# A Novel Bald Eagle Search Variant with Enhanced Global Search Capabilities

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**ABSTRACT:** The use of metaheuristics is one of the most encouraging methodologies for taking care of real-life problems. Bald eagle search (BES) algorithm is the latest swarm-intelligence metaheuristic algorithm inspired by the intelligent hunting behavior of bald eagles. In recent research works,

BES algorithm has performed reasonably well over a wide range of application areas such as chemical engineering, environmental science, physics and astronomy, structural modeling, global optimization, engineering design, energy efficiency, etc. However, it still lacks adequate searching efficiency and has a tendency to stuck in local optima which affects the final outcome. This paper proposes a novel enhancement strategy addressing these issues by incorporating three key improvements:

Eagle Velocity Inspired Exploration: Inspired by Particle Swarm Optimization (PSO), this mechanism introduces momentum into the search process, allowing agents to explore further afield and escape local optima.

Levy Flights: This stochastic search mechanism, inspired by real-world foraging patterns, facilitates exploration beyond areas identified by the standard BES approach.

Random Walks: This technique introduces random jumps within the search process, helping the algorithm escape potential local optima and explore new solution regions.

The efficiency of the BES3 algorithm is initially evaluated with 29 CEC2017. In addition, the practicality of the BES3 is tested with a real-world feature selection problem and three engineering design problems. Results of the BES3 algorithm are compared against a number of classical metaheuristic algorithms using statistical metrics, convergence

analysis, and the Wilcoxon rank sum test. In the case of composite CEC2017 test functions F21-F30, BES3 wins against compared algorithms in 70% test cases, whereas for the rest of the test functions, it generates good results in 100% cases.In the case of the feature selection problem, the BES3 also showed competitiveness with the compared algorithms. Results and observations for all tested optimization problems show the superiority and robustness of the proposed BES3 over the baseline metaheuristics. It can be safely concluded that the improvements suggested in the BES3 are proved

to be effective making it competitive enough to solve a variety of optimization problems.

# 1 INTRODUCTION

**Challenges in Optimization and Rise of Nature-Inspired Techniques**

Finding optimal solutions in artificial intelligence remains a significant hurdle (Sameer et al., 2019; Tariq et al., 2018; Zaidan et al., 2017). To address this, numerous algorithms have been developed.

However, two key challenges exist:

1. Identifying both global and local optima (best solutions overall and within specific regions).
2. Maintaining these optimal solutions throughout the search process (Qu et al., 2012).

In recent years, nature-inspired computation has gained significant traction among researchers. Nature offers a treasure trove of concepts, mechanisms, and ideas for designing artificial intelligence systems that can tackle complex mathematical problems (Barhen et al., 1997).

**Evolutionary Algorithms (EAs): Inspired by Natural Selection**

Living organisms must adapt to their surroundings to survive and reproduce, a process known as evolution (Fogel, 1995, 2009; Fogel et al., 1965; Schwefel, 1995; Michalewicz & Attia, 1994). This principle inspires ”evolutionary algorithms” (EAs), a prominent and successful category of optimization algorithms.

Several types of EAs exist, including:

- Genetic algorithms (Houck et al., 1995; Joines & Houck, 1994; Kazarlis & Petridis, 1998; Holland, 1992)
- Genetic programming (Koza, 1992)
- Evolutionary programming and evolutionary strategies (Yao et al., 1999; Rechenberg, 1994; Whitley, 2001; Yao et al., 1999b; Yao & Liu, 1997)

EAs are widely used for tackling problems like combinatorial and non-linear optimization (Fiacco & McCormick, 1968). They excel at handling diverse issues by incorporating prior knowledge into the search process, allowing for efficient exploration of the solution space.

Despite their advantages, EAs can sometimes struggle to find the absolute optimal solution. Consequently, researchers have explored merging them with other techniques for improved performance (Whitley, 2001).

**Swarm Intelligence (SI): Mimicking Collective Animal Behavior**

Swarm intelligence (SI) is another form of nature-inspired optimization technique. One prominent example is particle swarm optimization (PSO) (Birge, 2003; Shi & Eberhart, 1998; Kennedy & Eberhart, 1995), which mimics the flocking behavior of birds or schooling of fish (Li, 2003).

Other SI algorithms include:

- Ant colony optimization (Dorigo et al., 2006) - inspired by foraging and schooling behavior of ants
- Gravitational search (Rashedi et al., 2009)
- Grey wolf optimizer (GWO) (Mirjalili et al., 2014)

- Artificial bee colony (Karaboga & Basturk, 2007)
- Moth-flame optimization (Mirjalili, 2015)
- Whale optimization (Mirjalili & Lewis, 2016)
- Group search optimizer (He et al., 2006, 2009)
- Ant lion optimizer algorithms (Mirjalili, 2015)
- PSO variants like comprehensive learning particle swarm optimizer (CLPSO) (Liang et al., 2006), fitness-distance- ratio-based particle swarm optimization (FDR-PSO) (Peram et al., 2003), and ensemble particle swarm optimizer (EPSO) (Lynn & Suganthan, 2017)

SI tackles problems by simulating how animals move in search of resources. The effectiveness of this technique can be influenced by problem size and non-linearity.

While SI can achieve optimal solutions for computational and combinatorial problems, traditional analytical methods often struggle with convergence.

**Advantages and Limitations of SI**

SI offers several benefits:

- Individuals can improve search efficiency by moving between positions.
- The entire swarm can collectively improve by sharing information.

Unlike EAs, where weak individuals are replaced, SI allows all individuals to contribute and explore new areas, potentially leading to faster convergence towards global optima.

However, SI also has limitations:

- Collective movement can lead to premature convergence on a local optimum, hindering exploration of other regions.
- Individuals might get stuck in local optima and fail to escape.

**The Potential of Combining Evolutionary and Swarm Techniques**

The versatility of nature-inspired techniques, coupled with the possibility of combining EAs and SI

## 2 BES algorithm

### 2.1 Behaviour of bald eagle during hunting

**Bald Eagle Hunting Strategies**

Despite their name, bald eagles aren't truly bald. These opportunistic predators primarily target fish (alive or dead), especially salmon. They're skilled hunters, capable of spotting prey from vast distances. However, catching fish is challenging, with only 1 in 20 attempts succeeding.

**Hunting Process:**

1. **Search:** Eagles begin by searching for prey from perches or while flying. They may follow other birds to locate fish concentrations.

2. **Selection:** Once a promising area is identified (usually within 700 meters of their nest), they initiate a focused search.

3. **Attack:** Upon spotting prey, eagles dive swiftly to snatch the fish out of the water.

**Efficiency:**

Bald eagles prioritize energy conservation. They rest after hunting due to the high energy expenditure involved.

Additionally, they exploit wind currents to soar efficiently while searching for prey.

**Key Advantages:**

- Excellent eyesight allows them to spot fish from long distances.
- Binocular vision enables them to see forward and sideways simultaneously.
- Powerful eyesight surpasses human vision by four times.

This revised version condenses the original text, focusing on key points about bald eagle hunting behavior and their physical adaptations for success.

## 2.2 Brief description

**Bald Eagle Search (BES) Algorithm**

The BES algorithm mimics how Bald Eagles hunt for prey. Here's a breakdown of the three main stages:

1. **Selection Stage:**
   - Eagles randomly choose a search area based on prey concentration.
   - They update their position using Equation (1) to balance exploration (searching new areas) and exploitation (focusing on areas with more prey).

$$Pnew, \quad i = P_{best} + \alpha * r(P_{mean} - P_i)$$

2. **Search Stage:**
   - Eagles fly in a spiral pattern within the chosen area, searching for the best location to attack.
   - Equation (12) defines the mathematical model for this spiral flight path.

$$P_{i,new} = P_i + y(i) * (P_i - P_{i+1}) + x(i) * (P_i - P_{mean})$$

$$x(i) = \frac{xr(i)}{\max(|xr|)}, \quad y(i) = \frac{yr(i)}{\max(|yr|)} \quad \text{(a)}$$

$$xr(i) = r(i) * \sin(\theta(i)), \quad yr(i) = r(i) * \cos(\theta(i)) \quad \text{(b)}$$

3. **Swooping Stage:**
   - Eagles dive quickly from the best-found location to capture prey.
   - Other eagles in the population also move towards this best position to attack.
   - Equation (14) describes how eagle positions are updated during this swooping stage.

$$P_{i,new} = rand * P_{best} + x1(i) * (P_i - c1 * P_{mean}) + y1(i) * (P_i - c2 * P_{best})$$

$$x1(i) = \frac{xr(i)}{\max(|xr|)}, \quad y1(i) = \frac{yr(i)}{\max(|yr|)}$$

$$xr(i) = r(i) * \sinh[\theta(i))], \quad yr(i) = r(i) * \cosh[\theta(i))]$$

**Optimization in BES**

The core idea behind optimization algorithms is to improve the performance of an algorithm in terms of factors like speed, memory usage, and reliability. BES aims to optimize these aspects by mimicking the efficient hunting behavior of Bald Eagles.

# 3 Proposed Algorithm

**Limitations of the Classical BES Algorithm**

The standard BES algorithm, while inspired by nature, has some shortcomings:

- **Potential for Local Optima:** Like many swarm-based algorithms, BES can get trapped in suboptimal solutions (local optima) and miss the truly best solution (global optimum). This is particularly problematic for complex, high-dimensional problems.
- **Slow Convergence:** The position update process relies heavily on previous positions, leading to slow convergence towards the optimal solution

To address these limitations, we propose the modified Bald Eagle Search (BES3) algorithm. This improved version aims to achieve better exploration and a better balance between exploration and exploitation, ultimately leading to finding the global optimum more effectively.

Here's a small algorithm summarizing the BES3 function:

**1. Initialization:**

- Create a population of nPop eagles with random positions within the search space.
- Evaluate the fitness of each eagle using the objective function (fobj).
- Initialize the best solution found so far (BestSol).

**2. Main Loop (for each iteration):**

a. **Selection Stage (select space function):** - Update eagle positions by drawing on the best solution and average population position. - Evaluate the fitness of these new positions. - Update BestSol if a better solution is found.

b. **Update Velocity (update velocity function):** - Calculate a velocity vector for each eagle based on: - Attraction to the best solution (social term). - Attraction to its own best position (cognitive term).

c. **Update Positions (update position function):** - Update eagle positions based on their current position and velocity.
Ensure positions stay within the search space boundaries.

update velocity based on social and cognitive components

*social term = c1 * rand(1,dim) .* (BestSol.pos - pop.pos(i,:));*
*cognitive term = c2 * rand(1,dim) .* (BestSol.pos - pop.pos(i,:));*

d. **Search Stage (search space function):** - Update eagle positions using a spiral search pattern around the average population position. - Evaluate the fitness of these new positions. - Update BestSol if a better solution is found.

e. **Swooping Stage (swoop function):** - Update eagle positions using a combination of: - Attraction to the best solution. - Attraction to a "doubled" average population position. - Le´vy Flights for exploration (random jumps). - Random Walk for exploration (small random steps). - Evaluate the fitness of these new positions. - Update BestSol if a better solution is found.

*levy jump = (sin(pi*alpha*(1 + beta)) * exp((alpha - 1)*log(v))) / (beta * power(u, (1/beta)));*
*step size = 0.1;*
*ran- dom walk = pop.pos(i, :) + step size * (rand(1, dim) - 0.5);*
*newsol.pos = random walk + levy jump;*

f. **Optional: Track Convergence** - Store the cost of the best solution in each iteration for analysis. - Print the cost of the best solution periodically (optional).

**3. Return Results:**

Return the best solution found (BestSol), convergence curve (optional), and total execution time.

## BES3 Algorithm

```
BaldEagleSearchOptimization(nPop, MaxIt, low, high, dim, fobj)
  Initialize BestSol with maximum possible cost
  For every eagle in population
    Initialize position and velocity randomly
    Determine cost based on objective function
    Update BestSol if current eagle has lower cost
  End for
  For every iteration
    Select space for each eagle
    Update velocity for each eagle based on best solution
    Update position for each eagle based on velocity
    Search and swoop in space for better solutions
    Update Convergence_curve with BestSol
  End for
  Return BestSol, Convergence_curve, and total processing time
End
SelectSpace(fobj, pop, npop, BestSol, low, high, dim)
  Determine mean position of population
  For every eagle in population
    Determine new solution based on best solution and mean position
    Update population and BestSol if new solution has lower cost
  End for
End
UpdateVelocity(pop, BestSol, w, c1, c2, npop, dim)
  For every eagle in population
    Determine social and cognitive terms based on best solution
    Update velocity based on social and cognitive terms
  End for
End
UpdatePosition(pop, low, high, npop)
  For every eagle in population
    Update position based on velocity
    Ensure position is within defined boundaries
  End for
End
SearchSpace(fobj, pop, best, npop, low, high)
  Determine mean position of population
  For every eagle in population
    Determine a new solution based on current position, next position, and mean position
    Update population and BestSol if new solution has lower cost
  End for
End
Swoop(fobj, pop, best, npop, low, high, dim)
  Determine mean position of population
  For every eagle in population
    Determine new solution based on a Lévy flight and a random walk
    Update population and BestSol if new solution has lower cost
  End for
End
```

# 4 Results and Analysis

*4.1. Experimental series 1: CEC 2017*

Benchmark functions and Parameter settings The proposed BES is first tested on challenging CEC 2017 benchmark instances with 50 dimensions (D = 50). The CEC 2017 benchmark suite comprises 30 functions, which are grouped into four categories based on their characteristics. Benchmarks from 1 to 3 are unimodal, test functions from 4 to 10 are multimodal, benchmarks from 11 to 22 are hybrid functions and finally, test bed from 21 to 30 belongs to the category of composite functions. Details of these benchmark functions are given

The comparison results of all algorithms over Unimodal & Multimodal CEC2017 benchmark functions.

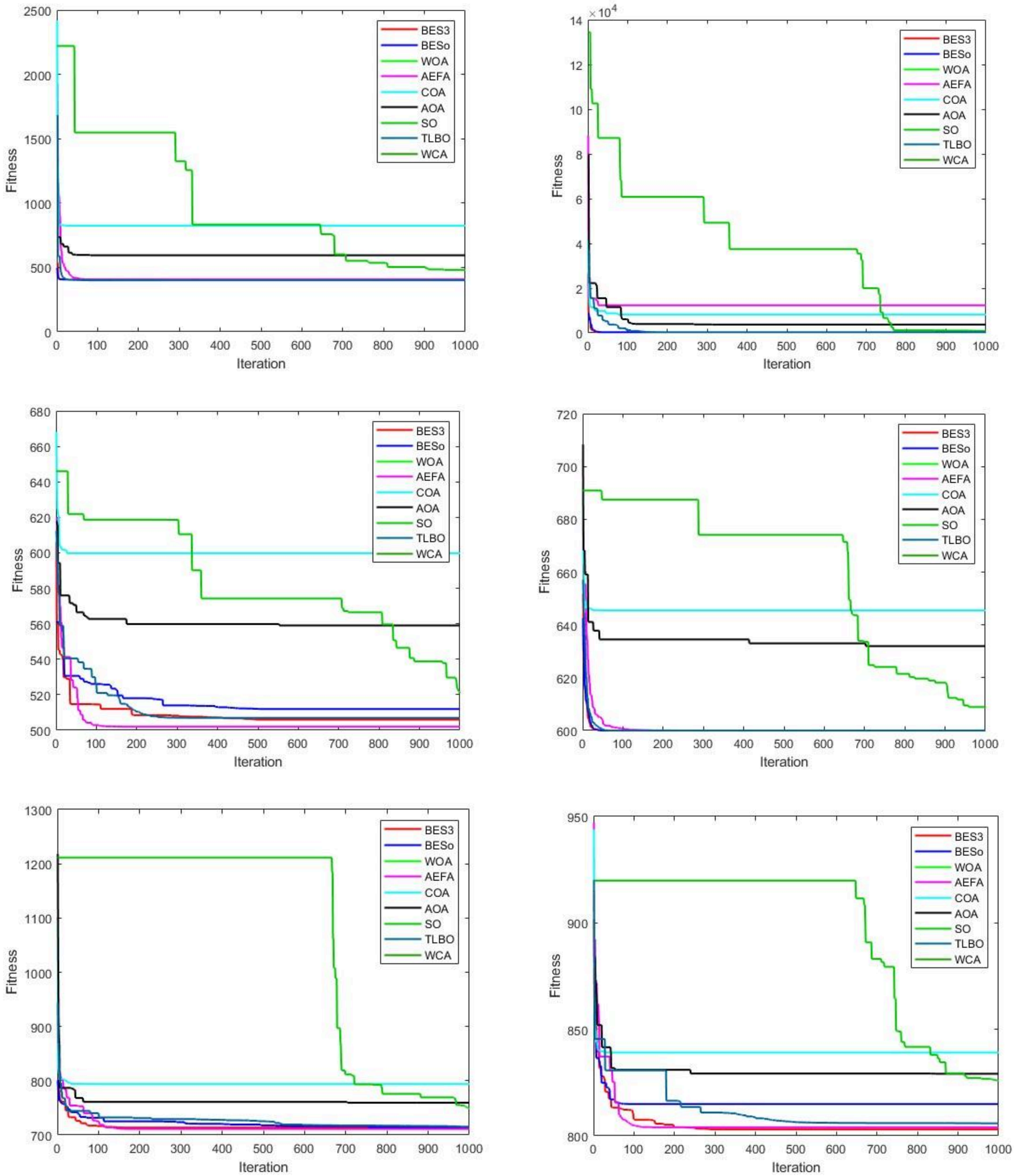
| F | | BES3 | BES | WOA | TLBO | SO | WCA | AEFA | COA | AOA |
|---|---|---|---|---|---|---|---|---|---|---|
| F1 | Best | 1.00E + 02 | 1.05E + 02 | 3.20E + 08 | 1.11E + 02 | 2.49E + 04 | 1.61E + 02 | 3.59E + 02 | 3.22E + 07 | 4.11E + 02 |
| | Mean | 1.02E + 02 | 3.74E + 03 | 1.64E + 09 | 4.35E + 03 | 2.53E + 05 | 7.55E + 03 | 1.04E + 06 | 8.67E + 07 | 2.32E + 04 |
| | Std | 7.61E + 00 | 4.62E + 03 | 1.05E + 09 | 4.62E + 03 | 2.57E + 05 | 7.43E + 03 | 4.88E + 06 | 3.97E + 07 | 3.57E + 04 |
| | Rank | 1 | 2 | 9 | 3 | 6 | 4 | 7 | 8 | 5 |
| F3 | Best | 3.00E + 02 | 3.00E + 02 | 1.51E + 05 | 6.09E + 03 | 2.70E + 04 | 5.61E + 02 | 7.75E + 04 | 4.32E + 04 | 2.55E + 04 |
| | Mean | 3.00E + 02 | 9.94E + 02 | 2.56E + 05 | 1.92E + 04 | 5.58E + 04 | 6.16E + 03 | 1.04E + 05 | 1.13E + 05 | 4.38E + 04 |
| | Std | 0.00E + 00 | 1.35E + 03 | 7.49E + 04 | 7.51E + 03 | 1.10E + 04 | 5.15E + 03 | 1.52E + 04 | 3.41E + 04 | 1.25E + 04 |
| | Rank | 1 | 2 | 9 | 4 | 6 | 3 | 7 | 8 | 5 |
| F4 | Best | 4.00E + 02 | 4.00E + 02 | 6.30E + 02 | 4.64E + 02 | 4.04E + 02 | 4.70E + 02 | 4.91E + 02 | 5.04E + 02 | 4.72E + 02 |
| | Mean | 4.00E + 02 | 4.40E + 02 | 8.58E + 02 | 4.96E + 02 | 4.89E + 02 | 5.11E + 02 | 5.37E + 02 | 5.65E + 02 | 4.82E + 02 |
| | Std | 0.00E + 00 | 3.60E + 01 | 1.58E + 02 | 2.24E + 01 | 2.59E + 01 | 2.43E + 01 | 2.34E + 01 | 3.87E + 01 | 8.23E + 00 |
| | Rank | 1 | 2 | 9 | 5 | 4 | 6 | 7 | 8 | 3 |
| F5 | Best | 5.01E + 02 | 5.95E + 02 | 7.17E + 02 | 5.63E + 02 | 5.57E + 02 | 5.62E + 02 | 5.68E + 02 | 5.81E + 02 | 5.78E + 02 |
| | Mean | 5.06E + 02 | 6.52E + 02 | 8.40E + 02 | | 6.30E + 02 | 6.01E + 02 | 5.91E + 02 | 6.33E + 02 | 6.19E + 02 |
| | Std | 3.39E + 00 | 3.03E + 01 | 6.26E + 01 | 2.52E + 01 | 3.67E + 01 | 2.04E + 01 | 1.49E + 01 | 2.53E + 01 | 2.03E + 01 |
| | Rank | 1 | 8 | 9 | 4 | 6 | 3 | 2 | 7 | 5 |
| F6 | Best | 6.00E + 02 | 6.12E + 02 | 6.62E + 02 | 6.04E + 02 | 6.05E + 02 | 6.37E + 02 | 6.14E + 02 | 6.02E + 02 | 6.05E + 02 |
| | Mean | 6.00E + 02 | 6.34E + 02 | 6.79E + 02 | 6.17E + 02 | 6.15E + 02 | 6.59E + 02 | 6.24E + 02 | 6.05E + 02 | 6.15E + 02 |
| | Std | 1.17E-02 | 1.14E + 01 | 1.10E + 01 | 7.77E + 00 | 6.83E + 00 | 1.17E + 01 | 6.36E + 00 | 1.57E + 00 | 7.50E + 00 |
| | Rank | 1 | 7 | 9 | 5 | 3 | 8 | 6 | 2 | 4 |
| F7 | Best | 7.10E + 02 | 8.59E + 02 | 1.09E + 03 | 8.15E + 02 | 7.90E + 02 | 8.28E + 02 | 7.51E + 02 | 8.50E + 02 | 8.05E + 02 |
| | Mean | 7.13E + 02 | 9.76E + 02 | 1.27E + 03 | 8.89E + 02 | 8.37E + 02 | 8.72E + 02 | 7.74E + 02 | 9.07E + 02 | 8.68E + 02 |
| | Std | 1.28E + 00 | 6.09E + 01 | 8.43E + 01 | 4.18E + 01 | 3.93E + 01 | 3.13E + 01 | 1.57E + 01 | 2.61E + 01 | 3.37E + 01 |
| | Rank | 1 | 8 | 9 | 6 | 3 | 5 | 2 | 7 | 4 |
| F8 | Best | 8.01E + 02 | 8.60E + 02 | 9.51E + 02 | 8.51E + 02 | 8.46E + 02 | 9.37E + 02 | 8.43E + 02 | 8.79E + 02 | 8.72E + 02 |
| | Mean | 8.05E + 02 | 9.13E + 02 | 1.04E + 03 | 8.87E + 02 | 8.96E + 02 | 9.91E + 02 | 8.73E + 02 | 9.38E + 02 | 9.05E + 02 |
| | Std | 1.69E + 00 | 2.81E + 01 | 5.29E + 01 | 1.79E + 01 | 2.13E + 01 | 3.85E + 01 | 1.36E + 01 | 2.48E + 01 | 1.83E + 01 |
| | Rank | 1 | 6 | 9 | 3 | 4 | 8 | 2 | 7 | 5 |
| F9 | Best | 9.00E + 02 | 1.74E + 03 | 6.11E + 03 | 9.68E + 02 | 1.27E + 03 | 3.48E + 03 | 1.11E + 03 | 1.67E + 03 | 1.25E + 03 |
| | Mean | 9.00E + 02 | 4.35E + 03 | 1.02E + 04 | 1.71E + 03 | 3.39E + 03 | 1.98E + 03 | 1.55E + 03 | 2.51E + 03 | 2.31E + 03 |
| | Std | 3.80E - 01 | 1.77E + 03 | 2.67E + 03 | 5.58E + 02 | 1.23E + 03 | 5.92E + 02 | 3.75E + 02 | 6.42E + 02 | 7.43E + 02 |
| | Rank | 1 | 8 | 9 | 2 | 7 | 4 | 3 | 6 | 5 |
| F10 | Best | 1.00 + 03 | 3.45E + 03 | 5.71E + 03 | 7.27E + 03 | 3.04E + 03 | 4.17E + 03 | 3.84E + 03 | 4.16E + 03 | 3.98E + 03 |
| | Mean | 1.42E + 03 | 5.57E + 03 | 7.23E + 03 | 8.11E + 03 | 3.72E + 03 | 5.97E + 03 | 4.81E + 03 | 5.04E + 03 | 5.21E + 03 |
| | Std | 2.18E + 02 | 1.31E + 03 | 7.77E + 02 | 3.64E + 02 | 4.59E + 02 | 8.18E + 02 | 6.16E + 02 | 4.67E + 02 | 7.72E + 02 |
| | Rank | 1 | 6 | 8 | 9 | 2 | 7 | 3 | 4 | 5 |

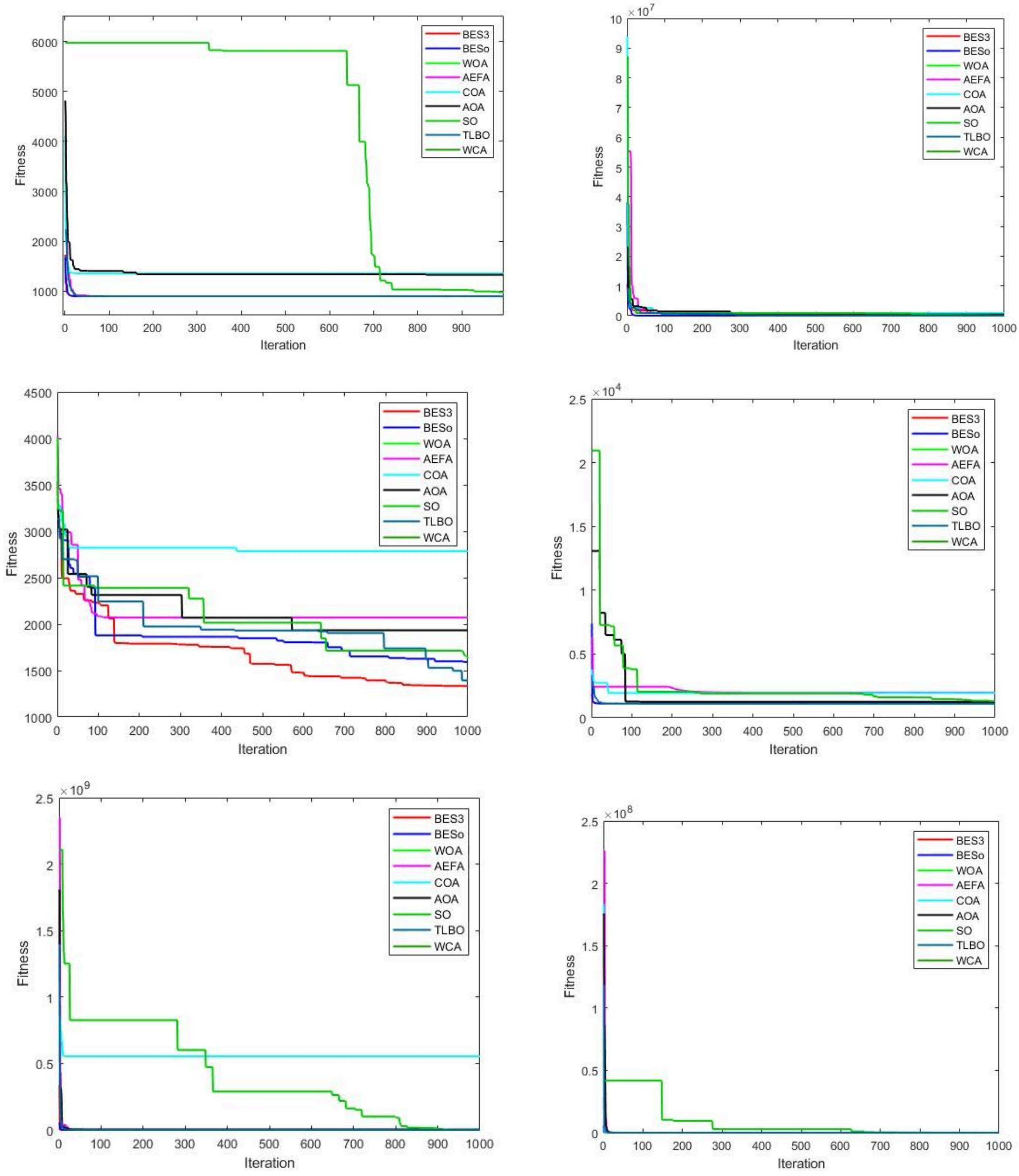
| F | | BES3 | BES | TLBO | WOA | SO | WCA | AEFA | COA | AOA |
|---|---|---|---|---|---|---|---|---|---|---|
| F11 | Best | 1.10E + 03 | 1.15E + 03 | 1.16E + 03 | 1.86E + 03 | 1.16E + 03 | 1.21E + 03 | 3.38E + 03 | 1.24E + 03 | 1.17E + 03 |
| | Mean | 1.11E + 03 | 1.22E + 03 | 1.25E + 03 | 6.07E + 03 | 1.30E + 03 | 1.35E + 03 | 7.07E + 03 | 1.45E + 03 | 1.26E + 03 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Std | 8.89E + 00 | 5.21E + 01 | 5.42E + 01 | 2.58E + 03 | 7.62E + 01 | 8.04E + 01 | 2.39E + 03 | 3.15E + 02 | 7.04E + 01 |
| | Rank | 1 | 2 | 3 | 8 | 5 | 6 | 9 | 7 | 4 |
| F12 | Best | 1.44E + 03 | 8.87E + 03 | 1.26E + 04 | 5.56E + 07 | 9.75E + 04 | 1.28E + 05 | 1.51E + 06 | 1.75E + 06 | 9.63E + 04 |
| | Mean | 3.77E + 03 | 3.07E + 04 | 9.87E + 04 | 3.03E + 08 | 1.38E + 06 | 1.03E + 06 | 1.13E + 07 | 7.60E + 06 | 1.06E + 06 |
| | Std | 4.61E + 03 | 1.84E + 04 | 8.10E + 04 | 2.03E + 08 | 1.31E + 06 | 9.85E + 05 | 7.68E + 06 | 5.69E + 06 | 7.97E + 05 |
| | Rank | 1 | 2 | 3 | 9 | 6 | 4 | 8 | 7 | 5 |
| F13 | Best | 1.30E + 03 | 1.96E + 03 | 2.07E + 03 | 2.44E + 05 | 6.04E + 03 | 5.87E + 03 | 9.90E + 03 | 5.61E + 04 | 1.60E + 03 |
| | Mean | 1.42E + 03 | 2.01E + 04 | 1.97E + 04 | 1.80E + 06 | 2.94E + 04 | 3.25E + 04 | 3.54E + 04 | 4.62E + 05 | 5.14E + 03 |
| | Std | 1.68E + 02 | 1.85E + 04 | 1.96E + 04 | 1.83E + 06 | 1.89E + 04 | 2.97E + 04 | 1.34E + 04 | 8.16E + 05 | 4.25E + 03 |
| | Rank | 1 | 4 | 3 | 9 | 5 | 6 | 7 | 8 | 2 |
| F14 | Best | 1.40E + 03 | 1.53E + 03 | 1.89E + 03 | 6.45E + 04 | 2.69E + 03 | 2.02E + 03 | 3.02E + 05 | 2.39E + 03 | 2.37E + 03 |
| | Mean | 1.43E + 03 | 1.82E + 03 | 1.45E + 04 | 2.28E + 06 | 3.80E + 04 | 2.43E + 04 | 1.15E + 06 | 5.14E + 04 | 3.21E + 04 |
| | Std | 1.48E + 01 | 1.81E + 02 | 1.46E + 04 | 2.22E + 06 | 2.87E + 04 | 4.05E + 04 | 5.45E + 05 | 7.89E + 04 | 3.79E + 04 |
| | Rank | 1 | 2 | 3 | 9 | 6 | 4 | 8 | 7 | 5 |
| F15 | Best | 1.50E + 03 | 1.83E + 03 | 1.74E + 03 | 7.82E + 04 | 2.29E + 03 | 2.07E + 03 | 6.02E + 03 | 2.37E + 03 | 1.59E + 03 |
| | Mean | 1.53E + 03 | 8.22E + 03 | 9.61E + 03 | 1.42E + 06 | 1.08E + 04 | 1.44E + 04 | 1.76E + 04 | 1.96E + 04 | 3.04E + 03 |
| | Std | 3.20E + 01 | 6.71E + 03 | 9.07E + 03 | 2.99E + 06 | 9.93E + 03 | 1.72E + 04 | 8.78E + 03 | 1.42E + 04 | 1.65E + 03 |
| | Rank | 1 | 3 | 4 | 9 | 5 | 6 | 7 | 8 | 2 |
| F16 | Best | 1.60E + 03 | 2.02E + 03 | 1.78E + 03 | 2.93E + 03 | 2.05E + 03 | 2.20E + 03 | 2.24E + 03 | 2.21E + 03 | 2.25E + 03 |
| | Mean | 1.61E + 03 | 2.51E + 03 | 2.45E + 03 | 4.23E + 03 | 2.54E + 03 | 2.92E + 03 | 2.95E + 03 | 2.73E + 03 | 2.72E + 03 |
| | Std | 3.52E + 01 | 3.66E + 02 | 3.11E + 02 | 6.28E + 02 | 2.62E + 02 | 3.30E + 02 | 3.87E + 02 | 2.40E + 02 | 2.62E + 02 |
| | Rank | 1 | 3 | 2 | 9 | 4 | 7 | 8 | 6 | 5 |
| F17 | Best | 1.71E + 03 | 1.78E + 03 | 1.81E + 03 | 2.17E + 03 | 1.90E + 03 | 2.04E + 03 | 1.93E + 03 | 1.88E + 03 | 1.77E + 03 |
| | Mean | 1.73E + 03 | 2.08E + 03 | 2.00E + 03 | 2.75E + 03 | 2.19E + 03 | 2.47E + 03 | 2.51E + 03 | 2.17E + 03 | 2.07E + 03 |
| | Std | 9.53E + 00 | 2.07E + 02 | 1.60E + 02 | 3.16E + 02 | 1.97E + 02 | 2.00E + 02 | 2.95E + 02 | 1.68E + 02 | 2.09E + 02 |
| | Rank | 1 | 4 | 2 | 9 | 6 | 7 | 8 | 5 | 3 |
| F18 | Best | 1.81E + 03 | 2.85E + 03 | 7.81E + 04 | 1.99E + 05 | 1.01E + 05 | 1.74E + 04 | 6.62E + 04 | 2.26E + 04 | 3.22E + 04 |
| | Mean | 1.89E + 03 | 4.30E + 04 | 3.83E + 05 | 1.03E + 07 | 6.37E + 05 | 3.45E + 05 | 1.96E + 06 | 5.25E + 05 | 1.65E + 05 |
| | Std | 6.40E + 01 | 4.28E + 04 | 3.25E + 05 | 1.07E + 07 | 4.88E + 05 | 4.43E + 05 | 2.36E + 06 | 1.18E + 06 | 1.09E + 05 |
| | Rank | 1 | 2 | 5 | 9 | 7 | 4 | 8 | 6 | 3 |
| F19 | Best | 1.90E + 03 | 2.00E + 03 | 2.05E + 03 | 1.38E + 05 | 2.14E + 03 | 2.32E + 03 | 7.00E + 04 | 2.53E + 03 | 2.36E + 03 |
| | Mean | 1.09E + 03 | 8.58E + 03 | 9.99E + 03 | 1.75E + 07 | 1.31E + 04 | 1.31E + 04 | 9.83E + 05 | 1.59E + 04 | 4.80E + 03 |
| | Std | 1.00E + 01 | 7.59E + 03 | 8.41E + 03 | 1.53E + 07 | 9.29E + 03 | 2.49E + 04 | 1.08E + 06 | 1.67E + 04 | 2.35E + 03 |
| | Rank | 1 | 3 | 4 | 8 | 5 | 4 | 7 | 6 | 2 |
| F20 | Best | 2.00E + 03 | 2.24E + 03 | 2.14E + 03 | 2.36E + 03 | 2.20E + 03 | 2.27E + 03 | 2.28E + 03 | 2.15E + 03 | 2.21E + 03 |
| | Mean | 2.21E + 03 | 2.46E + 03 | 2.42E + 03 | 2.83E + 03 | 2.48E + 03 | 2.78E + 03 | 2.69E + 03 | 2.43E + 03 | 2.45E + 03 |
| | Std | 3.51E + 01 | 2.24E + 02 | 1.72E + 02 | 2.09E + 02 | 1.48E + 02 | 2.47E + 02 | 2.38E + 02 | 1.80E + 02 | 1.89E + 02 |
| | Rank | 1 | 5 | 2 | 9 | 6 | 8 | 7 | 3 | 4 |

| F | | BES3 | BES | TLBO | WOA | SO | WCA | AEFA | COA | AOA |
|---|---|---|---|---|---|---|---|---|---|---|
| F21 | Best | 2.20E + 03 | 2.35E + 03 | 2.35E + 03 | 2.49E + 03 | 2.36E + 03 | 2.43E + 03 | 2.36E + 03 | 2.38E + 03 | 2.35E + 03 |
| | Mean | 2.29E + 03 | 2.41E + 03 | 2.39E + 03 | 2.61E + 03 | 2.39E + 03 | 2.50E + 03 | 2.40E + 03 | 2.43E + 03 | 2.40E + 03 |
| | Std | 3.76E + 01 | 3.07E + 01 | 2.27E + 01 | 4.52E + 01 | 1.57E + 01 | 5.78E + 01 | 2.32E + 01 | 2.50E + 01 | 2.30E + 01 |
| | Rank | 1 | 6 | 3 | 9 | 2 | 8 | 5 | 7 | 4 |
| F22 | Best | 2.20E + 03 | 2.30E + 03 | 2.30E + 03 | 2.69E + 03 | 2.30E + 03 | 2.30E + 03 | 2.30E + 03 | 2.35E + 03 | 2.30E + 03 |
| | Mean | 2.30E + 03 | 3.05E + 03 | 2.57E + 03 | 7.56E + 03 | 3.71E + 03 | 6.15E + 03 | 2.31E + 03 | 5.65E + 03 | 7.01E + 03 |
| | Std | 2.08E + 01 | 1.77E + 03 | 1.39E + 03 | 2.36E + 03 | 1.44E + 03 | 1.90E + 03 | 1.40E + 01 | 1.87E + 03 | 2.36E + 03 |
| | Rank | 1 | 4 | 3 | 9 | 5 | 7 | 2 | 6 | 8 |
| F23 | Best | 2.60E + 03 | 2.72E + 03 | 2.72E + 03 | 2.93E + 03 | 2.74E + 03 | 2.82E + 03 | 2.78E + 03 | 2.76E + 03 | 2.77E + 03 |
| | Mean | 2.61E + 03 | 2.81E + 03 | 2.79E + 03 | 3.13E + 03 | 2.81E + 03 | 2.98E + 03 | 2.86E + 03 | 2.79E + 03 | 2.87E + 03 |
| | Std | 6.45E + 01 | 4.72E + 01 | 4.64E + 01 | 1.06E + 02 | 4.22E + 01 | 9.54E + 01 | 4.41E + 01 | 1.94E + 01 | 6.27E + 01 |
| | Rank | 1 | 5 | 3 | 9 | 4 | 8 | 6 | 2 | 7 |
| F24 | Best | 2.50E + 03 | 2.87E + 03 | 2.91E + 03 | 3.08E + 03 | 2.88E + 03 | 2.94E + 03 | 2.88E + 03 | 2.92E + 03 | 2.91E + 03 |
| | Mean | 2.72E + 03 | 2.97E + 03 | 2.95E + 03 | 3.23E + 03 | 2.94E + 03 | 3.16E + 03 | 2.96E + 03 | 2.99E + 03 | 2.97E + 03 |
| | Std | 6.51E + 01 | 6.13E + 01 | 2.63E + 01 | 9.87E + 01 | 2.89E + 01 | 1.48E + 02 | 4.41E + 01 | 2.69E + 01 | 4.25E + 01 |
| | Rank | 1 | 7 | 3 | 9 | 2 | 8 | 5 | 4 | 6 |
| F25 | Best | 2.90E + 03 | 2.88E + 03 | 2.88E + 03 | 3.02E + 03 | 2.89E + 03 | 2.88E + 03 | 2.91E + 03 | 2.89E + 03 | 2.88E + 03 |
| | Mean | 2.92E + 03 | 2.90E + 03 | 2.91E + 03 | 3.13E + 03 | 2.91E + 03 | 2.90E + 03 | 2.99E + 03 | 2.94E + 03 | 2.90E + 03 |
| | Std | 2.35E + 01 | 1.82E + 01 | 2.27E + 01 | 6.77E + 01 | 1.57E + 01 | 2.11E + 01 | 3.28E + 01 | 2.51E + 01 | 1.74E + 01 |
| | Rank | 6 | 2 | 5 | 9 | 4 | 3 | 8 | 7 | 1 |
| F26 | Best | 2.90E + 03 | 2.90E + 03 | 2.80E + 03 | 4.68E + 03 | 3.56E + 03 | 5.73E + 03 | 2.80E + 03 | 3.50E + 03 | 2.80E + 03 |
| | Mean | 2.97E + 03 | 5.69E + 03 | 4.44E + 03 | 8.20E + 03 | 5.45E + 03 | 7.13E + 03 | 5.09E + 03 | 4.93E + 03 | 4.87E + 03 |
| | Std | 2.18E + 02 | 1.01E + 03 | 1.42E + 03 | 1.26E + 03 | 6.56E + 02 | 1.09E + 03 | 1.40E + 03 | 6.21E + 02 | 1.65E + 03 |
| | Rank | 1 | 7 | 2 | 9 | 6 | 8 | 5 | 4 | 3 |
| F27 | Best | 3.09E + 03 | 3.24E + 03 | 3.21E + 03 | 3.32E + 03 | 3.24E + 03 | 3.23E + 03 | 3.29E + 03 | 3.21E + 03 | 3.20E + 03 |
| | Mean | 3.10E + 03 | 3.28E + 03 | 3.26E + 03 | 3.46E + 03 | 3.29E + 03 | 3.31E + 03 | 3.40E + 03 | 3.23E + 03 | 3.20E + 03 |
| | Std | 1.22E + 01 | 3.62E + 01 | 4.18E + 01 | 1.25E + 02 | 2.70E + 01 | 8.21E + 01 | 6.51E + 01 | 1.05E + 01 | 2.34E-04 |
| | Rank | 1 | 5 | 4 | 9 | 6 | 7 | 8 | 3 | 2 |
| F28 | Best | 3.10E + 03 | 3.10E + 03 | 3.20E + 03 | 3.37E + 03 | 3.23E + 03 | 3.20E + 03 | 3.30E + 03 | 3.30E + 03 | 3.24E + 03 |
| | Mean | 3.25E + 03 | 3.16E + 03 | 3.23E + 03 | 3.56E + 03 | 3.30E + 03 | 3.23E + 03 | 3.35E + 03 | 3.34E + 03 | 3.30E + 03 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Std | 8.39E + 01 | 5.91E + 01 | 2.16E + 01 | 1.40E + 02 | 3.79E + 01 | 2.51E + 01 | 4.10E + 01 | 3.02E + 01 | 1.27E + 01 |
| | Rank | 4 | 1 | 2 | 9 | 6 | 3 | 8 | 7 | 5 |
| F29 | Best | 3.13E + 03 | 3.49E + 03 | 3.45E + 03 | 4.00E + 03 | 3.63E + 03 | 3.67E + 03 | 3.89E + 03 | 3.52E + 03 | 3.31E + 03 |
| | Mean | 3.16E + 03 | 3.96E + 03 | 3.81E + 03 | 5.34E + 03 | 4.06E + 03 | 4.37E + 03 | 4.65E + 03 | 3.85E + 03 | 3.66E + 03 |
| | Std | 3.00E + 01 | 2.95E + 02 | 1.77E + 02 | 5.39E + 02 | 2.21E + 02 | 2.59E + 02 | 3.27E + 02 | 1.79E + 02 | 2.09E + 02 |
| | Rank | 1 | 5 | 3 | 9 | 6 | 7 | 8 | 4 | 2 |
| F30 | Best | 3.40E + 03 | 5.10E + 03 | 5.75E + 03 | 2.35E + 06 | 9.57E + 03 | 7.19E + 03 | 1.66E + 06 | 9.46E + 03 | 3.24E + 03 |
| | Mean | 3.69E + 05 | 8.35E + 03 | 1.06E + 04 | 4.47E + 07 | 8.01E + 04 | 1.34E + 05 | 6.45E + 06 | 8.24E + 04 | 3.89E + 03 |
| | Std | 5.54E + 05 | 2.12E + 03 | 3.51E + 03 | 4.38E + 07 | 7.33E + 04 | 4.96E + 05 | 3.56E + 06 | 6.45E + 04 | 8.66E + 02 |
| | Rank | 7 | 2 | 3 | 9 | 4 | 6 | 8 | 5 | 1 |

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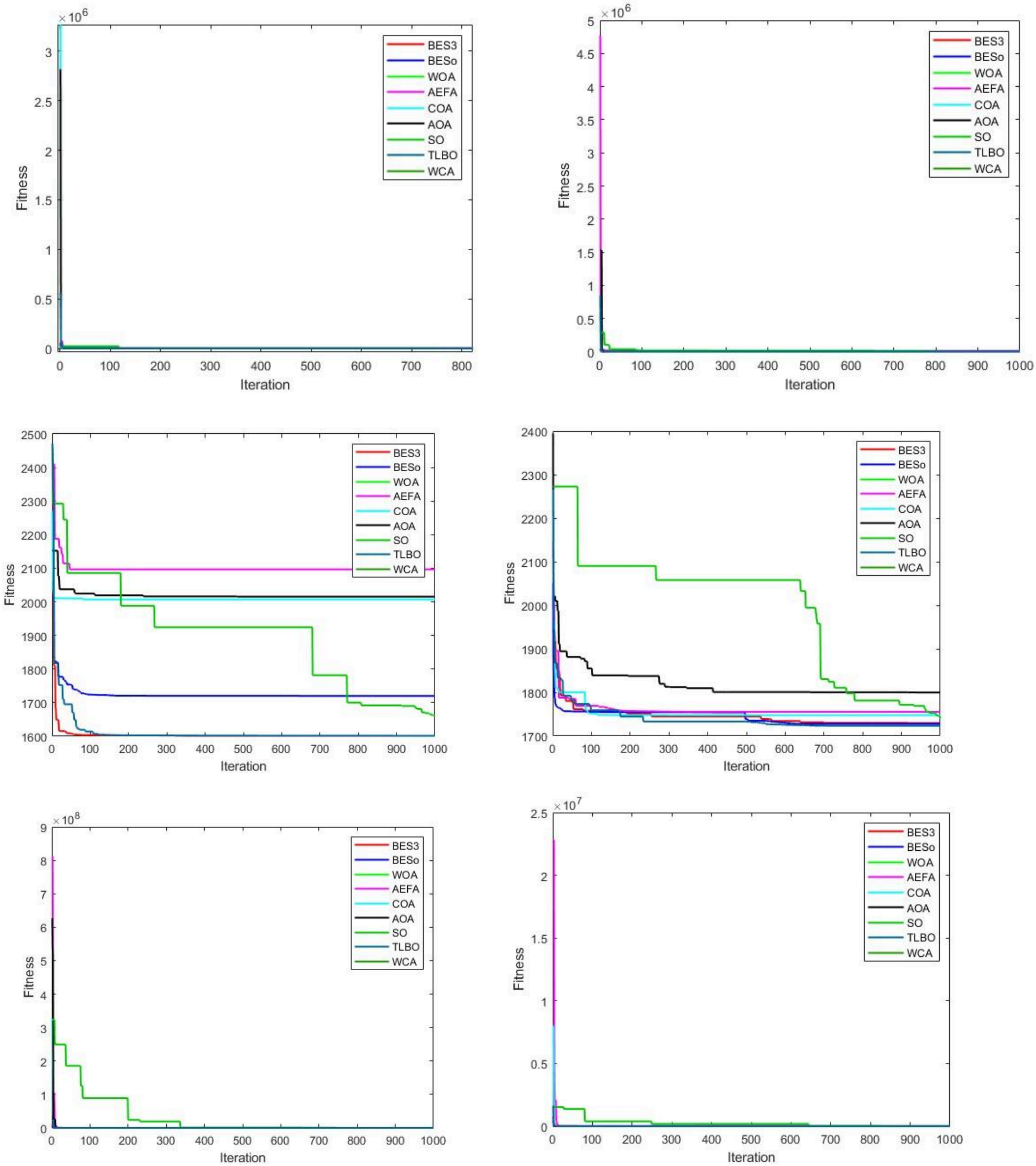
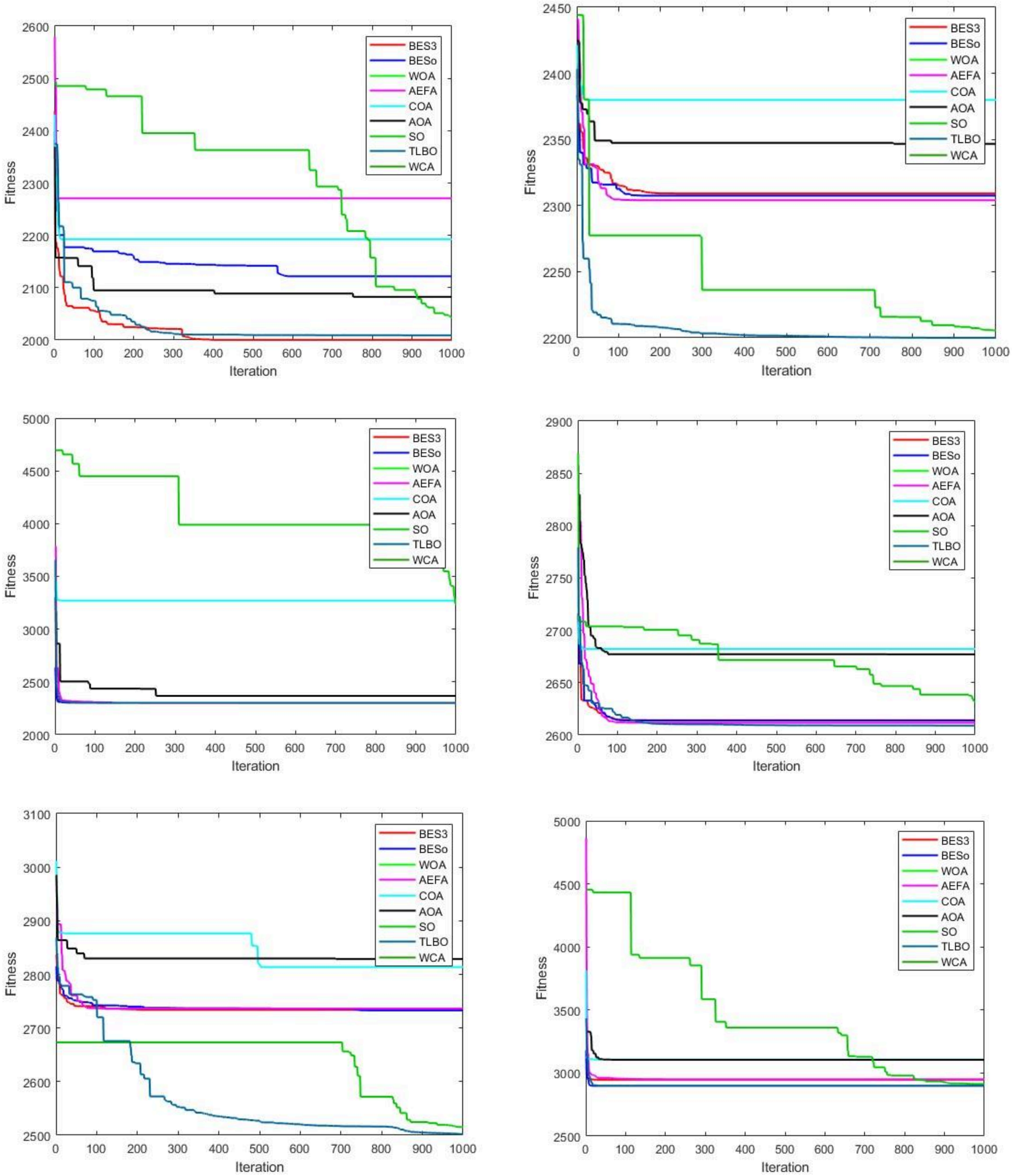
AEFA
COA
AOA
SO
TLBO
WCA
Fitness
Iteration



BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA
Fitness
Iteration
$\times 10^6$
$\times 10^8$
$\times 10^7$


BES3
BESo
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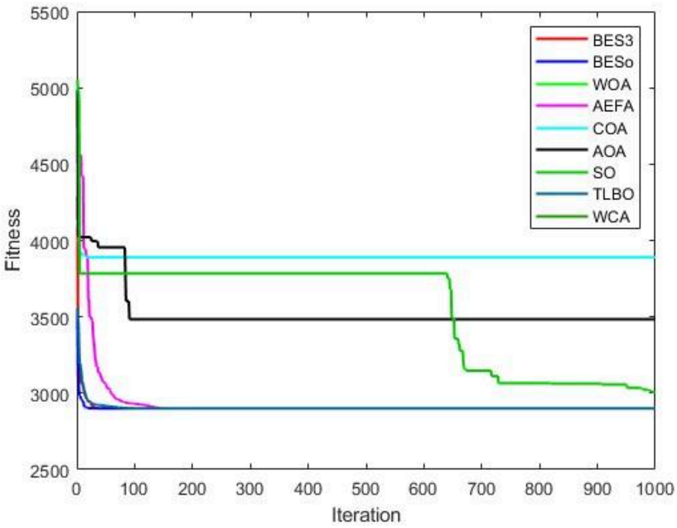
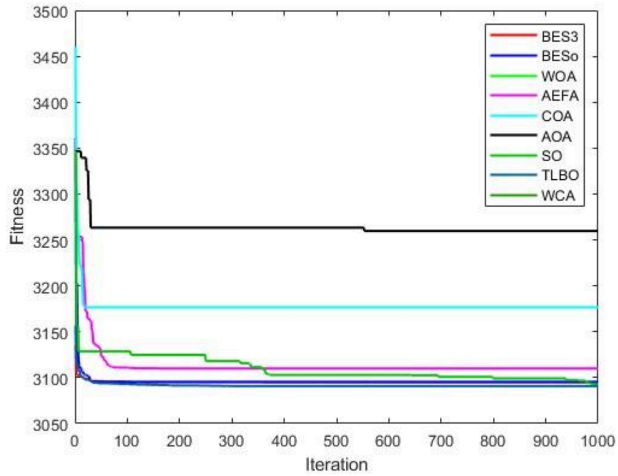
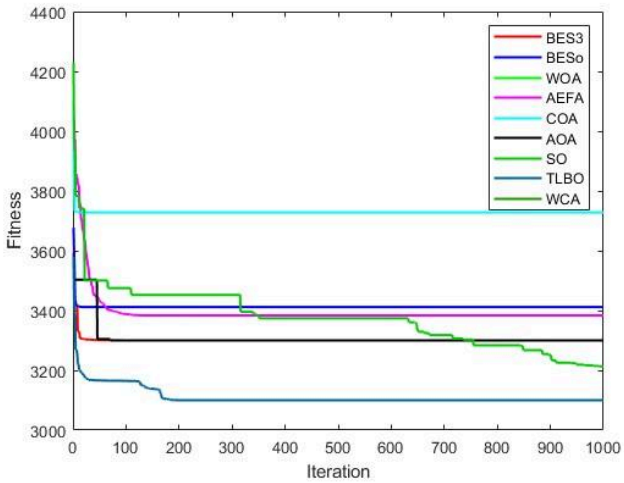
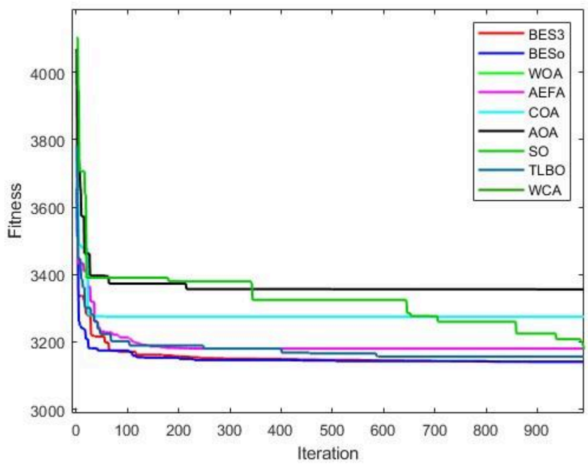
COA
AOA
SO
TLBO
WCA
Fitness
Iteration










## *4.1. Experimental series 1: CEC 2014*

Benchmark functions and Parameter settings The proposed BES is first tested on challenging CEC 2017 benchmark instances with 50 dimensions (D = 50). The CEC 2017 benchmark suite comprises 30 functions, which are grouped into four categories based on their characteristics. Benchmarks from 1 to 3 are unimodal, test functions from 4 to 10 are multimodal, benchmarks from 11 to 22 are hybrid functions and finally, test bed from 21 to 30 belongs to the category of composite functions. Details of these benchmark functions are given

| Problem | Statistics | BES | BES3 |
|---|---|---|---|
| F1 | Mean | *3.66E+03* | 114.3195257 |
| | STD | *3.61E+03* | 48.89758384 |
| | Best | *8.36E+01* | 1.41611 |
| | Winner | | |
| F2 | Mean | *1.40E-10* | 200 |
| | STD | *4.16E−10* | 0 |
| | Best | *2.84E−13* | 200 |
| | Winner | | |
| F3 | Mean | *2.57E−10* | 300 |
| | STD | *4.68E−10* | 0 |
| | Best | *3.41E−13* | 300 |
| | Winner | | |
| F4 | Mean | *2.31E+00* | *400* |
| | STD | *1.19E+01* | *0* |
| | Best | *5.26E−06* | *400* |
| | Winner | | |
| F5 | Mean | 20.86845 | *519.023546* |
| | STD | 0.061067 | *4.00137872* |
| | Best | 2.07E+01 | *500* |
| | Winner | | |
| F6 | Mean | 23.34192 | *601.308452* |
| | STD | 3.047271 | *1.155524012* |
| | Best | 15.6302 | *600* |
| | Winner | | |
| F7 | Mean | 0.019991 | *700.0428002* |
| | STD | 0.018088 | *0.041979404* |
| | Best | 1.25E−12 | *700* |
| | Winner | | |
| F8 | Mean | 87.95414 | *803.5027533* |
| | STD | 23.85544 | *1.71702309* |
| | Best | 45.76804 | *800* |
| | Winner | | |
| F9 | Mean | 107.5547 | *904.2717902* |
| | STD | 19.11751 | *1.926395917* |
| | Best | 76.61167 | *900.99496* |
| | Winner | | |
| F10 | Mean | 2395.764 | *1398.927014* |
| | STD | 670.425 | *186.5272094* |
| | Best | 1313.635 | *1078.3118* |
| | Winner | | |
| F11 | Mean | 3098.091 | *1673.318535* |
| | STD | 492.9754 | *268.937779* |
| | Best | 2165.823 | *1123.5966* |
| | Winner | | |
| F12 | Mean | 1.319921 | *1200.574875* |
| | STD | 0.246955 | *0.138437256* |
| | Best | 8.25E−01 | *1200.2809* |

| | | | |
|---|---|---|---|
| | Winner | | |
| F13 | Mean | *0.36879* | *1300.089441* |
| | STD | *0.073468* | *0.0199633* |
| | Best | *0.213096* | *1300.0474* |
| | Winner | | |
| F14 | Mean | *0.265301* | 1400.084178 |

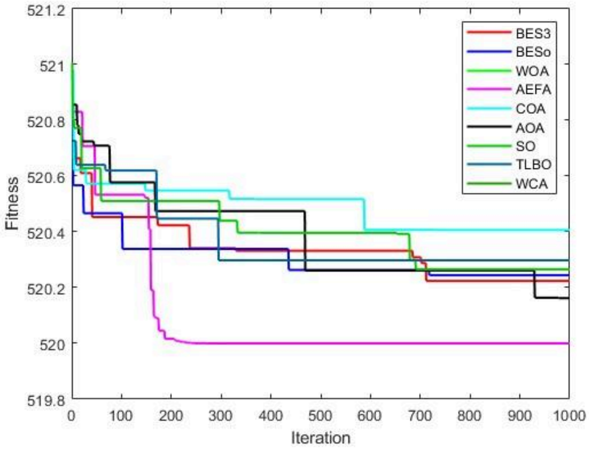
| Problems | Statistics | BES | BES3 |
|---|---|---|---|
| | STD | 0.115124 | 0.031090794 |
| | Best | 0.154129 | 1400.0327 |
| | Winner | | |
| F15 | Mean | 16.10439 | 1500.768296 |
| | STD | 6.990639 | 0.225823386 |
| | Best | 6.777028 | 1500.3907 |
| | Winner | | |
| F16 | Mean | 10.82171 | 1602.173608 |
| | STD | 0.603008 | 0.481940011 |
| | Best | 9.725559 | 1601.1272 |
| | Winner | | |

| problem | Statistics | BES | BES3 |
|---|---|---|---|
| F17 | Mean | 2210.814 | 1899.223063 |
| | STD | 1637.401 | 166.1689438 |
| | Best | 675.6954 | 1713.1573 |

| problem | Statistics | BES | BES3 |
|---|---|---|---|
| F17 | Mean | *2210.814* | |
| | STD | *1637.401* | |
| | Best | *675.6954* | |
| | Winner | | |
| F18 | Mean | *2608.092* | *1934.697169* |
| | STD | *2524.524* | *98.86049705* |
| | Best | *150.4415* | *1803.833* |
| | Winner | | |
| F19 | Mean | 15.21444 | *1901.101286* |
| | STD | 18.90164 | *0.90995047* |
| | Best | 5.489727 | *1900.0315* |
| | Winner | | |
| F20 | Mean | *220.9628* | *2014.642241* |
| | STD | *191.7257* | *15.99635389* |
| | Best | *96.52486* | *2001.006* |
| | Winner | | |
| F21 | Mean | *1550.811* | *2159.867386* |
| | STD | *839.2444* | *98.34820328* |
| | Best | *581.2392* | *2100.8639* |
| | Winner | | |

| | | | |
|---|---|---|---|
| F22 | Mean | *333.7552* | *2227.802071* |
| | STD | *152.6302* | *24.5740428* |
| | Best | *27.01889* | *2204.1918* |
| | Winner | | |

| problem | Statistics | BES | BES3 |
|---|---|---|---|
| F23 | Mean | *284.5123* | *2672.340257* |
| | STD | *51.83408* | *42.62962375* |
| | Best | *200* | *2591.1273* |
| | Winner | | |
| F24 | Mean | *200* | *2535.197292* |
| | STD | *9.41E−07* | *40.74913107* |
| | Best | *200* | *2506.2875* |
| | Winner | | |
| F25 | Mean | *200* | *2692.044214* |
| | STD | *0* | *20.27851838* |
| | Best | *200* | *2631.8265* |
| | Winner | | |
| F26 | Mean | *156.8014* | *2700.12512* |
| | STD | *50.24401* | *0.029995934* |
| | Best | *100.2133* | *2700.0722* |
| | Winner | | |
| F27 | Mean | 811.2298 | *3047.774104* |
| | STD | 208.9184 | *81.75399375* |
| | Best | 401.2241 | *2702.778* |
| | Winner | | |
| F28 | Mean | 1503.785 | *3155.220504* |
| | STD | 326.2962 | *38.00693342* |
| | Best | 1006.089 | *3134.4714* |
| | Winner | | |
| F29 | Mean | *289,353.1* | *4370.369114* |
| | STD | *1,578,238* | *8836.31387* |
| | Best | *738.8932* | *3113.1563* |
| | Winner | | |
| F30 | Mean | *2376.079* | *3349.101442* |
| | STD | *881.5484* | *78.93046155* |
| | Best | *1069.026* | *3261.9516* |
| | Winner | | |


521.2
521
520.8
520.6
520.4
520.2
520
519.8
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Iteration
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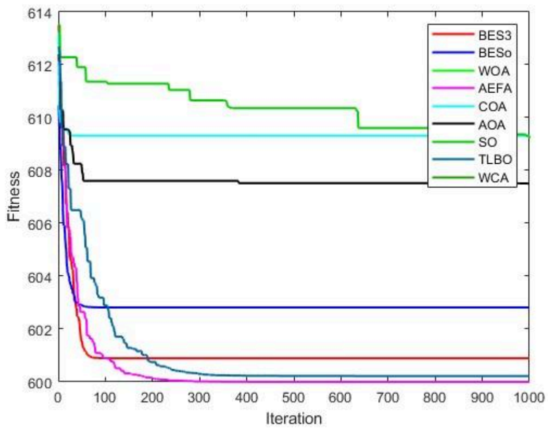
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA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602
600
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AEFA
COA
AOA
SO
TLBO
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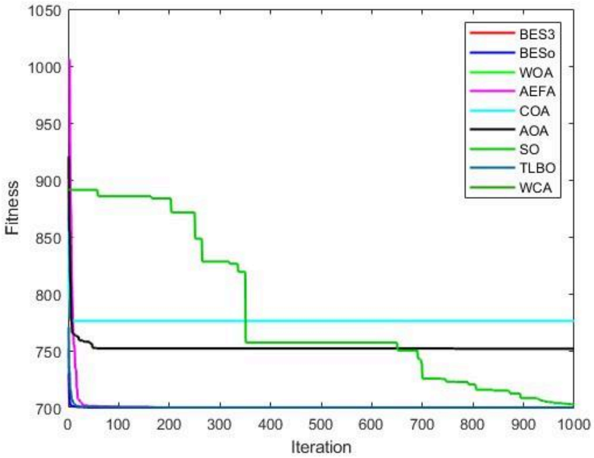
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850
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SO
TLBO
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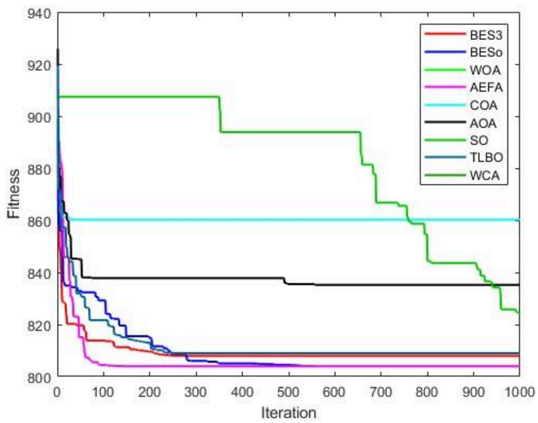
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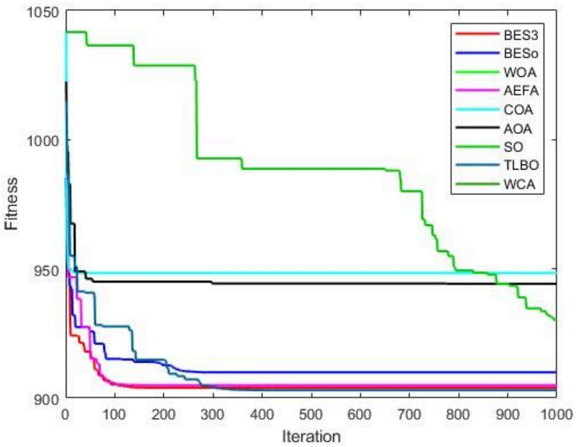
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TLBO
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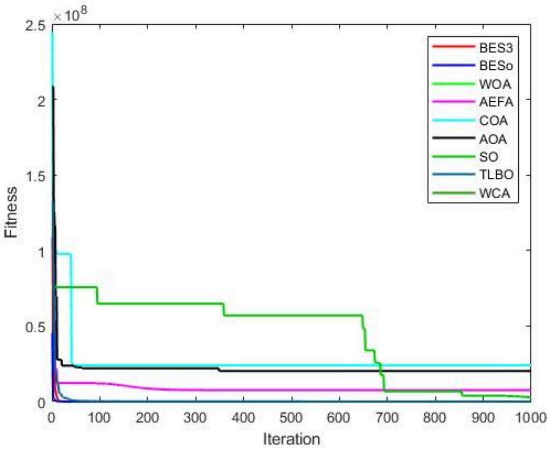
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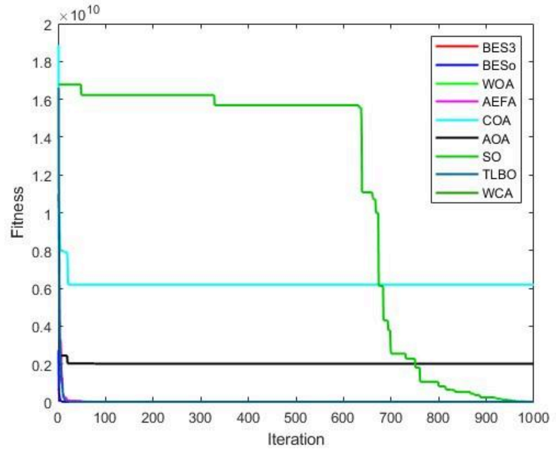
SO
TLBO
WCA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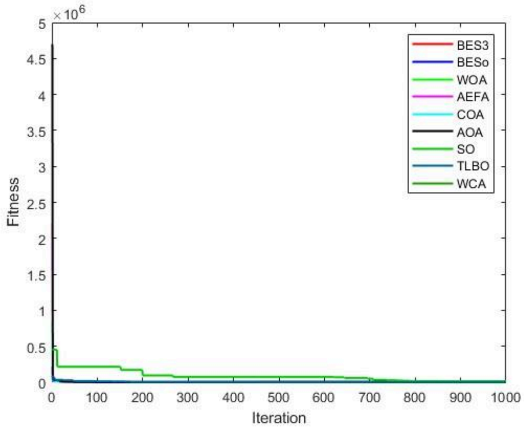
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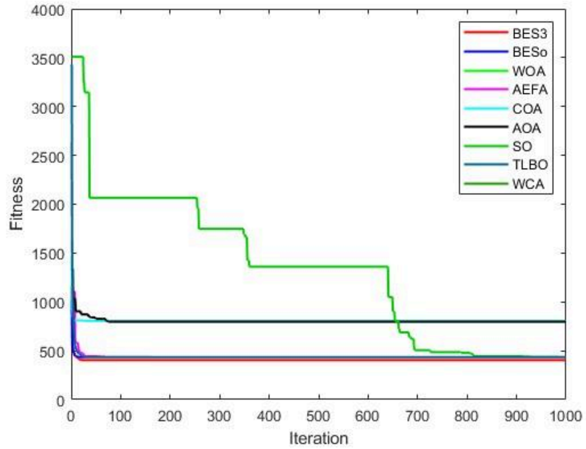
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Iteration
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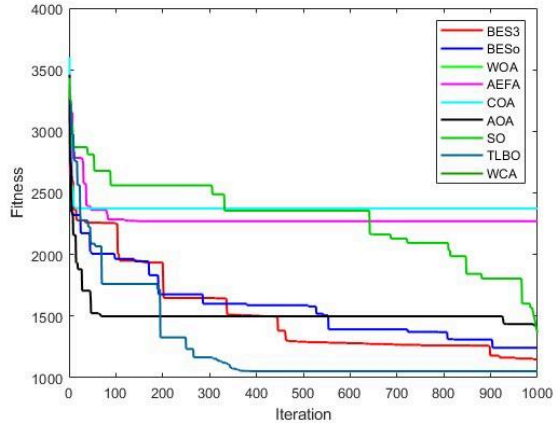
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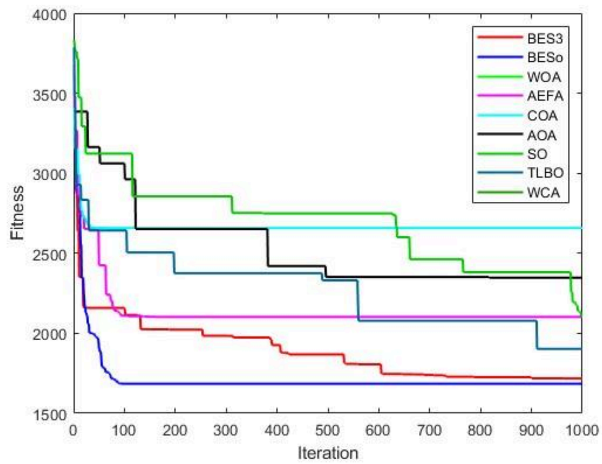
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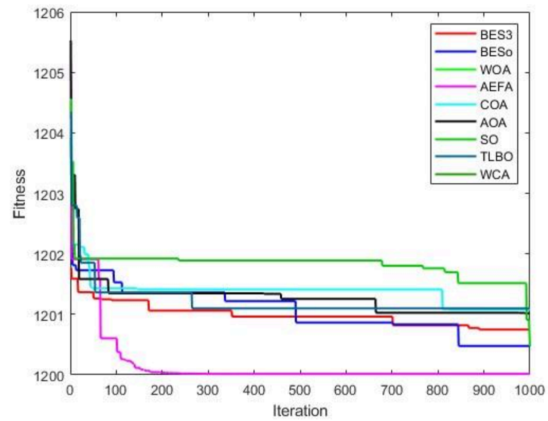
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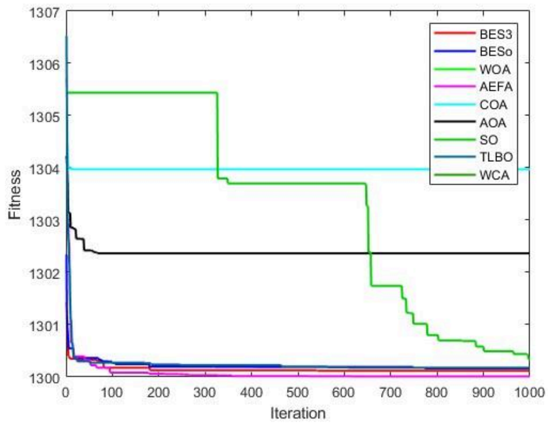
TLBO
WCA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1306
1305
1304
1303
1302
1301
1300
Fitness
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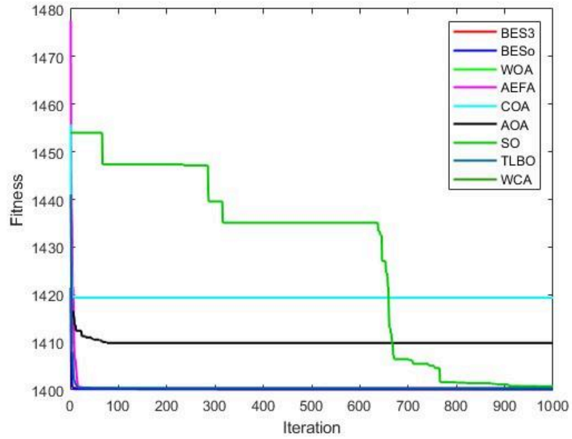
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA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1450
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Fitness
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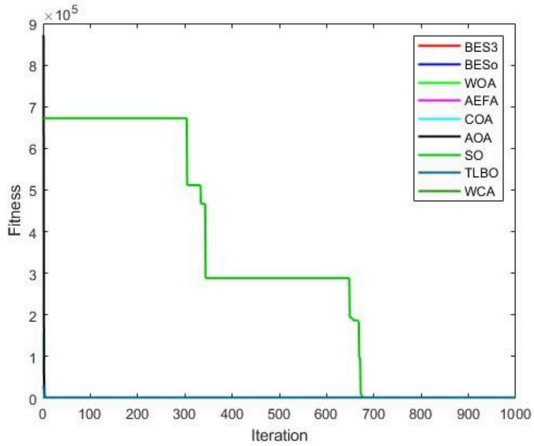
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA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8
7
6
5
4
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2
1
0
Fitness
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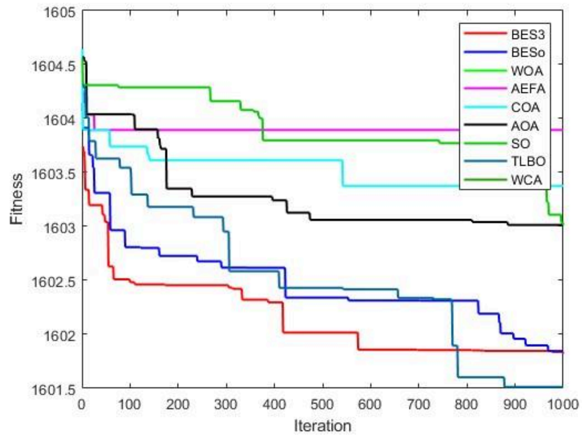
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA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1604
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1602.5
1602
1601.5
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Iteration
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BESo
WOA
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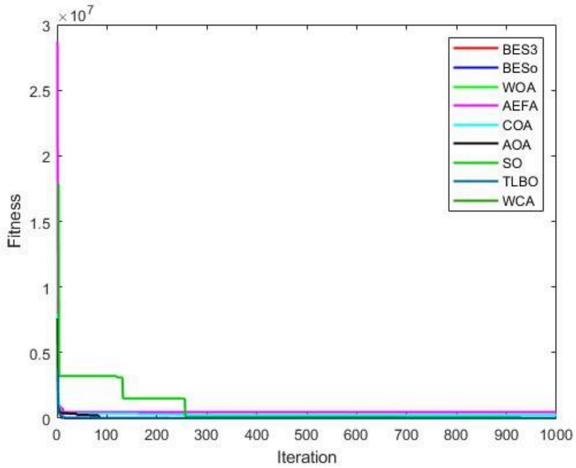
COA
AOA
SO
TLBO
WCA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BESo
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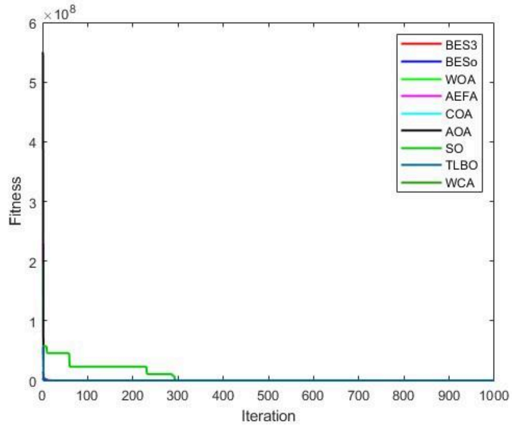
AEFA
COA
AOA
SO
TLBO
WCA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Iteration
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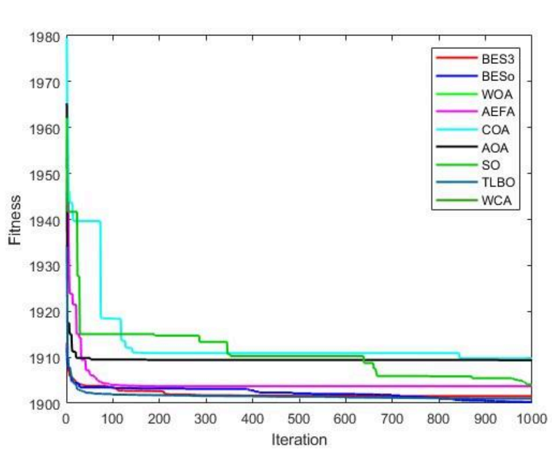
WOA
AEFA
COA
AOA
SO
TLBO
WCA


Fitness
Iteration
BES3
BESo
WOA
AEFA
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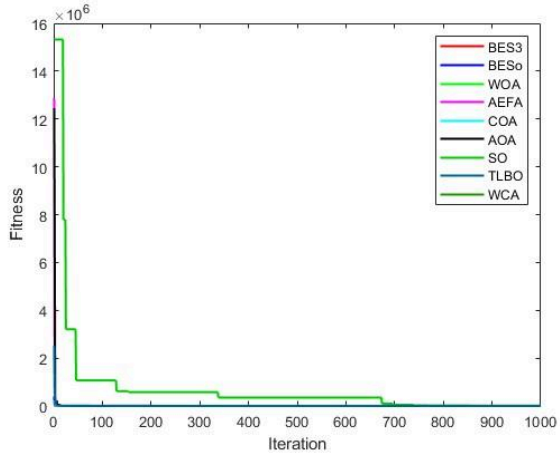
AOA
SO
TLBO
WCA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Fitness
Iteration
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WOA
AEFA
COA
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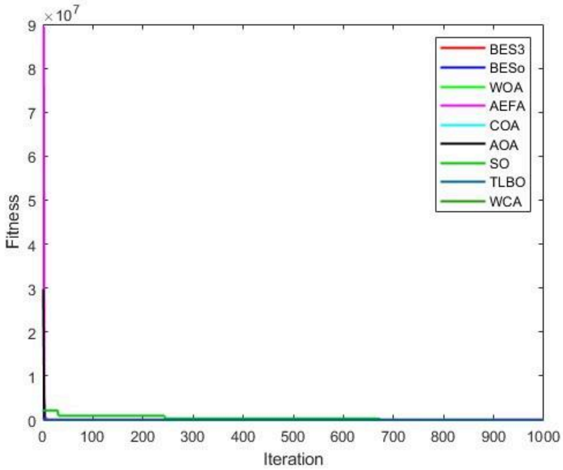
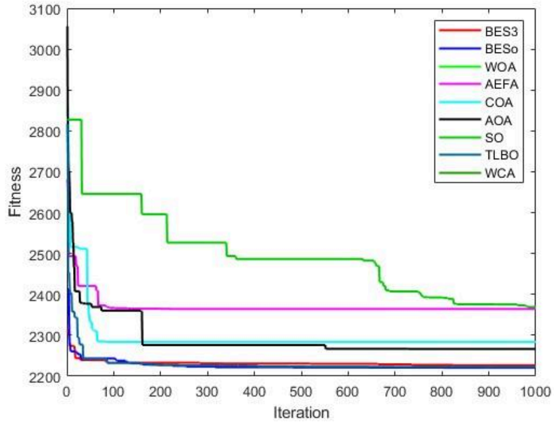
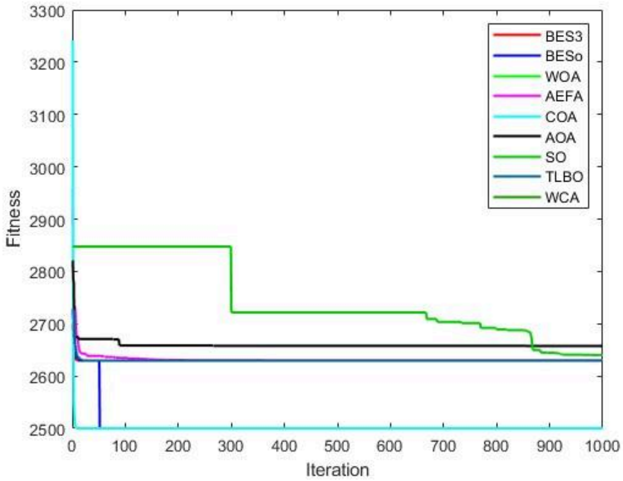
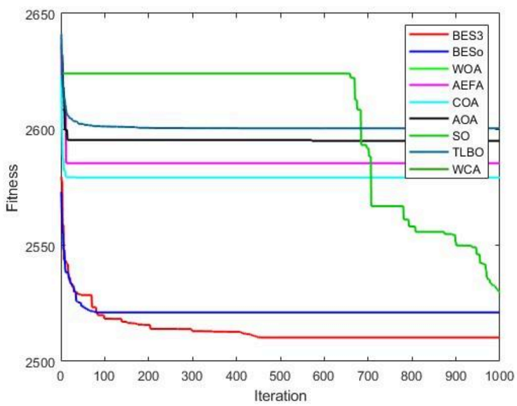
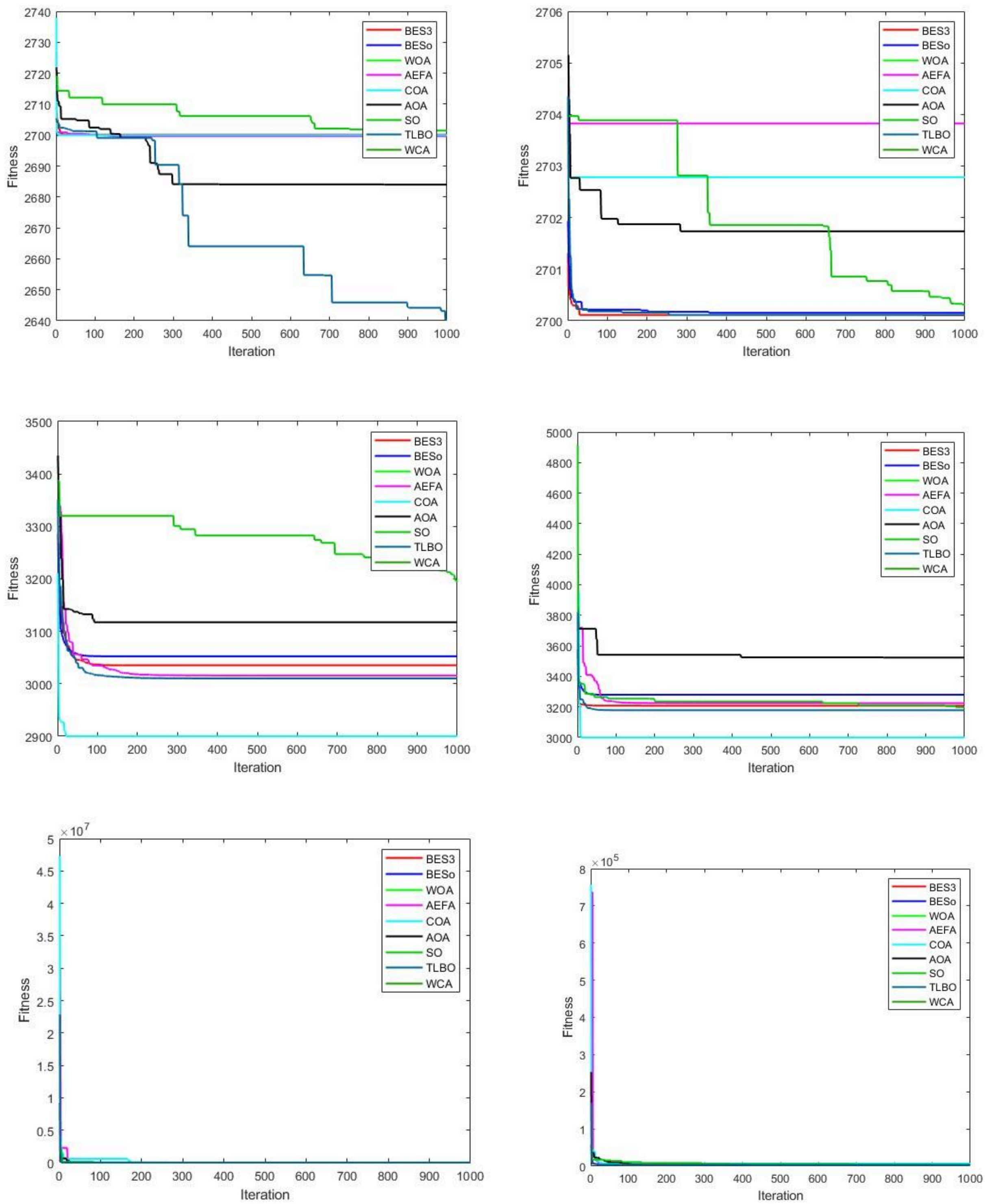
SO
TLBO
WCA


$\times 10^7$
Fitness
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA


Fitness
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA


Fitness
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA


Fitness
Iteration
BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA


BES3
BESo
WOA
AEFA
COA
AOA
SO
TLBO
WCA
Fitness
Iteration

*4.1. Experimental series : Engineering*

*4.3.1. Pressure vessel design* The 1st constrained engineering problem in this study known as pressure vessel design which aims to find the minimal production cost of material & cylindrical vessel welding. This problem has 4 different parameters and its mathematical model is described below

$$
\begin{array}{ll}
\text{Consider} & \vec{x} = [x_1 x_2 x_3 x_4], \\
\text{Minimize} & f(\vec{x}) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3, \\
\text{Subjectto} & g_1(x) = -x_1 + 0.0193 x_3 \leqslant 0, \\
 & g_2(x) = -x_2 + 0.00954 x_3 \leqslant 0, \\
 & g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \leqslant 0, \\
 & g_4(x) = x_4 - 240 \leqslant 0, \\
\text{Variablesrange} & 0 \leqslant x_1, x_2 \leqslant 100 \text{ and } 10 \leqslant x_3, x_4 \leqslant 200
\end{array}
$$

\

*4.3.3.Tension/compression spring design* Tension/Compression spring design is one of the most famous and well-known constrained problem.The aim of this problem is to select the best values for parameter  to minimize weight. This problem has three design variables and its mathematical formulation is described follows:

$$
\begin{array}{ll}
\text{Consider} & \vec{x} = [x_1 x_2 x_3] = [dDN], \\
\text{Minimize} & f(\vec{x}) = (x_3 + 2) x_2 x_1^2, \\
\text{Subjectto} & g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leqslant 0, \\
 & g_2(\vec{x}) = \frac{4x_2^2 - x_1 x_2}{12566\left(x_2 x_1^3 - x_1^4\right)} + \frac{1}{5108 x_1^2} \leqslant 0, \\
 & g_3(\vec{x}) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leqslant 0, \\
 & g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \leqslant 0, \\
\text{Variablesrange} & 0.05 \leqslant x_1 \leqslant 2 \\
 & 0.25 \leqslant x_2 \leqslant 1.30 \\
 & 2.00 \leqslant x_3 \leqslant 15
\end{array}
$$

*4.3.4. Cantilever Beam* The objective of this optimization task is to achieve minimization of the weight of a cantilever beam, which is constructed from hollow square blocks. The structure consists of  five such blocks, with the first block being fixed in position

Consider:

$$\vec{x} = [x_1 x_2 x_3 x_4 x_5].$$

Minimize:

$$f(\vec{x}) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5).$$

Subject to:

$$g(\vec{x}) = \frac{61}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0.$$

Variable range:

$$0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100.$$

Results of BES3 versus other metaheuristics on Tension compression spring design.

| | Best | x1 | x2 | x3 |
|---|---|---|---|---|
| BES3 | 0.012665 | 0.051689 | 0.356718 | 11.28897 |
| BES | 0.012665 | 0.051689 | 0.356718 | 11.28897 |
| TLBO | 0.012665 | 0.05173 | 0.357711 | 11.23098 |
| WOA | 0.012688 | 0.050587 | 0.330782 | 12.98892 |
| SO | 0.012665 | 0.051572 | 0.353908 | 11.45562 |
| WCA | 0.01267 | 0.052205 | 0.369246 | 10.59057 |
| AEFA | 0.012704 | 0.051165 | 0.343921 | 12.11018 |
| COA | 0.013026 | 0.05 | 0.316659 | 14.45492 |
| AOA | 0.012665 | 0.051803 | 0.359477 | 11.12903 |

Statistical results of BES3 versus other metaheuristics on Tension compression spring design.

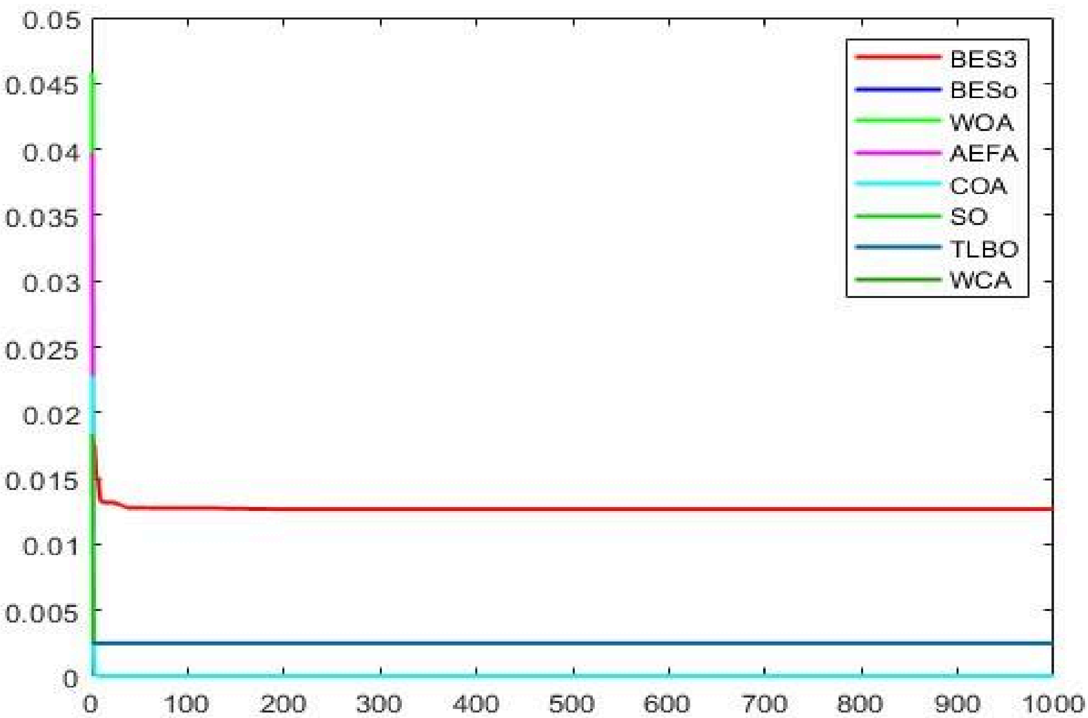
| Algorithms | min | max | mean | STD | Rank |
|---|---|---|---|---|---|
| BES3 | 0.012665 | 0.012671 | 0.012665 | 8.56E-07 | 2 |
| BES | 0.012665 | 0.012665 | 0.012665 | 3.14E-08 | 1 |
| TLBO | 0.012665 | 0.012712 | 0.01268 | 1.39E-05 | 3 |
| WOA | 0.012688 | 228120 | 7604.013 | 41648.82 | 9 |
| SO | 0.012665 | 0.013958 | 0.013063 | 0.000421 | 5 |
| WCA | 0.01267 | 0.017773 | 0.01409 | 0.002113 | 6 |
| AEFA | 0.012704 | 0.151567 | 0.017661 | 0.025293 | 8 |
| COA | 0.013026 | 0.029548 | 0.015977 | 0.004743 | 7 |
| AOA | 0.012665 | 0.014318 | 0.013044 | 0.000507 | 4 |

Results of BES3 versus other metaheuristics on Pressure vessel design problem.

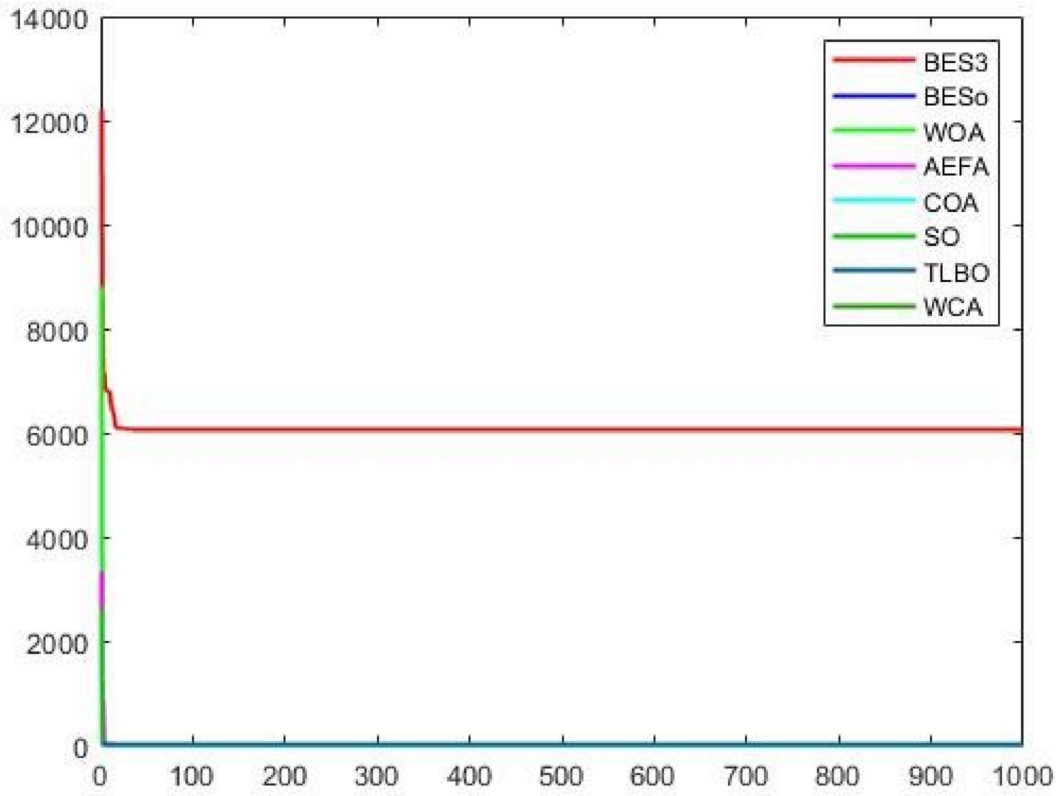
| | Best | x1 | x2 | x3 | x4 |
|---|---|---|---|---|---|
| BES3 | 6059.267 | 12.6681 | 6.7224 | 42.0984 | 176.6366 |
| BES | 6090.5262 | 13.8641 | 7.036 | 45.3368 | 140.2538 |
| TLBO | 6059.714335 | 0.75 | 0.375 | 38.86 | 221.37 |
| WOA | 7505.418 | 1.189947 | 0.604413 | 56.61231 | 53.23316 |
| SO | 5870.124 | 0.774549 | 0.383204 | 40.31962 | 199.2918 |
| WCA | 5916.0914 | 1.7958 | 0.3933 | 43.2310 | 187.6901 |
| AEFA | 6769.697 | 0.902527 | 0.445849 | 46.93715 | 146.8159 |
| COA | 5893.1336 | 0.783194 | 0. 3874305 | 40.5794 | 196.3323 |
| AOA | 6048.7844 | 0.8303737 | 0.4162057 | 42.75127 | 169.3454 |

Results of BES3 versus other metaheuristics on Cantilever design problem.

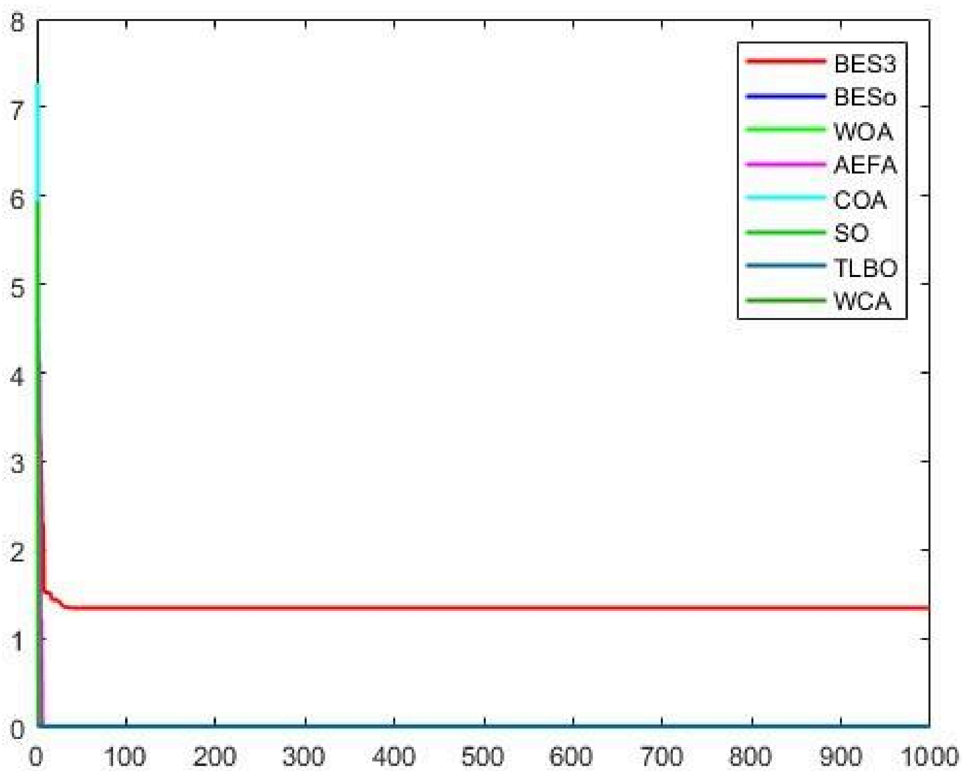
| | Best | x1 | x2 | x3 | x4 | x5 |
|---|---|---|---|---|---|---|
| BES3 | 1.3399564 | 6.016 | 5.3092 | 4.4943 | 3.5015 | 2.1527 |
| BES | 1.3399564 | 6.016 | 5.3092 | 4.4943 | 3.5015 | 2.1527 |
| TLBO | 52.68935 | 0.997376 | 0.2 | 0.264208 | 0.2 | 0.2 |
| WOA | 1.340371 | 6.048602 | 5.391859 | 5.213008 | 3.162458 | 1.719603 |
| SO | 13.0268 | 5.9832 | 4.7939 | 4.6247 | 3.4697 | 2.0584 |
| WCA | 1.30325382 | 5.9799 | 4.8821 | 4.4659 | 3.4733 | 2.1380 |
| AEFA | 1.303342 | 5.944606 | 4.865280 | 4.503500 | 3.492579 | 2.134620 |
| COA | 1.33996 | 6.017257314 | 5.307150983 | 4.491255551 | 3.50815678 | 2.14991302 |
| AOA | 1.340118523 | 6.018624846 | 5.291696407 | 4.5000009572 | 3.50955615 | 2.15433829 |




(A) Tension/Compression Design




(B) Pressure vessel design




© Cantilever Beam

## *4.1. Experimental series : Wilcoxon Test*

*Wilcoxon Signed-Rank Test:* A Non-Parametric Approach for Paired Data

The Wilcoxon signed-rank test is a non-parametric statistical test used to compare two related samples (paired data). It assesses whether there's a statistically significant difference between the medians of these two paired samples. Unlike parametric tests like the paired t-test, the Wilcoxon signed-rank test does not assume a normal distribution for the data.

Key Characteristics:

Non-parametric: Makes no assumptions about the underlying distribution of the data.

Paired data: Requires data points to be collected in pairs, where each pair represents measurements from the same subject or unit under different conditions.

Ordinal data: Applicable to data measured on an ordinal scale (ranked or ordered) or continuous data.

| BES3 vs. | BES | TLBO | WOA | SO | WCA | AEFA | COA | AOA |
|---|---|---|---|---|---|---|---|---|
| F1 | 0.020729 | 5.04E-16 | 7.07E-18 | 7.07E-18 | 6.32E-16 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F3 | 0.30265 | 0.23469 | 4.25E-19 | 1.14E-18 | 1.18E-13 | 5.20E-19 | 7.43E-19 | 1.63E-19 |
| F4 | 0.26113 | 4.65E-18 | 4.78E-18 | 4.40E-18 | 1.72E-18 | 3.47E-18 | 4.58E-18 | 4.17E-18 |
| F5 | 5.77E-06 | 0.31228 | 1.85E-17 | 1.70E-16 | 6.20E-17 | 6.86E-18 | 6.46E-18 | 6.54E-18 |
| F6 | 0.01021 | 3.03E-09 | 6.97E-18 | 7.03E-18 | 2.78E-14 | 7.05E-18 | 7.00E-18 | 6.97E-18 |
| F7 | 5.97E-16 | 3.54E-09 | 7.07E-18 | 7.07E-18 | 7.07E-18 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F8 | 2.11E-16 | 2.90E-06 | 1.68E-17 | 6.96E-18 | 1.05E-17 | 6.56E-18 | 5.58E-18 | 6.21E-18 |
| F9 | 0.000565 | 0.63922 | 2.97E-18 | 3.57E-18 | 8.59E-05 | 3.28E-18 | 2.67E-18 | 2.39E-18 |
| F10 | 0.016908 | 0.71226 | 4.25E-16 | 1.43E-08 | 1.95E-13 | 7.07E-18 | 8.46E-18 | 7.07E-18 |
| F11 | 0.21195 | 0.18438 | 2.05E-17 | 3.51E-17 | 6.67E-10 | 7.03E-18 | 6.99E-18 | 6.98E-18 |
| F12 | 0.091885 | 1.72E-12 | 7.07E-18 | 7.07E-18 | 4.81E-12 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F13 | 0.33621 | 2.71E-15 | 7.07E-18 | 7.07E-18 | 1.17E-09 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F14 | 0.10375 | 0.022702 | 1.08E-17 | 7.07E-18 | 3.13E-17 | 7.50E-18 | 1.29E-17 | 7.07E-18 |
| F15 | 0.58839 | 0.004656 | 7.07E-18 | 7.50E-18 | 1.45E-11 | 7.07E-18 | 1.29E-17 | 7.07E-18 |
| F16 | 0.000686 | 0.51031 | 5.02E-17 | 5.15E-13 | 1.31E-07 | 7.07E-18 | 8.99E-18 | 7.07E-18 |
| F17 | 0.010436 | 1.04E-05 | 1.64E-15 | 2.27E-13 | 6.20E-11 | 7.50E-18 | 8.46E-18 | 7.07E-18 |
| F18 | 0.089258 | 1.01E-17 | 8.46E-18 | 7.07E-18 | 0.000652 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F19 | 0.06929 | 3.25E-09 | 7.07E-18 | 1.01E-17 | 1.16E-13 | 7.07E-18 | 8.46E-18 | 7.07E-18 |
| F20 | 0.097885 | 0.13727 | 1.42E-16 | 6.12E-15 | 6.97E-13 | 7.01E-18 | 2.05E-17 | 7.01E-18 |
| F21 | 0.158 | 0.000315 | 0.00041 | 1.41E-06 | 0.000702 | 8.83E-16 | 0.000342 | 6.86E-18 |
| F22 | 0.19373 | 0.00262 | 1.20E-16 | 3.39E-09 | 8.77E-09 | 7.06E-18 | 7.05E-18 | 7.06E-18 |
| F23 | 0.002083 | 0.00038 | 6.72E-17 | 3.58E-16 | 4.44E-15 | 7.06E-18 | 7.07E-18 | 7.06E-18 |
| F24 | 0.58364 | 6.20E-11 | 1.05E-13 | 2.94E-13 | 1.23E-10 | 7.07E-18 | 3.42E-12 | 7.06E-18 |
| F25 | 0.45848 | 0.017876 | 4.16E-08 | 0.35024 | 0.361 | 7.03E-18 | 7.06E-18 | 7.06E-18 |
| F26 | 0.45345 | 0.49852 | 1.86E-09 | 4.91E-08 | 0.000293 | 3.19E-18 | 3.50E-15 | 2.44E-18 |
| F27 | 0.43998 | 0.002283 | 7.25E-14 | 0.42991 | 0.98625 | 7.07E-18 | 7.05E-18 | 7.06E-18 |
| F28 | 0.005722 | 0.16246 | 2.76E-05 | 0.449 | 0.188 | 5.67E-18 | 6.42E-18 | 7.03E-18 |
| F29 | 0.002447 | 0.061252 | 3.53E-17 | 4.36E-09 | 3.09E-13 | 7.07E-18 | 9.54E-18 | 7.07E-18 |
| F30 | 0.16685 | 0.051474 | 1.14E-05 | 0.47128 | 0.004656 | 7.07E-18 | 4.06E-14 | 7.07E-18 |

BES3 vs other meta-heuristics algorithms for CEC2017 in terms of p-values of the Wilcoxon rank sum test

| BES3 vs. | BES | TLBO | WOA | SO | WCA | AEFA | COA | AOA |
|---|---|---|---|---|---|---|---|---|
| F1 | 0.95327 | 7.07E-18 | 7.07E-18 | 7.07E-18 | 7.50E-18 | 7.07E-18 | 6.26E-19 | 7.07E-18 |
| F2 | 0.48052 | 6.88E-18 | 6.81E-18 | 6.60E-18 | 6.61E-18 | 6.82E-18 | 6.69E-18 | 6.76E-18 |
| F3 | 0.2938 | 6.71E-18 | 6.66E-18 | 6.54E-18 | 6.58E-18 | 6.79E-18 | 6.26E-19 | 6.60E-18 |
| F4 | 2.92E-06 | 0.026304 | 6.34E-08 | 1.02E-10 | 0.000126 | 7.05E-18 | 1.92E-18 | 7.05E-18 |
| F5 | 0.001876 | 0.002083 | 1.48E-10 | 5.37E-10 | 1.10E-12 | 3.13E-17 | 6.69E-18 | 7.07E-18 |
| F6 | 0.017882 | 5.27E-05 | 1.95E-17 | 1.01E-16 | 3.39E-11 | 7.05E-18 | 6.91E-18 | 7.05E-18 |
| F7 | 0.56481 | 0.009636 | 7.04E-18 | 7.03E-18 | 8.08E-13 | 7.06E-18 | 7.07E-18 | 7.05E-18 |
| F8 | 2.69E-06 | 0.025203 | 9.44E-18 | 9.74E-18 | 1.17E-11 | 6.38E-18 | 6.62E-18 | 7.07E-18 |
| F9 | 4.34E-13 | 0.40382 | 6.75E-18 | 7.11E-18 | 2.07E-17 | 6.70E-18 | 2.71E-18 | 6.74E-18 |
| F10 | 0.16064 | 9.45E-06 | 3.58E-05 | 7.31E-06 | 0.017553 | 7.07E-18 | 1.63E-17 | 7.07E-18 |
| F11 | 0.066179 | 0.19142 | 5.84E-15 | 4.93E-09 | 3.46E-14 | 7.07E-18 | 7.05E-18 | 7.07E-18 |
| F12 | 0.35025 | 5.86E-10 | 0.073623 | 0.018219 | 6.49E-11 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F13 | 0.073623 | 0.59793 | 1.37E-17 | 7.50E-18 | 3.11E-14 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F14 | 5.14E-10 | 4.44E-15 | 1.85E-13 | 7.07E-18 | 2.54E-16 | 7.07E-18 | 1.01E-17 | 7.07E-18 |
| F15 | 0.98625 | 0.000995 | 7.97E-18 | 7.07E-18 | 8.11E-15 | 7.07E-18 | 7.97E-18 | 7.07E-18 |
| F16 | 0.30923 | 0.19142 | 1.63E-17 | 1.95E-17 | 4.73E-17 | 7.07E-18 | 1.29E-17 | 7.07E-18 |
| F17 | 0.19142 | 6.53E-14 | 7.07E-18 | 7.07E-18 | 8.19E-12 | 7.07E-18 | 1.37E-17 | 7.07E-18 |
| F18 | 0.5327 | 2.47E-17 | 1.08E-17 | 7.07E-18 | 0.001181 | 7.07E-18 | 7.97E-18 | 7.07E-18 |
| F19 | 0.75377 | 0.004656 | 2.86E-15 | 7.30E-07 | 8.41E-07 | 7.07E-18 | 7.07E-18 | 7.07E-18 |
| F20 | 0.087967 | 5.33E-16 | 1.08E-17 | 1.08E-17 | 2.69E-11 | 7.07E-18 | 1.79E-17 | 7.07E-18 |
| F21 | 0.1586 | 0.32592 | 7.07E-18 | 7.50E-18 | 1.52E-09 | 7.07E-18 | 8.61E-05 | 7.07E-18 |
| F22 | 0.20097 | 5.86E-10 | 1.14E-17 | 1.37E-17 | 0.83885 | 7.07E-18 | 7.05E-18 | 7.07E-18 |
| F23 | 0.003045 | 0.000398 | 1.45E-16 | 1.23E-19 | 2.26E-17 | 1.63E-19 | 7.06E-18 | 6.63E-20 |
| F24 | 2.88E-12 | 0.004364 | 1.80E-16 | 2.95E-17 | 1.51E-16 | 7.06E-18 | 1.32E-08 | 7.07E-18 |
| F25 | 0.005624 | 8.68E-10 | 0.059353 | 1.04E-12 | 0.000546 | 2.78E-17 | 7.05E-18 | 7.07E-18 |
| F26 | 0.82809 | 1.58E-07 | 1.19E-14 | 1.14E-17 | 2.05E-13 | 7.07E-18 | 1.30E-13 | 7.07E-18 |
| F27 | 0.30596 | 0.019983 | 5.70E-13 | 1.63E-12 | 0.012214 | 9.38E-16 | 1.01E-16 | 7.06E-18 |
| F28 | 0.36465 | 0.056631 | 4.48E-06 | 5.30E-07 | 0.004178 | 7.07E-18 | 6.21E-18 | 7.06E-18 |
| F29 | 0.078175 | 1.52E-09 | 5.57E-14 | 9.92E-12 | 0.000721 | 7.07E-18 | 2.78E-17 | 7.07E-18 |
| F30 | 0.31917 | 0.000324 | 2.69E-16 | 0.005886 | 2.14E-06 | 7.07E-18 | 1.18E-10 | 7.07E-18 |

BES3 vs other meta-heuristics algorithms for CEC2017 in terms of p-values of the Wilcoxon rank sum test.

*Advantages:*

Robust to outliers: Less sensitive to the presence of extreme values in the data compared to parametric tests.

Fewer assumptions: No need to assume normality of the data distribution.

Wide applicability: Useful for analyzing ordinal or continuous data with paired samples.

*Disadvantages:*

Loss of information: Discards information about the direction of the difference (positive or negative) during the ranking step.

Less powerful than t-test for normal data: If data is normally distributed, the paired t-test might be more powerful.

## ***Conclusions and future directions***

In this research work, we developed an improved BES3 approach combining the original BES algorithm, Leverages Lévy Flights, which are large random jumps in the search space, for exploration. Lévy Flights help the eagles escape local optima and explore distant regions with potentially better solutions. for Lévy jump generation, with parameters alpha and beta controlling the distribution and magnitude of these jumps.

Introduces Random Walk, involving small random steps, for further exploration in the vicinity of the current position.

The combination of these strategies allows the eagles to exploit promising areas identified by the best solution and the average population, while also venturing into new regions through Lévy Flights and Random Walk.

To scrutinize the performance of the BES3 algorithm, we have considered 29 standard CEC2017 and 30 CEC2014 optimization benchmark functions, five engineering design problems, and a practical feature selection problem. The obtained results are compared in terms of mean, worst, and standard deviation values with several classical metaheuristics including original BES, TLBO, WOA, PSO, SO, WCA, AEFA, COA, and AOA algorithms. Comparative results clearly show the BES3's superiority and competitiveness among tested algorithms.

We further tested the performance of the proposed BES algorithm over 3 engineering design problems namely Pressure Vessel Design, , Tension/Compression Spring Design, and Cantilever Beam Design. The superior performance of the BES over compared algorithms indicate the fact that the BES method can effectively solve various types of practical optimization problems.

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