Task 2.(c)

From the RandSelect-2(A, n, k) function, we find that k^{th} position of A will be put at the k'^{th} location of A' which is equal to k X m/n.

Assuming a sorted array A, we will remap in a reverse fashion i.e. A' to A. Again, the kth element in A' will map to $(k \text{ X } n/m)^{th}$ position in A.

Assuming there are 1000 elements in A and we have to map the 100^{th} position of A' in A, then that will be equal to 100 * (100/1000) = 10^{th} position in A'

Similarly, if we have to map the kth location of A' to A, that will be equal to 10 * (1000/100) = 100^{th} position in A.

We have to bound the probability that the element lies between xright and xleft.
Let Xi= { 1, if the element's value lies in between xright and xleft,
0, otherwise

Here,
Xleft = k^{th} left smallest element of A' and
xright = k^{th} right smallest element of A'

We have calculated the value of $\Delta = kright - kleft$ in Task 2. (b)

Now, let us define new variables k'right and k'left which are the positions of xleft and xright in A respectively.

Therefore, Δ' = k'right – k'left

But, k'left = kleft X n/m
And k'right = kright X n/m

Therefore, Δ' = kright X n/m – kleft X n/m
= Δ X n/m

Calculating the expected value of Δ':
E[Δ'] = E[$\Delta\, X\, n/m$]
= (n/m) X E[Δ]
= (n/m) X μ
= μ'

Hence, we have to prove:

Pr(X $\leq$ 4($\sqrt{2m\, logn}$) loglogn) is high i.e w.h.p. in n

Initially we will prove that:

$$\Pr(X \geq 4(\sqrt{2mlogn})\ \text{loglogn})\ \text{is low i.e w.l.p. in n}$$

Using this we will prove the contrary.

Using Chernoff bound, we get:

$$(1+\delta)\mu' = 4(\sqrt{2mlogn}) * (\text{loglogn})$$

From above, we have:

$$\mu' = (n/m) * \mu$$

By substituting, we get:

$$(1+\delta)(\mathrm{n/m}) * \mu = 4(\sqrt{2mlogn})\ \text{loglogn})$$

Here,

$$m = \frac{n}{loglogn}$$

Substituting m, we get:

$$(1+\delta)(\mathrm{nloglogn/n}) * \mu = 4(\sqrt{2mlogn})\ \text{loglogn})$$

$$(1+\delta) * \mu = 4(\sqrt{2mlogn})$$

Substituting μ , we get:

$$(1+\delta) * 2\sqrt{mlogn})(\sqrt{1-k/n} + \sqrt{k/n}) = 4(\sqrt{2mlogn})$$

$$(1+\delta) = 2\sqrt{2}\ /\ \sqrt{1-k/n} + \sqrt{k/n}$$

$$\delta = 2\sqrt{2}\ /\ \sqrt{1-k/n} + \sqrt{k/n} - 1$$

Using this, we can show that:

$$\Pr(X \geq 4(\sqrt{2mlogn})\ \text{loglogn})$$

Using Chernoff's bound, we get:

$$\Pr(X \geq 4(\sqrt{2mlogn})\ \text{loglogn}) \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu'}$$

Consider:

$$X = \sqrt{1 - k/n} + \sqrt{k/n}$$

Then,

$$\Pr(X \geq 4(\sqrt{2mlogn}) \text{ loglogn}) \leq \left(\frac{e^{\frac{2\sqrt{2}}{x}-1}}{\left(\frac{2\sqrt{2}}{x}\right)^{\left(\frac{2\sqrt{2}}{x}\right)}}\right)^{(n/m) * \mu}$$

$$\Pr(X \geq 4(\sqrt{2mlogn}) \text{ loglogn}) \leq \left(\left(\frac{1}{e}\right)\frac{e^{\frac{2\sqrt{2}}{x}}}{\left(\frac{2\sqrt{2}}{x}\right)^{\left(\frac{2\sqrt{2}}{x}\right)}}\right)^{(n/m) * \mu}$$

Substituting μ in the terms of x, we get:

$$\Pr(X \geq 4(\sqrt{2mlogn}) \text{ loglogn}) \leq \left(\left(\frac{1}{e}\right)\frac{e^{\frac{2\sqrt{2}}{x}}}{\left(\frac{2\sqrt{2}}{x}\right)^{\left(\frac{2\sqrt{2}}{x}\right)}}\right)^{(n/m) * 2\sqrt{mlogn}(x)}$$

Simplifying the above equation, we get:

$$\Pr(X \geq 4(\sqrt{2mlogn}) \text{ loglogn}) \leq \left(\left(\frac{1}{e^x}\right)\frac{e^{2\sqrt{2}}}{\left(\frac{2\sqrt{2}}{x}\right)^{(2\sqrt{2})}}\right)^{(\text{nloglogn}/n) * 2\sqrt{logn(\frac{n}{loglogn})}}$$

As $(\frac{1}{e^x})$ is constant, we can substitute it with some constant c

$$\Pr(X \geq 4(\sqrt{2mlogn}) \text{ loglogn}) \leq \left((c)\left(\frac{e}{\frac{2\sqrt{2}}{x}}\right)^{2\sqrt{2}}\right)^{(2\sqrt{nlognloglogn})}$$

Here, The term $(\frac{e}{\frac{2\sqrt{2}}{x}})^{2\sqrt{2}})$ is less than 1.

As we know any number when multiplied by a number which is less than 1, will yield in an even smaller number.

Hence, by contrary we have proved that Pr(X $\geq$ $4(\sqrt{2mlogn})$ loglogn) occurs with a low probability in n.