M.Sc. in ‚Transportation Systems'

Applied Statistics in Transport
Exercises: Descriptive Statistics

Exercises with or without R:

1. A vehicle drives a distance of 25km. The parts 1 to 7 of the road are in different condition, so that different speed limits are imposed.

j	Distance D_i [km]	Speed s_j [km/h]
1	1	30
2	1	45
3	12	50
4	3	65
5	1	80
6	2	100
7	5	120
Total	25	

Determine the average speed over the whole distance of 25km, given that the speeds listed in the above table are driven. What is the overall travel time?

Solution:

The average speed is the harmonic mean of the respective speeds:

j	Distance D_i [km]	speed s_i [km/h]	Travel Time D_j/s_j [h]	s_i x f_i
1	1	30	0.033	30
2	1	45	0.022	45
3	12	50	0.240	600
4	3	65	0.046	195
5	1	80	0.013	80
6	2	100	0.020	200
7	5	120	0.042	600
Total	25		0.416	1750

$$\overline{s}_h = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{s_i}*f_i} = \frac{n}{\sum\frac{1}{s_i}*f_i} = \frac{25km}{0.416\frac{km}{km/h}} = 60.13\,\frac{km}{h}$$

2. Prove that the sum of the deviations of the numbers x_1, x_2, ..., x_n of the arithmetic mean $\overline{x}$ equals zero.

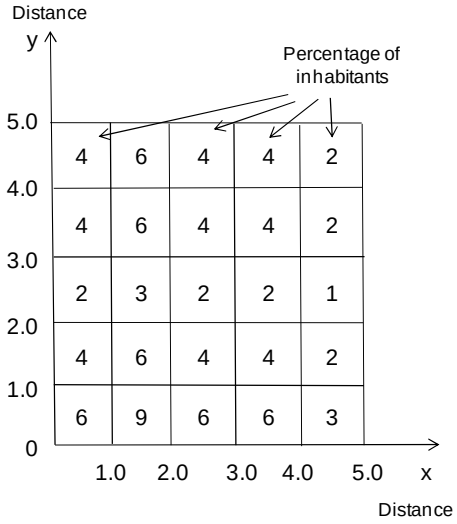
Solution:

Consider $d_1 = x_1 - \overline{x}, d_2 = x_2 - \overline{x}, \ldots, d_n = x_n - \overline{x}$ being the deviations of the numbers x_1, x_2, ..., x_n of the arithmetic mean $\overline{x}$. Then we can write:

Sum of the deviations

$$= \sum d_i = \sum (x_i - \overline{x}) = \sum x_i - n\overline{x} = \sum x_i - n\left(\frac{\sum x_i}{n}\right) = \sum x_i - \sum x_i = 0$$

3. Location decision: The following percentages of inhabitants are given for region A (distances are measured in kilometres)



For answering the following questions we assume that the inhabitants are uniformly distributed within the quadrates, i.e. the inhabitants are concentrated in the centre of the quadrates.

a) Determine the coordinates of the geographic centre GC for the whole region.
b) Determine the coordinates of the centre of gravity for the inhabitants IC for the whole region.
c) Where is the mode MOD located?
d) Determine the coordinates for the median point MED.
e) Determine the grids(quadrates) with the highest potential of inhabitants, that means the so called potential point POT. Use the weights below w(d) as a function of the distance d.

0	0	0	0	0	0	0
0	0	0.2	0.3	0.2	0	0
0	0.2	0.6	0.8	0.6	0.2	0
0	0.3	0.8	$\mathbf{P_i}$	0.8	0.3	0
0	0.2	0.6	0.8	0.6	0.2	0
0	0	0.2	0.3	0.2	0	0
0	0	0	0	0	0	0

f) Which of the five above determined measures of location do you consider to be most suitable for determining the location of:
   - Fire station
   - Television transmitter
   - Department store

Solution:

a) The geographic centre does not depend on the distribution of the inhabitants, GC=(1.5;1.5).

b) The centre of gravity for the inhabitants IC has the coordinates of the arithmetic means of the marginal distributions:

$$x_{IC} = \frac{20 * 0.5 + 30 * 1.5 + 20 * 2.5 + 20 * 3.5 + 10 * 4.5}{100} = 2.2$$

$$y_{IC} = \frac{30 * 0.5 + 20 * 1.5 + 10 * 2.5 + 20 * 3.5 + 20 * 4.5}{100} = 2.3$$

c) The modal point is the centre of the quadrate with the most inhabitants, MOD=(1.5;0.5)

d) Median point = the point with as much inhabitants living nearer to (0,0) and living further to (0,0). It is located in the intersection of the median values of the marginal distributions, MED=(2.0;2.0).

e) For POT the four quadrates around the median point are especially suitable:

P_1(1.5;1.5), P_2(2.5;1.5), P_3(1.5;2.5), P_4(2.5;2.5)

$$\mathrm{POT}_1 = 6 + 20 * 0.8 + 16 * 0.6 + 10 * 0.3 + 16 * 0.2 = 37.8$$
$$\mathrm{POT}_2 = 4 + 18 * 0.8 + 20 * 0.6 + 10 * 0.3 + 22 * 0.2 = 37.8$$
$$\mathrm{POT}_3 = 3 + 16 * 0.8 + 16 * 0.6 + 17 * 0.3 + 22 * 0.2 = 36.1$$
$$\mathrm{POT}_4 = 2 + 13 * 0.8 + 20 * 0.6 + 13 * 0.3 + 37 * 0.2 = 35.7$$

There are two points with the same potential of inhabitants:
POT(1.5;1.5) and POT(2,5;1.5)

f) The fire station should be located in the centre of gravity for the inhabitants IC. This centre minimises the distances to the inhabitants and short distances mean fast arrival at the fire area.
The television transmitter should be located in the geographic centre in order to achieve a constant service in the whole region.
The department store should be located in the potential point in order to have as many customers as possible.