

Class 12

Chapter 10 - Vector Algebra

1. If θ is the angle between two vectors \mathbf{a} and \mathbf{b} , then $\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a} \times \mathbf{b}\|$ when θ is equal to

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) π

Solution:

Given,

$$\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a} \times \mathbf{b}\| \quad (1)$$

Since, scalar product of two vectors is $\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta$ (2)

and vector product of two vector is $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta$ (3)

Substituting (3) (2) in (1), we get

$$\implies \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta \quad (4)$$

$$\implies \cos \theta = \sin \theta \quad (5)$$

$$\implies \tan \theta = 1 \quad (6)$$

$$\implies \theta = \frac{\pi}{4} \quad (7)$$

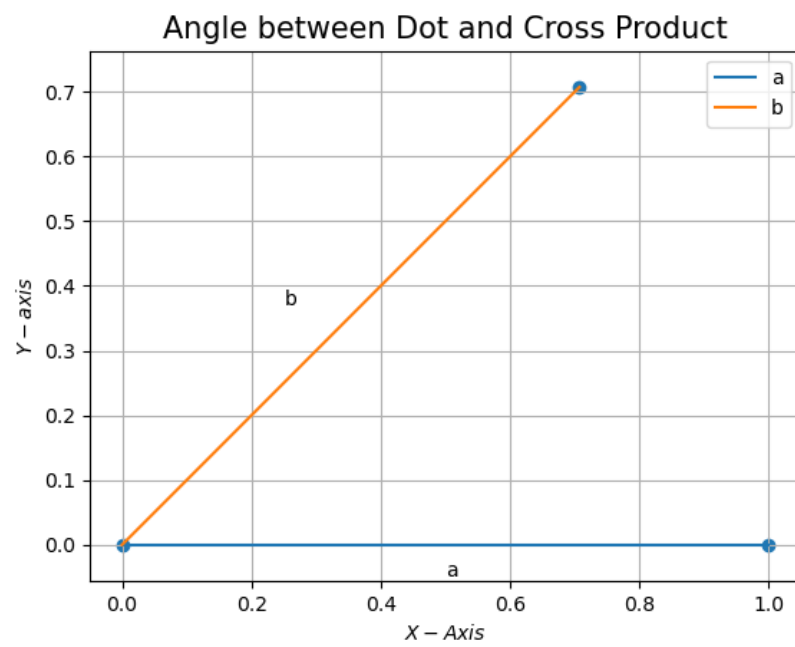


Figure 1: $\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a} \times \mathbf{b}\|$