# Applied Signal Processing and Computer Science
WS 11/12

## Tutorial 1: Complex Numbers

*1. Arithmetics of Complex Numbers:*

1.1. Evaluate the following complex numbers:
- $j^n$ for $n = 2, 3, 4, ..., 9$
- $j^{4n}$, $j^{4n+1}$, $j^{4n+2}$, $j^{4n+3}$ for $n \in \mathrm{N}$

1.2. Convert the following numbers into a $a + jb$ representation:
- $j - \frac{1}{j}$
- $(-j)^3 + 3j^3$
- $\left(j^9 - j^{14}\right)^2$

1.3. Evaluate:
- $6j \cdot 2j$
- $(3+4j)(2-j)$
- $(2-3j)(4j+2)(3-4j)$

*2. The Complex Plane*

2.1. Represent the following numbers in the complex plane by using vector sums, and write the respective result as a complex number:
- $(2+3j)+(1+2j)$
- $(2-3j)+(3+5j)$
- $(1+2j)+(2+j)+(1-j)$

2.2. Calculate the magnitude of $z_1 + z_2$ and $z_1 \cdot z_2$, with $z_1 = 0.6 + 0.8j$ and $z_2 = 1.2 + 1.6j$

2.3. Convert the following numbers into a Euler representation (magnitude and phase):
- $1 + j$
- $3 + \sqrt{3}j$
- $z_1 \cdot z_2$ and $\frac{z_1}{z_2}$, with $z_1 = \sqrt{3} - j$ and $z_2 = 1 + \sqrt{3}j$
- $z_1 \cdot z_2$ and $z_1 \cdot z_2^*$, with $z_1 = \frac{\sqrt{2}}{2}(1+j)$ and $z_2 = \frac{\sqrt{2}}{2}(1-j)$

2.4. Demonstrate the validity of the following Euler laws for all $\varphi \in \Re$:

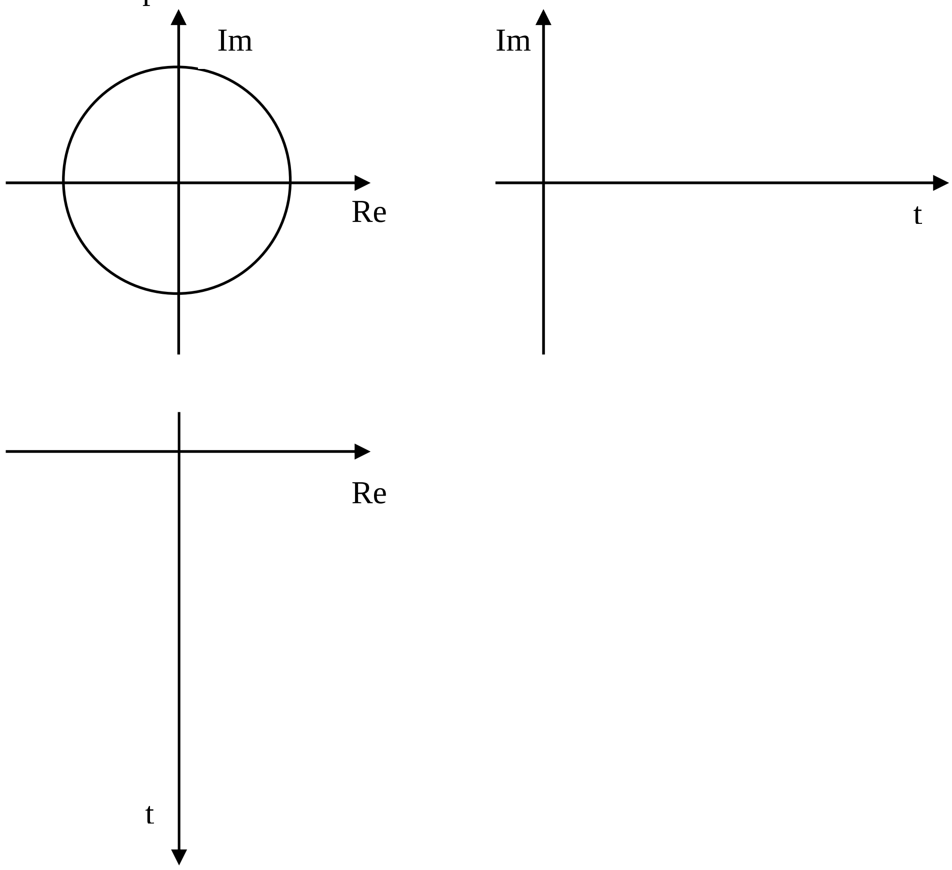
- $\left(e^{j\varphi}\right)^{*} = e^{-j\varphi}$
- $\frac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right) = \cos(\varphi)$
- $\frac{1}{2j}\left(e^{j\varphi} - e^{-j\varphi}\right) = \sin(\varphi)$

2.5. Raise the complex number $z_k = (1+j)^{k}$ to the power of $k$ for all $k$ = 0, ..., 4 and plot the results in the complex plane. Plot the function $z_\alpha = (1+j)^{\alpha}$, with $\alpha = 0 \ldots 4$ ($\alpha \in \Re$) in the complex plane.

### 3. *Complex Harmonic Oscillations*

3.1. Specify the complex harmonic oscillation $u(t)$ in a way, that its real part corresponds to $\mathrm{Re}\{u(t)\} = A\sin(2\pi ft + \pi/3)$. Plot, both, the real $\mathrm{Re}\{u(t)\}$, and imaginary part $\mathrm{Im}\{u(t)\}$ of $u(t)$ and $u(t=0)$ in the chart below.

Complex Plane



3.2. Draw the superimposed oscillation $u(t) = \sin(2\pi ft) + 2\cos(2\pi ft) = A\cos(2\pi ft + \varphi)$ after a previous complex expansion. Determine the magnitude $A$ and phase $\varphi$ of the superimposed oscillation by complex and/or real valued calculations.