Assignment 4

Due: July 7th, 2024

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Use pseudo-code in questions that need you to present your algorithms, but your pseudo-code can be "English-like" if the operation is obvious. Make sure that each line in your pseudo-code is numbered, and the indentation is correct.

1. Consider the following approach to the matrix-chain multiplication problem: Whenever there are at least 3 matrices ($A_i \dots A_j, j - i \geq 2$) remaining, always split the chain at k: $(A_i \dots A_k)(A_{k+1} \dots A_j)$, such that p_k is the smallest number among $\{p_i, \dots, p_{j-1}\}$. Prove that this algorithm doesn't always give the optimal way to calculate the product.

2. Given a set of k positive integers $\{a_1, a_2, \cdots, a_k\}$ that add up to N. Note that, there might exist $1 \leq i \neq j \leq k$ such that $a_i = a_j$ (in other words, these k integers might not be unique). Is there a subset $B \subseteq \{1,2, \dots, k\}$ such that the follow equation is satisfied?

$$\sum_{i \in B} a_i = \sum_{i \in \{1,2,\dots,k\} \setminus B} a_i$$

   a) If N is an odd number, what can we say about the problem?
   b) From now on, we assume that N is an even number. Try to create a recursive function $b(i, j)$ that represents the Boolean value whether there is a subset in $\{a_1, a_2, \cdots, a_i\}$ that add up to j.
   c) Create an algorithm using dynamic programming to solve this problem. You may use pseudo-code, or you may answer the following questions:
      a. How to create a memo to record the value of each $b(i, j)$?
      b. How to use the created memo to find the solution to the problem?
   d) What is the time complexity of your dynamic programming algorithm?

3. A subsequence is a $palindrome$ if it is the same when read left to right and right to left. A subsequence does not have to be contiguous. How to find the longest subsequence which is a palindrome in the given string $A = a_1 a_2 \cdots a_n$? For example, the string $abcab$ contains four palindromes of length 3 as subsequence: aba, aca, bcb, and bab, but no palindrome of length 4; thus, any palindrome of length 3 is an optimal solution.

   a) Create a recursive function $L(i, j)$ that represents the $length$ of the longest subsequence which is a palindrome in substring $a_i \cdots a_j$.
   b) Create an algorithm using dynamic programming to solve this problem. You may use pseudo-code, or you may answer the following questions:
      a. How to create a memo to record the value of each $L(i, j)$?
      b. So that one can find the longest subsequence which is a palindrome in A, what extra info is needed to be recorded in the memo?
      c. How to use the created memo to find an optimal solution to the problem?
   c) What is the time complexity of your dynamic programming algorithm?

4. In the Art Gallery Guarding Problem we are given a straight line L that represents a long hallway in an art gallery, and we are given a set $X = \{x_1, x_2, \ldots, x_n\}$ of real numbers that specify the positions of paintings in this hallway. Suppose that a single guard can protect all the paintings within distance at most 1 of his position on both sides.
    a) Design a greedy algorithm for finding a placement of guards that uses the minimum number of guards to protect all the paintings with positions in X. A guard can be placed at any real number position in the hallway.
    b) Show the correctness of your algorithm (aka, show greedy choice property).