MA0314-Probability Statistics & Numerical Analysis

Tutorial- Unit-3 INTERPOLATION

Q. (1) Use newton's forward interpolation formula find the approximate value of f(2.3) from the following data:

x	2	4	6	8
f(x)	4.2	8.2	12.2	16.2

Let,

$x = 2.3$, $x_0 = 2$, $h = 2$

$r = \frac{x - x_0}{h}$

$= \frac{2.3 - 2}{2}$

$= \frac{0.3}{2}$

$\boxed{r = 0.15}$

Difference Table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
2	4.2		
		4	
4	8.2		0
		4	
6	12.2		0
		4	
8	16.2		

By Newton's Interpolation formula,

$f(x) = f(x_0) + r\Delta f(x_0) + \frac{r(r-1)}{2!}\Delta^2 f(x_0)$

$f(2.3) = 4.2 + (0.15)(4) + \frac{0.15(0.15-1)}{2!}.(0)$

$= 4.2 + 0.6$

$= 4.8$

Q.(2) Use newton's forward interpolation formula find the value of $f(218)$ from the following data :

x	100	150	200	250	300	350	400
$f(x)$	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Let,

$x = 218$, $x_0 = 100$, $h = 50$

$$r = \frac{x - x_0}{h}$$

$$= \frac{218 - 100}{50}$$

$$= \frac{118}{50}$$

$$\boxed{r = 2.36}$$

Difference table :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
100	10.63	2.4	-0.39	0.15	-0.07	0.02	0.02
150	13.03	2.01	-0.24	0.08	-0.05	0.04	
200	15.04	1.77	-0.16	0.03	-0.01		
250	16.81	1.61	-0.13	0.02			
300	18.42	1.48	-0.11				
350	19.90	1.37					
400	21.27						

By Newton's forward interpolation formula,

$$f(x) = f(x_0) + r\Delta f(x_0) + \frac{r(r-1)}{2!}\Delta^2 f(x_0) + \frac{r(r-1)(r-2)}{3!}\Delta^3 f(x_0) +$$

$$\frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 f(x_0) + \frac{r(r-1)(r-2)(r-3)(r-4)}{5!}\Delta^5 f(x_0)$$

$$+ \frac{r(r-1)(r-2)(r-3)(r-4)(r-5)}{6!}\Delta^6 f(x_0)$$

$$f(218) = 10.63 + 2.36(2.4) + \frac{2.36(2.36-1)}{2!}(-0.39) + \frac{2.36(2.36-1)(2.36}{3!}$$

$$(0.15) + \frac{2.36(2.36-1)(2.36-2)(2.36-3)}{4!}(-0.07) +$$

$$\frac{2.36(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{5!}(0.02) +$$

$$\frac{2.36(2.36-1)(2.36-2)(2.36-3)(2.36-4)(2.36-5)}{6!}(0.02)$$

$$= 10.63 + 5.664 - 0.6259 + 0.0578 + 0.0130 + 0.0049 + -0.0027$$

$$= 15.7141$$

Q.(3) The population of a town is given below. Estimate the population for the year 1895 and 1930 using suitable interpolation.

Year x	1891	1901	1911	1921	1931
Population Y	46	66	81	93	101

Let, $x = 1895$, $x_0 = 1891$, $h = 10$

$$r = \frac{x - x_0}{h}$$

$$= \frac{1895 - 1891}{10}$$

$$= 0.4$$

Difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Using Networn forward interpolation formula,

$$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$y(1895) = 46 + 0.4(20) + \frac{0.4(0.4-1)}{2}(-5) + \frac{0.4(0.4-1)(0.4-2)}{3}(2) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4}(-3)$$

$= 46 + 8 + 0.6 + 0.128 + 0.1248$

$= 54.8528$

Q.(4) Using Stirling's formula, estimate the value of tan 16°.

x	0°	5	10°	15°	20°	25°	30°
y = tan x	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Let, $x = 16°$; $x_0 = 15°$; $h = 5°$

$$\gamma = \frac{x - x_0}{h} = \frac{16-15}{5} = \frac{1}{5}$$

$$\boxed{\gamma = 0.2}$$

Difference table,

x	γ	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	-3	0						
			0.0875					
5	-2	0.0875		0.0013				
			0.0888		0.0015			
10	-1	0.1763		0.0028		0.0002		
			0.0916		0.0017		-0.0002	
15	0	0.2679		0.0045		0		0.0011
			0.0961		0.0017		0.0009	
20	1	0.3640		0.0062		0.0009		
			0.1023		0.0026			
25	2	0.4663		0.0088				
			0.1111					
30	3	0.5774						

Using Stirling's formula,

$$y(x) = y_0 + \gamma\left(\frac{\Delta y_{-1} + \Delta y_0}{2}\right) + \frac{\gamma^2}{2!}\Delta^2 y_{-1} + \frac{\gamma(\gamma^2-1)}{3!}\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right)$$

$$+ \frac{\gamma^2(\gamma^2-1)}{4!}\Delta^4 y_{-2} + \frac{\gamma(\gamma^2-1)(\gamma^2-4)}{5!}\left(\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2}\right)$$

$$+ \frac{\gamma^2(\gamma^2-1)(\gamma^2-4)}{6!}\Delta^6 y_{-3}$$

$$y(16) = 0.2679 + 0.2\left(\frac{0.0916 + 0.0961}{2}\right) + \frac{(0.2)^2}{2!}(0.0045)$$

$$+ \frac{(0.2)(0.2^2-1)}{3!}\left(\frac{0.0017 + 0.0017}{2}\right) + 0 + \frac{(0.2)(0.2^2-1)(0.2^2-4)}{5!}$$

$$\left(\frac{-0.0002 + 0.0009}{2}\right) + \frac{(0.2)^2(0.2^2-1)(0.2^2-4)}{6!}(0.0011)$$

$$= 0.2679 + 0.0188 + (9 \times 10^{-5}) - (5.44 \times 10^{-5}) + 0 + (2.2176 \times 10^{-6}) + (2.3232 \times 10^{-7})$$

$$= 0.2867$$

Q.(5) Compute f(4) from the tabular values given:

x	2	3	5	7
f(x)	0.1506	0.3001	0.4517	0.6259

By Lagrange's Interpolation Formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$f(4) = \frac{(4-3)(4-5)(4-7)}{(2-3)(2-5)(2-7)}(0.1506) + \frac{(4-2)(4-5)(4-7)}{(3-2)(3-5)(3-7)}(0.3001)$$

$$+ \frac{(4-2)(4-3)(4-7)}{(5-2)(5-3)(5-7)}(0.4517) + \frac{(4-2)(4-3)(4-5)}{(7-2)(7-3)(7-5)}(0.6259)$$

$$= \frac{(1)(-1)(-3)}{(-1)(-3)(-5)}(0.1506) + \frac{(2)(-1)(-3)}{(1)(-2)(-4)}(0.3001) + \frac{(2)(1)(-3)}{(3)(2)(-2)}(0.4517)$$

$$+ \frac{(2)(1)(-1)}{(5)(4)(2)}(0.6259)$$

$$= \frac{3}{-15}(0.1506) + \frac{6}{8}(0.3001) + \frac{-6}{-12}(0.4517) + \frac{(-2)}{40}(0.6259)$$

$$= -0.0301 + 0.2250 + 0.2258 - 0.03129$$

$$= -0.0301 + 0.2251 + 0.2259 - 0.0313$$

$$= 0.3896$$

Q.(6) Find the Lagrange interpolation polynomial from the following data :

x	0	1	4	5
f(x)	1	3	24	39

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$= \frac{(x-1)(x-4)(x-5)}{(0-1)(0-4)(0-5)} (1) + \frac{(x-0)(x-4)(x-5)}{(1-0)(1-4)(1-5)} (3) +$$

$$\frac{(x-0)(x-1)(x-5)}{(4-0)(4-1)(4-5)} (24) + \frac{(x-0)(x-1)(x-4)}{(5-0)(5-1)(5-4)} (39)$$

$$= \frac{(x-1)(x-4)(x-5)}{(-1)(-4)(-5)} + \frac{(x)(x-4)(x-5)}{(1)(-3)(-4)} (3) + \frac{(x)(x-1)(x-5)}{(4)(3)(-1)} (24) +$$

$$\frac{(x)(x-1)(x-4)}{(5)(4)(1)} (39)$$

$$= \frac{(x-1)(x-4)(x-5)}{(-20)} + \frac{x(x-4)(x-5)}{4} + (-2)(x)(x-1)(x-5) + \frac{39(x)(x-1)(x}{20}$$

$$= \cancel{x^3+0}$$

$$= -\frac{x^3-10x^2+29x-20}{20} + \frac{x^3-9x^2+20x}{4} - (2x^3-12x^2+10x)$$

$$+ \frac{39(x^3-5x^2+4x)}{20}$$

$$= \frac{1}{20}(3x^3+10x^2+27x+20)$$

Q.(7) Using Newton's divided difference interpolation eleucte f(9) using the following table :

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Divided Difference table,

x	$f(x)$	First Divided Difference	Second Divided Difference	Third Difference	Fourth Difference
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

$$f(x) = f(x_0) + (x-x_0)[x_0,x_1] + (x-x_0)(x-x_1)[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)[x_0,x_1,x_2,x_3] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)[x_0,x_1,x_2,x_3,x_4]$$

$$f(9) = (150) + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1) + (9-5)(9-7)(9-11)(9-13)(0)$$

$$= (150) + 484 + 192 - 16$$

$$= 150 + 484 + 192 - 16$$

$$= 810$$

Q.(8) Using Newton's divided difference interpolation find a polynomial from the given data :

x	1	2	4	7
$f(x)$	10	15	67	430

By Newton's divided difference formula,

$$f(x) = f(x_0) + (x - x_0)[x_0, x_1] + (x - x_0)(x-1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3]$$

Difference table,

x	$f(x)$	first	Second	Third
1	10			
		5		
2	15		7	
		26		2
4	67		19	
		121		
7	430			

$$f(x) = f(x_0) + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3]$$

$$= (10) + (x-1)(5) + (x-1)(x-2)(7) + (x-1)(x-2)(x-4)(2)$$

$$= 10 + 5x - 5 + 7x^2 - 21x + 14 + (x^3 - 7x^2 + 10x + 8)(2)$$

$$\cancel{= x^3 + 7x^2 - 7x^2 + 5x - 21x + 10x + 10 - 5 + 14 + 8}$$

$$\cancel{= x^3 - 6x + 27}$$

$$= 10 + 5x - 5 + 7x^2 - 21x + 14 + 2x^3 - 14x^2 + 28x - 16$$

$$= 2x^3 + 7x^2 - 14x^2 + 5x - 21x + 28x + 10 - 5 + 14 - 16$$

$$= 2x^3 - 7x^2 + 12x + 3$$

Q.(9) Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking $h=0.2$ using trapezodial Rule

Here,

$a = 1, b = 2, h = 0.2$

$n = \frac{b-a}{h}$

$= \frac{2-1}{0.2} = \frac{1}{0.2}$

$= 5$

Dividing the interval into 5 parts

x	0	1	2	3	4	5
$f(x)$	1	0.5	0.2	0.1	0.058	0.0385
	y_0	y_1	y_2	y_3	y_4	y_5

By using trapezodial rule,

$$\int_1^2 \frac{dx}{1+x^2} = \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$= \frac{0.2}{2}[(1 + 0.0385) + 2(0.5 + 0.2 + 0.1 + 0.058)]$$

$$= \frac{0.2}{2}[1.0385 + 2(0.858)]$$

$$= \frac{0.2}{2}(1.0385 + 1.716)$$

$$= \frac{0.2}{2}(2.7545)$$

$$= 0.27545$$

Q. (10). Calculate $\int_0^1 2e^x \, dx$ with $n=10$ using trapezodial rule.

Let,

$a = 0$, $b = 1$, $n = 10$

$h = \frac{b-a}{n}$

$= \frac{1-0}{10}$

$= \frac{1}{10}$

$h = 0.1$ $\qquad y = f(x) = 2e^x$

~~x~~	~~0~~	~~0.1~~	~~0.2~~	~~0.3~~	~~0.4~~	~~0.5~~	~~0.6~~	~~0.7~~	~~0.8~~	~~0.9~~	~~0.10~~
~~f(x)~~	~~2~~	~~2.2103~~	~~2.4428~~								
	~~y_0~~	~~y_1~~	~~y_2~~	~~y_3~~	~~y_4~~	~~y_5~~	~~y_6~~	~~y_7~~	~~y_8~~		

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	2	2.2103	2.4428	2.6997	2.9836	3.2974	3.6442	4.0275	4.4510	4.9192	5.4365
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

Using trapezodial rule,

$$\int_0^1 2e^x dx = \frac{h}{2} [(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9$$

$$= \frac{0.1}{2} [(2 + 5.4365) + 2(2.2103 + 2.4428 + 2.6997 + 2.9836 + 3.2974 + 3.6442 + 4.0275 + 4.4510 + 4.9192)]$$

$$= (0.05)[(7.4365) + 2(30.6757)]$$

$$= (0.05)(7.4365 + 61.3514)$$

$$= (0.05)(68.7879)$$

$$= 3.43939 \text{ or } 3.4394$$

Q.(11) Elevate $\int_0^5 \frac{dx}{4x+5}$ by using Simpson's 1/3 rule, taking $n=10$.

Let,

$a=0$, $b=5$, $n=10$

$$h = \frac{b-a}{n}$$
$$= \frac{5-0}{10}$$
$$= \frac{5}{10}$$
$$h = 0.5$$

$$y = f(x) = \frac{1}{4x+5}$$

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.2	0.1428	0.1111	0.0909	0.0769	0.0666	0.0588	0.0526	0.0476	0.0437	0.04
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

Using Simpson's 1/3 rule,

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 3(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{0.5}{3}\left[(0.2+0.04) + 4(0.1428 + 0.0909 + 0.6666 + 0.0526 + 0.0437) + 3(0.1111 + 0.0769 + 0.0588 + 0.0476)\right]$$

$$= \frac{0.5}{3}\left[(0.24) + 4(0.9966) + 3(0.2944)\right]$$

$$= \frac{0.5}{3}\left[(0.24) + 3.9864 + 0.8832\right]$$

$$= \frac{0.5}{3}(0.24 + 3.9864 + 0.8832)$$

$$= \frac{0.5}{3}(5.1096)$$

$$= 0.4026$$

Q. (12) Elevate $\int_0^6 \frac{dx}{1+x^2}$ by using i) trapezodial rule ii) Simpon's 1/3 rule and iii). Simpson's 4/3 rule.

Let,

$a = 0, b = 6$,

Dividing the intervals into six parts,

$n = 6$

$$h = \frac{b-a}{n}$$
$$= \frac{6-0}{6}$$
$$= 1$$

$$y = f(x) = \frac{1}{1+x^2}$$

x	0	1	2	3	4	5	6
f(x)	1	0.5	0.2	0.1	0.0588	0.0384	0.0270
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

i). trapezodial rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)]$$

$$= \frac{1}{2}[(1+0.0270)+2(0.5+0.2+0.1+0.0588+0.0384)]$$

$$= \frac{1}{2}[(1.027)+2(0.8972)]$$

$$= \frac{1}{2}(1.027+1.7944)$$

$$= \frac{1}{2}(2.8214)$$

$$= 1.4107 \text{ or } 1.4108$$

ii). By simpson's 1/3 rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.0270) + 4(0.5+0.1+0.0385) + 2(0.2+0.0588)\right]$$

$$= \frac{1}{3}\left[(1.027) + 4(0.6385) + 2(0.2588)\right]$$

$$= \frac{1}{3}\left[1.027 + 2.554 + 0.5176\right]$$

$$= \frac{1}{3}(4.0986)$$

$$= 1.3662$$

iii). By simpson's 3/8 rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3}{8}\left[(1+0.027) + 2(0.1) + 3(0.5+0.2+0.0588+0.0385)\right]$$

$$= \frac{3}{8}\left[(1.027) + (0.2) + 3(0.7973)\right]$$

$$= \frac{3}{8}(1.027 + 0.2 + 2.3919)$$

$$= \frac{3}{8}(3.6189)$$

$$= \frac{10.8567}{8}$$

$$= 1.3570 \text{ or } 1.3571$$

Q(13). Given the following table of x and y.

x	1	1.05	1.10	1.15	1.20	1.25	1.30
y	1.000	1.025	1.049	1.072	1.095	1.118	1.40

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.05$; Applying Newton Forward differentiation formula.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	1						
		0.025	−0.001				
1.05	1.025						
		0.024		0	0.001		
1.10	1.049		−0.001	0.001		−0.002	
		0.023					
1.15	1.072		0		−0.001		0.262
		0.023		0			
1.20	1.095		0			0.26	
		0.023					
1.25	1.118			0.259	0.259		
		0.282	0.259				
1.30	1.40						

i). $$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}\left(\Delta y_0 - \left[\left(\frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3}\right) - \left(\frac{\Delta^4 y_0}{4} + \frac{\Delta^5 y_0}{5}\right)\right]\right)$$

Here,

$x = 1.05$, $x_0 = 1.05$ and $h = 0.5$

$$\left[\frac{dy}{dx}\right]_{x=1.05} = \frac{1}{0.5}\left(0.024 - \left[\left(\frac{(-0.001)}{2} + \frac{(0.001)}{3}\right) - \left(\frac{(-0.001)}{4} + \frac{0.26}{5}\right.\right.\right.$$

$$= \frac{1}{0.5}\left(0.024 - [(-0.0005 + 0.0003) - (-0.00025 + 0.052\right.$$

$$= (2)(0.024 - [(-0.0002) - (0.05175)]$$

$$= (2)(0.024 + 0.05195)$$

$$= (2)(0.07595)$$

$$= 0.1519$$

ii). $\left[\frac{d^2y}{dx^2}\right]_{x=x_0} = \frac{1}{h^2}\left(\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right)$

$\left(\frac{d^2y}{dx^2}\right)_{x=1.05} = \frac{1}{(0.5)^2}\left[(-0.001) - (0.001) + \frac{11}{12}(-0.001) - \frac{5}{6}(0.26)\right]$

$= \frac{1}{0.25}\left[(-0.001) - (0.001) + \cancel{\frac{-0.011}{12}} - 0.0009 - 0.2166\right]$

$= \frac{1}{0.25}(-0.001 - 0.001 - 0.0009 - 0.2166)$

$= \frac{1}{0.25}(-0.2195)$

$\left(\frac{d^2y}{dx^2}\right) = -0.878$

Q.(14) Give the following table of x and y.

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.000	1.025	1.049	1.072	1.095	1.118	1.140

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.25$; Applying Netwon Backward differentiation

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
1.00	1.000	0.025	-0.001				
1.05	1.025	0.024		0	0.001		
1.10	1.049	0.023	-0.001			-0.002	
1.15	1.072	0.023	0	0.001			0.002
1.20	1.095	0.023	0	0	-0.001	0	
1.25	1.118	0.022	-0.001	-0.001	-0.001		
1.30	1.140						

i) $\frac{dy}{dx} = \frac{1}{h}\{(\nabla y_0 + \frac{1}{2}\cdot\nabla^2 y_0 + \frac{1}{3}\cdot\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 + \frac{1}{5}\nabla^5 y_0\}$

$= \frac{1}{0.5}((0.023) + \frac{1}{2}(0) + \frac{1}{3}(0) + \frac{1}{4}(-0.001) + \frac{1}{5}(-0.002))$

$= \frac{1}{0.5}(0.023 + 0 + 0 - 0.00025 - 0.0004)$

$= (2)(0.02235)$

$= 0.0447$

ii). $\frac{d^2y}{dx^2} = \frac{1}{h^2}(\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \nabla^5 y_0$

$= \frac{1}{(0.5)^2}((0) + (0) + \frac{11}{12}(-0.001) + (-0.002))$

$= \frac{1}{0.25}(0 + 0 - 0.00091 - 0.002)$

$= \frac{1}{0.25}(-0.00291)$

$= -0.01164$