Applied Signal Processing & Computer Science

Chapter 2: Systems

Xiaoxiang Zhu

xiao.zhu@dlr.de

Remote Sensing Technology
TU München
&
Remote Sensing Technology Institute
German Aerospace Center (DLR)
Oberpfaffenhofen

Chapter 2: Systems

2.1 Systems

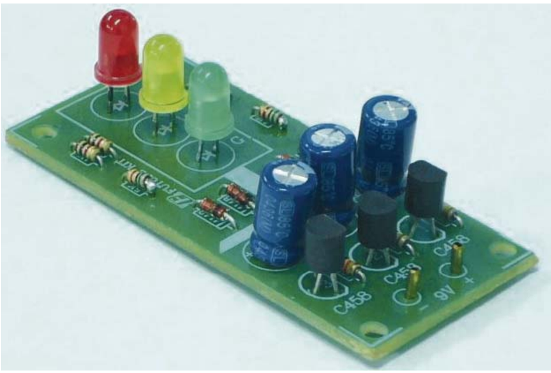
2.2 System Classes

2.3 Examples of Typical Physical Systems

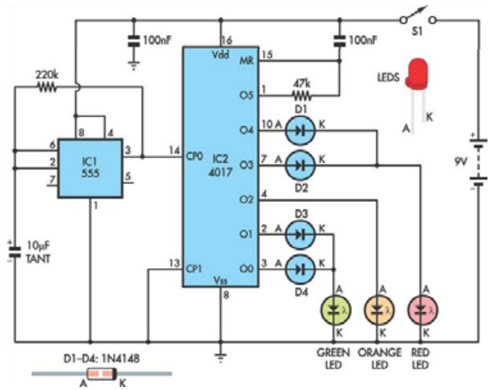
2.1 Systems

- Physically…a system is something that “takes in”one or more input signals and “produces” one or more output signals…
  - Maybe it is a circuit
  - Maybe it is a mechanical thing
  - Maybe it’s an algorithm
  - Maybe it is...

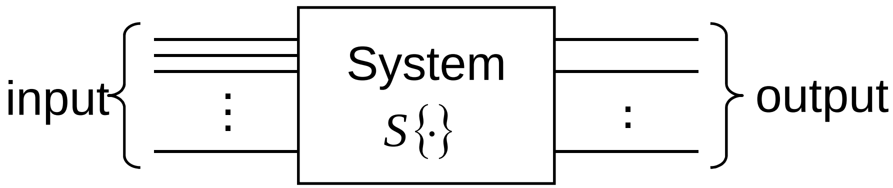
- Example: traffic light



Physical view



Schematic view

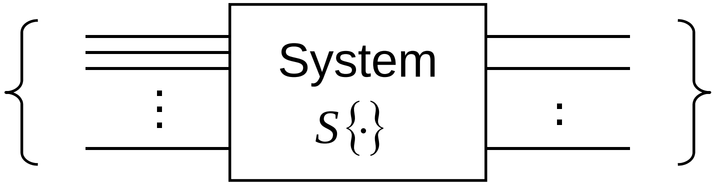


System view

Systems

System $\equiv$ Operator $\equiv$ Mapping

input variables



output variables = $S\{$input variables$\}$

Input and output variables are mostly physically measurable signals:
voltage, field strength, brightness, acoustic pressure, temperature, traffic density, ...

Notation:

Input signals: 1-D: $u_1(t), u_1(x), ..., u_1[i]$

n-D: $u_1(x,y,z,t), u_1(t_1,t_2), u_1(\lambda_1,\lambda_2,...), ..., u_1[i,k]$

Output signals: $u_2(t), u_2(x,y,z,t), u_2(t_1,t_2), ..., u_2[i]$

or in general: $u(\underline{x})$ with $\underline{x} = (x_1, x_2, x_3, x_4, ..., x_n)$

dimensionality ‚n‘ of a signal = number of variables with *several* inputs and outputs (*multivariable* system):

$$\underline{u}_1(\underline{x}) = \begin{pmatrix} u_{1,1}(\underline{x}) \\ u_{1,2}(\underline{x}) \\ u_{1,3}(\underline{x}) \\ \vdots \end{pmatrix} \qquad \underline{u}_2(\underline{x}) = \begin{pmatrix} u_{2,1}(\underline{x}) \\ u_{2,2}(\underline{x}) \\ u_{2,3}(\underline{x}) \\ \vdots \end{pmatrix}$$

multivariable system: $\underline{u}_2(\underline{x}) = S\{\underline{u}_1(\underline{x})\}$

- A system is characterized by its input, its output and its mathematical model

- The study of systems consists of **three major areas**:

  - **Mathematical modeling**: How to derive the mathematical description of a system

  - **Analysis:** How to determine the system outputs for a given inputs and a given mathematical model of a system

  - **Design:** How to construct a system which will produce a desired set of outputs for the given inputs

Our primary interest: cause-and-effect relation

Typical signal sources:

- Microphone
- Measuring instruments (Seismograph, Photomultiplier, Magnetometer, ...)
- Receivers (Radio, TV, Radar, GPS, ...)
- (CCD-)Cameras
- Spectrometer

2.2 System Classes

- <u>deterministic</u> vs. random
- 1-D vs. n-D
- scalar vs. multivariable (single vs. multiple inputs/outputs)
- memory-less vs. memory
- <u>linear</u> vs. non-linear: $S\left\{\sum_i a_i \, \underline{u}_{1,i}(\underline{x})\right\} = \sum_i a_i \, S\{\underline{u}_{1,i}(\underline{x})\} = \sum_i a_i \, \underline{u}_{2,i}(\underline{x})$
- time- (space-, shift-) <u>invariant</u> vs. -variant: $S\{\underline{u}_1(\underline{x} - \underline{x}_0)\} = \underline{u}_2(\underline{x} - \underline{x}_0)$

"LTI" linear time invariant

System Classes (cont.)

- causal vs. non-causal: $u_1(t) \equiv 0 \quad \forall \quad t \leq t_0 \quad \Rightarrow \quad u_2(t) \equiv 0 \quad \forall \quad t \leq t_0$

- stable vs. unstable:
  - a system is stable, if every bounded input produces a bounded output
  - necessary and sufficient condition for stability (h: impulse response function, explained later): $\int_{-\infty}^{\infty} \left| h(t) \right| dt < \infty$

- no internal sources vs. sources: $S\{0\} \equiv 0$
  (zero input response=0)

System Classes (cont.)

- Exercise: Are the following systems linear and/or time-invariant?
  - Differentiator $u_2(t) = d / dt\, u_1(t)$
  - Multiplication of a signal with a carrier $u_2(t) = u_1(t) \cdot \cos(2\pi f_0 t)$
  - Dilation of a signal $u_2(t) = u_1(k\, t)$ with $k \in \Re$
  - Deformation of a signal by a static transfer characteristic $u_2(t) = g(u_1(t))$, where $g(x) = x^2$

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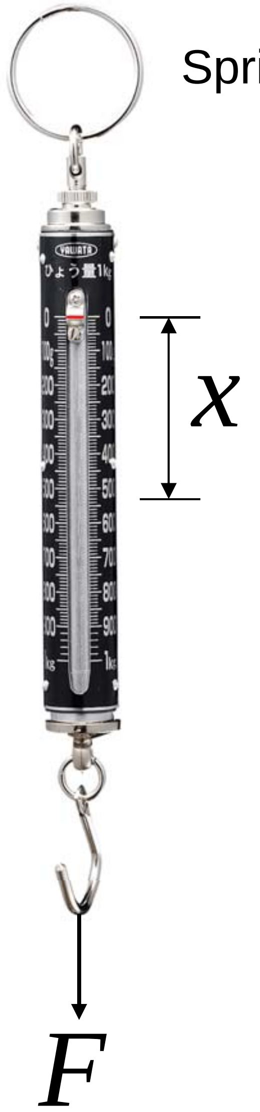
2.3 Examples: Spring Scale

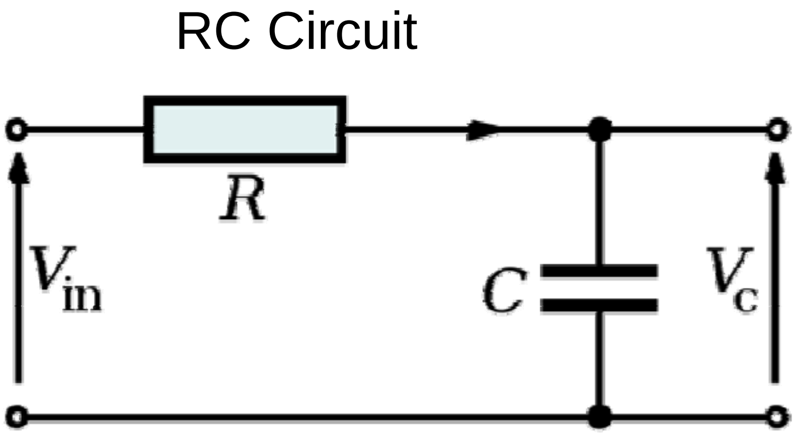
Spring Scale



$$F \rightarrow x?$$

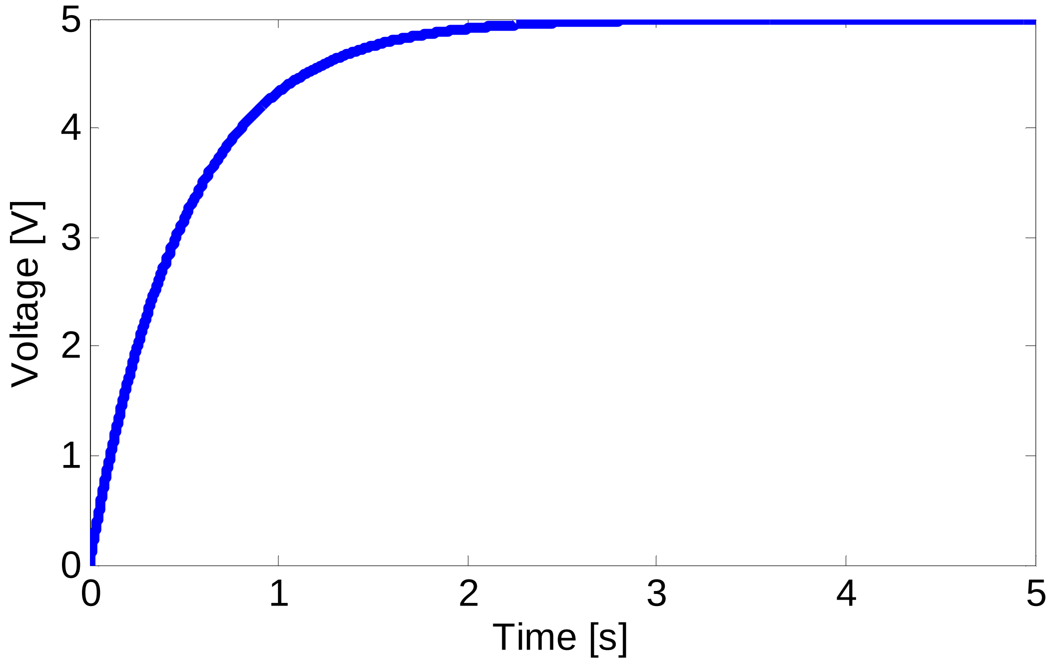
$$x = -\frac{F}{k}$$

2.3 Examples: RC Circuit

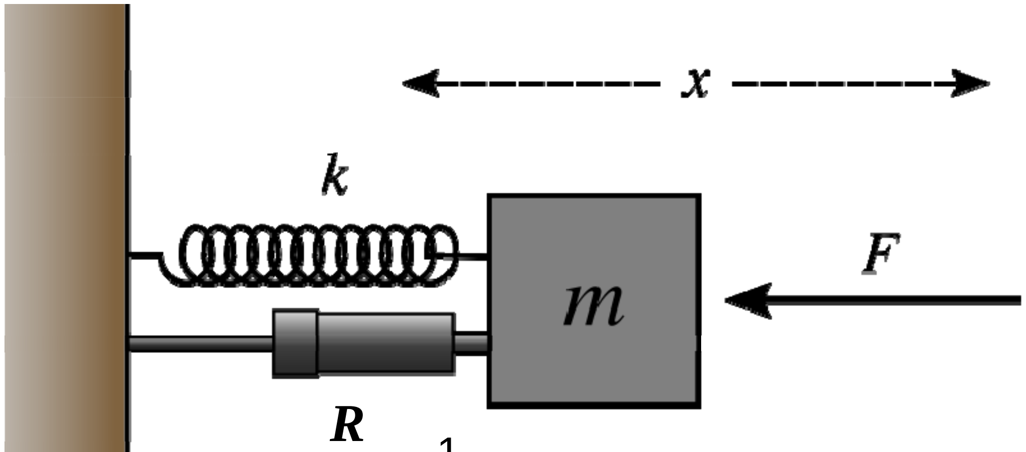


$$V_{in} \rightarrow V_C\ ?$$

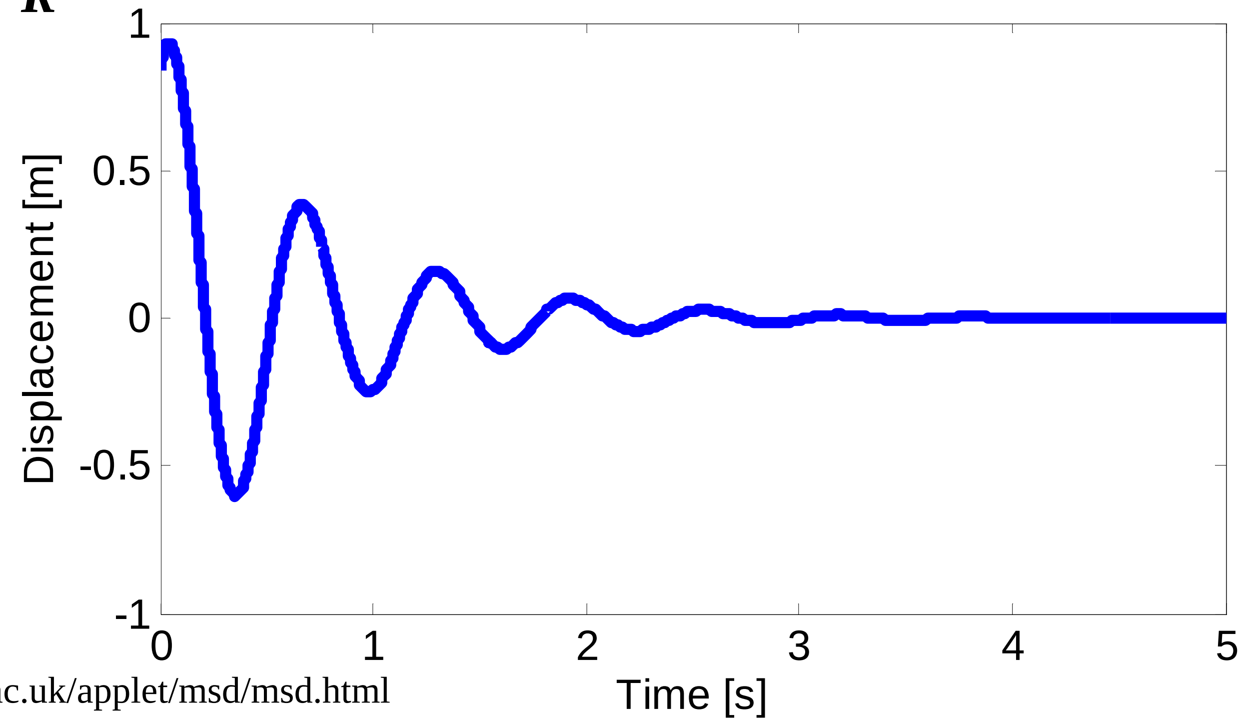
$$V_C = V_{in}(1 - e^{-\frac{t}{RC}})$$



2.3 Example: Mass-Spring Damper System



$$m\ddot{x} = -kx - R\dot{x}$$



http://mathinsite.bmth.ac.uk/applet/msd/msd.html

2.3 Examples: Camera

2.3 Examples: Earth System