**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans = B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Mean = 38

SD= 6

Z score for 44 = (44-38)/6

=1

= 84.13%

People above 44 age = 100- 84.13

= 15.87%

= 63 out of 400

Z score for 38 = (38- 38)/6

=0/6

=50%

Hence people between 38 and 44 age = 84.13 – 50

= 34.13%

137 out of 400

Therefore the statement is false .

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
2. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

= if X ∼ N (µ1, σ12), and Y ∼ N (µ2, σ22) are two independent random variables then

X + Y ∼ N (µ1 + µ2, σ12 + σ22), and X − Y ∼ N (µ1 − µ2, σ12 + σ22).

Similarly if Z = aX + bY, where X and Y are as defined above, i.e. Z is linear combination of X and Y, then Z ∼ N (aµ1 + bµ2, a2 σ12 + b2 σ22).

Therefore in the question

2X1~ N (2 u, 4 σ2) and

X1+X2 ~ N (µ + µ, σ2 + σ2) ~ N (2 u, 2σ2)

2X1-(X1+X2) = N (4µ, 6 σ2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

We need to find two values a and b

The probability of getting value between a and b should be 0.99

Probability left from a = -0.005

Probability right from b = +0.005

So we have the probabilities of a and b need to calculate X the random variable at a and b which has got these probabilities

Z=(X- µ)/𝞼

From Z table Z value is -2.57

Z \* 𝞼+ µ = X

Z (-0.0.005) \* 20+100= - (-2.57)\*20+100= 151.4

Z (-0.0.005) \* 20+100= (-2.57)\*20+100 = 48.6

So option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains a 95% probability for the annual profit of the company.

Range is Rs. (99.008, 980.991) in millions

1. Specify the 5th percentile of profit (in Rupees) for the company

5th percentile of profit is 170.0 (in million rupees)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Probability of division 1 making loss= 0.0477

Probability of division 2 making a loss= 0.04005

Answer: we convert the dollars into rupees

Profit 1 (22.5, 144)

Profit 2(31.5, 189)

First, determine the mean and variance for the company

μ=﻿ ﻿μ1+μ2

= 22.5+31.5

= 54﻿

σ21+σ22=144+189=333﻿

σ=333​=18.248﻿

Second, we calculate the interval with a 95% confidence level, apply the empirical rule, add and subtract 2 deviations from the mean

﻿(μ−2σ, μ+2σ)= (54−2∗18.248, 54+2∗18.246) = (17.504, 90.496)﻿

Third, we determine the fifth percentile

﻿ Given, μ=54

σ= 18.248

P (x ≤ x) =0.05

First, we computed z

z= −1.64

Second, we can calculate x using the formula,

z= x−μ/σ third, x can be calculated as follows:

X=Zσ + μ

Fourth, putting the given values, we have

x= −1, 64∗18.248+54= 24.07328 ≈ 24.07

Fifth, hence, the required x is24.07​

Fourth, we determine which division is most likely to lose.

(A). profit 1

﻿Given use standardized normal distributions = x − μ​/σ

μ1​= 22.5, σ1​= 12.0, x = 0,

The probability can be calculated as

P(x<0) =p (x − μ/σ​<0 − 22.5​/12) =p (z < −1.88) =0.0301

Hence, the required the probability

P(x<0) =0.0301​

(B). profit 2

﻿Given: - use standardized normal distribution

Z = x − μ​/σ

μ2 = 31.5, σ2​= 13.748, x = 0

The probability can be calculated as

P(x < 0) = p (x − μ/σ < 13.7480 − 31.5) =p (z < −2.29) =0.011

Hence, the required the probability

P(x < 0) =0.011​

(C). Profit1 is more likely to lose in a year.