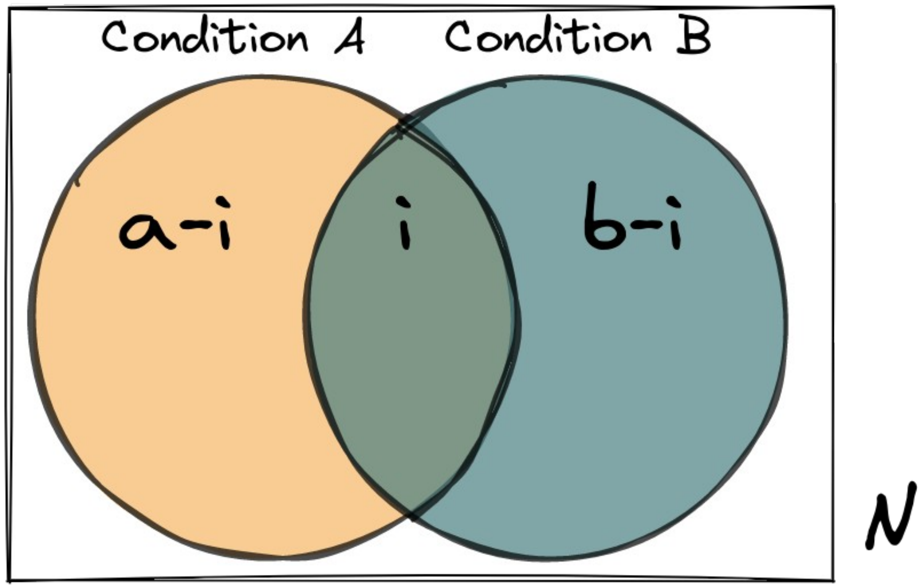
Bayes' Theorem



Week 14

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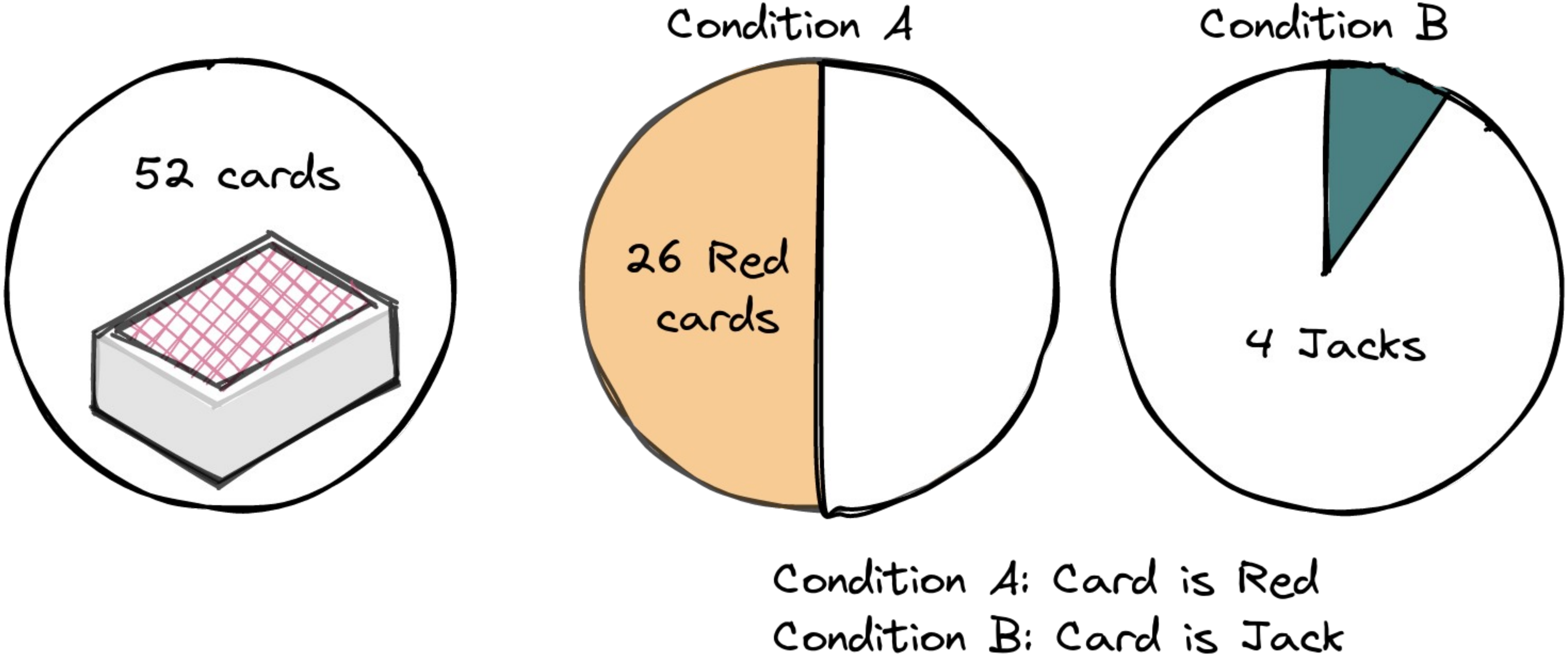
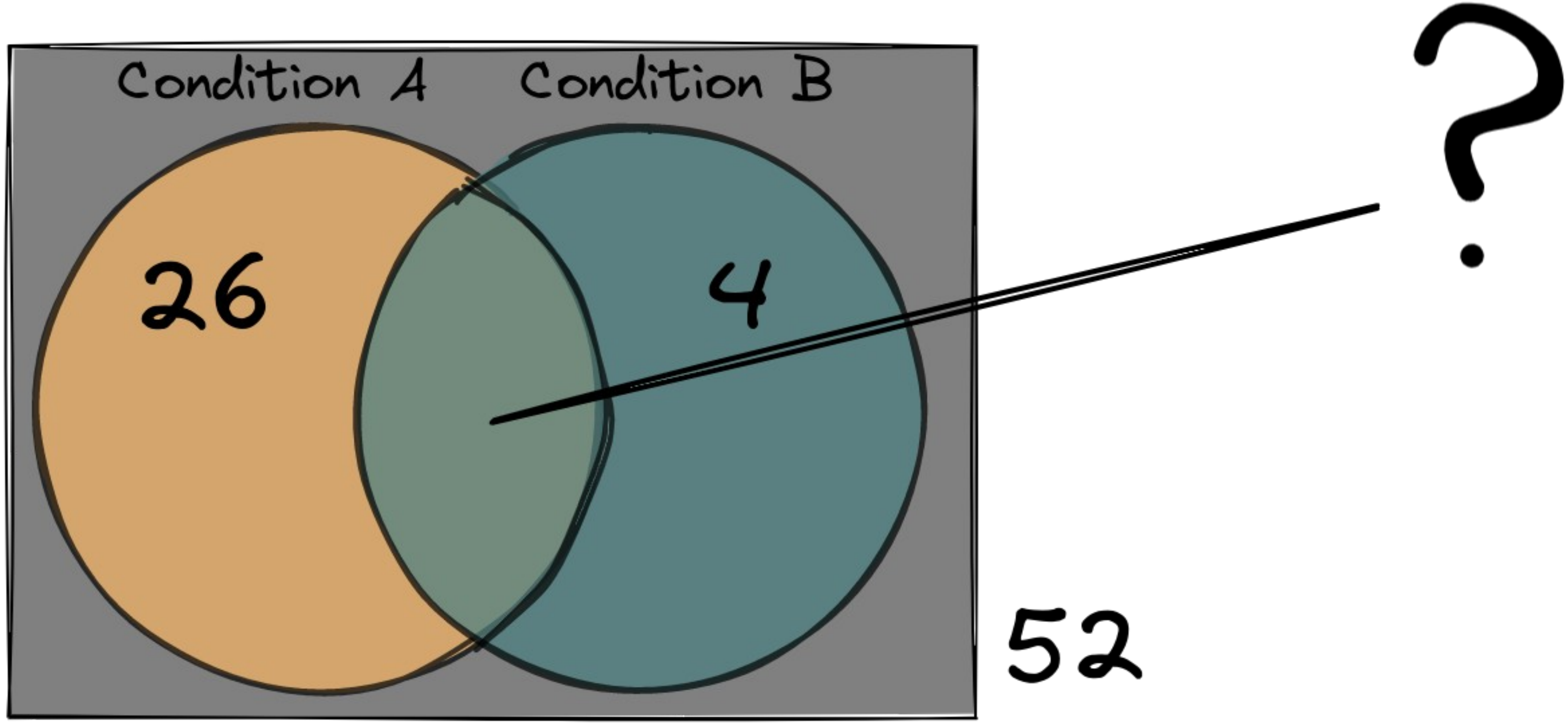
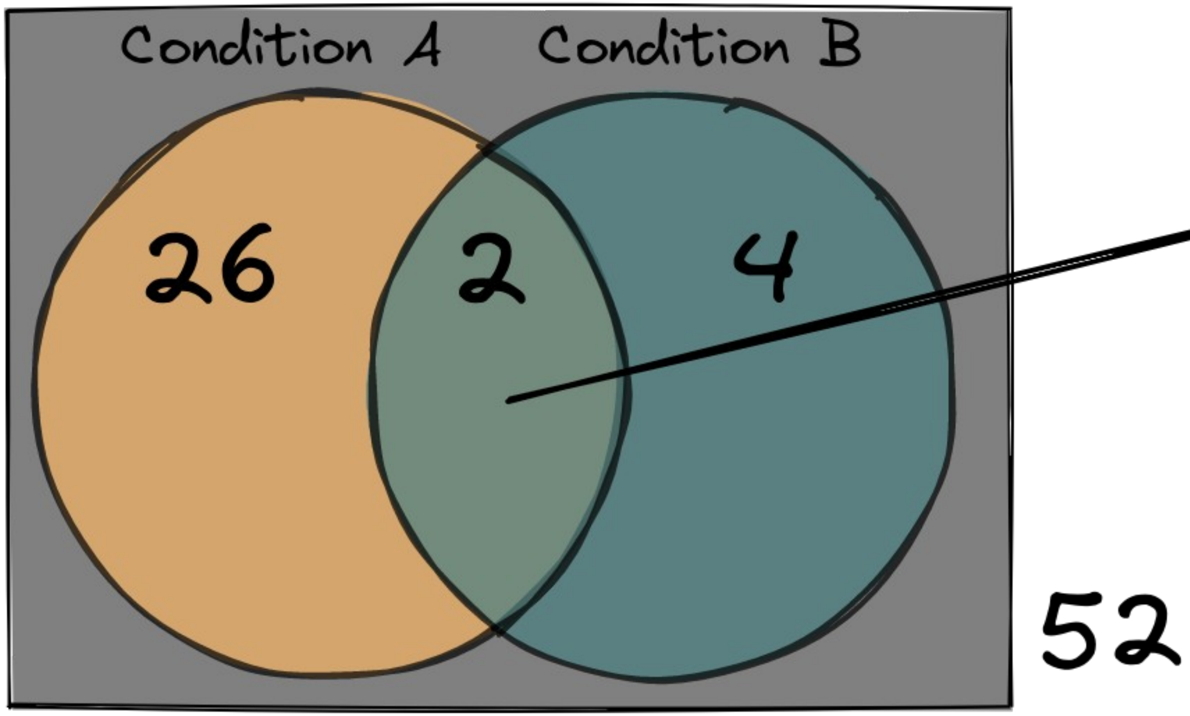
Card example: conditions



Card example: intersection



Card example: intersection



Answer: Jack of Hearts,
Jack of Diamonds

J♡ J◇

Probabilities

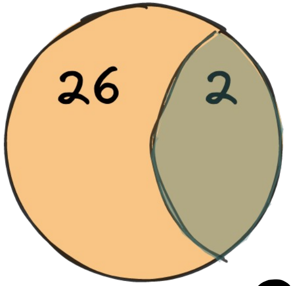
Marginal probability:

$$P(\text{Red}) = \frac{26}{52} = \frac{1}{2}$$

$$P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$$

Conditional probability:

What is the probability of Jack if (*condition*) card is Red?



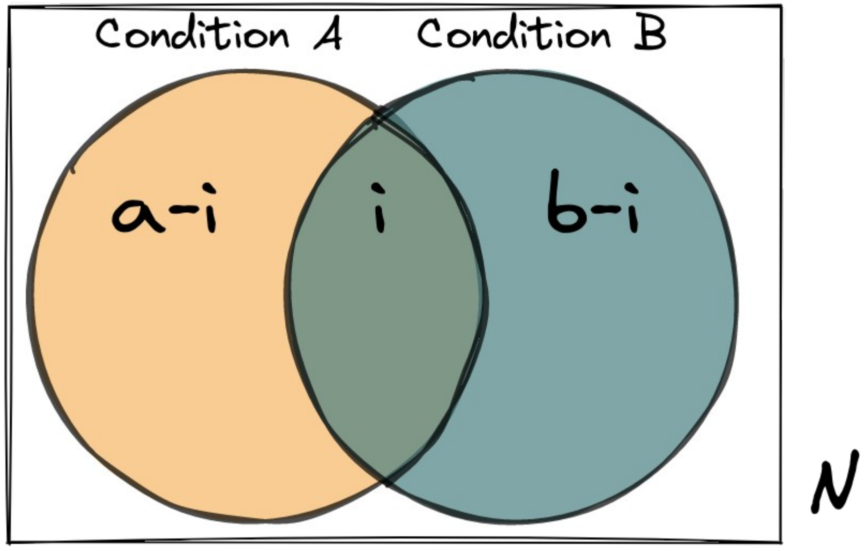
$$P(\underbrace{\text{Jack} \mid \text{Red}}_{\text{condition}}) = \frac{2}{26} = \frac{1}{13}$$

Joint probability:

What is the probability of Jack and Red?

$$P(\underbrace{\text{Red} \cap \text{Jack}}_{\text{intersection}}) = \frac{2}{52} = \frac{1}{26}$$

Bayes' Theorem



What is the probability of A if B happened?

$$P(A \mid B) = \frac{i}{b}$$

What is the probability of A and B happening?

$$P(A \cap B) = \frac{i}{N}$$

What is the probability of B happening?

$$P(B) = \frac{b}{N}$$

$$\Rightarrow \boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Bayes' Theorem

The probability of A if B happened:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

More often, we calculate the probability of both A and B happening:

$$P(A \cap B) = P(B) * P(A \mid B)$$

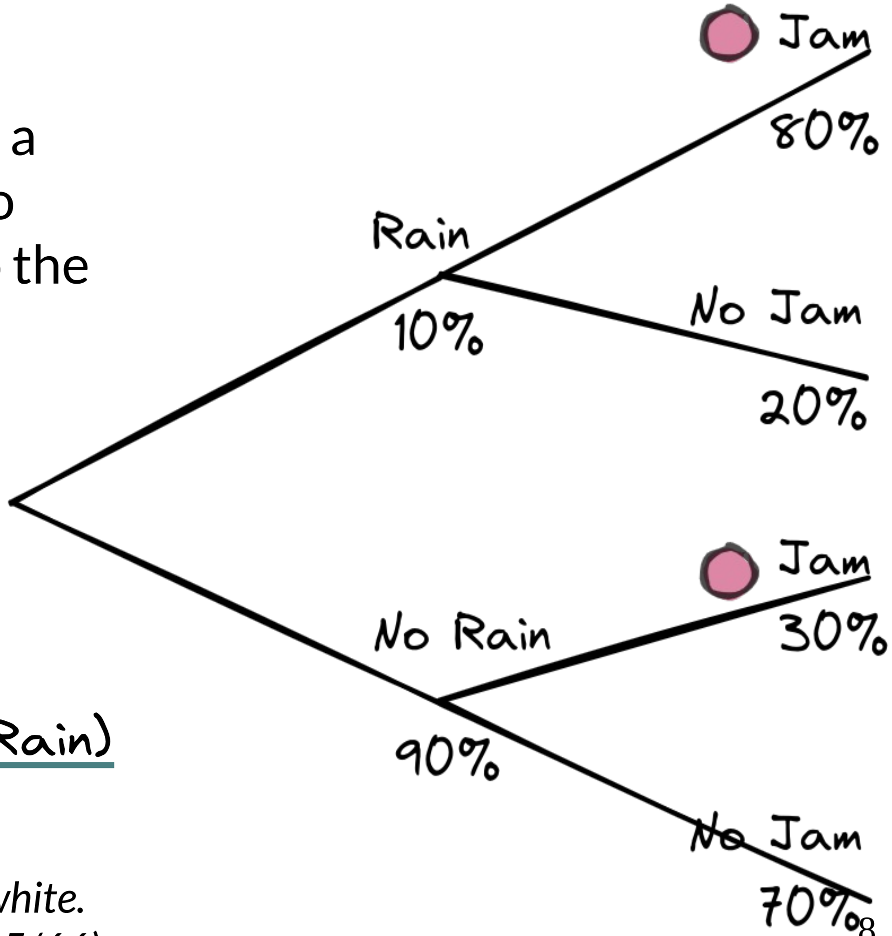
Probability Tree

What is the probability of us getting stuck in a jam today? Intuition suggests that in order to calculate this probability, we need to sum up the following multiplication results:

$$P(\text{Jam}) = 0.1 * 0.8 + 0.9 * 0.3 = 0.35$$

In fact, we are applying Bayes' theorem:

$$P(\text{Jam}) = \underline{P(\text{Rain}) * P(\text{Jam} \mid \text{Rain})} + \underline{P(\text{No Rain}) * P(\text{Jam} \mid \text{No Rain})}$$



DS Interview question: you have 12 beads – 7 black and 5 white. What is the probability of taking 1b+1w simultaneously? (35/66)

Independence of events

What if one would like to learn if two events are independent? To answer it, we need to understand what is independent? Intuition hints to us that events A and B are independent if the probability that event B will occur remains the same, regardless of whether event A has occurred or not:

$$P(B) = P(B \mid A)$$

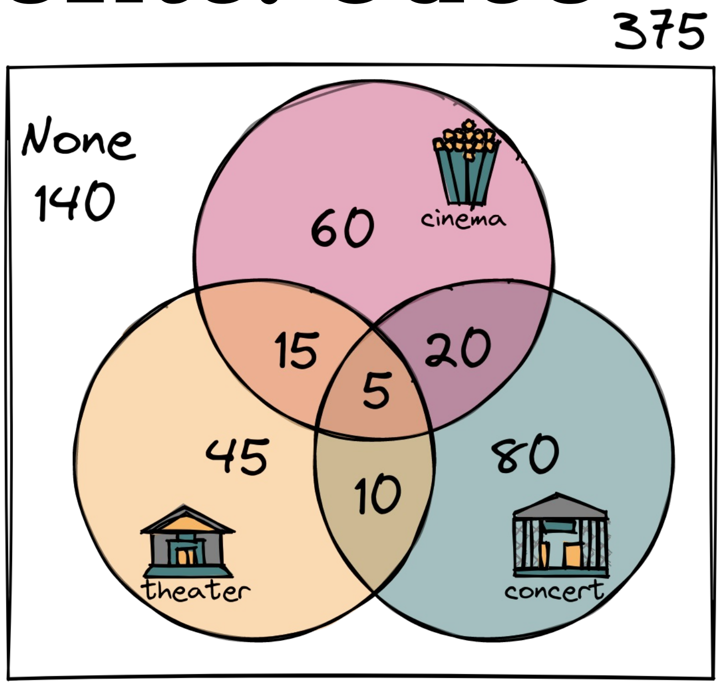
We could've achieved the above formula from the following ones:

independent events: $P(A \cap B) = P(A) * \boxed{P(B)}$

conditional events: $P(A \cap B) = P(A) * \boxed{P(B \mid A)}$

Independence of events: Case

Suppose we took a survey from 375 people and asked if they've visited theater, cinema or concert in the last year at least once:



Let's check if visiting cinema and concert are independent events:

P (cinema) * P (concert) = 100/375 *
* 115/375 = 0.8177

P (cinema ∩ concert) = 25/375 = 0.666

Events are dependent, as 0.8177 ≠ 0.666

Now let's check if visiting theater and cinema are independent events:

P (theater) * P (cinema) = 75/375 * 100/375 = 0.5333

P (cinema ∩ concert) = 20/375 = 0.5333

Events are independent, as 0.5333 == 0.5333