M.Sc. in ‚Transportation Systems'

Applied Statistics in Transport

Exercises: Distributions

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1. Given the probability density function $f(x) = \frac{2x+1}{25}$ $(x = 0,1,2,3,4)$; determine the following probabilities:

   a) $\mathrm{P}(\mathrm{X} = 4)$

   b) $\mathrm{P}(\mathrm{X} \leq 1)$

   c) $\mathrm{P}(2 \leq \mathrm{X} < 4)$

   d) $\mathrm{P}(\mathrm{X} > -10)$

2. The length of train parts are measured to the nearest tenth of a millimetre. The lengths are uniformly distributed, with values at every tenth of a millimetre starting at 590.0 and continuing through 590.9. Determine the probability density function. Compute P(X≤590.5). Determine the mean and the variance of length.

3. There is a chance that a bit transmitted through a digital transmission channel is incorrectly received. Let X equal the number of incorrectly received bits in the next four transmitted bits. The possible values for X are {0,1,2,3,4}. Suppose that the probabilities are

$$P(X = 0) = 0.65; P(X = 1) = 0.29; P(X = 2) = 0.04; P(X = 3) = 0.0036; P(X = 4) = 0.0001$$

   Determine the mean and the variance for the probability distribution.

4. Determine the probability to roll exactly one 6 with 8 dice.

5. From income and age statistics it is known that 2 percent of the inhabitants in city A are retired persons with a monthly income of less than 800 Euro. What is the probability that at least 5 persons from this group are represented in a random sample of 200 persons?

6. A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. If four parts are selected randomly and without replacement, what is the probability they are all from the local supplier? What is the probability that two or more parts in the sample are from the local supplier? What is the probability that at least one part in the sample is from the local supplier?

7. Queuing space: A maximum of 20 vehicles can stand in the queuing space between a railway crossing gate and the next intersection. The average number of arriving vehicles is 150 vehicles per hour. The gate is closed for a maximum of 4 minutes.
   a) What is the probability that the vehicles jam up to the next intersection?
   b) Due to changes in the timetable the closing time of the gate goes up from 4 to 6 minutes. What is now the probability that the vehicles jam up to the next intersection?

8. Determine the probability to get three to six times the head when throwing an ideal coin 10 times. Use (a) the binomial distribution and (b) the normal distribution:

9. General continuous distributions: Suppose that $f(x) = x/8$ for 3<x<5. Determine the following probabilities:
a) $P(X < 4)$
b) $P(X > 3.5)$
c) $P(4 < X < 5)$
d) $P(X < 4.5)$
e) $P(X < 3.5 \; or \; X > 4.5)$

10. Suppose that $f(x) = 0.125x$ for 0<x<4. Determine the mean and variance of X:

11. For two alternative routes the following means μ and standard deviations σ of the travel time x are known:

    Route A through the city centre: $\mu_A = 27\ min, \sigma_A = 5\ min$

    Route B using a tangent: $\mu_B = 29\ min, \sigma_B = 2\ min$

    For which route the risk to come late to an important date is lower, if the departure time is (Case 1) 28 minutes and (Case 2) 32 minutes before the date?

    Remark: We assume the travel times to be normally distributed.