NAGARJUNA COLLEGE OF ENGINEERING AND TECHNOLOGY
(An autonomous institution under VTU)
Department Of Mathematics

Academic Year 2023-24

Fourth Semester

Discrete Mathematics and Graph Theory (22CDT45A)

CCE-1: Module-1 Questions

1. A) Construct a truth table for the following compound propositions $[(p \wedge q) \vee (\neg r)] \leftrightarrow p$.

B) Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore", respectively. Express each of these compound propositions as an English sentence.
a) $\neg q$ b) $p \wedge q$ c) $\neg p \vee q$ d) $p \rightarrow \neg q$ e) $\neg q \rightarrow p$ f) $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ h) $\neg p \wedge (p \vee \neg q)$.

2. A) Construct a truth table for the following compound propositions $q \leftrightarrow (\neg p \vee \neg q)$.

B) Let *p*, *q*, and *r* be the propositions
p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.
Write these propositions using *p*, *q*, and *r* and logical connectives (including negations).
a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
e) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

3. A) Write the truth tables for the compound statement $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$.

B) Let *s*, *t*, and *u* denote the following primitive statements:
s: Phyllis goes out for a walk.
t: The moon is out.
u: It is snowing.
(i) Translate each of the symbolic form into an English sentence.
a) $(t \wedge \neg u) \rightarrow s$
b) $t \rightarrow (\neg u \rightarrow s)$
c) $\neg(s \leftrightarrow (u \vee t))$

(ii) Write the following in symbolic form.

a) Phyllis will go out walking if and only if the moon is out.

b) If it is snowing and the moon is not out, then Phyllis will not go out for a walk.

c) It is snowing but Phyllis will still go out for a walk.

4. A) Show that $[p \to (q \vee r)] \equiv [\neg r \to (p \to q)]$ are logically equivalent using truth table.

B) Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

5. A) Use truth tables to verify the following logical equivalence.

$$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r).$$

B) Establish the validity of the argument

$$\begin{array}{l} (\neg p \vee q) \to r \\ r \to (s \vee t) \\ \neg s \wedge \neg u \\ \underline{\neg u \to \neg t} \\ \therefore p \end{array}$$

6. A) Prove that, for any propositions p, q, r the compound propositions $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

B) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

7. A) Show that

(i) $p \to (p \vee q)$ is a tautology

(ii) $p \wedge (\neg p \wedge q)$ is a contradiction.

B) Provide the reasons for the steps verifying the following argument.

$$\begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ \underline{p \wedge t} \\ \therefore\ u. \end{array}$$

<u>Step</u>

1. $p \to q$
2. $q \to (r \wedge s)$
3. $p \to (r \wedge s)$
4. $p \wedge t$

5. p
6. $r \wedge s$
7. r
8. $\neg r \vee (\neg t \vee u)$
9. $\neg(r \wedge t) \vee u$
10. t
11. $r \wedge t$
12. $\therefore u$

8. A) Consider the open statements. $p(t)$: t has two sides of equal length. $q(t)$: t is an isosceles triangle. $r(t)$: t has two angles of equal measure. Then the arguments:

In the triangle XYZ there is no pair of angles of equal measure.
If a triangle has two sides of equal length, then it is isosceles.
If a triangle is isosceles, then it has two angles of equal measure.
Therefore triangle XYZ has no two sides of equal length.
Write the arguments symbolically and validate the arguments.

B) Give a direct proof for each of the following.

a) for all integers k and l, if k, l are both even, then $k + l$ is even.
b) for all integers k and l, if k, l are both even, then kl is even.

9. A) Find whether the following argument is valid:

No engineering student of first or second semester studies logic.
Anil is an engineering student who studied Logic.
____
∴ Anil is not in second semester.

B) Give (i) an indirect proof (ii) a contradiction proof, of the following statement.

"For every integer n, if n^2 is odd, then n is odd."

10. A) Let $j(x)$, $s(x)$, and $p(x)$ be open statements that are defined for a given universe. Establish the validity of the argument.

$$\begin{array}{l} \forall x\, [(j(x) \vee s(x)) \rightarrow \neg p(x)] \\ \underline{p(m) \qquad\qquad\qquad\qquad} \\ \therefore \neg s(m) \end{array}$$

B) Give (i) a direct proof (ii) an indirect proof (iii) a contradiction proof, of the following statement. "If m is an even integer, then $m + 7$ is odd."

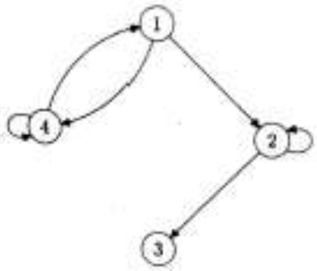
CCE-1: Module-2 Questions

11. A) Let $A = \{1,2,3,4\}$, $B = \{2,5\}$, $c = \{3,4,7\}$. Write down the following:
$A \times B$, $B \times A$, $A \cup (B \times C)$, $(A \cup B) \times C$, $(A \times C) \cup (B \times C)$.

B) Let $\mathcal{R}$ and $\mathcal{S}$ be relations on a set A. Prove the following:
(1) If $\mathcal{R}$ and $\mathcal{S}$ are reflexive, so are $\mathcal{R} \cap \mathcal{S}$ and $\mathcal{R} \cup \mathcal{S}$.
(2) If $\mathcal{R}$ and $\mathcal{S}$ are symmetric, so are $\mathcal{R} \cap \mathcal{S}$ and $\mathcal{R} \cup \mathcal{S}$.

12. A) For any non-empty sets A, B, C, prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

B) Find the relation represented by the digraph given below. Also, write down its matrix, and determine the in-degrees and out-degrees of the vertices in the diagraph.



13. A) For any non-empty sets A, B, C, prove that $A \times (B - C) = (A \times B) - (A \times C)$.

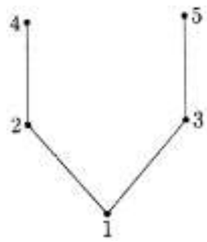
B) Let $A = \{1,2,3,4\}$. On A, define the relation $\mathcal{R}$ by $x\mathcal{R}y$ if and only if 'x divides y'. Prove that $(A, \mathcal{R})$ is a poset. Draw the Hasse diagram for this relation and matrix of $\mathcal{R}$.

14. A) Let $A = \{1,2,3,4,5,6\}$ and $B = \{6,7,8,9,10\}$. If a function $f: A \to B$ is defined by $f = \{(1,7), (2,7), (3,8), (4,6), (5,9), (6,9)\}$, determine $f^{-1}(6)$ and $f^{-1}(9)$. If $B_1 = \{7,8\}$ and $B_2 = \{8,9,10\}$, find $f^{-1}(B_1)$ and $f^{-1}(B_2)$.

B) Let A, B, C, D be non-empty sets. Prove that $(A \times B) \subseteq (C \times D)$ if and only if $A \subseteq C$ and $B \subseteq D$.

15. A) Let $f: R \longrightarrow R$ be defined by
$$f(x) = \begin{cases} 3x - 5, & x > 0 \\ -3x + 1, & x \leq 0 \end{cases}$$
Determine $f(0)$, $f\left(\frac{-5}{3}\right)$, $f^{-1}(1)$, $f^{-1}(-3)$, $f^{-1}([-5,5])$.

B) Determine the matrix and digraph of the partial order whose Hasse diagram is given below:



16. A) Let $A = B = C = R$, and $f: A \to B$ and $g: B \to C$ be defined by $f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in A, \forall b \in B$. Compute $g \circ f$ and show that $g \circ f$ is invertible. What is $(g \circ f)^{-1}$?

B) For any non-empty sets A, B, C, prove the following results:
$A \times (B \cap C) = (A \times B) \cap (A \times C)$.

17. A) Let f and g be the function from $\mathcal{R}$ to $\mathcal{R}$ defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(g \circ f\ )(x) = 9x^2 - 9x + 3$, determine a, b.

B) Draw the Hasse diagram representing the positive divisors of 36.

18. A) Consider the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1$, $\forall x \in \mathcal{R}$. Find $g \circ f$, $f \circ g$, $f^2$and g^2.

B) Let $A = \{1,2,3,4\}$ and $\mathcal{R}$ be a relation on A defined by $x\mathcal{R}y$ if and only if "$y = 2x$",
(a) Write down $\mathcal{R}$ as a set of ordered pairs.
(b) Draw the digraph of $\mathcal{R}$.
(c) Determine the in-degrees and out-degrees of the vertices in the diagraph.

19. A) Determine the number of relations on $A = \{a, b, c, d, e\}$ that are (a) reflexive (b) symmetric (c) reflexive and symmetric (d) antisymmetric (e) asymmetric (f) irreflexive.

B) A function $f: A \to B$ is invertible if and only if it is one-to-one and onto.

20. A) If $A = \{1, 2, 3, 4\}$, give an example of a relation $\mathcal{R}$ on A that is
a) Reflexive and symmetric but not transitive
b) Reflexive and transitive but not symmetric
c) Symmetric and transitive but not reflexive.

B) Let $X \to Y$ be a function and A and B be arbitrary nonempty subsets of X. Then,
(1) If $A \subseteq B$, then $f(A) \subseteq f(B)$.
(2) $f(A \cup B) = f(A) \cup f(B)$.

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