M.Sc. in ‚Transportation Systems‘

Applied Statistics in Transport

Exercises: Hypotheses Testing, Statistical Tests

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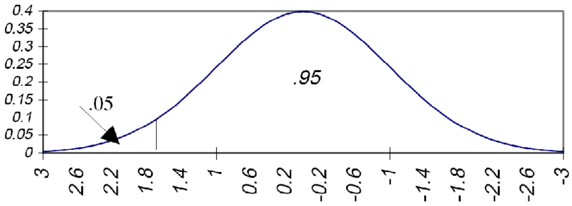
1. In road construction for highways, the pavement is specified to be at least 0.5 centimetres thick by one of the many quality measures. The thickness is known to be N(μ, σ=0.2). The quality control system uses a significance level of 0.05. In 50 samples on one day, the mean thickness was 0.43 centimetres. Should the operator adjust the machine or is the thickness sufficient?

Solution:

a) The engineer would act by adjusting the machines only if the pavement was too thin. H_0: $\mu \geq \mu_0 = 0.5\ cm$, H_1: $\mu < \mu_0$ (if 0.5cm are not reached, re-adjustment is necessary)
b) Choose a significance level α. $\alpha = 0.05$
c) Decision Rule: Reject H_0: $\mu \geq \mu_0 = 0.5\ cm$ in favour of H_1: $\mu < \mu_0$ if $z \leq z_{0.05} = -1.645$ (one sided hypothesis).
d) $\hat{z} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{0.43-0.5}{0.2/\sqrt{50}} = -2.4749$, (0.43-0.5)/(0.2/sqrt(50))
e) Reject H_0 and adjust the machine, the pavement is too thin.



2. In a nuclear power plant, the cold start procedure consists of bringing the reactor to 35% of power, and then to 65% of power, before full operation. At each stage, engineers make measurements of several critical reactor attributes. If the binding energy does not have an exact mean rate of 11.5 MeV at 35% power, then the reactor could cascade into a critical configuration at subsequent power levels. Set up the hypothesis for a decision system at the 35% power level stage using $\alpha = 0.05$. It is known that the population of measurement errors is normal with standard deviation $\sigma = 1.5$. On this day's power-up, the sample mean of nine observations is 10.2 MeV. What should the operators of the reactor do?

Solution:

a) The plant operators would not continue to power up the reactor if the binding energy did not meet specification. The action to be taken would be to shut down. H_0: $\mu = \mu_0 = 11.5$, H_1: $\mu \neq \mu_0$ (stop if 11.5 are not reached)
b) Choose a significance level α. $\alpha = 0.05$
c) Decision Rule: Reject H_0: $\mu = \mu_0 = 11.5$ in favour of H_1: $\mu \neq \mu_0$ if $|z| \geq z_{0.025} = 1.96$ (two sided hypothesis).
d) $\hat{z} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{10.2-11.5}{1.5/\sqrt{9}} = -2.6000$
e) Reject H_0 and shut down the power-up.

3. Compute these two t-tests including the p-value and the confidence interval without the command t.test:

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> t.test(1:10,7:20)

        Welch Two Sample t-test

data:  1:10 and 7:20
t = -5.4349, df = 21.982, p-value = 1.855e-05
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -11.052802  -4.947198
sample estimates:
mean of x mean of y
      5.5      13.5
> t.test(1:10)

        One Sample t-test

data:  1:10
t = 5.7446, df = 9, p-value = 0.0002782
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 3.334149 7.665851
sample estimates:
mean of x
      5.5
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Solution:
length(1:10)=10, length(7:20)=14,means: 5.5,13.5
sd(1:10)= 3.027650, sd(7:20)= 4.1833 (R always divides by n-1)
var(1:10) #9.166667, sqrt(9.166667)=3.027650
var(7:20) #17.5, sqrt(17.5)=4.1833
  #variance of the means of the differences (unequal variances):
sqrt((var(1:10)/length(1:10))+(var(7:20)/length(7:20))) # 1.47196
sqrt((9.166667/9)+(17.5/13)) # 1.47196
(t<-(5.5-13.5)/1.47196) #-5.43493
Reject H_0 if $|t| \leq t_{0.05,22} = -2.074$

Conclusion: The difference between the two means is significant at the 95% level, we cannot accept the null hypothesis.

p-value: 2*pt(-5.4349,21.982) #gives 1.855473e-05

  #CI:
  ((mean(1:10)-mean(7:20))-(qt(0.975,22)*(1.47196))) #-11.05266
  ((mean(1:10)-mean(7:20))+(qt(0.975,22)*(1.47196))) #-4.947342

However, we should have used the argument var.equal=T because the variances of the two sample do not differ significantly:

We can assume equal variances because of the F-test:
> var.test(7:20,1:10)
        F test to compare two variances
  data: 7:20 and 1:10
  F = 1.9091, num df = 13, denom df = 9, p-value = 0.3343

alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
0.4983797 6.3229710
sample estimates:
ratio of variances
1.909091

#variance of the means of the differences (equal variances):
t.test(1:10,c(7:20),var.equal=T)
(var_mean_equal_var<-sqrt((((9*(3.027650^2))+(13*(4.1833^2)))/22)*sqrt(1/10+1/14)) #= 1.554215
(t<-(13.5-5.5)/1.554215) #5.147293
2*pt(-5.147293,length(1:10)+length(7:20)-2) #p-value: gives 3.690576e-05 (woher hatte ich hier die exakte reale Zahl für die d.f.?
#CI:
((mean(1:10)-mean(7:20))-(qt(0.975,22)*(1.554215))) #-11.22324
((mean(1:10)-mean(7:20))+(qt(0.975,22)*(1.554215))) #-4.776755
Conclusion: The difference between the two means is significant at the 95% level, we cannot accept the null hypothesis.

The command t.test(1:10) tests the hypothesis whether the sample mean significantly differs from zero:
t.test(1:10)
length(1:10) #=10
mean(1:10) #3.5
sd(1:10) #3.027650
var(1:10) #9.166667, sqrt(9.166667)=3.027650
(t_value<-(mean(1:10)-0)/(sqrt(var(1:10)/length(1:10)))) #5.744563
qt(0.975,9) #2.262157
2*pt(-5.744563,9) #0.0002781959=(1-pt(5.744563,9))*2, two-tailed test
#CI:
mean(1:10)-(qt(0.975,9)*(sqrt(var(1:10)/length(1:10)))) #3.334149
mean(1:10)+(qt(0.975,9)*(sqrt(var(1:10)/length(1:10)))) #7.66585
#The mean lies between 3.33 and 7.67 in 95% of sampling.