CS430 Assignment 2
Due: June 9th, 2024

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Use pseudo-code in questions that need you to present your algorithms, but your pseudo-code can be "English-like" if the operation is obvious. Make sure that each line in your pseudo-code is numbered, and the indentation is correct.

1. In the algorithm **SELECT**, we achieved a worst-case linear time by grouping elements in groups of size 5. What is the worst-case time complexity of an alternative version of **SELECT** where we group elements in groups of size $2m + 1$ (where $m \geq 2$ is an integer constant)? Show your work.

2. Give an $O(n \lg k)$-time algorithm to merge k sorted arrays into one sorted array, where n is the total number of elements in all the input arrays. (Hint: Use a min-heap for k-way merging.)

3. Provide pseudo-code for the operation $\mathbf{MAX-HEAP-DELETE}$ (A, i) that deletes the element in $A[i]$ from a binary max-heap A. In your code, you can call $\mathbf{MAX-HEAPIFY}$ and/or $\mathbf{HEAP-INCREASE-KEY}$ from the textbook/lecture notes directly if you want to. Analyze the time complexity of your algorithm.

4. Professor Bunyan thinks he has discovered a remarkable property of binary search trees. Suppose that the search for key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Professor Bunyan claims that any three keys $a \in A, b \in B$, and $c \in C$ must satisfy $a \leq b \leq c$. Give the smallest possible (with fewest nodes) counterexample to the professor's conjecture.

   As an aside, think about how to prove that your counterexample is the smallest, you don't need to show this proof in your solution.