Math Computing

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NCERT 9.7.1.7

This question is from class 9 ncert chapter 7.triangles

1. **AB** is a line segment and **P** is its mid-point. **D** and **E** are points on the same side of **AB** such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that
   (a) $\triangle\mathbf{DAP} \cong \triangle\mathbf{EBP}$
   (b) $\mathbf{AD} = \mathbf{BE}$

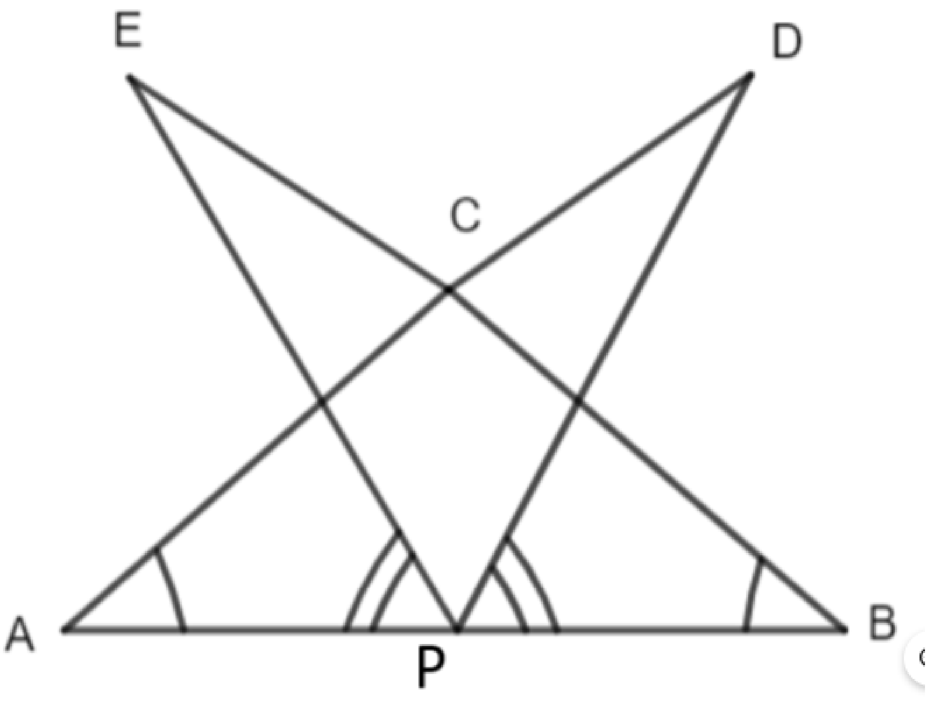

Figure 1: $\triangle\mathbf{DAP} \quad and \quad \triangle\mathbf{EBP}$

Construction steps:

(i) Let point $\mathbf{A}$ be the reference point whose coordinates are at origin.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

(ii) Let the distance between point $\mathbf{A}$ and $\mathbf{B}$ be x, and also considering the point $\mathbf{B}$ on same axis .

$$\|A - B\| = x \tag{2}$$

So,the coordinates of point $\mathbf{B}$ be,

$$\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{3}$$

(iii) Given the point $\mathbf{P}$ is the mid-point of line segment $\mathbf{AB}$,

$$\mathbf{P} = \left(\tfrac{A+B}{2}\right) \tag{4}$$

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{5}$$

(iv) Let the coordinate points of $\mathbf{D}$ and $\mathbf{E}$ are,

$$\mathbf{D} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{6}$$

$$\mathbf{E} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \tag{7}$$

(v) Let assume the distance between point $\mathbf{A}, \mathbf{D}$ and $\mathbf{B}, \mathbf{E}$ be $\mathbf{r}$,and the line $\mathbf{AB}$ makes an angle θ anticlock-wise from point $\mathbf{A}$ clockwise from point $\mathbf{B}$ with the line $\mathbf{AD}$, $\mathbf{BE}$.

$$\|A - D\| = \mathbf{r} = \|B - E\| \tag{8}$$

$$\angle BAD = \theta = \angle ABE \tag{9}$$

$\therefore$ Now the coordinates of point $\mathbf{D}, \mathbf{E}$ are,

$$\mathbf{D} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{10}$$

$$\mathbf{E} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{11}$$

(vi) Similarly, the mid-point **P** also makes an angle θ with the points **D** and **E**

$$\angle BAD = \theta = \angle ABE \tag{12}$$

(vii) Let assume,

Symbol	**Value**	**Description**
θ_1	$30°$	$\angle BAD = \angle ABE$
θ_2	$60°$	$\angle EPA = \angle DPB$
r	5	Length of **AB**
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Reference point at origin
B	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$	point **B** on the same axis of **A**
P	$\begin{pmatrix} 2.5 \\ 0 \end{pmatrix}$	Mid-point of **AB**
D	$\begin{pmatrix} 5\cos 30° \\ 5\sin 30° \end{pmatrix}$	From point **A** makes an angle θ_1 in anticlock-wise with line $\mathbf{AB}, \mathbf{AD}$
E	$\begin{pmatrix} -5\cos 30° \\ 5\sin 30° \end{pmatrix}$	From point **B** makes an angle θ_1 in cloock-wise with line $\mathbf{AB}, \mathbf{BE}$
D	$\begin{pmatrix} 5\cos 60° \\ 5\sin 60° \end{pmatrix}$	From point **P** makes an angle θ_2 in anticlock-wise with line $\mathbf{BP}, \mathbf{PD}$
E	$\begin{pmatrix} -5\cos 60° \\ 5\sin 60° \end{pmatrix}$	From point **P** makes an angle θ_2 in clock-wise with line $\mathbf{AP}, \mathbf{PE}$

Table 1: Parameters

(viii) on calculating we get,

$$\mathbf{r} = 5, \tag{13}$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{14}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \tag{15}$$

$$\mathbf{P} = \begin{pmatrix} 2.5 \\ 0 \end{pmatrix}, \tag{16}$$

$$\mathbf{D} = \begin{pmatrix} 4.330127 \\ 2.5 \end{pmatrix}, \tag{17}$$

$$\mathbf{E} = \begin{pmatrix} -4.330127 \\ 2.5 \end{pmatrix}, \tag{18}$$

$$\mathbf{D} = \begin{pmatrix} 2.5 \\ 4.330127 \end{pmatrix}, \tag{19}$$

$$\mathbf{E} = \begin{pmatrix} -2.5 \\ 4.330127 \end{pmatrix} \tag{20}$$

Joining these points forms the required figure

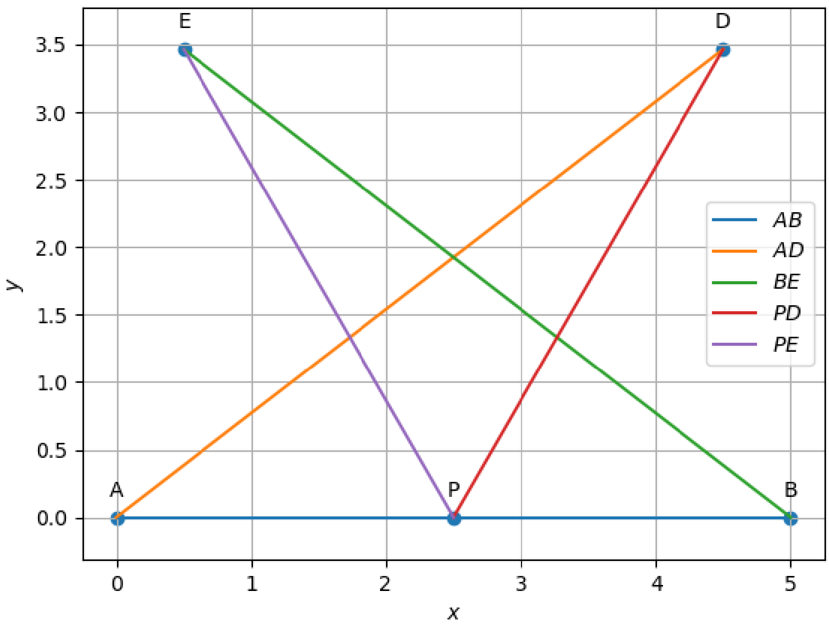

Figure 2: $\triangle\mathbf{DAP}$ *and* $\triangle\mathbf{EBP}$