Math Computing

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NCERT 9.7.1.7

This question is from class 9 ncert chapter 7.triangles

1. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that
   (a) $\triangle DAP \cong \triangle EBP$
   (b) $AD = BE$

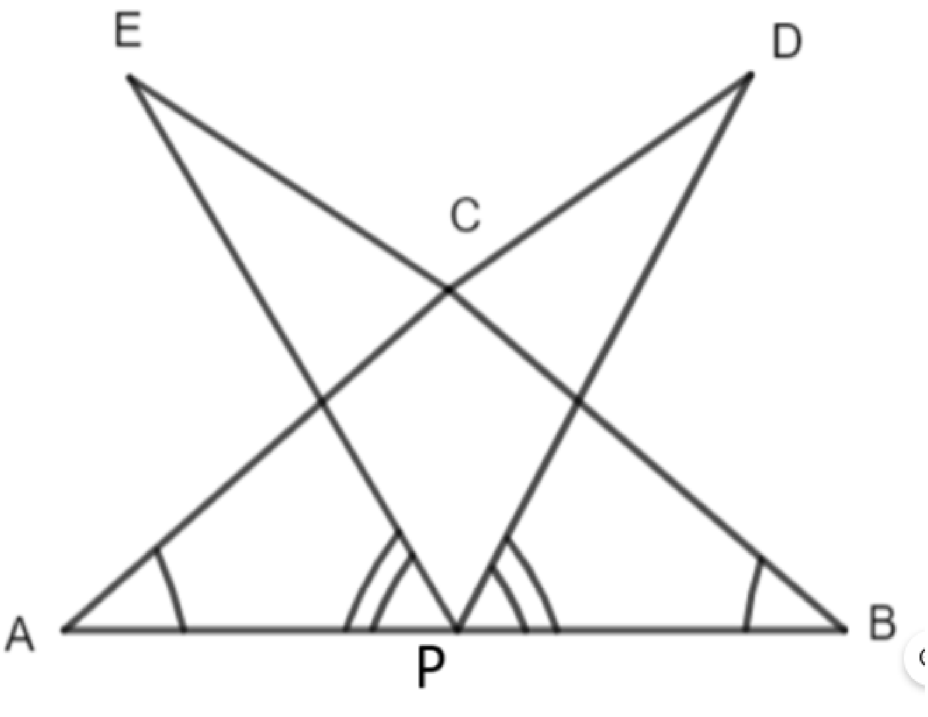

Figure 1: $\triangle DAP \quad and \quad \triangle EBP$

Construction steps:

(i) Let point A be the reference point whose coordinates are at origin.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

(ii) Let the distance between point A and B be x, and also considering the point B on same axis .

$$\|A - B\| = x \tag{2}$$

So,the coordinates of point B be,

$$\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{3}$$

(iii) Given the point P is the mid-point of line segment AB,

$$\mathbf{P} = \left(\tfrac{A+B}{2}\right) \tag{4}$$

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{5}$$

(iv) Let the coordinate points of D and E are,

$$\mathbf{D} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{6}$$

$$\mathbf{E} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \tag{7}$$

(v) Let assume the distance between point A, D and B, E be r ,and the line AB makes an angle θ anticlock-wise from point A clock-wise from point B with the line AD BE.

$$\|A - D\| = \mathbf{r} = \|B - E\| \tag{8}$$

$$\angle BAD = \theta = \angle ABE \tag{9}$$

$\therefore$ Now the coordinates of point D, E are,

$$\mathbf{D} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{10}$$

$$\mathbf{E} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{11}$$

(vi) Similarly, the mid-point P also makes an angle θ with the points D and E

$$\angle BAD = \theta = \angle ABE \tag{12}$$

(vii) Now the coordinates of A, B, P, D and E are ,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{13}$$

$$\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix}, \tag{14}$$

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix}, \tag{15}$$

$$\mathbf{D} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{16}$$

$$\mathbf{E} = \begin{pmatrix} -r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{17}$$

(viii) Let assume,

$$x = 5 \tag{18}$$

$$r = 4 \tag{19}$$

$$\theta_1 = 30°(\text{for the angle BAD and ABE}) \tag{20}$$

$$\theta_2 = 60°(\text{for the angle EPA and DPB}) \tag{21}$$

(ix) on substituting the values,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{22}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \tag{23}$$

$$\mathbf{P} = \begin{pmatrix} 2.5 \\ 0 \end{pmatrix}, \tag{24}$$

$$\mathbf{D} = \begin{pmatrix} 4\cos 30° \\ 4\sin 30° \end{pmatrix}, \tag{25}$$

$$\mathbf{E} = \begin{pmatrix} -4\cos 30° \\ 4\sin 30° \end{pmatrix}, \tag{26}$$

$$\mathbf{D} = \begin{pmatrix} 4\cos 60° \\ 4\sin 60° \end{pmatrix}, \tag{27}$$

$$\mathbf{D} = \begin{pmatrix} -4\cos 60° \\ 4\sin 60° \end{pmatrix} \tag{28}$$

(x) on calculating we get ,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{29}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \tag{30}$$

$$\mathbf{P} = \begin{pmatrix} 2.5 \\ 0 \end{pmatrix}, \tag{31}$$

$$\mathbf{D} = \begin{pmatrix} 3.464101 \\ 2 \end{pmatrix}, \tag{32}$$

$$\mathbf{E} = \begin{pmatrix} -3.464101 \\ 2 \end{pmatrix}, \tag{33}$$

$$\mathbf{D} = \begin{pmatrix} -2 \\ 3.464101 \end{pmatrix}, \tag{34}$$

$$\mathbf{E} = \begin{pmatrix} -2 \\ 3.464101 \end{pmatrix} \tag{35}$$

Joining these points forms the required figure

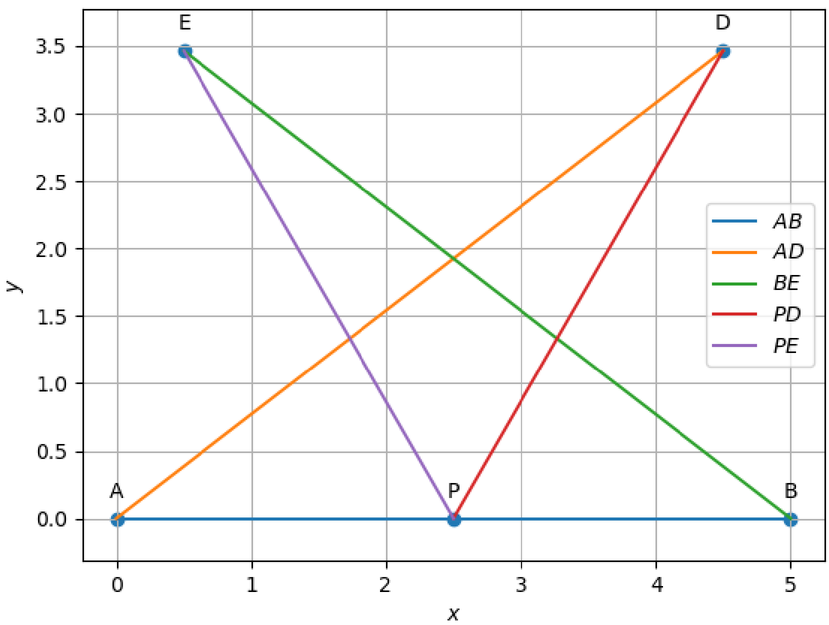

Figure 2: $\triangle DAP \quad and \quad \triangle EBP$