Ukkonen's suffix tree algorithm in plain English

Ask Question

I feel a bit thick at this point. I've spent days trying to fully wrap my head around suffix tree construction, but because I don't have a mathematical background, many of the explanations elude me as they start to make excessive use of mathematical symbology. The closest to a good explanation that I've found is *Fast String Searching With Suffix Trees*, but he glosses over various points and some aspects of the algorithm remain unclear.

A step-by-step explanation of this algorithm here on Stack Overflow would be invaluable for many others besides me, I'm sure.

For reference, here's Ukkonen's paper on the algorithm: http://www.cs.helsinki.fi/u/ukkonen/SuffixT1withFigs.pdf

My basic understanding, so far:

- I need to iterate through each prefix P of a given string T
- I need to iterate through each suffix S in prefix P and add that to tree
- To add suffix S to the tree, I need to iterate through each character

in S, with the iterations consisting of either walking down an existing branch that starts with the same set of characters C in S and potentially splitting an edge into descendent nodes when I reach a differing character in the suffix, OR if there was no matching edge to walk down. When no matching edge is found to walk down for C, a new leaf edge is created for C.

The basic algorithm appears to be $O(n^2)$, as is pointed out in most explanations, as we need to step through all of the prefixes, then we need to step through each of the suffixes for each prefix. Ukkonen's algorithm is apparently unique because of the suffix pointer technique he uses, though I think *that* is what I'm having trouble understanding.

I'm also having trouble understanding:

- exactly when and how the "active point" is assigned, used and changed
- what is going on with the canonization aspect of the algorithm
- Why the implementations I've seen need to "fix" bounding variables that they are using

Here is the completed **C#** source code. It not only

works correctly, but supports automatic canonization and renders a nicer looking text graph of the output. Source code and sample output is at:

> **https://gist.github.com/2373868**

Update 2017-11-04

After many years I've found a new use for suffix trees, and have implemented the algorithm in **JavaScript**. Gist is below. It should be bug-free. Dump it into a js file, `npm install chalk` from the same location, and then run with node.js to see some colourful output. There's a stripped down version in the same Gist, without any of the debugging code.

> **https://gist.github.com/axefrog/c347bf0f5e0723cbd09b1aaed6ec6fc6**

javascript c# algorithm search suffix-tree

edited Nov 4 '17 at 2:39

asked Feb 26 '12 at 11:30

Nathan Ridley
17.7k 23 100 178

protected by Mysticial Aug 15 '12 at 4:56

This question is protected to prevent "thanks!", "me too!", or

spam answers by new users. To answer it, you must have earned at least 10 reputation on this site (the association bonus does not count).

2 Did you take a look at the description given in Dan Gusfield's book? I found that to be helpful. – jogojapan Feb 27 '12 at 2:10

3 The gist does not specify the license - can I change your code and republish under MIT (obviously with attributions)? – Yurik Oct 11 '12 at 4:26

1 Yep, go for your life. Consider it public domain. As mentioned by another answer on this page, there's a bug that needs fixing anyway. – Nathan Ridley Oct 11 '12 at 15:27

1 maybe this implementation will help others, goto code.google.com/p/text-indexing – cos Dec 3 '13 at 8:40

2 @JamesYoungman I think you're overthinking things. The fact that I said it's public domain means I wouldn't have a leg to stand on in any court of law. Even so, here you go: "I declare that the suffix tree code that I wrote and linked to from this StackOverflow thread is MIT-licensed. Do whatever you want with it." – Nathan Ridley May 22 '16 at 23:00

|

6 Answers

The following is an attempt to describe the Ukkonen algorithm by first showing what it does when the string is simple (i.e. does not contain any repeated characters), and then extending it to the full algorithm.

First, a few preliminary statements.

1. What we are building, is *basically* like a search trie. So there is a root node, edges going out of it leading to new nodes, and further edges going out of those, and so forth
2. **But**: Unlike in a search trie, the edge labels are not single characters. Instead, each edge is labeled using a pair of integers `[from,to]`. These are pointers into the text. In this sense, each edge carries a string label of arbitrary length, but takes only O(1) space (two pointers).

Basic principle

I would like to first demonstrate how to create the suffix tree of a particularly simple string, a string with no repeated characters:

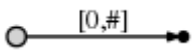
```
abc
```

The algorithm **works in steps, from left to right**. There is **one step for every character of the string**.

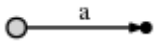
Each step might involve more than one individual operation, but we will see (see the final observations at the end) that the total number of operations is O(n).

So, we start from the *left*, and first insert only the single character `a` by creating an edge from the root node (on the left) to a leaf, and labeling it as `[0,#]`, which means the edge represents the substring starting at position 0 and ending at *the current end*. I use the symbol `#` to mean *the current end*, which is at position 1 (right after `a`).

So we have an initial tree, which looks like this:



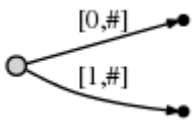
And what it means is this:



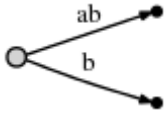
Now we progress to position 2 (right after `b`). **Our goal at each step** is to insert **all suffixes up to the current position**. We do this by

- expanding the existing `a`-edge to `ab`
- inserting one new edge for `b`

In our representation this looks like



And what it means is:

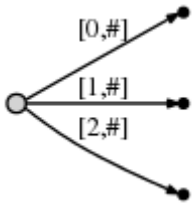


We observe two things:

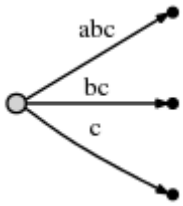
- The edge representation for `ab` is **the same** as it used to be in the initial tree: `[0,#]`. Its meaning has automatically changed because we updated the current position `#` from 1 to 2.
- Each edge consumes O(1) space, because it consists of only two pointers into the text, regardless of how many characters it represents.

Next we increment the position again and update the tree by appending a `c` to every existing edge and inserting one new edge for the new suffix `c`.

In our representation this looks like



And what it means is:



We observe:

- The tree is the correct suffix tree *up to the*

current position after each step

- There are as many steps as there are characters in the text
- The amount of work in each step is O(1), because all existing edges are updated automatically by incrementing `#`, and inserting the one new edge for the final character can be done in O(1) time. Hence for a string of length n, only O(n) time is required.

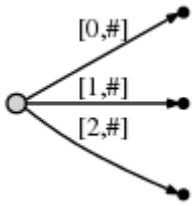
First extension: Simple repetitions

Of course this works so nicely only because our string does not contain any repetitions. We now look at a more realistic string:

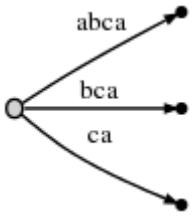
```
abcabxabcd
```

It starts with `abc` as in the previous example, then `ab` is repeated and followed by `x`, and then `abc` is repeated followed by `d`.

Steps 1 through 3: After the first 3 steps we have the tree from the previous example:



Step 4: We move `#` to position 4. This implicitly updates all existing edges to this:



and we need to insert the final suffix of the current step, `a`, at the root.

Before we do this, we introduce **two more variables** (in addition to `#`), which of course have been there all the time but we haven't used them so far:

- The **active point**, which is a triple `(active_node,active_edge,active_length)`
- The `remainder`, which is an integer indicating how many new suffixes we need to insert

The exact meaning of these two will become clear soon, but for now let's just say:

- In the simple `abc` example, the active point was always `(root,'\0x',0)`, i.e. `active_node` was the root node, `active_edge` was specified as the null character `'\0x'`, and `active_length` was zero. The effect of this was that the one new edge that we inserted in every step was inserted at the root node as a freshly created edge. We will see soon why a triple is necessary to represent this information.

- The `remainder` was always set to 1 at the beginning of each step. The meaning of this was that the number of suffixes we had to actively insert at the end of each step was 1 (always just the final character).

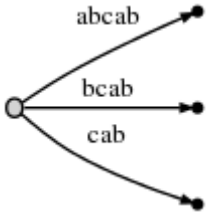
Now this is going to change. When we insert the current final character `a` at the root, we notice that there is already an outgoing edge starting with `a`, specifically: `abca`. Here is what we do in such a case:

- We **do not** insert a fresh edge `[4,#]` at the root node. Instead we simply notice that the suffix `a` is already in our tree. It ends in the middle of a longer edge, but we are not bothered by that. We just leave things the way they are.
- We **set the active point** to `(root,'a',1)`. That means the active point is now somewhere in the middle of outgoing edge of the root node that starts with `a`, specifically, after position 1 on that edge. We notice that the edge is specified simply by its first character `a`. That suffices because there can be *only one* edge starting with any particular character (confirm that this is true after reading through the entire description).

- We also increment `remainder`, so at the beginning of the next step it will be 2.

Observation: When the final **suffix we need to insert is found to exist in the tree already**, the tree itself is **not changed** at all (we only update the active point and `remainder`). The tree is then not an accurate representation of the suffix tree *up to the current position* any more, but it **contains** all suffixes (because the final suffix `a` is contained *implicitly*). Hence, apart from updating the variables (which are all of fixed length, so this is O(1)), there was **no work** done in this step.

Step 5: We update the current position `#` to 5. This automatically updates the tree to this:



And **because `remainder` is 2**, we need to insert two final suffixes of the current position: `ab` and `b`. This is basically because:

- The `a` suffix from the previous step has never been properly inserted. So it has *remained*, and since we have progressed one step, it has now grown from `a` to `ab`.

- And we need to insert the new final edge `b`.

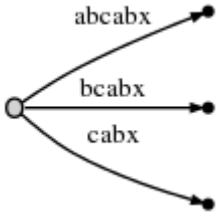
In practice this means that we go to the active point (which points to behind the `a` on what is now the `abcab` edge), and insert the current final character `b`. **But:** Again, it turns out that `b` is also already present on that same edge.

So, again, we do not change the tree. We simply:

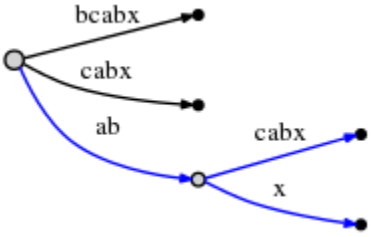
- Update the active point to `(root,'a',2)` (same node and edge as before, but now we point to behind the `b`)
- Increment the `remainder` to 3 because we still have not properly inserted the final edge from the previous step, and we don't insert the current final edge either.

To be clear: We had to insert `ab` and `b` in the current step, but because `ab` was already found, we updated the active point and did not even attempt to insert `b`. Why? Because if `ab` is in the tree, **every suffix** of it (including `b`) must be in the tree, too. Perhaps only *implicitly*, but it must be there, because of the way we have built the tree so far.

We proceed to **step 6** by incrementing `#`. The tree is automatically updated to:



Because **`remainder` is 3**, we have to insert `abx`, `bx` and `x`. The active point tells us where `ab` ends, so we only need to jump there and insert the `x`. Indeed, `x` is not there yet, so we split the `abcabx` edge and insert an internal node:



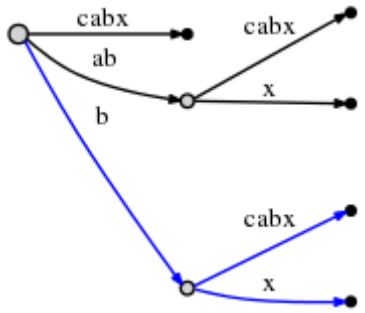
The edge representations are still pointers into the text, so splitting and inserting an internal node can be done in O(1) time.

So we have dealt with `abx` and decrement `remainder` to 2. Now we need to insert the next remaining suffix, `bx`. But before we do that we need to update the active point. The rule for this, after splitting and inserting an edge, will be called **Rule 1** below, and it applies whenever the `active_node` is root (we will learn rule 3 for other cases further below). Here is rule 1:

> After an insertion from root,
>
> - `active_node` remains root

> - `active_edge` is set to the first character of the new suffix we need to insert, i.e. `b`
> - `active_length` is reduced by 1

Hence, the new active-point triple `(root,'b',1)` indicates that the next insert has to be made at the `bcabx` edge, behind 1 character, i.e. behind `b`. We can identify the insertion point in O(1) time and check whether `x` is already present or not. If it was present, we would end the current step and leave everything the way it is. But `x` is not present, so we insert it by splitting the edge:

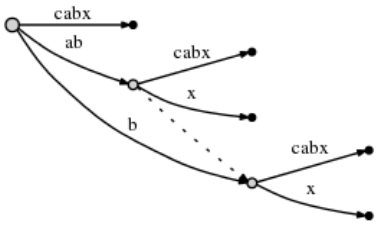


Again, this took O(1) time and we update `remainder` to 1 and the active point to `(root,'x',0)` as rule 1 states.

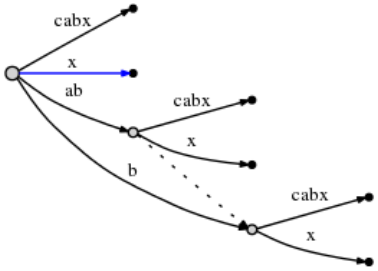
But there is one more thing we need to do. We'll call this **Rule 2:**

> If we split an edge and insert a new node, and if that is *not the first node* created during the current step, we connect the previously inserted node and the new node through a special pointer,

> a **suffix link**. We will later see why that is useful. Here is what we get, the suffix link is represented as a dotted edge:



We still need to insert the final suffix of the current step, `x`. Since the `active_length` component of the active node has fallen to 0, the final insert is made at the root directly. Since there is no outgoing edge at the root node starting with `x`, we insert a new edge:



As we can see, in the current step all remaining inserts were made.

We proceed to **step 7** by setting `#` =7, which automatically appends the next character, `a`, to all leaf edges, as always. Then we attempt to insert the new final character to the active point (the root), and find that it is there already. So we end the current step without inserting anything and update the active point to `(root,'a',1)`.

In **step 8**, `#` =8, we append `b`, and as seen before, this only means we update the active point to `(root,'a',2)` and increment `remainder` without doing anything else, because `b` is already present. **However,** we notice (in O(1) time) that the active point is now at the end of an edge. We reflect this by re-setting it to `(node1,'\0x',0)`. Here, I use `node1` to refer to the internal node the `ab` edge ends at.

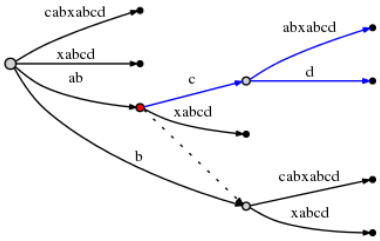
Then, in **step `#` =9**, we need to insert 'c' and this will help us to understand the final trick:

Second extension: Using suffix links

As always, the `#` update appends `c` automatically to the leaf edges and we go to the active point to see if we can insert 'c'. It turns out 'c' exists already at that edge, so we set the active point to `(node1,'c',1)`, increment `remainder` and do nothing else.

Now in **step `#` =10**, `remainder is 4`, and so we first need to insert `abcd` (which remains from 3 steps ago) by inserting `d` at the active point.

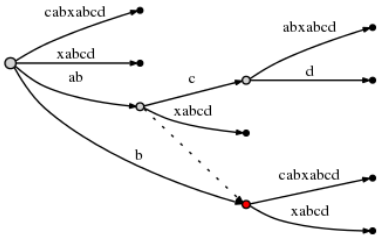
Attempting to insert `d` at the active point causes an edge split in O(1) time:



The `active_node`, from which the split was initiated, is marked in red above. Here is the final rule, **Rule 3:**

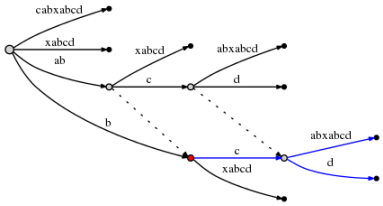
> After splitting an edge from an `active_node` that is not the root node, we follow the suffix link going out of that node, if there is any, and reset the `active_node` to the node it points to. If there is no suffix link, we set the `active_node` to the root. `active_edge` and `active_length` remain unchanged.

So the active point is now `(node2,'c',1)`, and `node2` is marked in red below:



Since the insertion of `abcd` is complete, we decrement `remainder` to 3 and consider the next remaining suffix of the current step, `bcd`. Rule 3 has set the active point to just the right node and edge so inserting `bcd` can be done by simply inserting its final character `d` at the active point.

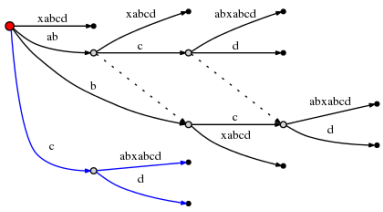
Doing this causes another edge split, and **because of rule 2**, we must create a suffix link from the previously inserted node to the new one:



We observe: Suffix links enable us to reset the active point so we can make the next *remaining insert* at O(1) effort. Look at the graph above to confirm that indeed node at label `ab` is linked to the node at `b` (its suffix), and the node at `abc` is linked to `bc`.

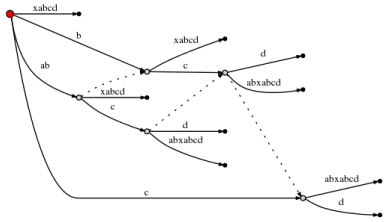
The current step is not finished yet. `remainder` is now 2, and we need to follow rule 3 to reset the active point again. Since the current `active_node` (red above) has no suffix link, we reset to root. The active point is now `(root,'c',1)`.

Hence the next insert occurs at the one outgoing edge of the root node whose label starts with `c`: `cabxabcd`, behind the first character, i.e. behind `c`. This causes another split:



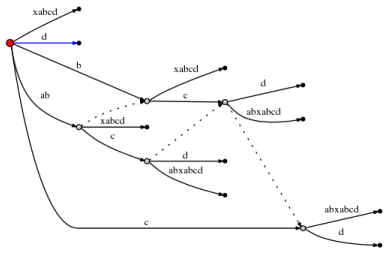
And since this involves the creation of a new internal node,we follow rule 2 and

set a new suffix link from the previously created internal node:



(I am using Graphviz Dot for these little graphs. The new suffix link caused dot to re-arrange the existing edges, so check carefully to confirm that the only thing that was inserted above is a new suffix link.)

With this, `remainder` can be set to 1 and since the `active_node` is root, we use rule 1 to update the active point to `(root,'d',0)`. This means the final insert of the current step is to insert a single `d` at root:



That was the final step and we are done. There are number of **final observations**, though:

- In each step we move `#` forward by 1 position. This automatically updates all leaf nodes in O(1) time.
- But it does not deal with a) any suffixes *remaining* from previous steps, and b) with the

one final character of the current step.

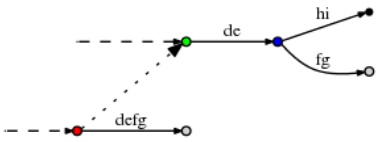
- `remainder` tells us how many additional inserts we need to make. These inserts correspond one-to-one to the final suffixes of the string that ends at the current position `#`. We consider one after the other and make the insert. **Important:** Each insert is done in O(1) time since the active point tells us exactly where to go, and we need to add only one single character at the active point. Why? Because the other characters are *contained implicitly* (otherwise the active point would not be where it is).
- After each such insert, we decrement `remainder` and follow the suffix link if there is any. If not we go to root (rule 3). If we are at root already, we modify the active point using rule 1. In any case, it takes only O(1) time.
- If, during one of these inserts, we find that the character we want to insert is already there, we don't do anything and end the current step, even if `remainder` >0. The reason is that any inserts that remain will be suffixes of the one we just tried to make. Hence they are all *implicit* in the current

tree. The fact that `remainder` >0 makes sure we deal with the remaining suffixes later.

- What if at the end of the algorithm `remainder` >0? This will be the case whenever the end of the text is a substring that occurred somewhere before. In that case we must append one extra character at the end of the string that has not occurred before. In the literature, usually the dollar sign `$` is used as a symbol for that. **Why does that matter?** --> If later we use the completed suffix tree to search for suffixes, we must accept matches only if they *end at a leaf*. Otherwise we would get a lot of spurious matches, because there are *many* strings *implicitly* contained in the tree that are not actual suffixes of the main string. Forcing `remainder` to be 0 at the end is essentially a way to ensure that all suffixes end at a leaf node. **However,** if we want to use the tree to search for *general substrings*, not only *suffixes* of the main string, this final step is indeed not required, as suggested by the OP's comment below.
- So what is the complexity of the entire algorithm? If the text is

n characters in length, there are obviously n steps (or n+1 if we add the dollar sign). In each step we either do nothing (other than updating the variables), or we make `remainder` inserts, each taking O(1) time. Since `remainder` indicates how many times we have done nothing in previous steps, and is decremented for every insert that we make now, the total number of times we do something is exactly n (or n+1). Hence, the total complexity is O(n).

- However, there is one small thing that I did not properly explain: It can happen that we follow a suffix link, update the active point, and then find that its `active_length` component does not work well with the new `active_node`. For example, consider a situation like this:



(The dashed lines indicate the rest of the tree. The dotted line is a suffix link.)

Now let the active point be `(red,'d',3)`, so it points to the place behind the `f` on the `defg` edge. Now assume we made the necessary updates and now follow the suffix link to

update the active point according to rule 3. The new active point is `(green,'d',3)`. However, the `d`-edge going out of the green node is `de`, so it has only 2 characters. In order to find the correct active point, we obviously need to follow that edge to the blue node and reset to `(blue,'f',1)`.

In a particularly bad case, the `active_length` could be as large as `remainder`, which can be as large as n. And it might very well happen that to find the correct active point, we need not only jump over one internal node, but perhaps many, up to n in the worst case. Does that mean the algorithm has a hidden $O(n^2)$ complexity, because in each step `remainder` is generally O(n), and the post-adjustments to the active node after following a suffix link could be O(n), too?

No. The reason is that if indeed we have to adjust the active point (e.g. from green to blue as above), that brings us to a new node that has its own suffix link, and `active_length` will be reduced. As we follow down the chain of suffix links we make the remaining inserts, `active_length` can only decrease, and the number of active-point adjustments we can make on the way can't be larger than `active_length` at any given time. Since `active_length` can never be larger than `remainder`, and `remainder` is O(n) not only in every single step, but the total sum

of increments ever made to `remainder` over the course of the entire process is O(n) too, the number of active point adjustments is also bounded by O(n).

edited Jun 5 '17 at 8:19

Web_Designer
32.4k 74 175 237

answered Mar 1 '12 at 9:13

jogojapan
50.4k 7 71 107

63 Sorry this ended up a little longer than I hoped. And I realize it explains an number of trivial things we all know, while the difficult parts might still not be perfectly clear. Let's edit it into shape together. – jogojapan Mar 1 '12 at 9:14

61 And I should add that this is *not* based on the description found in Dan Gusfield's book. It's a new attempt at describing the algorithm by first considering a string with no repetitions and then discussing how repetitions are handled. I hoped that would be more intuitive. – jogojapan Mar 1 '12 at 9:16

6 Thanks @jogojapan, I was able to write a fully-working example thanks to your explanation. I've published the source so hopefully somebody else may find it of use: gist.github.com/2373868 – Nathan Ridley Apr 13 '12 at 5:17

3 @NathanRidley Yes (by the way, that final bit is what Ukkonen calls canonicize). One way to trigger it is to make sure

there is a substring that appears three times and ends in a string that appears one more time in yet a different context. E.g. `abcdefabxybcdmnabcdex`. The initial part of `abcd` is repeated in `abxy` (this creates an internal node after `ab`) and again in `abcdex`, and it ends in `bcd`, which appears not only in the `bcdex` context, but also in the `bcdmn` context. After `abcdex` is inserted, we follow the suffix link to insert `bcdex`, and there canonicize will kick in. – jogojapan Apr 14 '12 at 10:04

6 Ok my code has been completely rewritten and now works correctly for all cases, including automatic canonization, plus has a much nicer text graph output. gist.github.com/2373868 – Nathan Ridley Apr 15 '12 at 0:17

|

I tried to implement the Suffix Tree with the approach given in jogojapan's answer, but it didn't work for some cases due to wording used for the rules. Moreover, I've mentioned that nobody managed to implement an absolutely correct suffix tree using this approach. Below I will write an "overview" of jogojapan's answer with some modifications to the rules. I will also describe the case when we forget to create ***important*** suffix links.

Additional variables used

1. **active point** - a triple (active_node; active_edge; active_length), showing from where we must start inserting a new suffix.
2. **remainder** - shows the number of suffixes we must add *explicitly*. For instance, if our word is 'abcaabca', and remainder = 3, it means we must process 3 last suffixes: **bca**, **ca** and **a**.

Let's use a concept of an **internal node** - all the nodes, except the *root* and the *leafs* are **internal nodes**.

Observation 1

When the final suffix we need to insert is found to exist in the tree already, the tree itself is not changed at all (we only update the `active point` and `remainder`).

Observation 2

If at some point `active_length` is greater or equal to the length of current edge (`edge_length`), we move our `active point` down until `edge_length` is strictly greater than `active_length` .

Now, let's redefine the rules:

Rule 1

> If after an insertion from the *active node* = *root*,

> the *active length* is greater than 0, then:
>
> 1. *active node* is not changed
> 2. *active length* is decremented
> 3. *active edge* is shifted right (to the first character of the next suffix we must insert)

Rule 2

> If we create a new *internal node* **OR** make an inserter from an *internal node*, and this is not the first **SUCH** *internal node* at current step, then we link the previous **SUCH** node with **THIS** one through a *suffix link*.

This definition of the `Rule 2` is different from jogojapan', as here we take into account not only the *newly created* internal nodes, but also the internal nodes, from which we make an insertion.

Rule 3

> After an insert from the *active node* which is not the *root* node, we must follow the suffix link and set the *active node* to the node it points to. If there is no a suffix link, set the *active node* to the *root* node. Either way, *active edge* and *active length* stay unchanged.

In this definition of `Rule 3` we also consider the inserts of leaf nodes (not only split-nodes).

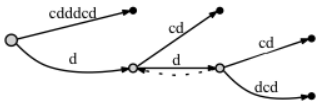
And finally, Observation 3:

When the symbol we want to add to the tree is already on the edge, we, according to `Observation 1`, update only `active point` and `remainder`, leaving the tree unchanged. **BUT** if there is an *internal node* marked as *needing suffix link*, we must connect that node with our current `active node` through a suffix link.

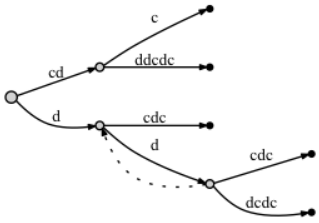
Let's look at the example of a suffix tree for **cdddcdc** if we add a suffix link in such case and if we don't:

1. If we **DON'T** connect the nodes through a suffix link:

    - before adding the last letter **c**:

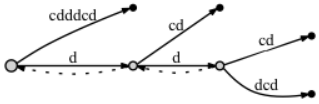


    - after adding the last letter **c**:

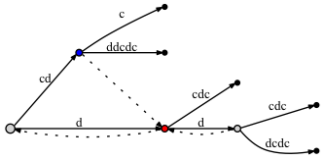


2. If we **DO** connect the nodes through a suffix link:

    - before adding the last letter **c**:



- after adding the last letter **c**:



Seems like there is no significant difference: in the second case there are two more suffix links. But these suffix links are *correct*, and one of them - from the blue node to the red one - is very **important** for our approach with **active point**. The problem is that if we don't put a suffix link here, later, when we add some new letters to the tree, we might omit adding some nodes to the tree due to the `Rule 3`, because, according to it, if there's no a suffix link, then we must put the `active_node` to the root.

When we were adding the last letter to the tree, the red node had **already existed** before we made an insert from the blue node (the edge labled **'c'**). As there was an insert from the blue node, we mark it as *needing a suffix link*. Then, relying on the **active point** approach, the `active node` was set to the red node. But we don't make an insert from the red node, as the letter **'c'** is already on the edge. Does it mean that the blue node must be left without a suffix link? No, we must connect the blue node with the red one through a suffix link.

Why is it correct? Because the **active point** approach guarantees that we get to a right place, i.e., to the next place where we must process an insert of a **shorter** suffix.

Finally, here are my implementations of the Suffix Tree:

1. Java
2. C++

Hope that this "overview" combined with jogojapan's detailed answer will help somebody to implement his own Suffix Tree.

edited Oct 10 '16 at 23:15

answered Jan 29 '13 at 9:57

makagonov

1,352 1 8 11

3 Thanks very much and +1 for you effort. I am sure you are right.. although I don't have the time to think about the details right away. I'll check later and possibly modify my answer then as well. – jogojapan Jan 30 '13 at 1:30

Thanks so much, it really helped. Though, could you be more specific on Observation 3? For instance, giving the diagrams of the 2 steps that introduce the new suffix link. Is the node linked the active node? (as we don't actually insert the 2nd node) – dyesdyes Jun 8 '14 at 16:15

@makagonov Hey can

you help me build suffix tree for your string "cdddcdc" I am bit confused doing so (the starting steps). – tariq zafar Nov 10 '14 at 6:20

1 As for rule 3, a smart way is to set the suffix link of root to root itself, and (in default) set the suffix link of every node to root. Thus we can avoid the conditioning and just follow the suffix link. – sqd May 1 '15 at 9:41

Great explanation! Just one thing I'd like to add. I am still trying to understand, but as far as I understood, this explanation skips one edge case: "abca". The case where the last digit was repeated in the string before. In that case we need to append an end character to the string, sth like '$'. Check the 5th paragraph in cise.ufl.edu/~sahni/dsaaj/enrich/c16/suffix.htm – elif Dec 21 '15 at 7:54

|

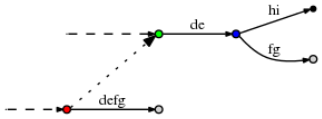
Thanks for the well explained tutorial by **@jogojapan**, I implemented the algorithm in Python.

A couple of minor problems mentioned by @jogojapan turns out to be more **sophisticated** than I have expected, and need to be treated very carefully. It cost me several days to get my implementation **robust enough** (I suppose). Problems and solutions are listed below:

1. **End with** `Remainder > 0` It turns out this

situation can also happen **during the unfolding step**, not just the end of the entire algorithm. When that happens, we can leave the remainder, actnode, actedge, and actlength **unchanged**, end the current unfolding step, and start another step by either keep folding or unfolding depending on if the next char in the original string is on the current path or not.

2. **Leap Over Nodes:** When we follow a suffix link, update the active point, and then find that its active_length component does not work well with the new active_node. We have to **move forward** to the right place to split, or insert a leaf. This process might be **not that straightforward** because during the moving the actlength and actedge keep changing all the way, when you have to move back to the **root node**, the **actedge** and **actlength** could be **wrong** because of those moves. We need additional variable(s) to keep that information.



The other two problems have somehow been pointed out by **@managonov**

3. **Split Could Degenerate** When trying to split an edge, sometime you'll find the split operation is right on a node. That case we only need add a new leaf to that node, take it as a standard edge split operation, which means the suffix links if there's any, should be maintained correspondingly.
4. **Hidden Suffix Links** There is another special case which is incurred by *problem 1* and *problem 2*. Sometimes we need to hop over several nodes to the right point for split, we might **surpass** the right point if we move by comparing the remainder string and the path labels. That case the suffix link will be neglected unintentionally, if there should be any. This could be avoided by **remembering the right point** when moving forward. The suffix link should be maintained if the split node already exists, or even the *problem 1* happens during a unfolding step.

Finally, my implementation in **Python** is as follows:

- **Python**

> **Tips:** *It includes a naive **tree printing** function in the code above, which is very important while debugging*. It saved me a

> lot of time and is convenient for locating special cases.

edited Aug 2 '17 at 14:38

answered Aug 2 '17 at 2:35

mutux
169 2 9

My intuition is as follows:

After k iterations of the main loop you have constructed a suffix tree which contains all suffixes of the complete string that start in the first k characters.

At the start, this means the suffix tree contains a single root node that represents the entire string (this is the only suffix that starts at 0).

After len(string) iterations you have a suffix tree that contains all suffixes.

During the loop the key is the active point. My guess is that this represents the deepest point in the suffix tree that corresponds to a proper suffix of the first k characters of the string. (I think proper means that the suffix cannot be the entire string.)

For example, suppose you have seen characters 'abcabc'. The active point would represent the point in the tree corresponding to the suffix 'abc'.

The active point is represented by (origin,first,last). This means that you are currently at the point in the tree that you get to by starting at node origin and then feeding in the characters in string[first:last]

When you add a new character you look to see whether the active point is still in the existing tree. If it is then you are done. Otherwise you need to add a new node to the suffix tree at the active point, fallback to the next shortest match, and check again.

Note 1: The suffix pointers give a link to the next shortest match for each node.

Note 2: When you add a new node and fallback you add a new suffix pointer for the new node. The destination for this suffix pointer will be the node at the shortened active point. This node will either already exist, or be created on the next iteration of this fallback loop.

Note 3: The canonization part simply saves time in checking the active point. For example, suppose you always used origin=0, and just changed first and last. To check the active point you would have to follow the suffix tree each time along all the intermediate nodes. It makes sense to cache the result of following this path by recording just the distance from the last node.

Can you give a code example of what you mean by "fix" bounding variables?

Health warning: I also found this algorithm particularly hard to understand so please realise that this intuition is likely to be incorrect in all important details...

edited Feb 26 '12 at 20:41

answered Feb 26 '12 at 20:16

Peter de Rivaz
26.2k 4 28 56

One of the academic papers defines "proper" as meaning the "proper suffix" of a string doesn't contain its first character. Sometimes you call a whole substring a "suffix", but when defining the algorithm the terms "string" and "substring" and "suffix" get thrown around liberally and sometimes you need to be very clear what you mean by "suffix", so the term "proper suffix" excludes calling the whole thing a suffix. So a suffix substring of a string can be any legitimate substring and can have a proper suffix that isn't the same suffix. Because logic. – Blair Houghton May 23 '16 at 16:26

Hi i have tried to implement the above explained implementation in ruby , please check it out. it seems to work fine.

the only difference in the implementation is that , i

have tried to use the edge object instead of just using symbols.

its also present at https://gist.github.com/suchitpuri/9304856

```
    require 'pry'


class Edge
    attr_accessor :data ,
    def initialize data
        @data = data
        @edges = []
        @suffix_link = nil
    end

    def find_edge element
        self.edges.each do
            return edge if
        end
        return nil
    end
end

class SuffixTrees
    attr_accessor :root ,
:last_split_edge , :remain

    def initialize
        @root = Edge.new n
        @active_point = {
0}
        @remainder = 0
        @pending_prefixes
        @last_split_edge =
        @remainder = 1
    end

    def build string
        string.split("").e


            add_to_edges @

            update_pending
            add_pending_el
            active_length

            # if(@active_p
&& @active_point[:active_e
@active_point[:active_edge
1])
            #   @active_po
@active_point[:active_edge
            #   @active_po
Edge.new(@active_point[:ac
            # end

            if(@active_poi
@active_point[:active_edge
```

```
                @active_po
                @active_po
@active_point[:active_node
                @active_po
            end
        end
    end

    def add_pending_elemen

        to_be_deleted = []
        update_active_leng
        # binding.pry
        if( @active_point[
            @active_point[
            @active_point[
@active_point[:active_node
== nil
            @remainder = @
            return
        end



        @pending_prefixes.

            # binding.pry

            if @active_poi
@active_point[:active_node

                @active_po

            else

                @active_po
@active_point[:active_edge

                data = @ac
                data = dat

                location =


                # binding.
                if(data[0.
@active_point[:active_node


                else #tree
                    split_
                end

            end
        end
    end



    def update_pending_pre
        if @active_point[:
            @pending_prefi
            return

        end
```

```
        @pending_prefixes

        length = @active_p
        data = @active_poi
        @remainder.times d
                @pending_p
        end

        @pending_prefixes.

    end

    def split_edge data ,
        location = @active
        old_edges = []
        internal_node = (@

        if (internal_node)
            old_edges = @a
            @active_point[
        end

        @active_point[:act
        @active_point[:act
Edge.new(data[location..da


        if internal_node
            @active_point[
        else
            @active_point[
        end

        if internal_node
            @active_point[
        end


        #setup the suffix
        if @last_split_edg
@active_point[:active_edge

            @last_split_ed
        end

        @last_split_edge =

        update_active_poin

    end


    def update_active_poin
        if(@active_point[:
            @active_point[
            @remainder = @
            @active_point[
@active_point[:active_node
        else
            if @active_poi
                @active_po
@active_point[:active_node
            else
                @active_po
```

```
            end
            @active_point[
@active_point[:active_node
            @remainder = @
        end
    end

    def add_to_edges root
        return if root ==
        root.data = root.d
        root.edges.each do
            add_to_edges e
        end
    end
end

suffix_tree = SuffixTrees.
suffix_tree.build("abcabxa
binding.pry
```

edited Mar 3 '14 at 3:32

answered Mar 2 '14 at 10:54

Suchit Puri

354 1 7

@jogojapan you brought awesome explanation and visualisation. But as @makagonov mentioned it's missing some rules regarding setting suffix links. It's visible in nice way when going step by step on http://brenden.github.io/ukkonen-animation/ through word 'aabaaabb'. When you go from step 10 to step 11, there is no suffix link from node 5 to node 2 but active point suddenly moves there.

@makagonov since I live in Java world I also tried to follow your implementation to grasp ST building workflow but it was hard to me because of:

- combining edges with nodes

- using index pointers instead of references
- breaks statements;
- continue statements;

So I ended up with such implementation in Java which I hope reflects all steps in clearer way and will reduce learning time for other Java people:

```
import java.util.Arrays;
import java.util.HashMap;
import java.util.Map;

public class ST {

  public class Node {
    private final int id;
    private final Map<Char
    private Node slink;

    public Node(final int
        this.id = id;
        this.edges = new H
    }

    public void setSlink(f
        this.slink = slink
    }

    public Map<Character,
        return this.edges;
    }

    public Node getSlink()
        return this.slink;
    }

    public String toString
        return new StringB
                .append("{
                .append("\
                .append(":
                .append(th
                .append(",
                .append("\
                .append(":
                .append(th
                .append(",
                .append("\
                .append(":
                .append(ed
                .append("}
                .toString(
    }

    private StringBuilder
        final StringBuilde
        edgesStringBuilder
```

```
        for(final Map.Entr
            edgesStringBui
                    .appen
                    .appen
                    .appen
                    .appen
                    .appen
        }
        if(!this.edges.isE
            edgesStringBui
        }
        edgesStringBuilder
        return edgesString
    }

    public boolean contain
        return !suffix.isE
                && this.ed
                && this.ed
    }
}

public class Edge {
    private final int from
    private final int to;
    private final Node nex

    public Edge(final int
        this.from = from;
        this.to = to;
        this.next = next;
    }

    public int getFrom() {
        return this.from;
    }

    public int getTo() {
        return this.to;
    }

    public Node getNext()
        return this.next;
    }

    public int getLength()
        return this.to - t
    }

    public String toString
        return new StringB
                .append("{
                .append("\
                .append(":
                .append("\
                .append(wo
                .append("\
                .append(",
                .append("\
                .append(":
                .append(th
                .append("}
                .toString(
    }
```

```
    public boolean contain
        if(this.next == nu
            return word.su
        }
        return suffix.star
                this.to))
this.from));
    }
  }

  public class ActivePoint
    private final Node act
    private final Characte
    private final int acti

    public ActivePoint(fin
                       fin
                       fin
        this.activeNode =
        this.activeEdgeFir
        this.activeLength
    }

    private Edge getActive
        return this.active
    }

    public boolean pointsT
        return this.active
    }

    public boolean activeN
        return this.active
    }

    public boolean activeN
        return this.active
    }

    public boolean activeN
        return this.active
    }

    public boolean pointsT
{
        return word.charAt
character;
    }

    public boolean pointsT
        return this.getAct
    }

    public boolean pointsA
        return this.getAct
    }

    public ActivePoint mov
        return new ActiveP
    }

    public ActivePoint mov
        return new ActiveP
    }
```

```
    public ActivePoint mov
        return new ActiveP
                this.activ
                this.activ
    }

    public ActivePoint mov
        return new ActiveP
this.activeLength);
    }

    public ActivePoint mov
        return new ActiveP
                this.activ
                this.activ
    }

    public ActivePoint mov
node,

character) {
        return new ActiveP
    }

    public ActivePoint mov
index) {
        return new ActiveP
                word.charA
this.getActiveEdge().getLe
                this.activ
    }

    public void addEdgeToA
        this.activeNode.ge
    }

    public void splitActiv



        final Edge activeE
        final Edge splitte
                activeEdge
                nodeToAdd)
        nodeToAdd.getEdges
this.activeLength),
                new Edge(a
                        ac
                        ac
        nodeToAdd.getEdges
null));
        this.activeNode.ge
splittedEdge);
    }

    public Node setSlinkTo

        if(previouslyAdded
            previouslyAdde
        }
        return node;
    }

    public Node setSlinkTo
previouslyAddedNodeOrAdded
```

```
        return setSlinkTo(
    }
  }

  private static int idGen

  private final String wor
  private final Node root;
  private ActivePoint acti
  private int remainder;

  public ST(final String w
    this.word = word;
    this.root = new Node(i
    this.activePoint = new
    this.remainder = 0;
    build();
  }

  private void build() {
    for(int i = 0; i < thi
        add(i, this.word.c
    }
  }

  private void add(final i
    this.remainder++;
    boolean characterFound
    Node previouslyAddedNo
    while(!characterFoundI
        if(this.activePoin
            if(this.active
                activeNode
previouslyAddedNodeOrAdded
                characterF
            }
            else {
                if(this.ac
                    rootNo
                }
                else {
                    previo
internalNodeHasNotEdgeStar

                }
            }
        }
        else {
            if(this.active
                activeEdge
                characterF
            }
            else {
                if(this.ac
                    previo
edgeFromRootNodeHasNotChar


                }
                else {
                    previo
edgeFromInternalNodeHasNot


                }
```

```
            }
        }
    }
  }

  private void activeNodeH

previouslyAddedNodeOrAdded
    this.activePoint.setSl
    this.activePoint =
this.activePoint.moveToEdg
    if(this.activePoint.po
        this.activePoint =
    }
  }

  private void rootNodeHas
char character) {
    this.activePoint.addEd
this.word.length(), null))
    this.activePoint = thi
    this.remainder--;
    assert this.remainder
  }

  private Node internalNod


previouslyAddedNodeOrAdded
    this.activePoint.addEd
this.word.length(), null))
    previouslyAddedNodeOrA
this.activePoint.setSlinkT
    if(this.activePoint.ac
        this.activePoint =
    }
    else {
        this.activePoint =
    }
    this.remainder--;
    return previouslyAdded
  }

  private void activeEdgeH
    this.activePoint = thi
    if(this.activePoint.po
        this.activePoint =
    }
  }

  private Node edgeFromRoo


previouslyAddedNodeOrAdded
    final Node newNode = n
    this.activePoint.split
    previouslyAddedNodeOrA
this.activePoint.setSlinkT
    this.activePoint =
this.activePoint.moveToEdg
            this.word.char
    this.activePoint = wal
    this.remainder--;
    return previouslyAdded
  }
```

```
  private Node edgeFromInt


previouslyAddedNodeOrAdded
    final Node newNode = n
    this.activePoint.split
    previouslyAddedNodeOrA
this.activePoint.setSlinkT
    if(this.activePoint.ac
        this.activePoint =
    }
    else {
        this.activePoint =
    }
    this.activePoint = wal
    this.remainder--;
    return previouslyAdded
  }

  private ActivePoint walk
    while(!this.activePoin
            && (this.activ
this.activePoint.pointsAft
        if(this.activePoin
            this.activePoi
this.activePoint.moveToNex
        }
        else {
            this.activePoi
        }
    }
    return this.activePoin
  }

  public String toString(f
    return this.root.toStr
  }

  public boolean contains(
    return this.root.conta
  }

  public static void main(
    final String[] words =
            "abcabcabc$",
            "abc$",
            "abcabxabcd$",
            "abcabxabda$",
            "abcabxad$",
            "aabaaabb$",
            "aababcabcd$",
            "ababcabcd$",
            "abccba$",
            "mississipi$",
            "abacabadabaca
            "abcabcd$",
            "00132220$"
    };
    Arrays.stream(words).f
        System.out.println
        final ST suffixTre
        System.out.println
        for(int i = 0; i <
            assert suffixT
```

```
            }
        });
    }
}
```

edited Apr 29 '17 at 17:02

answered Apr 21 '17 at 14:22

Kamil

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