Applied Signal Processing and Computer Science

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Solution 5: Fourier Transform

1. Proof of laws of the Fourier Transform

1.1

- Shift:

$$\int_{-\infty}^{\infty} u(t-t_0)e^{-j2\pi ft}dt \xrightarrow{t^{'}=:t-t_0} = \int_{-\infty}^{\infty} u(t^{'})e^{-j2\pi f(t^{'}+t_0)}dt^{'} = e^{-j2\pi f_0 t}\int_{-\infty}^{\infty} u(t^{'})e^{-j2\pi ft^{'}}dt^{'}$$

$$= U(f)e^{-j2\pi f_0 t}$$

- Derivatives:

$$u(t) = \int_{-\infty}^{\infty} U(f)e^{j2\pi ft}df$$

$$\underbrace{\frac{d}{dt}u(t)}_{k(t)} = \frac{d}{dt}(\int_{-\infty}^{\infty} U(f)e^{j2\pi ft}df) = \int_{-\infty}^{\infty} U(f)\frac{d}{dt}(e^{j2\pi ft})df = \int_{-\infty}^{\infty} \underbrace{(j2\pi f)\cdot U(f)}_{K(f)}e^{j2\pi ft}df$$

$$=> \quad k(t) \quad \leftrightarrow \quad K(f)$$

$$=> \frac{d}{dt}u(t) \quad \leftrightarrow \quad j2\pi f\, U(f)$$

- Convolution:

$$\int_{-\infty}^{\infty} u(t)*h(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}[\int_{-\infty}^{\infty} u(t^{'})h(t-t^{'})dt^{'}]e^{-j2\pi ft}dt$$

$$= \int_{-\infty}^{\infty} u(t^{'})[\int_{-\infty}^{\infty} h(t-t^{'})e^{-j2\pi ft}dt]dt^{'}$$

$$= \int_{-\infty}^{\infty} u(t^{'})[H(f)e^{-j2\pi ft^{'}}]dt^{'}$$

$$= H(f)\int_{-\infty}^{\infty} u(t^{'})e^{-j2\pi ft^{'}}dt^{'}$$

$$= U(f)\cdot H(f)$$

2. Fourier Series

2.1

$$u_1(t) = rect(\frac{t}{\Delta t}) \quad \leftrightarrow \quad U_1(f) = \Delta t \sin c(\Delta t\cdot f)$$

$$u_2(t) = rect(\frac{t}{\Delta t}-\frac{1}{2}) \quad \leftrightarrow \quad U_2(f) = \Delta t \sin c(\Delta t\cdot f)\cdot e^{-j\pi f\Delta t}$$

$$|U_1(f)| = |U_2(f)|$$

2.2 $u(t) = \sin c(\frac{t}{T}) \cdot e^{j2\pi \frac{t}{T}}$

$$\sin c(\frac{t}{T}) \quad \leftrightarrow \quad T\ rect(Tf)$$

$$according\ to: \quad u(t)e^{j2\pi f_0 t} \quad \leftrightarrow \quad U(f - f_0)$$

$$=> \qquad \sin c(\frac{t}{T}) \cdot e^{j2\pi \frac{t}{T}} \quad \leftrightarrow \quad T\ rect(T(f - \frac{1}{T}))$$