

3. a) Originally, Sam would compute $B^a = g^{ab} \pmod{p}$ and Naum would compute $A^b = g^{ab} \pmod{p}$.
However, after Malcolm's interception, Sam computes $C^a = g^{ac} \pmod{p}$ and Naum computes $C^b = g^{bc} \pmod{p}$.

The ^{values} ~~multiplication~~ of the two integers b and c is what is hard to compute in efficient time, but since Malcolm already knows g^c ,

$$A = g^a$$

$$A^c = g^{ac} = g^{ca} = C^a$$

$$A^c = C^a. \text{ Malcolm already knows } A, c, \text{ and } C$$

$$a = c \log_c A$$

$$\text{Similarly, } b = c \log_c B$$

Hence, calculating the secret keys becomes very easy.

- b) Since Sam replaced both keys A and B with C , the encrypted message m' will be $E(A, m)$ will be replaced with $E(C, m)$ before the message reaches Naum and vice versa.

Decrypting m' is quite easy for Malcolm no matter who the sender is because he already know the secret key. Decrypting A or B with C will cause both of them to have an incorrect shared key.

- c) The keys ~~to~~ A and B were only shared once at the beginning, hence, Malcolm already intercepted it at that time and replaced it with C .
Therefore, although they will be able to successfully communicate, the information sent and received will automatically be modified by the key C .