Class 12

Chapter 10 - Vector Algebra

1. If θ is the angle between and two vectors $\mathbf{a}$ and $\mathbf{b}$, then $\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a} \times \mathbf{b}\|$ when θ is equal to

   a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π

Solution:
Given,

$$\|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a} \times \mathbf{b}\| \tag{1}$$

$$\text{Since, scalar product of two vectors is } \|\mathbf{a}^\top \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta \tag{2}$$

$$\text{and vector product of two vector is } \|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta \tag{3}$$

Substituting (3) (2) in (1), we get

$$\implies \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta \tag{4}$$

$$\implies \cos\theta = \sin\theta \tag{5}$$

$$\implies \tan\theta = 1 \tag{6}$$

$$\implies \theta = \frac{\pi}{4} \tag{7}$$

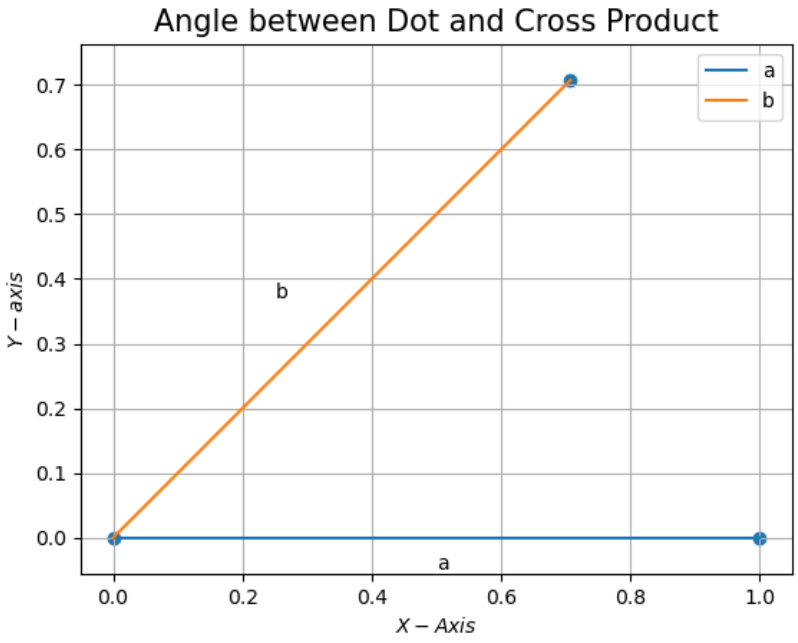

Figure 1: The vectors $\mathbf{a} = 1$ and $\mathbf{b} = \mathrm{n}$