M.Sc. in ‚Transportation Systems‘

Applied Statistics in Transport

Exercises: Theory of Probability

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1. A digital scale is used that provides distances to the nearest meter. Let A denote the event that a distance exceeds 11 meters, let B denote the event that a distance is less than or equal to 15 meters, and let C denote the event that a distance is greater than or equal to 8 meters and less than 12 meters.

   a) What is the sample space for this experiment?

   b) $A \cup B$

   c) $A \cap B$

   d) $\overline{A}$

   e) $A \cup B \cup C$

   f) $\overline{(A \cup C)}$

   g) $A \cap B \cap C$

   h) $\overline{B} \cap C$

   i) $A \cup (B \cap C)$

Solution:

a) S={0,1,2,3,...}

b) S

c) {12,13,14,15}

d) {0,1,2,...,11}

e) S

f) {0,1,2,3,4,5,6,7}

g) ∅

h) ∅

i) {8,9,10,...}

2. Data entry by students is completed correctly by 25% of the students, completed with a minor error by 70% and completed with a major error by 5%.

   a) If a student is selected randomly to complete the data, what is the probability it is entered without error?

   b) What is the probability that the data is entered with either a minor or a major error?

Solution: $P(C) = 0.25, P(Mi) = 0.7, P(Ma) = 0.05$

   a) $P(C) = 0.25$

   b) $P(Mi \cup Ma) = 0.7 + 0.05 + 0.75$

3. A, B and C are mutually exclusive events. Can this statement be true for P(A)=0.3, P(B)=0.4 and P(C)=0.5? Why or why not?

Solution: No, since P(A)+P(B)+P(C)>1

4. Let E_1, E_2 and E_3 denote the samples that conform to a percentage of solids specification, a molecular weight specification and a colour specification, respectively. A total of 240 samples are classified by the E_1, E_2 and E_3-specifications, where yes indicates that the sample conforms.

E_3 yes

		E_2		
		Yes	No	Total
E_1	Yes	200	1	201
	No	5	4	9
Total		205	5	210

E_3 No

		E_2		
		Yes	No	Total
E_1	Yes	20	4	24
	No	6	0	6
Total		26	4	30

   a) Are E_1, E_2 and E_3 mutually exclusive events?

   b) Are $\overline{E}_1$, $\overline{E}_2$ and $\overline{E}_3$ mutually exclusive events?

   c) What is P($\overline{E}_1$ or $\overline{E}_2$ or $\overline{E}_3$)? (At least one fails)

   d) What is the probability that a sample conforms to all three specifications?

   e) What is the probability that a sample conforms to the E_1 or E_3 specification?

   f) What is the probability that a sample conforms to the E_1 or E_2 or E_3 specification?

   g) Are E_1 and E_2 independent?

Solution:

a) No, if E_3 yes, also E_1 and E_2 happens, sum of the probabilities is higher than 1.

b) No, since A is no.

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for three events:

$P(E'_1) + P(E'_2) + P(E'_3) - P(E'_1 \cap E'_2) - P(E'_1 \cap E'_3) - P(E'_2 \cap E'_3)$
$+P(E'_1 \cap E'_2 \cap E'_3) = \frac{15+9+30-4-6-4+0}{240} = \frac{40}{240} = 0.167$

d) $P(E_1 \cap E_2 \cap E_3) = \frac{200}{240} = 0.83$ (=1-prob. in c))

e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3) = (225 + 210 - 200)/240 = \frac{235}{240} = 0.98$

f) 240/240=1, sample space, can be calculated with addition rule for three events

g) $P(E_1) = \frac{225}{240} = 0.9375, P(E_2) = \frac{231}{240} = 0.9625$

$P(E_1) * P(E_2) = 0.9023\ but\ P(E_1 \cap E_2) = \frac{220}{240} = 0.9167$

Instead:

$P(E_1 \cap E_2) = P(E_2) * P(E_1 \mathrm{l}\, E_2) = \frac{231}{240} * \frac{220}{231} = P(E_1) * P(E_2 \mathrm{l}\, E_1) = \frac{225}{240} * \frac{220}{225} = \frac{220}{240} = 0.9167$ or direct calculation from the table: $P(E_1 \cap E_2) = \frac{220}{240}$

5. Mr. Dark writes on 3 subsequent days on overhead transparencies for preparing a presentation. Every morning he takes one pen out of five. All pens look absolutely equally. He writes with the pen all day long and

a) Puts it back in the evening

b) Does not put it back in the evening

On the 4th day Mr. Dark discovers some mistakes on his transparencies. He wants to correct the mistakes by wiping and newly writing the transparencies. Unfortunately, only 2 of the 5 pens were washable. What is probability of the events that:

A) He has written on the first day with a washable pen?

B) He has written on none of the days with a washable pen?

In both cases either for case a) and for case b).

Solution:

$P(Aa) = P(Ab) = \frac{2}{5} = 0.4$, $P(Ba) = \frac{3}{5} * \frac{3}{5} * \frac{3}{5} = 0.216, P(Bb) = \frac{3}{5} * \frac{2}{4} * \frac{1}{3} = 0.1$

6. A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		Evidence of Gas Leaks	
		Yes	No
Evidence of	Yes	55	17
Electrical failure	No	32	3

Find the probability:

a) That a failure involves a gas leak

b) That there is evidence of electrical failure given that there was a gas leak

c) That there is evidence of a gas leak given that there is evidence of electrical failure

Solution:

a) $P(GL) = \frac{87}{107} = 0.813$

b) $P(EF\ I\ GL) = \frac{55/107}{87/107} = 0.632$

c) $P(GL\ I\ EF) = \frac{55/107}{72/107} = 0.764$

7. In an automated filling operation, the probability of an incorrect fill when the process is operated at low speed is 0.001. When the process is operated at high speed, the probability of an incorrect fill is 0.01. Assume that 30% of the containers are filled when the process is operated at high speed and the remainder are filled when the process is operated at low speed.

a) What is the probability of an incorrectly filled container?

b) In an incorrectly filled container is found, what is the probability that it was filled during the high-speed operation?

Solution:

a) $P(IF) = P(IF\ I\ LS) * P(LS) + P(IF\ I\ HS) * P(HS) = 0.001 * 0.7$

$+0.01 * 0.3 = 0.0037$

b) $P(IF \cap LS) = P(LS) * P(IF\ I\ LS) = 0.7 * 0.001 = 0.0007$

$P(IF \cap HS) = P(HS) * P(IF\ I\ HS) = 0.3 * 0.01 = 0.003$

$P(HS\ I\ IF) = \frac{P(IF \cap HS)}{P(IF)} = \frac{0.003}{0.0037} = 0.811$

8. Incoming calls to a customer service centre are classified as complaints (75% of call) or requests for information (25% of calls). Of the complaints, 40% deal with computer equipment that does not respond and 57% deal with incomplete software installation; and in the remaining 3% of complaints the user has improperly followed the installation instructions. The requests for information are evenly divided on technical questions (50%) and requests to purchase more products (50%).

   a) What is the probability that an incoming call to the customer service centre will be from a customer who has not followed installation instructions properly?

   b) Find the probability that an incoming call is a request for purchasing more products.

Solution:

a) $P(C) = 0.75, P(Info) = 0.25, P(Equ\ I\ C) = 0.4, P(Incom\ I\ C) = 0.57$

$P(Instr\ I\ C) = 0.03, P(Tech\ I\ Info) = 0.5, P(Purch\ I\ Info) = 0.5$

$P(Instru \cap C)0.03 * 0.75 = 0.025$

b) $P(Purch \cap Info)0.5 * 0.25 = 0.125$

9. A new analytical method to detect pollutants in water is being tested. This new method of chemical analysis is important because, if adopted, it could be used to detect three different contaminants—organic pollutants, volatile solvents, and chlorinated compounds—instead of having to use a single test for each pollutant. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.

   a) What is the probability that the test will signal?

   b) If the test signals, what is the probability that chlorinated compounds are present?

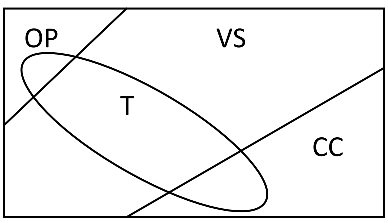
Solution: $P(T\ I\ OP) = 0.997, P(T\ I\ VS) = 0.9995, P(T\ I\ CC) = 0.897$

$P(OP) = 0.6, P(VS) = 0.27, P(CC) = 0.13$

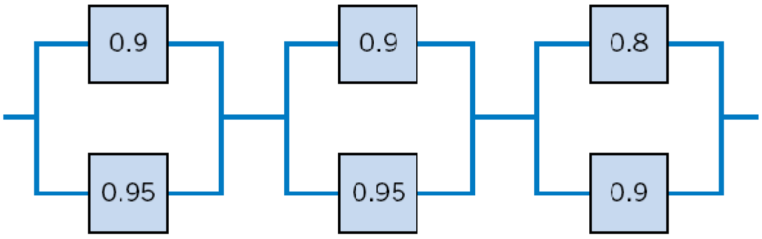
a) $P(T) = P(T\ I\ OP) * P(OP) + P(T\ I\ VS) * P(VS) + P(T\ I\ CC) * P(CC)$

$P(T) = 0.997 * 0.6 + 0.9995 * 0.27 + 0.897 * 0.13 = 0.9847$

b) $P(CC\ I\ T) = \frac{P(T\ I\ CC)*P(CC)}{P(T)} = \frac{0,897*0,13}{0,9847} = 0,1184$



10. The following circuit operates if and only if there is a path of functional devices from left to right. The probability each device functions is as shown. Assume that the probability that a device functions does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Solution:

Divide it into three segments, we are looking for

$P(S1 \cap S2 \cap S3) = P(S1) * P(S2) * P(S3)$

First segment: $P(T \cup B) = 1 - P\left[(T \cup B)'\right] = 1 - P(T' \cap B')$

$P\left(T' \cap B'\right) = P\left(T'\right) * P\left(B'\right) = (1 - 0.9) * (1 - 0.95) = 0.005$

$P(T \cup B) = 1 - P\left(T' \cap B'\right) = 1 - 0.005 = 0.995 = P(S1)$

Second segment (is exactly the same as S1, hence also 0,995):

$P(T \cup B) = 1 - P\left[(T \cup B)'\right] = 1 - P(T' \cap B')$

$P\left(T' \cap B'\right) = P\left(T'\right) * P\left(B'\right) = (1 - 0.9) * (1 - 0.95) = 0.005$

$P(T \cup B) = 1 - P\left(T' \cap B'\right) = 1 - 0.005 = 0.995 = P(S2)$

Third segment: $P(T \cup B) = 1 - P\left[(T \cup B)'\right] = 1 - P(T' \cap B')$

$P\left(T' \cap B'\right) = P\left(T'\right) * P\left(B'\right) = (1 - 0.8) * (1 - 0.9) = 0.02$

$P(T \cup B) = 1 - P\left(T' \cap B'\right) = 1 - 0.02 = 0.98 = P(S3)$

Synthesis: $P(S1 \cap S2 \cap S3) = 0.995 * 0.995 * 0.98 = 0.9702$

11. What is the number of permutations for the word statistics?

Solution: $\frac{10!}{3!3!1!2!1!} = 50{,}400$

12. A local parliament consists of 7 radicals, 5 conservatives, and 3 socialists. How many different commissions can be established given that they have to be composed of 2 radicals, 2 conservatives and 1 socialist?

Solution: Combination, order does not matter,

$\binom{7}{2} * \binom{5}{2} * \binom{3}{1} = \frac{7!}{5!2!} * \frac{5!}{3!2!} * \frac{3!}{1!2!} = \frac{6*7}{1*2} * \frac{4*5}{1*2} * \frac{3}{1} = 21 * 10 * 3 = 630$

13. A chain of supermarkets has eight different locations in which a branch/store can be placed. If four different stores are to be placed in the region, how many different alternatives are possible?

Solution: Each alternative consists of selecting a location from the eight locations for the first store, a location from the remaining seven for the second store, a location from the remaining six for the third store, and a location from the remaining five for the fourth store. Therefore,

$P_4^8 = 8 \times 7 \times 6 \times 5 = \frac{8!}{4!} = 1680$ different designs are possible.

$= 8!/4! = (8 * 7 * 6 * 5 * 4 * 3 * 2 * 1)/(1 * 2 * 3 * 4) = 1680$