system dynamic equations are linear, thereby precluding the need for arduous linearization and they are safe because the gains, K and covariance, P are calculated and stored prior to running the system. This guarantees that the covariance matrix remains symmetric and positive definite, the gains and covariance will reach a steady state and the filter will provide correct estimates of the sates.

With practical problems getting difficult, combinations of the continuous and discrete forms are being used with great success. The linearized Kalman filter and the extended Kalman filter are two forms which utilize the combination. Both were developed to handle problems where the nonlinearities were too expensive to ignore.

A better understanding of the Kalman filtering recursive process can be obtained from the graphical form. Enter the prior estimate and its error covariance $\hat{X}_k^-$ and P_k^-

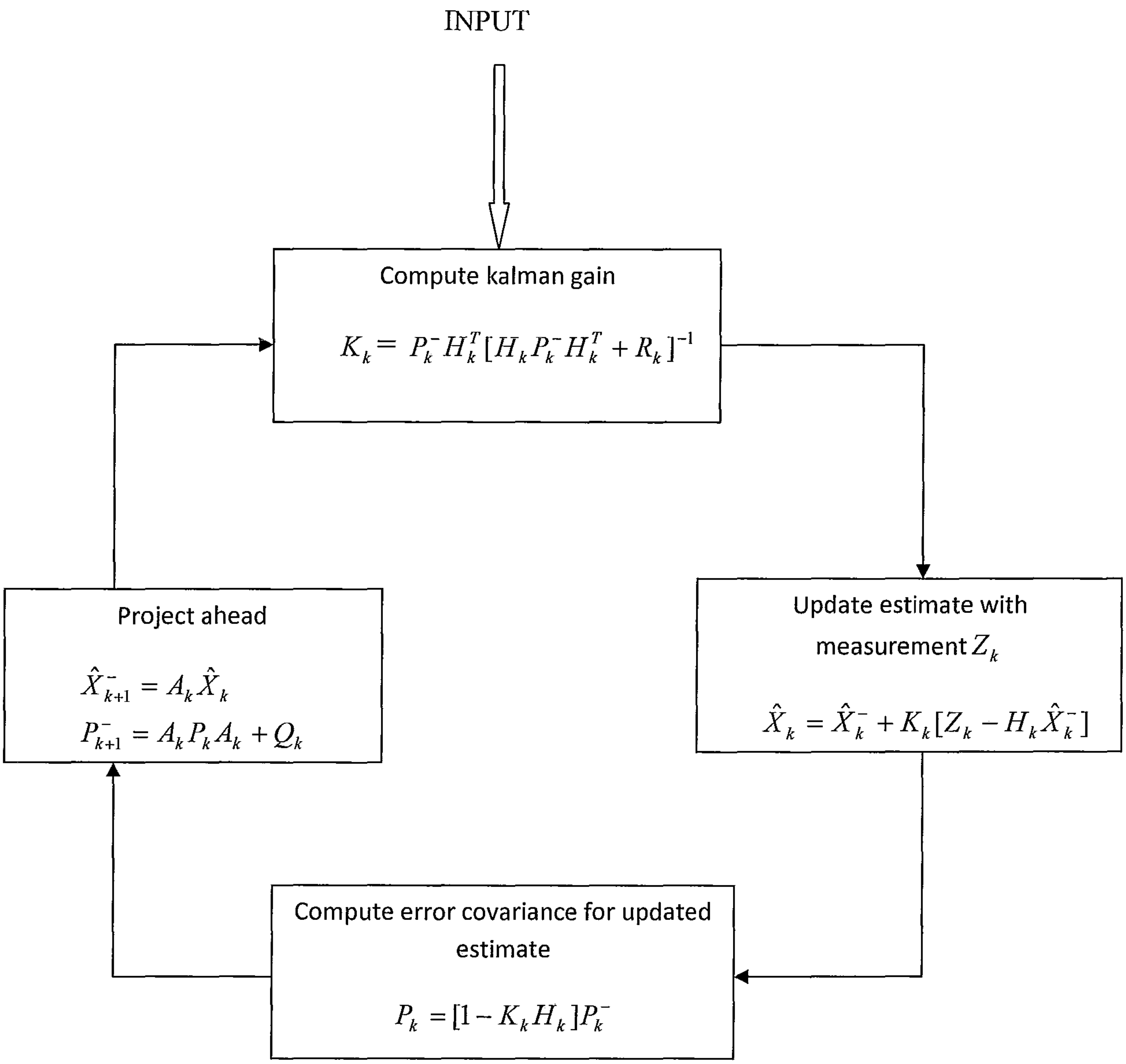

Figure 2.1 The Kalman filter loop

2.3 LINEARIZED KALMAN FILTER

The linearized Kalman filter linearizes the system equations either the differential equations or the state equations about a nominal trajectory. This chosen trajectory is the one of many in an ensemble of possible trajectories. If the conditions are favorable, the filter provides good results. Otherwise the filter fits the right data to the wrong curve and never recovers. The advantage of using a linearized Kalman filter is that it is independent of measurement data; therefore the gains and covariance can be recomputed and stored for later use.

2.4 EXTENDED KALMAN FILTER

It has become common in engineering practice to use an extended Kalman filter whenever the system dynamics and measurements are nonlinear and real time accuracy is required. The extended Kalman filter is the nonlinear version of the Kalman filter which linearizes about the current mean and covariance. The extended Kalman filter propagates the states and errors based on the most recent measurements. This method produces more accurate results than the discrete Kalman filter or the linearized Kalman filter because the estimated trajectory is constantly being refined after each measurement. However, there are disadvantages of using the extended Kalman filter.

Two disadvantages are the real time calculations of the error covariance matrix and the filter gains and divergence. Calculating the errors and gains in real time consumes valuable computing time. Filter divergence occurs when the error covariance predicted by the filter are grossly different from the actual error. These errors emerge from several sources, but three common difficulties arise from modeling errors, round off errors and lack of observability. At one time, the extended Kalman filter might have been considered the de facto standard in the theory of nonlinear state estimation, navigation systems and Gps. However, with the introduction of the unscented Kalman filter, the extended Kalman filter might no longer claim that title, for the reason that the unscented Kalman filter tends to be more robust and more accurate in its estimation in error.

Modeling errors are the designer's problem. If the designer does not have a thorough understanding of the problem to be solved, this inadequacy can appear in the filter. The filter would then try to fit measurement data to the wrong curve and after a period of time, the information becomes useless. To compensate for three unknown parameters, it is best to add some process noise into each state variable. This result in a suboptimal filter but it is one that follows the trajectory.

Round off errors in on line calculations can create numerical instability. To overcome this difficulty, either double precision or floating point arithmetic should be used instead of fixed point arithmetic. If this is not possible, filter simulations should be run several times with different conditions to insure that the solutions are bounded and that the P matrix remains symmetric and positive definite.

The least common divergence problem is the lack of observability of one or more of the states. Clearly if a state cannot be measured and it is unstable, linear combinations of that state will be unstable as well. As a general rule, many states are not observable. In practice, this is sometimes due to hardware constraints and budget restraints. To work within these restrictions, the Kalman gain matrix, K can be viewed as an observer that estimates the values of unmeasured states from those measured. Because these estimates affect system performance, care should be exercised during modeling and simulation.

To minimize the probability of divergence due to observability, the main diagonal elements of the P matrix of the filter simulations should not increase without bound. If there is no round off error problems but the P matrix increases without bound, an observability problem exists.

Divergence aside, the continuous discrete extended Kalman filter equations given below are similar to the discrete EKF propagates the states and covariance continuously but accepts measurements and calculates updates at discrete points in time. Assuming that the system and measurement models are of the form

$$\begin{aligned} \dot{X}(t) &= f(X(t),t) + W(t) \\ Z_k &= H_k(X(t_k)) + V_k \end{aligned} \qquad 2.14$$

The equations for estimating the state and covariance are

$$\begin{aligned} \dot{\hat{X}}(t) &= f(\hat{X}(t),t) \\ \dot{P}(t) &= F(\dot{X}(t),t)P(t) + P(t)F^T(\hat{X}(t),t) + Q(t) \end{aligned} \qquad 2.15$$

where

$$F(\hat{X}(t),t) = \left. \frac{\partial f(X(t),t)}{\partial X(t)} \right|_{X(t)=\hat{X}(t)}$$

The time varying Kalman gain matrix and the matrices that update the predicted estimates are given below

$$K_k = P_k^- H_k^T(\hat{X}_k^-)[H_k(\hat{X}_k^-)P_k H_k^T(\hat{X}_k^-) + R_k]^{-1} \tag{2.16}$$

$$\begin{aligned} \hat{X}_k^+ &= \hat{X}_k^- + K_k[Z_k - H_k(\hat{X}_k^-)] \\ P_k^+ &= [1 - K_k H(\hat{X}_k^-)]P_k^- \end{aligned} \tag{2.17}$$

The continuous discrete extended Kalman filter is only one of the forms. As stated before, continuous discrete EKF means that the error covariance matrix is propagated continuously while measurements are collected at discrete points in time. The different types of extended Kalman filter possible are continuous-continuous, continuous-discrete, discrete-continuous and discrete-discrete.

Continuous-discrete LKF and continuous-discrete EKF has no explanation as to when it is advantageous to use a certain combination. This implies that the choice depends on many factors such as application, the hardware, the required accuracy, the complexity of the dynamic equations and even the designer's whimsical tastes. EKF yields better estimates than the LKF because the nominal trajectory chosen for the LKF is usually not as close to the actual trajectory as is the constantly refined trajectory of the EKF.

The extended Kalman filter is the designer's choice for non linear systems requiring real time accuracy. In exchange for this accuracy is filter divergence. This occurs because the physical world is inherently nonlinear and Kalman filter requires a linear model. To minimize the likelihood of divergence, the model should be as accurate as possible, floating point arithmetic should be used and many states as possible should be observable. When only a few states are observable, as is the case in practical applications, filter simulations should be run many times to ensure that divergence due to observability does not occur.

CHAPTER 3

ANTENNA TRACKING SYSTEM

3.1 ANTENNA

An antenna is a transducer designed to transmit or receive electromagnetic waves. In other words, antennas convert electromagnetic waves into electrical waves and vice versa. They are used with waves in the radio part of the electromagnetic spectrum, that is, radio waves and are a necessary part of all radio equipment. Antennas are used in systems such as radio and television broadcasting, point to point radio communication, wireless LAN, cell phones, radar and spacecraft communication. Antennas are most commonly employed in air or outer space, but can also be operated under water or even though soil and rock at certain frequencies for short distances [6].

Physically, an antenna is an arrangement of one or more conductors, usually called elements in this context. In transmission, an alternating current is created in the elements by applying a voltage at the antenna terminals, causing the elements to radiate an electromagnetic field. In reception, the reverse occurs. An electromagnetic field from another source induces an alternating current in the current and a corresponding voltage at the antenna terminals. Some receiving antennas incorporate shaped reflected surfaces to collect electromagnetic waves from free space and direct them onto actual conductive elements.

3.2 TRANSPONDER

Transponder is an automatic device that receives, amplifies and retransmits a signal on a different frequency. It is an automatic device that transmits a predetermined a message in response to a predefined received signal. It is a receiver transmitter that will generate a reply signal upon proper electronic interrogation.

A communications satellite's channels are called transponders, because each is a separate transceiver. Most comsats are microwave radio relay stations in orbit and carry dozens of transponders, each with a bandwidth of tens of megahertz. Most transponders operate on a "bent pipe" principle referring to the sending back of what goes into the conduit with only amplification and a shift from uplink to downlink frequency, as opposed to a regenerative system whereby the signal is used to remake and remodulate the signal.

With data compression and multiplexing, several video and audio channels may travel through a single transponder on a single wideband carrier. Original analog video only has one channel per transponder with subcarriers for audio and automatic transmission identification service. Non-multiplexed stations also travel in single mode with multiple carriers per transponder. This allows each station to transmit directly to the satellite rather than paying for a whole transponder or send it to an earth station.

3.3 CONTROLLER

A controller is a device which monitors and affects the operational conditions of a given dynamical system. The operational conditions are typically referred to as output variables of the system which can be affected by adjusting certain input variables. The notion of controllers can be extended to more complex systems. In the natural world, individual organisms also appear to be equipped with controllers that assure the homeostasis necessary for survival of each individual. Both human made and natural systems exhibit collective behaviors amongst individuals in which the controllers seek some form of equilibrium.

3.4 ANTENNA SYSTEM

The antenna system described here is used to track a subset of the earth resources technology satellites. These types of satellites take pictures of the earth's surface and transmit imaging information to different receiving stations. After the images are processed, for example, one can know how much moisture it's going to be and cartographers can see the actual contours of the earth's surface.

Fundamental to the satellite is a power generation unit, solar cells for storing energy, an on board antenna system and transponder. The power generator or storage units must be capable of generating power for the entire life cycle of the satellite. These units are usually in a redundant configuration so that if one fails, the system can still operate with hundred percent efficiency. Most times, these power units are designed to work efficiently with up to fifty percent of their power capacity diminished.

The on board antenna system includes one or more dishes, receivers, transmitters and a computer system. Transmission and reception can occur only in fixed frequency bands. Suppose that the satellite is designed to receive at frequencies between 1.7 and 1.85 GHz and transmit at frequencies between 2.2 and 2.4 GHz. If the on board computer or someone on the ground inadvertently sent a command to transmit information at a frequency of 1.7 GHz, the equipment could not. Id the equipment is designed well, though the equipment can provide self diagnostic tests, recharge its batteries, isolate faults and failures and perform corrective action when necessary.

Transponders have the unenviable task of accepting signals weakened by miles of travel and corrupted by noise encountered along the way though the earth's atmosphere. It then filters the information from the noise, amplifies the result and sends it back to the earth stations.

The earth station antenna system is a ground based receiver which consists of a pedestal, antenna and controller. As an integrated unit, this system must be conFigured to receive signals transmitted by the landsat. It must also be packaged to withstand different types of environmental conditions. It should also be safe. In a good design, interlock circuitry and fail safe mechanisms are incorporated into the system to enhance reliability, maintainability and system availability. Most of the interlocks are in pedestal.

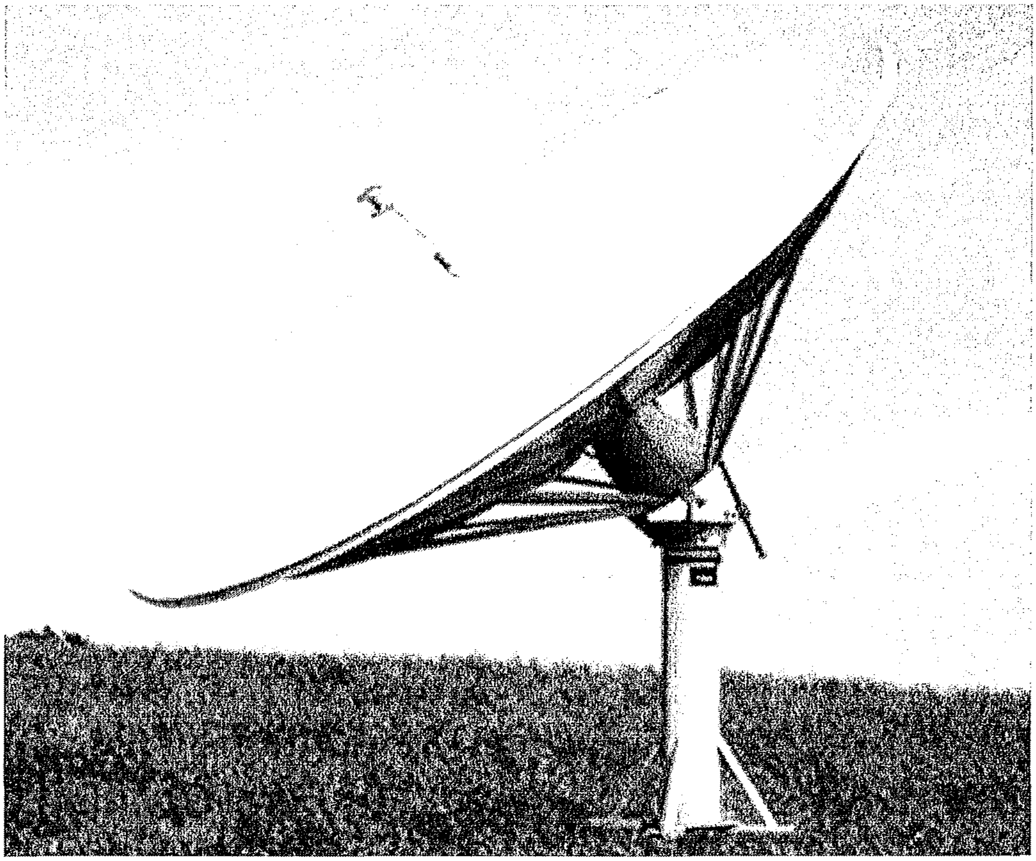

Figure 3.1 Tracking Antenna [11]

The most commonly used pedestal is an elevation over azimuth type mount which is used to position the antenna. It houses motors, servomechanisms, synchros or decoders, bearings, gear trains, junction boxes, other electromechanical parts, the cabling and the wiring requisite for power and signal continuity.

Two servos are used to drive the antenna, one to position the pedestal in the azimuth axis and the other to position the pedestal in the elevation axis. Servo compensation is either incorporated into the hardware or is implemented digitally. Either way the compensation is accomplished, the servos must be stable before the kalman filter can be applied.

The azimuth servo rotates the antenna and pedestal about a vertical axis. Some applications require azimuth travel to be from 0^0 to 360^0 or some other finite position while others demand continuous azimuth travel. The former usually requires the use of a cable wrap, which if not unwound periodically a wrap around until taut. The latter uses slip rings, a series or conductive rings which eliminate the wrap around problem but has a higher monetary cost.

The elevation servo rotates the antenna relative to the horizon. Elevation travel is usually limited from 0^0 to 180^0 with electrical and mechanical stops located 5 to 10 degrees beyond the normal operating range. The elevation stops when it reaches the range, should activate the brakes. The mechanical stops are rubber bumpers which are normally not used unless there is a brake failure or sudden loss of power. When operating normally, the elevation servo rotates the antenna within the pre-selected range. Since there has not been a need to track satellites through the center of the earth, elevation travel is limited to the horizon and above.

A standard antenna consists of a parabolic reflector, the feed electronics and the feed. This unit is mechanically balanced atop the elevation axis. The diameter of the reflector is determined by the frequency of the operation, amount of desired signal gain, the voltage standing wave ratio and other tracking. Several computer models are simulated before choosing the reflector which meets the performance requirements and has the added bonus of meeting meager budgets and impossible schedules.

The antenna feed continuously accepts signals at the selected frequency and bandwidth when focused on the dish. The signals received by the antenna system are first transmitted from the satellite and then reflected by the parabolic dish to a focus at the feed. After manipulations b the feed electronics like down conversion, mixing, demodulation, etc, the outputs to the tracking control electronics are the auto tracking errors. These two error measurements are essential to satellite auto tracking.

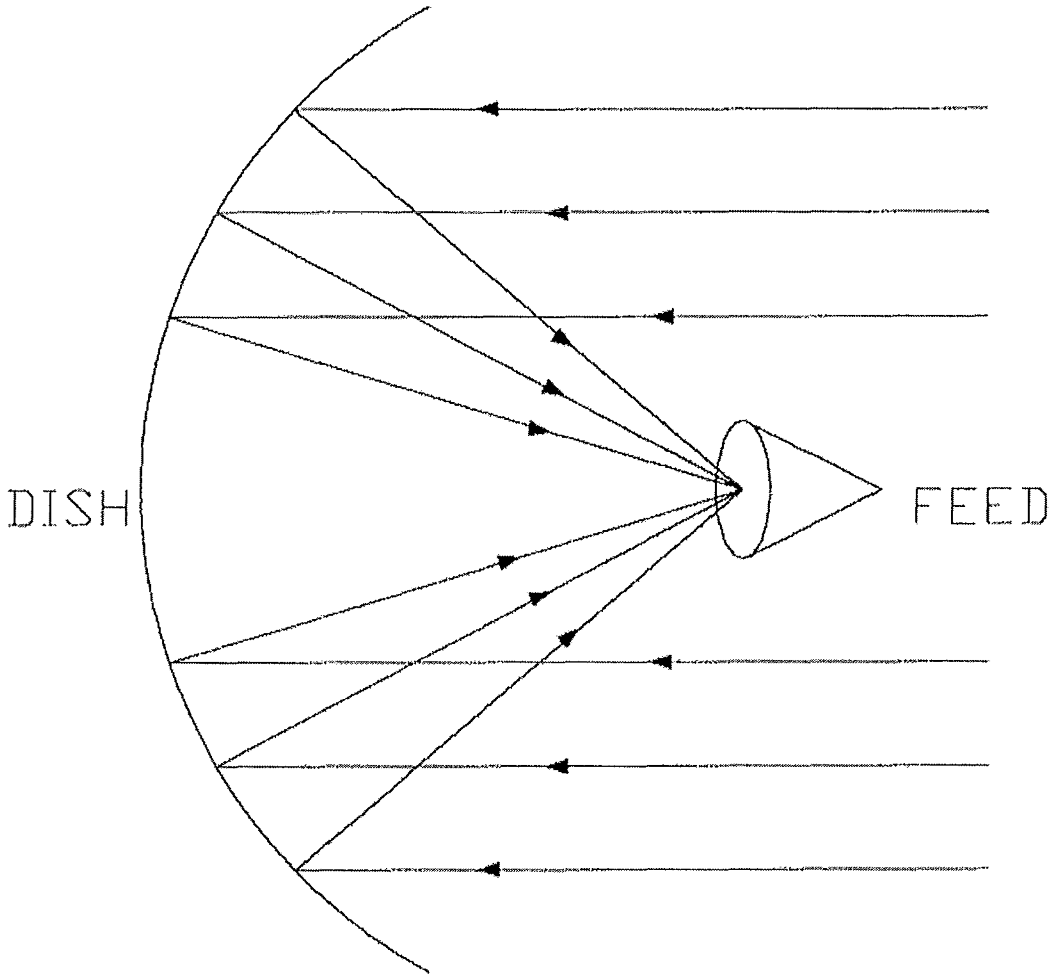

Figure 3.2 Signal receptions using a feed

The tracking controller serves as the interface between the operator and the pedestal hardware. It consists of a keypad for entering data, a display, a microprocessor electronics in which the controller software, including the kalman filter and other with a remote host computer. With an IEEE-488 interface, the controller can communicate with almost any piece of equipment including a receiver, data recorder, demodulator or bit synchronizer and spectrum analyzer.

A satellite must first be acquired before a satellite can be tracked automatically. The antenna system positions itself where the satellite ought to be at a certain time and scans that sector of space. The satellite's most likely position at any given time is determined by the satellite ephemeris loaded into the system. The ephemeris consists of an orbital element set which completely describe the size, shape and orientation of an orbit.

Without the satellite ephemeris, the system could only find the satellite by chance while continuously scanning the skies. Needless to say this is a difficult and time consuming effort which decreases the lifetime of the system by unnecessary wear of the bearings, gears and motors.

With the satellite ephemeris, the system can acquire the satellite efficiently by positioning the antenna at the most likely position of the satellite and waiting for it to appear. If the earth was spherical, had no atmosphere and was isolated from other bodies in the solar system, the orbit of a satellite would be an ellipse of constant size and shape in a plane whose direction remained fixed relative to the stars. This simple though rather insipid situation is upset by the effects of the earth's oblateness, atmospheric drag and the perturbations due to the sun and moon. This tells us why it is the most likely position and not the actual position is best explained.

Although the ephemeris by no means provides an absolute satellite position and velocity, it does not provide a window of possibilities. Automatic tracking can begin once the satellite has been acquired. The standard tracking loop is shown below

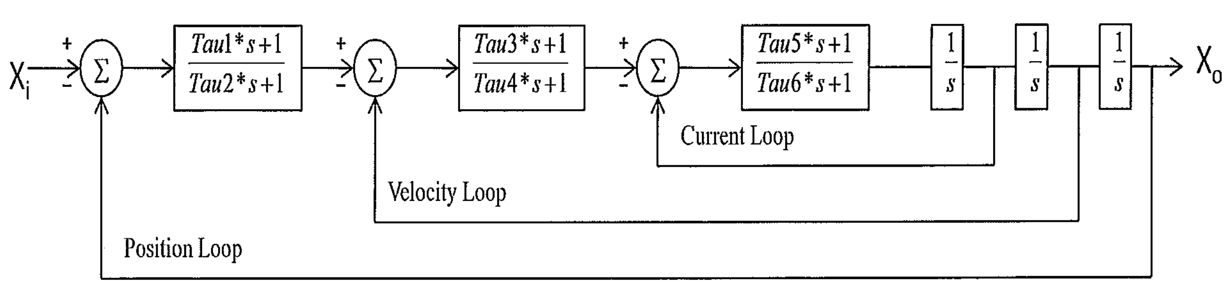

Figure 3.3 Block diagram of servo positioning

The time constraints of this third order system are chosen to meet design requirements such as servo bandwidth, wind torque, acceleration and pedestal velocity. The motor current, tachometer and the pedestal are controlled by the current, velocity and the position loops. For the simulation, the actual pedestal position is compared to the command position and the difference is the tracking error.

During auto tracking, actual position is not important, only the auto tracking error signals measured by the feed are of importance. When required, true satellite position is obtained by adding the antenna position to all the error sources, which includes the elevation error and the deflection error. This position can then be sent to the controller display unit.

Errors measured by the feed are in the deflection elevation coordinates instead of the azimuth elevation coordinates of the antenna. The elevation error signal is fed into the elevation servo as the elevation position tracking error but the deflection error signal, $\Delta \theta_{DEF}$ must be converted to an azimuth error before it can be fed into the azimuth error, secant correction is required as follows

$$\Delta\theta_{AZ} = \frac{1}{\cos\theta_{EL}}\Delta\theta_{DEF} \qquad 3.1$$

This correction is necessary in order that a constant gain crossover frequency can be maintained in the tracking loop.

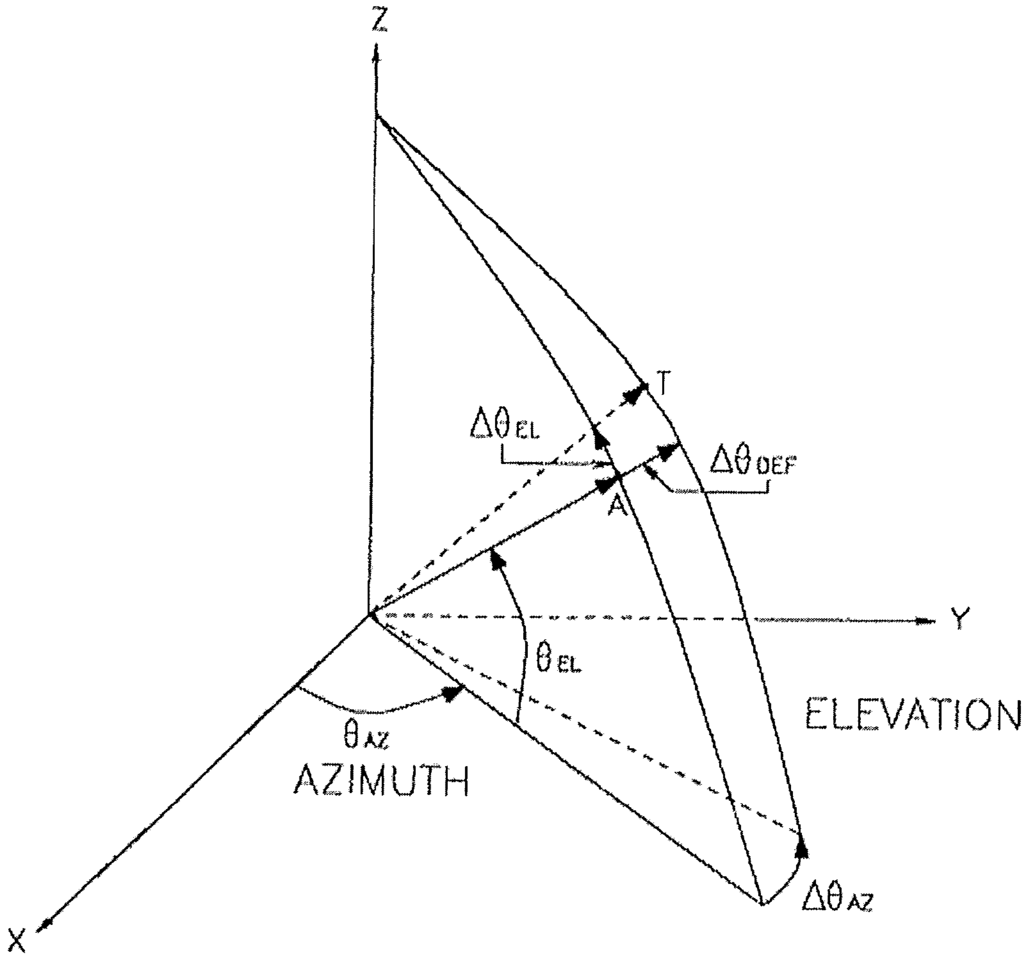

Figure 3.4 Azimuth elevation and deflection elevation coordinates

In conclusion, the antenna system described is used for tracking satellites orbiting the earth low altitudes. These satellites are self supporting units containing power generation and storage subsystems, an antenna subsystem and an on board computer. They operate efficiently because they have backup systems and they can perform some corrective actions. They transmit and receive within fixed frequency bands and cannot deviate.

The earth ground station antenna tracking system consists of an antenna, controller and a pedestal. Sitting atop the pedestal, the antenna dish and the feed combine to accept the signals sent by the satellite. Servos motors and other components combine to point the pedestal at the satellite so that the antenna can intercept the signals. The controller finally processes the data for display or storage.

These three subsystems work together to first acquire the satellite and then to track it automatically. Acquisition is easier with the pre-loaded satellite ephemeris but it is not a necessity. Auto tracking is independent of actual satellite position but is highly dependent on the tracking errors obtained from the feed. After some manipulation, these errors are fed into the servo loops as position tracking errors. For a visual display of actual satellite position, the feed errors plus other errors are added to the position measured by synchros or encoders.

CHAPTER 4

KALMAN FILTER AND ITS APPLICATION ON SATELLITE TRACKING

4.1 Kalman filter and its application

This paper focuses on tracking a low orbit satellite using an antenna tracking system. Conditions are determined and simulations are run to show that the Kalman filter can be used in this type of application. The Kalman filter discussing for the crossover course application is the extended Kalman filter. With this filter, it is possible to update the estimates with each new measurement. This refinement of the trajectory yields more accurate results than the linearized Kalman filter which assumes a set trajectory.

A target moving in a straight line constant velocity course at a constant altitude is the crossover course. The point of nearest approach between the target and the tracking station is the point where the azimuth angle is zero. This point is called the crossover point and hence the name crossover course.

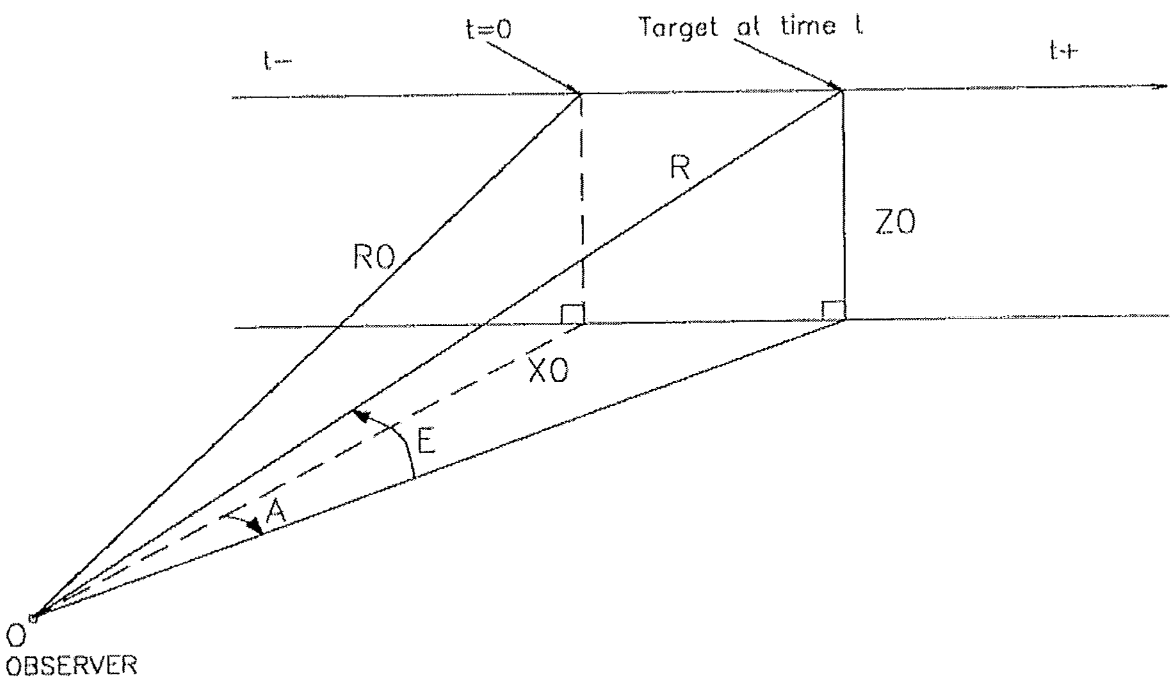

Figure 4.1 The crossing course concept

This course is used to simulate the worst case condition of the satellite trajectory. Although, the actual path may be circular or elliptical, the worst case antenna dynamics are seen during an overhead or near overhead pass, that is the azimuth angle goes to zero as the elevation angle approaches 90^0.

The benefits of using a crossover course are many. One reason to use it is its versatility. The model is applicable to low earth orbiting satellites, airplanes and missiles. Another reason is its simplicity. To model the actual trajectory of a satellite, the analyst needs an understanding of orbital mechanics, Kepler's equations and the so called J coefficients. On the other hand, the crossing course requires only basic mathematical manipulations.

When confronted with problems similar to this one, some designers have both a range and an angle filter. Others have used range and its derivatives as one of the states and elevation angle and its derivatives as the other. The antenna system where this filter us to be applied measures elevation and azimuth angles only therefore the chosen states are the elevation and azimuth angles.

In the design of most tracking systems, it is of utmost importance to reduce the computational complexity. To reach this end, some consideration should be given to the choice of coordinate systems. It is not simply a choice between Cartesian and polar or rotating and fixed.

A rule of thumb is to use the coordinate system that matches the measurements. For example, this antenna system measures position in azimuth and elevation angles, therefore the analysis is performed in the azimuth elevation coordinate system. Now suppose that the designer was told that only range and bearing were to be measured. The equations in azimuth elevation would have to be transformed into bearing range in order that the measurements are incorporated into the extended Kalman filter.

Bear in mind that the same system can appear to have different characteristics in different coordinate systems. When there is no choice in coordinates, the designer can look to Escobal for help in transformations. If the designer has a choice, the coordinate to choose is the one which requires the least number of transformations and the least number of assumptions. The former minimizes the number of computations while the latter decreases the number of errors introduced into the system. Whenever possible, the problem should be analyzed in the coordinates provided by the system's measurement sensors.

For the situation presented, let

V= the satellite velocity

X_0= the horizontal distance

Z_0= the altitude above the horizon

$$a = \frac{V}{X_0} \qquad 4.1$$

$$b = \frac{Z_0}{X_0} \qquad 4.2$$

$$\theta_{AZ} = \tan^{-1} at \qquad 4.3$$

$$\theta_{EL} = \tan^{-1} \frac{b}{\sqrt{1+(at)^2}} \qquad 4.4$$

The dynamic equations of motions in the antenna reference frame are

$$\ddot{\theta} = -a\dot{\theta}_{AZ} \sin 2\theta_{AZ} + w_1(t) \qquad 4.5$$

$$\ddot{\theta}_{EL} = \dot{\theta}_{EL}\{\frac{1}{t} + \frac{\dot{\theta}_{El}}{\cos^2 \theta_{Elt}}[\frac{3}{\tan\theta_{El}} - \sin 2\theta_{EL}]\} + w_2(t) \qquad 4.6$$

Another set of equations can be written for the range, R, from the antenna to the target. In this application, the range equations are ignored because the typical antenna system described use sensors which provide measurements of angles only. It would be a waste of computational effort to include the range equations when the antenna system is incapable of using them. On the other hand, this particular system is adequately described by the angle equations.

Range tracking information is generally at least 100 times more accurate than target position information derived from the angular tracking data according to Biernson. For this reason and to decrease the overall complexity of the filter, the range equations are not used. By eliminating the range equations, the concentration of efforts are placed where the errors are greatest, that is in azimuth and elevation.

Although the satellite is assumed to be moving with a constant velocity to the antenna, it appears to be accelerating. From the antenna reference frame, the satellite accelerates and decelerates depending on whether it is moving towards the antenna or away from it. To compensate for this effect, the Singer model is used.

Singer's model assumes that the target accelerates with a uniform probability distribution between +A_{MAX} and –AMAX. It also assumes a white or uncorrelated noise input. In most practical applications, the driving function of the system is correlated in time. To whiten the noise, the original driving function could first be sampled at random intervals before being fed into the system. It could also be whitened by passing it through a low pass filter. This assumes that passage of the unity white noise through the shaping filter results in a Gauss-Markov process. The autocorrelation function is

$$R(\tau) = \sigma^2 e^{\frac{|\tau|}{\beta}} \qquad 4.7$$

The corresponding differential equation is

$$\dot{x} = -\frac{1}{\beta} + \sqrt{\frac{2\sigma^2}{\beta}}\, w(t) \qquad 4.8$$

Either way, the white noise must be related to the original driving inputs via the differential equations.

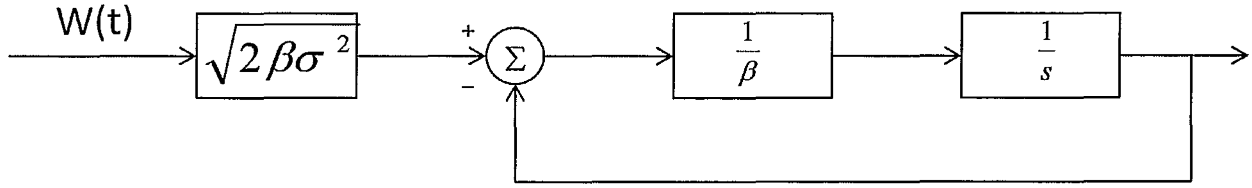

Figure 4.2 Process of whitening the input noise

This whitening process known as the state vector augmentation which creates additional states that must be integrated into the original system equations. Although the number of states increases complexity, the benefit of doing so is an increase in system controllability.

Assigning the states to be $\theta_{AZ} = X_1, \dot{\theta}_{AZ} = X_{2,} \theta_{EL} = X_4, \dot{\theta}_{EL} = X_5$ and adding two more states for the acceleration noise, the system equations are

$$\dot{X} = X_2 \tag{4.9}$$

$$\dot{X}_2 = -aX_2 \sin 2X_1 + X_3 \tag{4.10}$$

$$\dot{X}_3 = -\frac{1}{\beta_{AZ}} X_3 + G_{AZ} W_1(t) \tag{4.11}$$

$$\dot{X}_4 = X_5 \tag{4.12}$$

$$\dot{X}_5 = -\frac{a^2}{b^2} \cos^2 X_4 \tan^3 X_4 - \frac{a}{b^2} \tan X_1 \tan^2 x_4 [3 - 2\sin^2 X_4] X_5 + X_6 \tag{4.13}$$

$$\dot{X}_6 = -\frac{1}{\beta_{EL}} X_6 + G_{EL} W_2(t) \tag{4.14}$$

The dynamic equations requires for the system are now in the form

$$\dot{X}(t) = f(X(t), t) + G(X(t), t) W(t) \tag{4.15}$$

Measurements must also be written in state space format. Measurements of the positions are sensed by synchros or angle encoders for this antenna system. Two other measurements are possible with this equipment. The errors measured by the feed, $\Delta\theta_{EL}$ and $\Delta\theta_{DEF}$. To map the deflection error into an azimuth error, the deflection error is multiplied by a gain, $\frac{1}{\cos\theta_{EL}}$. Adding some noise into the measurement results in the expression

$$\theta_{AZ} = \theta_{AZ} + \frac{\Delta\theta_{DEF}}{\cos\theta_{EL}} + V_1 \qquad 4.16$$

$$\theta_{EL} = \theta_{EL} + \Delta\theta_{EL} + V_2 \qquad 4.17$$

With each measurement, the Kalman filter gains, error covariance's and state estimates are recalculated. New estimates of the states are produced from

$$f_1 = \hat{X}_2 \qquad 4.18$$

$$f_2 = -a\hat{X}_2 \sin 2\hat{X}_1 + \hat{X}_3 \qquad 4.19$$

$$f_3 = -\frac{1}{\beta_{AZ}}\hat{X}_3 \qquad 4.20$$

$$f_4 = \hat{X}_5 \qquad 4.21$$

$$f_5 = -\frac{a^2}{b^2}\cos^2\hat{X}_4 \tan^3\hat{X}_4 - \frac{a^2}{b^2}\tan\hat{X}_1 \tan^2\hat{X}_4[3 - 2\sin^2\hat{X}_4]\hat{X}_5 + \hat{X}_6 \qquad 4.22$$

$$f_6 = -\frac{1}{\beta_{EL}}\hat{X}_6 \qquad 4.23$$

Propagating the state error covariance matrix requires more complexity. The state equations must be linearized about the most recent measurement, $\widetilde{X}(t)$, and the process noise matrix must be formed. If the system equations are written in the form

$$X^* = F(\widetilde{X}(t), t)X^* \qquad 4.24$$

Where $F(\widetilde{X}(t),t)$ is the4 linearized system dynamics matrix, $\dot{X}^*$ and X^* are small perturbations from $\dot{X}$ a66nd X, then the linearized matrix is

$$F(\widetilde{X}(t),t) = \left.\frac{\partial f(X(t),t)}{\partial X(t)}\right|_{X(t)=\widetilde{X}(t)} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} & f_{26} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} \end{bmatrix} \qquad 4.25$$

and the non zero elements of $F(\widetilde{X}(t),t)$ are

$$f_{12} = 1$$
$$f_{21} = -2a\widetilde{X}_2 \cos 2\widetilde{X}_1$$
$$f_{22} = -a\sin 2\widetilde{X}_1$$
$$f_{23} = 1$$
$$f_{33} = -\frac{1}{\beta_{AZ}}$$
$$f_{45} = 1$$
$$f_{51} = -\frac{a\tan^2 \widetilde{X}_4}{b^2 \cos^2 \widetilde{X}_1}[3 - 2\sin^2 \widetilde{X}_4]\widetilde{X}_5$$
$$f_{54} = -\frac{a^2}{b^2}(\tan^2 \widetilde{X}_4 + 2\sin^4 \widetilde{X}_4) - \frac{2a}{b^2}\tan \widetilde{X}_1 \tan \widetilde{X}_4\left(\frac{1}{\cos^2 \widetilde{X}_4} - 2\sin^2 \widetilde{X}_4 + 2\right)\widetilde{X}_5$$
$$f_{55} = -\frac{a}{b^2}\tan \widetilde{X}_1 \tan^2 \widetilde{X}_4 [3 - 2\sin^2 \widetilde{X}_4]$$
$$f_{56} = 1$$
$$f_{66} = -\frac{1}{\beta_{EL}}$$

Most elements of the white noise sequence matrix are zero. The nonzero entries g_{33} and g_{66}, their values are from the low pass filter used to whiten the process noise. These values are

$$g_{33} = \sqrt{\frac{2\sigma^2}{\beta_{AZ}}}$$

$$g_{66} = \sqrt{\frac{2\sigma^2}{\beta_{EL}}}$$

Assuming that the process noise spectral density matrix has no cross correlation terms, this six-state filter has two nonzero elements, q_{33} and q_{66}. These are the random noise in the two acceleration states. Again using Singer's model for acceleration, these values are

$$q_{33} = \frac{A^2_{MAX\_AZ}}{3}$$
$$q_{66} = \frac{A^2_{MAX\_EL}}{3}$$

where A_{MAX} is the maximum acceleration of the satellite.

A time varying H matrix defines the Extended Kalman filter. Without the assimilation of new measurements into the recursive process, the filter could be either an Linearized Kalman filter or Discrete Kalman filter. For the satellite application of this paper, the measurements are azimuth and elevation angles, therefore

$$H(X(t),t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad 4.26$$

and

$$Z = \begin{bmatrix} \theta_{AZ} \\ \theta_{EL} \end{bmatrix} \qquad 4.27$$

with a noise spectral density matrix

$$R = \begin{bmatrix} \sigma^2_{AZ} & 0 \\ 0 & \sigma^2_{EL} \end{bmatrix} \qquad 4.28$$

The matrix P_0 is a six by six diagonal matrix which defines half of the initial conditions required by the filter and the other half being the initial states. Although it is not necessary to have the off diagonal elements equal to zero, it simplifies the calculations. This assumption results in a loss in state estimation accuracy, but the benefits of simplification outweighs this minimal loss.

For real time applications, the matrices previously defined are all that is required for a successful continuous-discrete Kalman filter. Simulations on the other hand require more manipulations and assumptions. For the purpose of this paper, the Continuous-Discrete Extended Kalman filter is simulated as a Discrete –Discrete Linearized Kalman filter.

The Discrete time Kalman filter equations from chapter 2 are recalled as

$$X_k = A_{k-1}X_{k-1} + B_k U_{k-1} + W_{k-1}$$

$$Z_k = H_k X_k + V_k$$

$$\hat{X}_k^- = A_{k-1}\,\hat{X}_{k-1}^+$$

$$P_k^- = A_{k-1}P_{k-1}^+ A_{k-1}^T + Q_{k-1}$$

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1}$$

$$\hat{X}_k^+ = \hat{X}_k^- + K_k[Z_k - H_k \hat{X}_k^-]$$

$$P_k^+ = [1 - K_k H_k]P_k^-$$

The Continuous-Discrete Extended Kalman filter equations are

$$\dot{X}(t) = f(X(t),t) + W(t)$$
$$Z_K = H_K(X(t_K)) + V_K$$
$$\dot{\hat{X}} = f(\hat{X}(t),t)$$
$$\dot{P}(t) = F(\hat{X}(t),t)P(t) + P(t)F^T(\hat{X}(t),t) + Q(t)$$
$$F(\hat{X}(t),t) = \left.\frac{\partial f(X(t),t)}{\partial X(t)}\right|_{X(t)=\hat{X}(t)}$$
$$K_K = P_K^- H_K^T(X_K^-)[H_K(\hat{X}_K^-)P_K^- H_K^T(\hat{X}_K^-) + R_K]^{-1}$$
$$\hat{X}_K^+ = \hat{X}_K^- + K_K[Z_K - H_K(\hat{X}_K^-)]$$
$$P_K^+ = [1 - K_K H(\hat{X}_K^-)]P_K^-$$

In the Discrete-Discrete linearized model, the noise spectral density matrix, Q(t), of the Continuous-Discrete Extended Kalman filter is replaced by the noise covariance matrix, Q_K.

$$Q_K = G(t)Q(t)G^T(t) \tag{4.29}$$

The value of G(t) can be calculated from the process for whitening the input noise. When the state equation for this filter is written in the form, $\dot{X} = FX + GW$. The white noise sequence is represented by

$$\dot{X} = -\frac{1}{\beta}X + \sqrt{\frac{2\sigma^2}{\beta}}W(t) \tag{4.30}$$

Clearly for each axis $G = \sqrt{\frac{2\sigma^2}{\beta}}$, therefore

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{2\sigma^2}{\beta_{EL}}\right)\left(\frac{A_{MAX\_EL}^2}{3}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(\frac{2\sigma^2}{\beta_{AZ}}\right)\left(\frac{A_{MAX\_AZ}^2}{3}\right) \end{bmatrix}$$

The measurement noise covariance is the same as the measurement noise spectral density matrix

$$R_K = \begin{bmatrix} \sigma_{AZ}^2 & 0 \\ 0 & \sigma_{EL}^2 \end{bmatrix}$$

The equations which comprise the Discrete-Discrete Linearized Kalman Filter to be used in the simulation are

$$\begin{aligned}
\dot{X}(t) &= f(X(t),t) + W(t) \\
Z_K &= HX(t) + V_K \\
K_K &= P_K^- H^T [HP_k^- H^T + R]^{-1} \\
\hat{X}_K^+ &= \hat{X}_K^- + K_K[Z_K - H] \\
P_K^+ &= [1 - K_K H]P_K^- \\
\dot{\hat{X}}(t) &= f(\hat{X}(t),t) \\
\dot{P}(t) &= F(\hat{X}(t),t)P(t) + p(t)F^T(\hat{X}(t),t) + GQG^T
\end{aligned}$$

The Continuous-Discrete Extended Kalman Filter is now ready for implementation as a Discrete-Discrete Linearized Kalman Filter.

As a simplification tool, the crossover course provides diversity and flexibility. It's use as an approximation for the worst case portion of the trajectory of a low orbit Earth satellite is common practice. In this paper, the system equations are modeled for use in angle coordinates. A Continuous-Discrete Extended Kalman Filter is applied to these equations and the matrix elements are determined. For the simulation, a discrete-discrete Linearized Kalman filter model is assumed and all the matrices needed to run the simulation are defined.

4.2 FILTER SIMULATION

The crossover course is used by a satellite communications industry to approximate a portion of a satellite orbit. Whether the orbit is a circular or elliptical, the difference between the crossover course and actual orbit calculations of position and velocity is negligible in some cases. This technique is particularly useful during the proposal phase of a project when there is very little time allowed for complex simulations but a high degree of accuracy is still required.

This filter estimates the antenna position and velocity required to track the target satellite. To accomplish this, the filter needs initial conditions and noise variances. A point known to be on the trajectory is chosen as the initial condition, $\theta_{AZ} = 0$ was the starting point. With the azimuth angle equal to zero, the elevation angle was forced to 80^0. The azimuth velocity was calculated to be the satellite velocity divided by the horizontal distance to the satellite ($\frac{V}{X_0}$) and the elevation velocity was 0^0/sec. The accelerations were calculated to be zero for both angles.

Variances for the measurement error (σ_R^2) and plant noise (σ_A^2) were either measured or calculated. The measurement error variance is a measure of the accuracy of the measuring instruments. For the Landsat system considered, the instruments have an accuracy of 0.005^0. This means that the target position can be measured to within 0.005^0 of the actual position. There are better systems with greater accuracy available but this number is normal for these types of systems. The system process noise variances were derived using basic calculus techniques.

Both the measurement error and process noise variances were used in the diagonal P_0 matrix to start the Kalman filter. They accounted for the range and acceleration terms but there was no clear cut method for finding velocity variances. To compensate for this inadequacy, random numbers between σ_R^2 and σ_A^2 were tried. An average value of these runs was finally accepted. These trials showed that once σ_R^2 and σ_A^2 were established. There was little change in the results if σ_V^2 was between σ_R^2 and σ_A^2. On the other hand, the filter loses its positive definiteness and diverged if σ_V^2 were larger than σ_R^2.

Obtaining the noise variance (σ^2) was similarly as unscientific as finding the velocity variances for the P matrix. It began with the assumption that the unity white noise entering the low pass filter becomes an uncorrelated Gauss-Markov process after the filter. A Gauss-Markov process has an autocorrelation function $R(\tau) = \sigma^2 e^{\frac{|\tau|}{\beta}}$.

Brown says that the β and σ^2 are parameters which may be difficult to determine in a real life problem.

For this simulation, β was fixed to 1. This is the value Singer suggests for maneuvers due to atmospheric disturbances. Because the noise was entered into the acceleration state, σ^2 was set equal to σ_A^2. The results are the Kalman filter estimates are shown in Figure 4.6.

Figures 4.7 through 4.10 superimpose the Kalman filter estimate with the actual angles and positions. Figure 4.7 shows that the filter follows a trajectory without error for about twelve seconds and then slowly starts to deviate. The deviation becomes more pronounced at the higher angles but eventually settles at a constant. For practical purposes, these results are excellent. Indulging in the pursuit of excellence leads to the question, why does it not match the actual curve? Why does it settle at a few degrees more than the actual trajectory?

To answer these questions, different values of σ^2 were tried. The larger values of σ^2 resulted in a divergent P matrix which, as expected, causes incorrect estimates. This is shown in Figures 4.11 through 4.14 with all variables are the same except that the value of σ^2 is equal to 10. The smaller the value of σ^2 leads to better estimates as shown in Figures 4.15 through 4.18. In both cases, there were no noticeable changes in the azimuth position trajectory at the beginning at the pass, around t=0. The effects were more pronounced at the end of the run where values reached their steady state. This would imply that the noise has little effect on the trajectory at the beginning even though the term $\sqrt{\frac{2\sigma^2}{\beta\Delta t}}$ is used to simulate the random noise. However, that does not explain the glitch at the beginning of the run shown on the azimuth and elevation velocity curves in Figures 4.4 through 4.6.

It is clear from the truth state equations containing random numbers that the initial conditions chosen, the noise is a significant contributor to future values of the states. For example, with the truth states being

```
XTDOT (1) = XT (2);
XTDOT (2) = -a*(sin (2*XT (1)))*XT (2) + XT (3);
        XTDOT (3) = -XT (3)/Beta_AZ + ........
                        sqrt (2*VAR_N_AZ)/(Beta_AZ*DT))*AZNOISE;
XTDOT (4) = XT (5);
XTDOT (5) = - ((a/b) ^2)*(cos (XT (4)))) ^2*(tan (XT (4))) ^3......
                        - (a/b) ^2)*tan (XT (1))* (tan (XT (4))) ^2*(3 - .....

                        2*(sin (XT (4))) ^2)*XT(5) + XT (6);

XTDOT (6) = -XT (6)/Beta_EL +

                sqrt(2*VAR_N_EL/ (Beta_EL*DT))*ELNOISE;

XT = XT + XTDOT*DT;
```

and the initial condition is equal to the acceleration is pure noise and this affects the velocity which in turn affects the position. This noise is probably the jerkiness visible in the azimuth velocity plot of Figure 4.4 and the elevation velocity plot of Figure 4.6. With a different set of initial conditions, the noise may not make a significant contribution to the estimates at the beginning of the run and the glitches may disappear.

$$\begin{bmatrix} 0 \\ a \\ 0 \\ 1.396 \\ 0 \\ 0 \end{bmatrix}$$

To decrease the steady state error to zero as time progresses may require increasing the value of the Q matrix. It may be that the P matrix approaches steady state too quickly and more process noise is required to force it to work until the end. Another option would be adjusting the time correlation constant.

The simulations for this filter were run on a compatible computer using MATLAB, by the Mathworks Inc. in the Microsoft Windows environment. Calculations and measurements were made once every ten milliseconds.

The satellite to be tracked is assumed to be a Landsat moving at an altitude of 463 km above the surface of the earth. We are interested in an 80^0 pass, which means that the maximum elevation tracking angle is 80^0.

Constants

$i = 80^0$

$\mu = 3.98601*10^5 km^3/s^2$

$A_{MAX\_AZ} = 0.649*a^2$

$A_{MAX\_EL} = 0.001478$

$X_0 = h*\cos(i)$

$Z_0 = h*\sin(i)$

$R_e = 6378.145km$

$h = 463km$

$$V = \sqrt{\frac{\mu}{R_e + h}}$$

$$a = \frac{V}{X_0}$$

$$b = \frac{Z_0}{X_0}$$

As states earlier in this paper, good modeling techniques are essential for accurately following the trajectory. This simulation shows that the Kalman filter offers the designer flexibility when absolute values are unknown and only engineering estimates are available. It also shows that the filter structure's rigidity makes even the simplest problems complex. Once the complex portions are complete and under control though, an endless number of "what if" conditions could be simulated. This flexibility is invaluable.

The simulation also shows that the Kalman filter estimates the satellite position and velocity with minimal error. These errors could possibly be improved with a more accurate initial error covariance matrix, a better estimate of the time correlation for the white noise model or a different arrangement of the variables before linearization. Another consideration could be increasing the sampling time or increasing the amount of process noise. For all practical applications, the results are good enough.

4.3 SIMULATION RESULTS

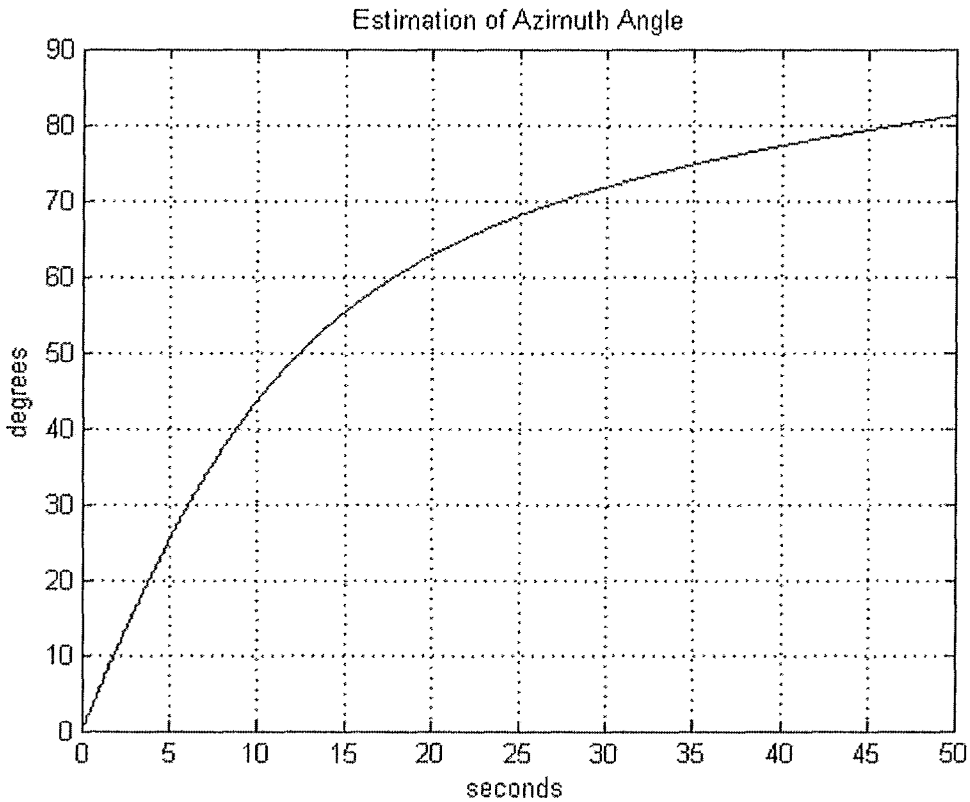

Figure 4.3 Estimation of Azimuth Angle

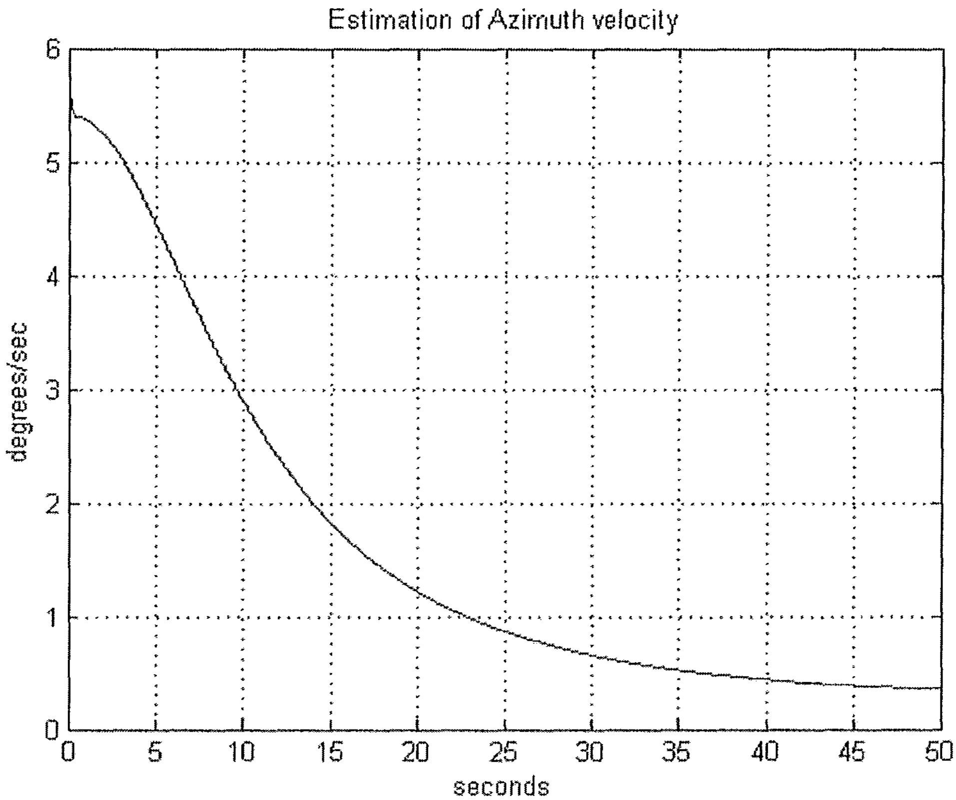

Figure 4.4 Estimation of Azimuth Velocity

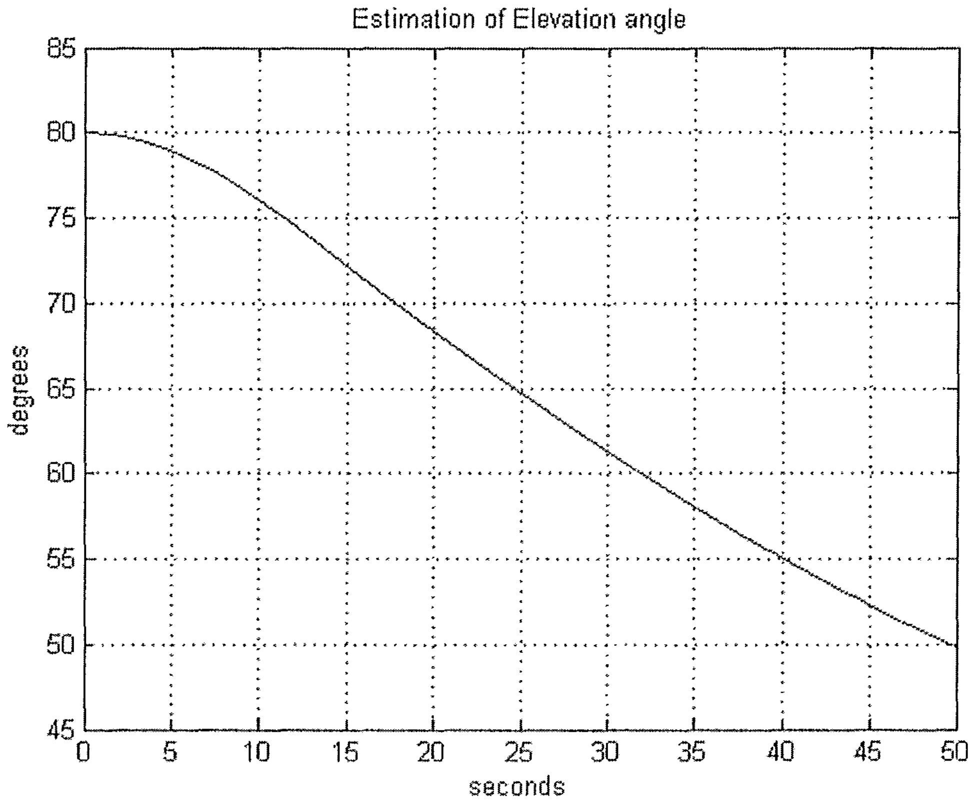

Figure 4.5 Estimation of Elevation Angle

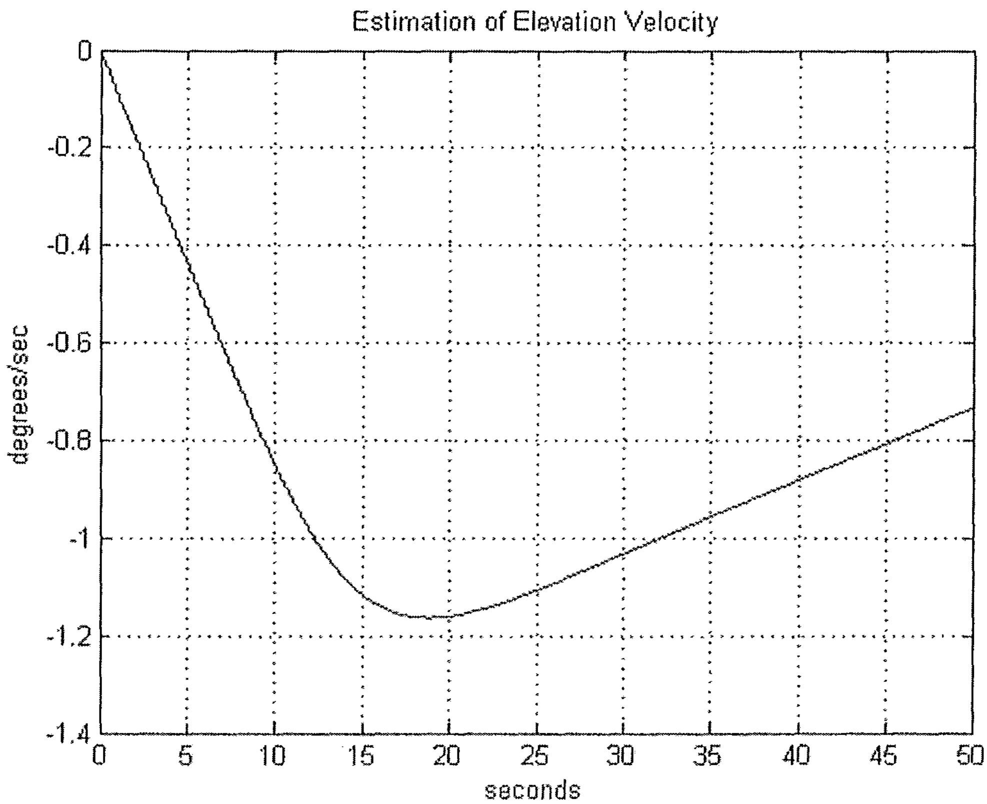

Figure 4.6 Estimation of Elevation Velocity

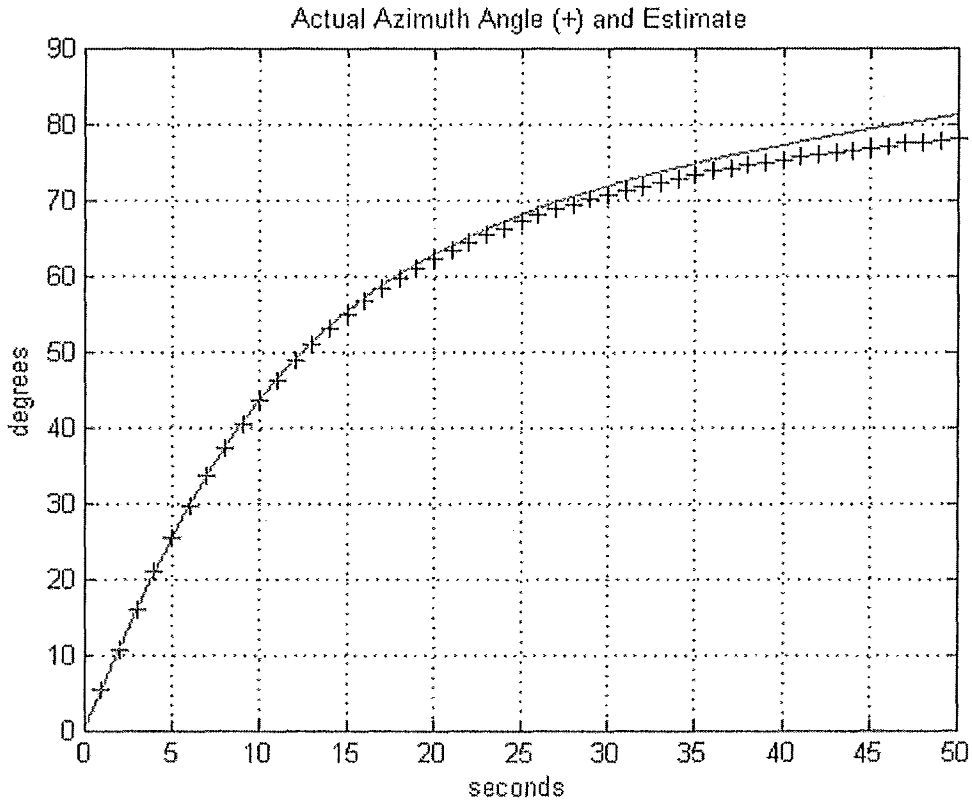

Figure 4.7 Actual Azimuth Angle (+) and Estimate

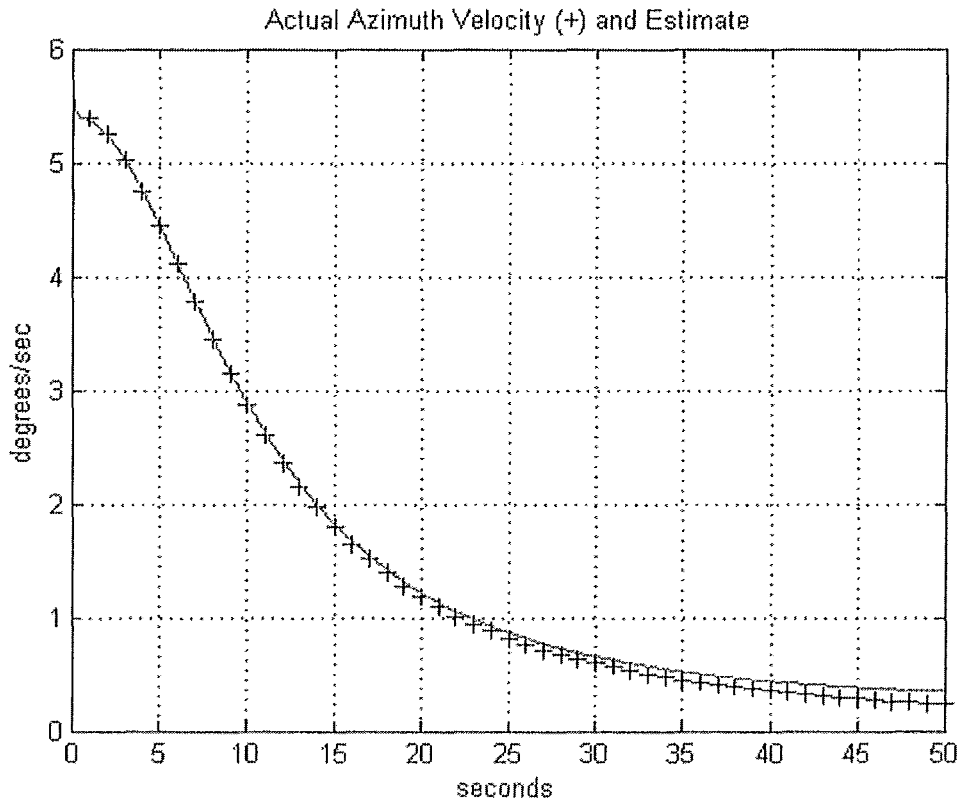

Figure 4.8 Actual Azimuth Velocity (+) and Estimate

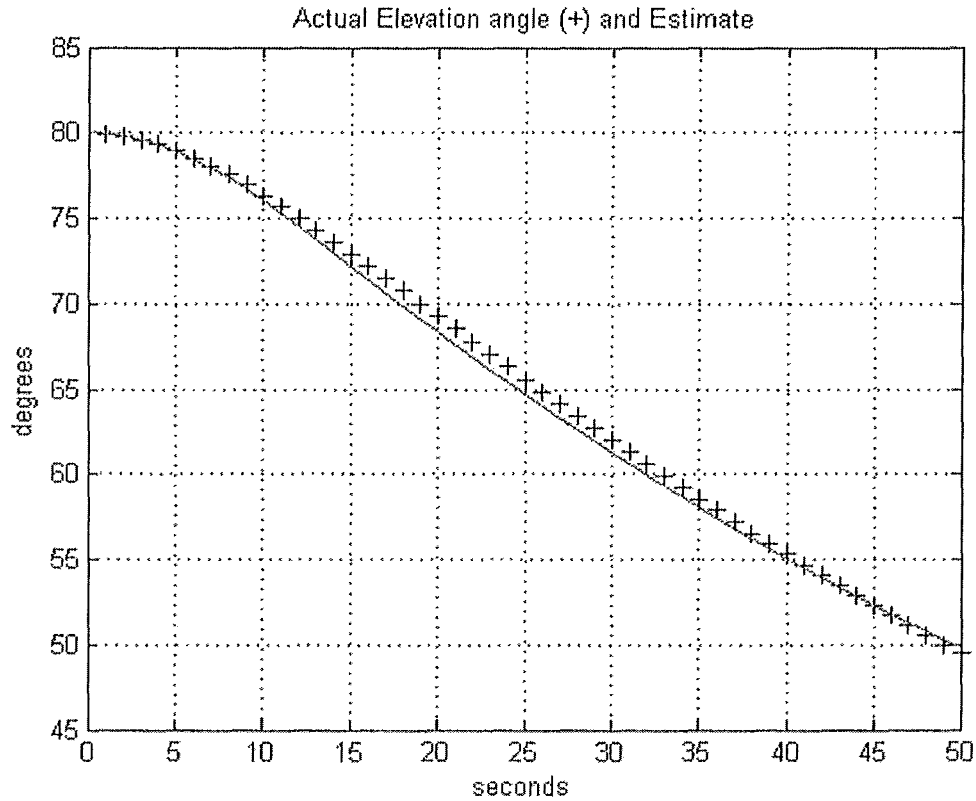

Figure 4.9 Actual Elevation Angle (+) and Estimate

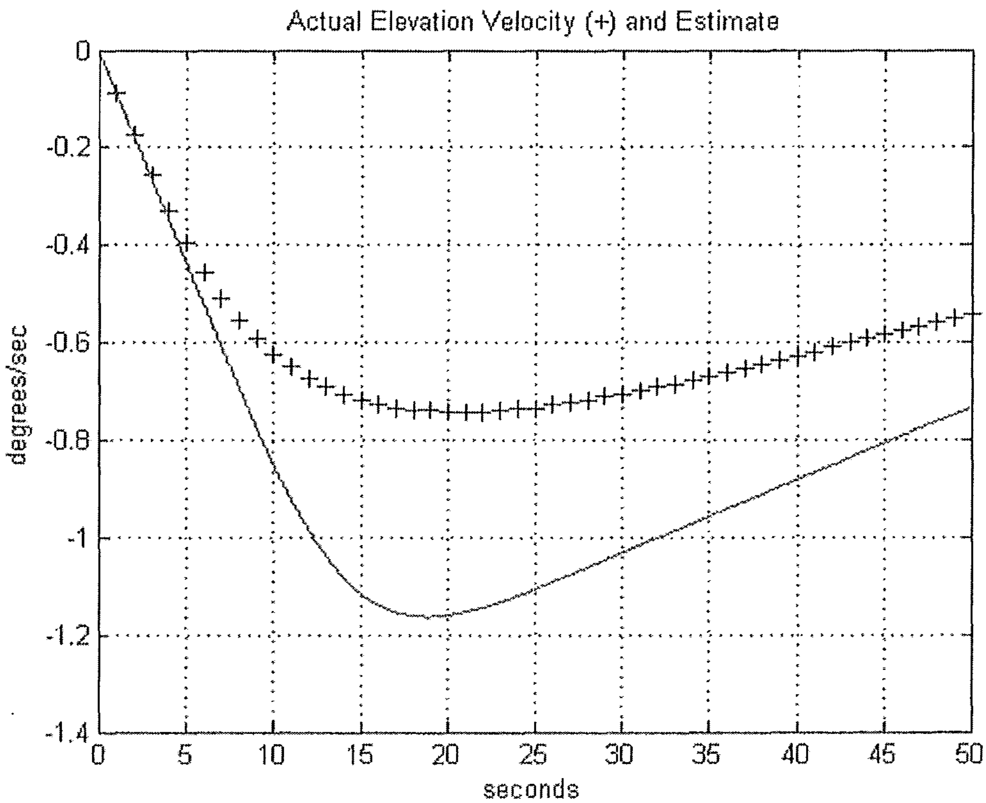

Figure 4.10 Actual Elevation Velocity (+) and Estimate

CHAPTER 5

FUTURE CONSIDERATIONS

The primary purpose of this project was to develop a Kalman filter to be used in satellite tracking applications starting with the crossover course. It was reasoned that the best way to proceed was to build a Kalman filter from the bottom up using a relatively simple trajectory. The relatively simple trajectory did not have a simple linear model for use in the Kalman filter equations and they had to be linearized.

This linearization turned out to be a cumbersome undertaking in order that the maximum number of states stood apart. The elevation acceleration is

$$\ddot{\theta}_{EL} = \dot{\theta}_{EL}\left\{\frac{1}{t} + \frac{\dot{\theta}_{EL}}{\cos^2\theta_{EL}}\left[\frac{3}{\tan\theta_{EL}} - \sin 2\theta_{EL}\right]\right\} + w_2(t) \tag{5.1}$$

If we delete the time t from the equation, the $\ddot{\theta}$ term then became a squared term. Remember, the goal is to have equations in the form $\dot{X} = AX$ but with the least amount of manipulative linearization.

To improve upon this filter, one may try several alternatives. First of all, one can rearrange the accelerations equations so that each derivative is in terms of one variable, either θ_{AZ} or θ_{EL}. For example, the azimuth acceleration equation is in terms of θ_{AZ} and $\ddot{\theta}_{AZ}$, the elevation acceleration equation is in terms of θ_{AZ}, θ_{EL} and $\ddot{\theta}_{AZ}$. Perhaps having a single variable can allow compression of the equations into a more compact form for easier linearization. It may also possible to have a polynomial in terms of $\sin\theta$ where $\sin\theta$ can be approximated as θ for small angles.

Secondly one can use a batch least square filter to start up the Kalman filter. If P_0 and X_0 matrices are unavailable, one can jump start the Kalman filter by slightly different equations. The P matrix is

$$P_0 = \left(P_\infty^{-1} + H^T R^{-1} H\right)^{-1} \quad 5.2$$

where P_∞ is set to a very large number times the identity matrix. Ideally it would be best to have $P_\infty = \infty I$ but a computer still has only limited capacity, even though that the capacity is constantly being extended.

To find X_0,

$$X_0 = P_0\left(\hat{X}_0^- + H^T R^{-1} Z\right) \quad 5.3$$

When no prior knowledge of X is known, set $\hat{X}_0^-$ equal to zero. Use these values of P_0 and X_0 to enter the Kalman filter loop. We can suggest that these values can be used at $t = 0$ and for $t_1 \ldots t_k$ one should use values generated by standard Kalman filter equations.

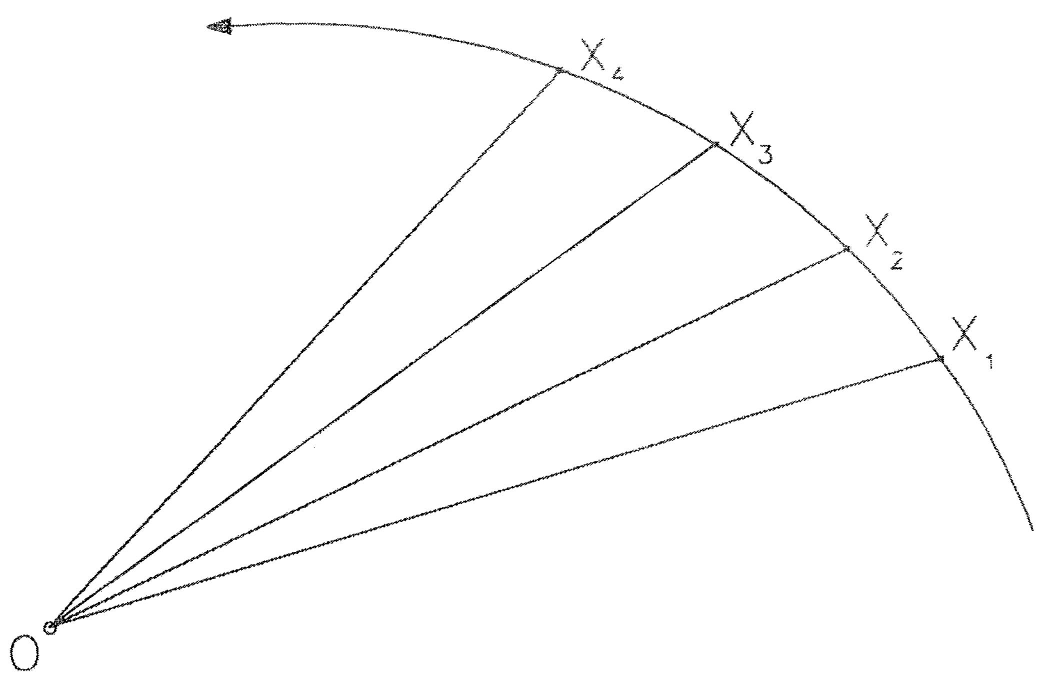

Figure 5.1 Sample Trajectory

Another method for starting the filter is from the stochastic control. This method allows the designer to choose the number of initialization points to combine into one point for entrance into the Kalman filter loop. Suppose that the target is moving in a trajectory as shown in Figure 5.1.

The points $X_1 X_2 .... X_n$ correspond to certain elevation angles measured at certain times. To estimate X_1 from Z_n measurements, one has

$$Z_1 = HX_1$$

$$Z_2 = HX_2 = H\phi X_1 = H_2 X_1 \text{ where } H_2 = H\phi$$

$$Z_3 = HX_3 = H\phi(\phi X_1) = H\phi^2 X_1 = H_3 X_1$$

$$\vdots$$

$$Z_n = HX_n = H\phi^{n-1} X_1 = H_n X_1 \qquad 5.4$$

A state transition matrix, ϕ is now required. It is defined as $\phi = e^{-At}$ where A is from $\dot{X} = AX$. For practical problems this form will be almost impossible to attain due to nonlinearities but it should be relatively simpler to get close. Remember that this is ϕ and used for not F, therefore the system equations should not be linearized at this point. The system equations should be arrange and rearranged for the best possible fit into $\dot{X} = AX$.

After determination of the A matrix, a second order Euler approximation can be used for ϕ, that is

$$\phi = 1 + A\Delta T + \frac{1}{2} A^2 (\Delta T)^2 \qquad 5.5$$

If the sampling time is small enough, a first order approximation should provide reasonably accurate results. Both should be studied to see whether the differences are major or minor. If the differences are insignificant, why perform the excess calculations? If the second order provides better results, it is likely to be worth the computational excess.

Although knowing the first value can be of use, it is better to estimate the position where the target is going to be X_n. This is accomplished using

$$\begin{aligned}
&Z_n = HX_n \\
&Z_{n-1} = HX_{n-1} = H\phi^{-1}X_n \\
&Z_{n-2} = HX_{n-2} = H\phi^{-1}\phi^{-1}X_n = H\left(\phi^{-1}\right)^2 X_1 \qquad 5.6 \\
&\vdots \\
&Z_1 = HX_1 = H\left(\phi^{-1}\right)^{n-1} X_n = H_1 X_n
\end{aligned}$$

clearly, the nth state can be estimated from the first measurement. Depending upon the number of measurements needed, the P_0 and X_0 matrices can be calculated using

$$\begin{aligned}
&P_0 = \left\{ P_\infty^{-1} + \sum_{i=1}^{n} \left[H\left(\phi^{-1}\right)^{n-i} \right]^T R^{-1} \left[H\left(\phi^{-1}\right)^{n-i} \right] \right\}^{-1} \\
&X_0 = P_0 \left\{ \sum_{i=1}^{n} \left[H\left(\phi^{-1}\right)^{n-i} \right]^T R^{-1} Z_i \right\}
\end{aligned} \qquad 5.7$$

When we tried the aforementioned method early in the progression of the project, the result was a divergent Kalman filter. The P matrix increased without bound and ceased to be positive definite. We did not try the method later on after changing the sampling time from one second to 0.01 seconds, the measurement variance from 0.25 to 0.005 degrees and several other parameters.

Now that the Kalman filter methodology has been shown to work, another exercise is to use the auto tracking feed errors $\Delta\theta_{EL}$ and $\Delta\theta_{DEF}$ as measurements instead of the actual elevation and azimuth angles. This creates more difficulties because in addition to the system dynamic equations being nonlinear, so are the measurements.

A different twist on the same problem would be to perform a coordinate transformation. We can use XYZ coordinates instead of using azimuth and elevation. This would provide states which may not need extensive linearizations. The measurement mapping matrix would definitely be nonlinear on the other hand.

Te next logical step is to use the actual satellite orbit instead of the crossing course approximation of a portion of the trajectory. Beyond that, it may be possible to use the Kalman filter to generate accurate satellite ephemeris. Presently, satellite ephemeris is downloaded every two to three days to ensure the accuracy of the satellite position at a certain time. If the time between data reloading is extended, this feature could prove beneficial to the satellite communications industry.

CHAPTER 6

CONCLUSION

Kalman filtering is a new approach to the classic least squares problem which has found many uses since its introduction. It has been used in missile applications, the latter of which was the subject of the paper. The paper focused on tracking a low orbit satellite using an antenna tracking system. Conditions were determined and simulations run to show that the Kalman filter could be used in this type of application.

The antenna tracking system consists of an Elevation over Azimuthally type which houses the servomechanisms, motors, bearings and other mechanical parts, a properly sized parabolic dish, a feed and feed electronics and the controller which provides the command center and display unit. Unlike filters of the past, the Kalman filter itself resides in the microprocessor of the control unit.

Kalman filter began with the discrete form which allowed only linear systems to be analyzed. In the years since its introduction, the basic Kalman filtering has had a continuous form, a linearized form and the extended form for real time filtering. To understand the problem at hand, the discrete Kalman filter equations were discussed in depth before similarities were drawn to formulate the extended Kalman filter, the continuous-discrete extended Kalman filter and the discrete-discrete linearized Kalman filter.

Combinations of the continuous and discrete formats abound depending upon whether the measurement or propagation is continuous or discrete. The simulation is of a discrete- discrete linearized Kalman filter which assumes that measurements are taken and errors are propagated at discrete points in time.

The Kalman filter discussed for the crossover course application is the extended Kalman filter. With this filter, it is possible to update the estimates with each new measurement. This refinement of the trajectory yields more accurate results than the linearized Kalman filter which assumes a set trajectory. In order that the benefits are reaped from the extended Kalman filter, some characteristic flaws must be addressed.

The extended Kalman filter comes equipped with its own problems, the primary one being divergence. This threat can be mitigated by better modeling techniques, use of floating point arithmetic and increasing the number of observable states whenever possible. When this is not possible, simulations should be run several times to be sure that the system is bounded. Once the P matrix increases without bound, it estimates states which quickly become useless.

Extended Kalman filters work in real time but not in simulations. Therefore to estimate the accuracy of the extended Kalman filter, a linearized Kalman filter was used in the simulation. After the problem was defined, that is satellite height, inclination angle, measurement error, initial conditions and noise had to be produced. A Gaussian random number generator was used to simulate the white noise and engineering estimates were used where used when no available information was obtained.

The goal of the project is to create a Kalman filter using antenna tracking system for the crossover course using azimuth and elevation coordinates.Overall, the Kalman filter provides good estimates of the position and velocity of the satellite trajectory.

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Retrieval date: 03/20/2010

APPENDIX

SIMULATION CODE

% Program Name - AJAY.M

% This program is of a Continuous-Discrete Extended Kalman Filter implemented as a Discrete -% Discrete Linearized Kalman filter which is applied to the crossover course in tracking an earth % orbiting satellite.

% Define constants

% i = Inclination angle at which satellite travels.
i=80*(pi/180);

% u = GM where G is the universal gravitational constant and M is the mass
%   of the earth.
u=3.98601*10^5;

% Re = The equatorial radius of the earth.
Re=6378.145;

% h = Satellite altitude above the surface of the earth.
h=463;

% v = Velocity of the satellite.
V=sqrt(u/(Re+h));

% X0 = Shortest horizontal distance from the antenna of the satellite.
X0=h*cos(i);

% Z0 = Corrected altitude.

```
Z0=h*sin(i);
a=V/X0;
b=Z0/X0;

% AMAX = Maximum accelration of the satellite.
AMAX_AZ=0.649*a^2;
AMAX_EL=0.001478;



% DT = The filter time step size in seconds. Measurements and updates are
%      taken every DT seconds.
DT=0.01;

% Initialize the state vectors.
% X = a 6x1 state vector of the estimate.
% XT = a 6x1 state vector of the true dynamics.
X=[0 0 0 0 0 0]';
XDOT=[0 0 0 0 0 0]';
XT=[0 0 0 0 0 0]';
XTDOT=[0 0 0 0 0 0]';
Y=0;
n=5000;
k=(1 n);
xout=[k;k;k;k;k;k]';

% X(1) = Satellite Azimuth angle.
% X(1) = AZ.
X(1)=0.0;

% X(2) = Satellite velocity in azimuth fron the antenna system.
% X(2) = AZDOT.
```

```
X(2)=a;

% X(3) = Satellite acceleration in azimuth.
% X(3) = AZACC.
X(3)=0.0;

% X(4) = Satellite elevation angle.
% X(4) = EL.
X(4)=80*(pi/180);
% X(5) = Satellite velocity in elevation from the antenna system.
% X(5) = ELDOT.
X(5)=0.0;

% X(6) = Satellite acceleration in elevation.
% X(6) = ELACC.
X(6)=0.0;

% Constants for the white noise model.

% Beta_AZ = time correlation in azimuth.
Beta_AZ=1;

% Beta_EL = time correlation in elevation.
Beta_EL=1;

% Constants for the Singer model.

% VAR_Q_AZ = Variance of the accel noise in azimuth.
VAR_Q_AZ=AMAX_AZ*AMAX_AZ/3.0;

% VAR_Q_EL = Variance of the accel noise in elevation.
```

```
VAR_Q_EL=AMAX_EL*AMAX_EL/3.0;

% VAR__N = Variance of the Gauss Markov process noise.
VAR_N_AZ= VAR_Q_AZ;
VAR_N_EL= VAR_Q_EL;

% Z = a 2x1 measurement vector.
Z=[0.0 0.0]';

% H = Measurement matrix relating the states to the measurements. Measures
%     elevation and azimuth angle.
H=[1 0 0 0 0 0;
 0 0 0 1 0 0];

% R = a 2x2 Measurement error matrix.
VAR_R_AZ=(0.005*(pi/180))^2;
VAR_R_EL=(0.005*(pi/180))^2;
R=[VAR_R_AZ 0.0;
  0.0    VAR_R_EL];

% Initialize the matrices. eye(6) is a 6x6 identity matrix.
I=eye(6);
P=0*eye(6);
F=P;
G=P;
Q=P;
G(3,3)=sqrt(2.0*VAR_N_AZ/Beta_AZ);
G(6,6)=sqrt(2.0*VAR_N_EL/Beta_EL);
Q(3,3)=VAR_Q_AZ;
Q(6,6)=VAR_Q_EL;
VAR_V_AZ=9.9547e-7;
```

```
VAR_V_EL=6.3544e-8;

% Initial truth state vector values.
XT(1)=X(1);
XT(2)=X(2);
XT(3)=X(3);
XT(4)=X(4);
XT(5)=X(5);
XT(6)=X(6);
P=[VAR_R_AZ 0 0 0 0 0;
  0 VAR_V_AZ 0 0 0 0;
  0 0 VAR_Q_AZ 0 0 0;
  0 0 0 VAR_R_EL 0 0;
  0 0 0 0 VAR_V_EL 0;
  0 0 0 0 0 VAR_Q_EL];
m=Y+1;
for  k=m n
k;
rand('state');
AZNOISE =sqrt(Q(3,3))*rand;
rand('state');
ELNOISE=sqrt(Q(6,6))*rand;
XTDOT(1) =XT(2);
XTDOT(2)=-a*(sin(2*XT(1)))*XT(2)+ XT(3);
XTDOT(3)=-XT(3)/Beta_AZ+ ...
      sqrt(2*VAR_N_AZ/(Beta_AZ*DT))*AZNOISE;
XTDOT(4)= XT(5);
XTDOT(5)=-((a/b)^2)*(cos(XT(4)))^2*(tan(XT(4)))^3 ...
      -(a/b^2)*tan(XT(1))*(tan(XT(4)))^2*(3- ...
      2*(sin(XT(4)))^2)*XT(5)+XT(6);
XTDOT(6)=-XT(6)/Beta_EL+ ...
```

```
    sqrt(2*VAR_N_EL/(Beta_EL*DT))*ELNOISE;
XT=XT+XTDOT*DT;

% Truth states X(1), the azimuth angle and X(4), the elevation angle are
% corrupted by measurement noise.
rand('state');
Z(1)=XT(1)+sqrt(VAR_R_AZ)*rand;
rand('state');
Z(2)=XT(4)+sqrt(VAR_R_EL)*rand;

% Compute the Kalman Gain.
K=P*H'*inv(H*P*H'+R);

% Update state estimate with new measurement.
X=X+K*(Z-H*X);
for j= 1 6
  xout(k,j)=X(j)*180/pi;
end

% Compute the error covariance for the updated estimate.
P=(I-K*H)*P;

%project ahead
XDOT(1)=X(2);
XDOT(2)=-a*(sin(2*X(1)))*X(2)+X(3);
XDOT(3)=-X(3)/Beta_AZ;
XDOT(4)=X(5);
XDOT(5)=-((a/b)^2)*(cos(X(4)))^2*(tan(X(4)))^3-...
-(a/b^2)*tan(X(1))*(tan(X(4)))^2*(3-...
2*(sin(X(4)))^2)*X(5)+X(6);
XDOT(6)=-X(6)/Beta_EL;
```

```
X=X+XDOT*DT;

%F=(partials of XTDOT)/(partials of X)
F(1,2)=1.0;
F(2,1)=-2*a*XT(2)*cos(2*XT(1));
F(2,2)=-a*sin(2*XT(1));
F(2,3)=1.0;
F(3,3)=-1.0/Beta_AZ;
F(4,5)=1.0;
F(5,1)=-(a/b^2)*(1/(cos(XT(1)))^2)*(tan(XT(4)))^2 ...
*(3-2*(sin(XT(4)))^2)*XT(5);
F(5,4)=-((a/b)^2)*((tan(XT(4)))^2+2*...
    (sin(XT(4)))^2)-2*(a/b^2)*tan(XT(1))*...
tan(XT(4))*((1/(cos(XT(4)))^2) - ...
2*(sin(XT(4)))^2+2)*XT(5);
F(5,5)=-(a/b^2)*tan(XT(1))*(tan(XT(4)))^2* ...
    (3-2*(sin(XT(4)))^2);
F(5,6)=1.0;
F(6,6)=-1.0/Beta_EL;
PDOT=F*P+P*F'+G*Q*G';
P=P+PDOT*DT;
end

for t=1 50
  A=atan(a*t);
  aout(t)=A*180/pi;
  Adot=a*(cos(A))^2;
  adout(t)=Adot*180/pi;
  E=atan(b/sqrt(1+(a*t)^2));
  eout(t)=E*180/pi;
  Edot=-a^2*b*t*(cos(E))^2/(1+(a*t)^2)^1.5;
```

```
  edout(t)=Edot*180/pi;
end

% keyboard
 k=m n;
 t=1 50;
 plot(k*0.01,xout(m n,1))
 grid
 xlabel('seconds')
 ylabel('degrees')
 title('Estimation of Azimuth Angle')

% keyboard
 plot(k*0.01,xout(m n,2))
 grid
 xlabel('seconds')
 ylabel('degrees/sec')
 title('Estimation of Azimuth velocity')

%keyboard
 plot(k*0.01,xout(m n,4))
 grid
 xlabel('seconds')
 ylabel('degrees')
 title('Estimation of Elevation angle')

%keyboard
 plot(k*0.01,xout(m n,5))
 grid
 xlabel('seconds')
 ylabel('degrees/sec')
```

```
title('Estimation of Elevation Velocity')

%keyboard
plot(t,aout,'+',k*0.01,xout(m n,1))
grid
xlabel('seconds')
ylabel('degrees')
title('Actual Azimuth Angle (+) and Estimate')

%keyboard
plot(t,adout,'+',k*0.01,xout(m n,2))
grid
xlabel('seconds')
ylabel('degrees/sec')
title('Actual Azimuth Velocity (+) and Estimate')

%keyboard
plot(t,eout,'+',k*0.01,xout(m n,4))
grid
xlabel('seconds')
ylabel('degrees')
title('Actual Elevation angle (+) and Estimate')

%keyboard
plot(t,edout,'+',k*0.01,xout(m n,5))
grid
xlabel('seconds')
ylabel('degrees/sec')
title('Actual Elevation Velocity (+) and Estimate')
```