Applied Signal Processing & Computer Science

Chapter 4: Time Domain Analysis

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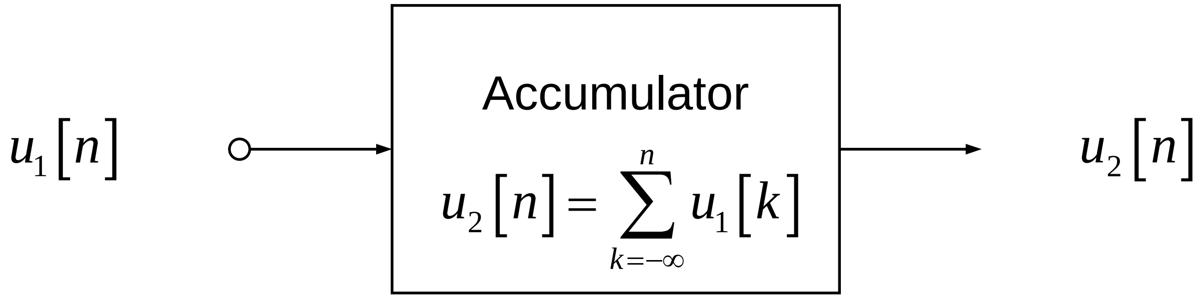
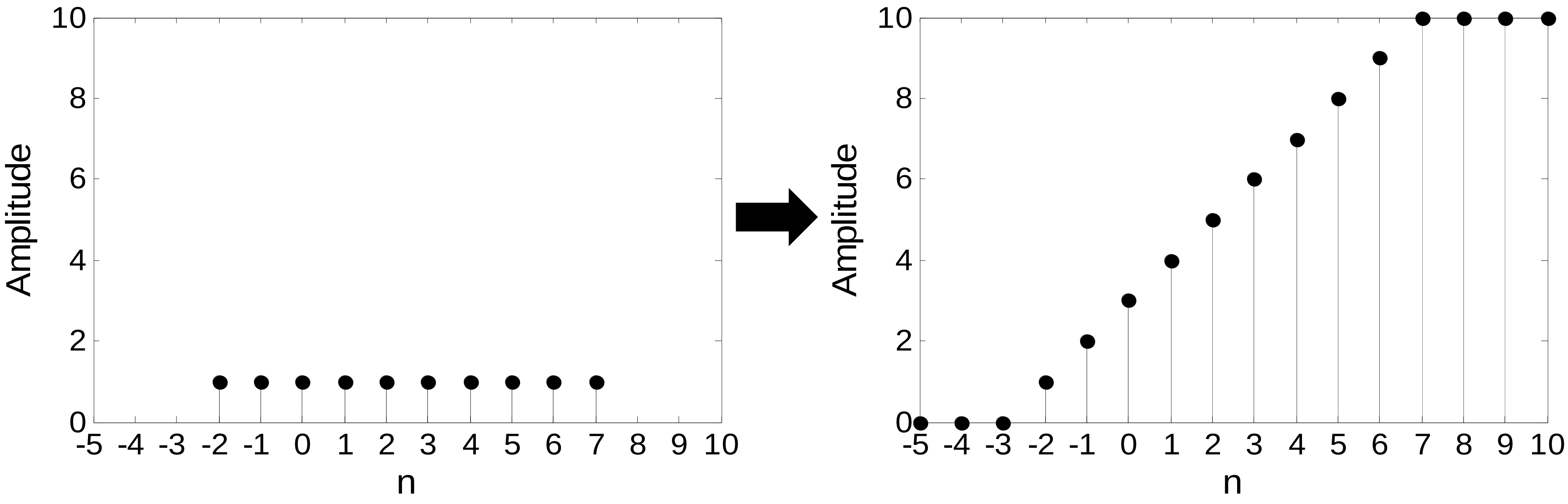
Chapter 4: Time Domain Analysis

Discrete-Time Systems: Accumulator

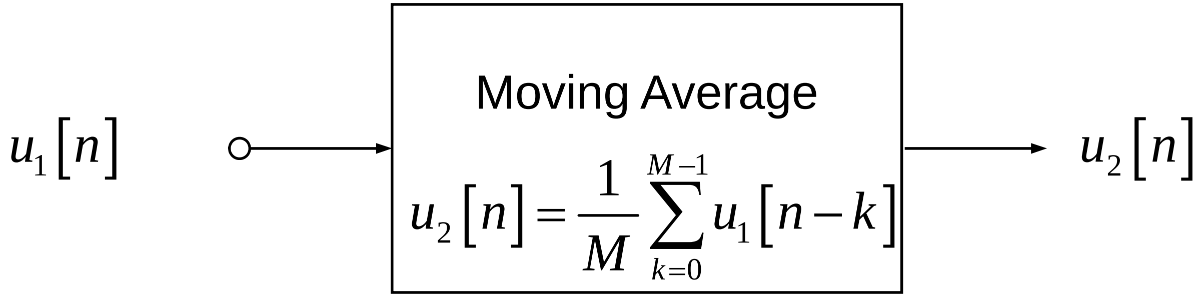
The output is the **sum** of all previous input samples from $-\infty$ to the instant n

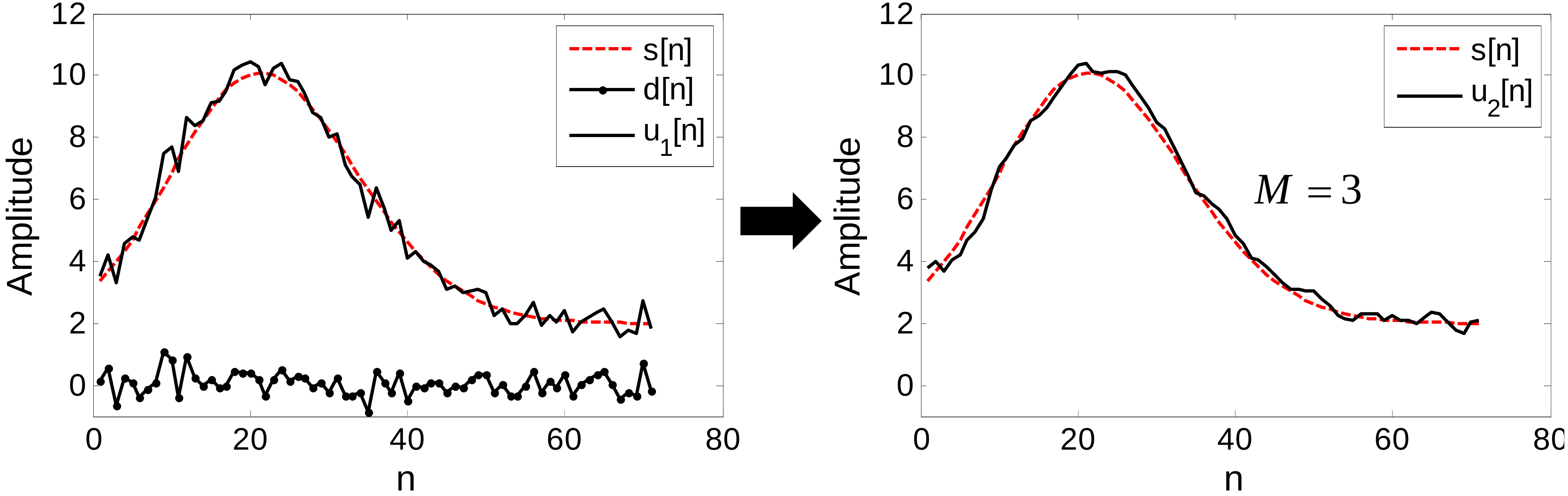




Discrete-Time Systems: Moving Average Filter

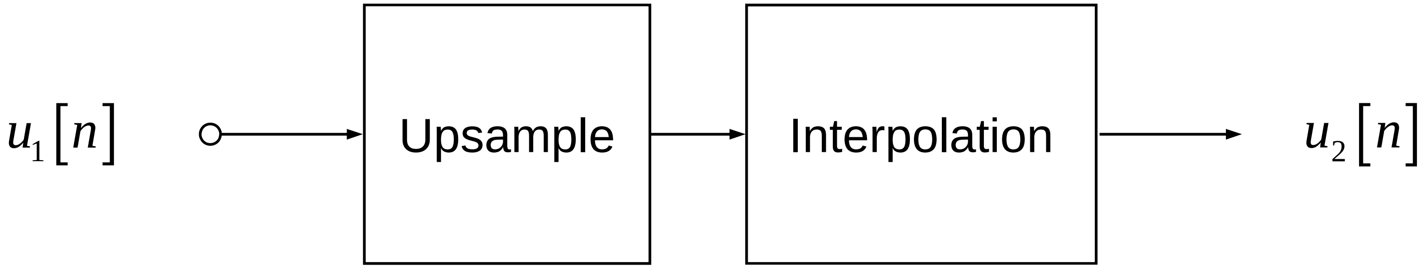
The output is M-point mean of the input samples from the instant *n-M+1* to *n*

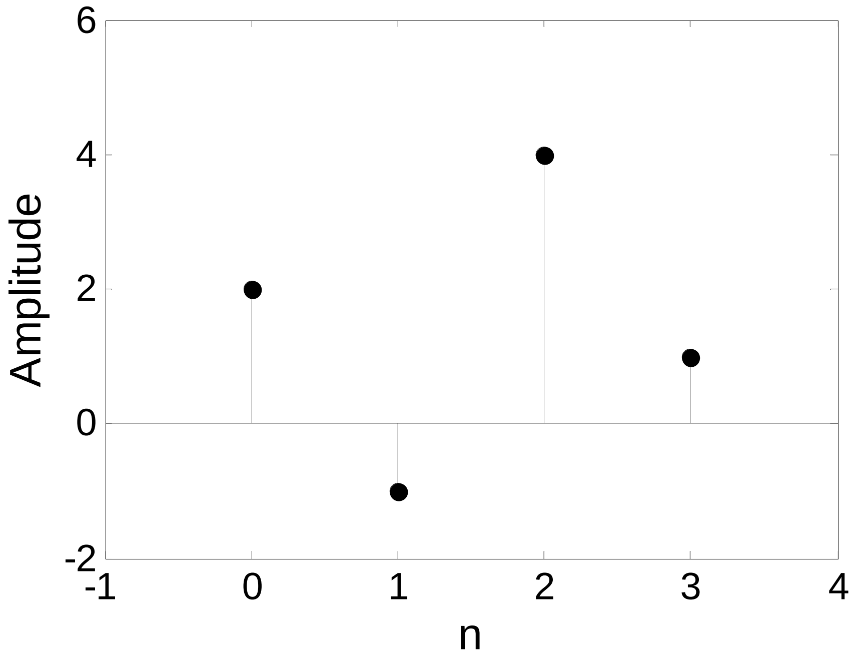


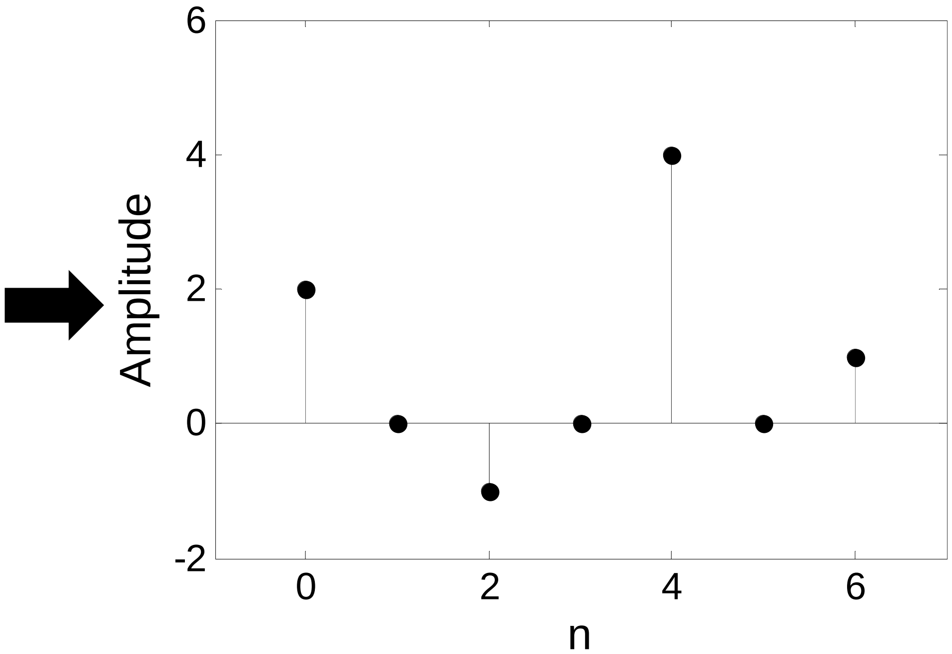


Discrete-Time Systems: Bilinear Interpolator

The output is the mean of a pair of upsampled adjacent values

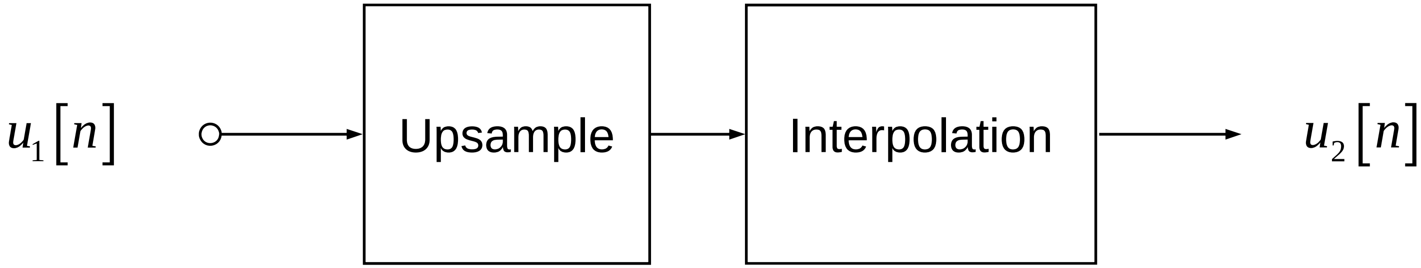


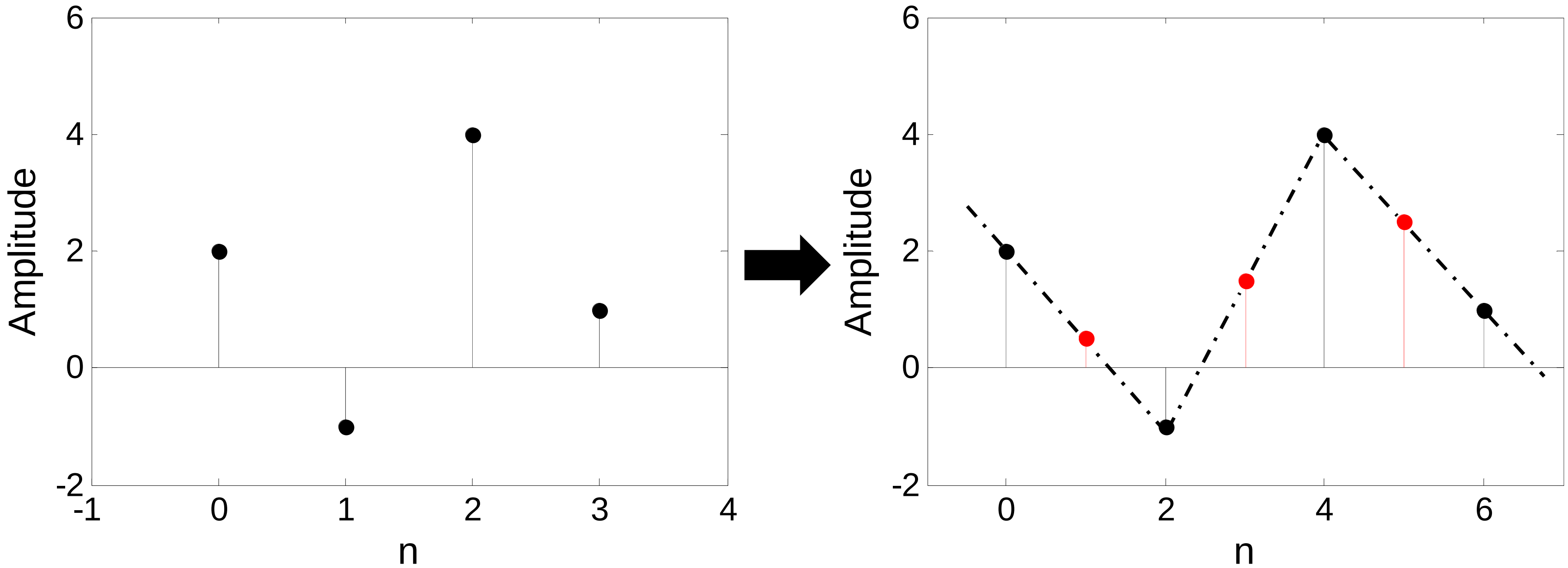




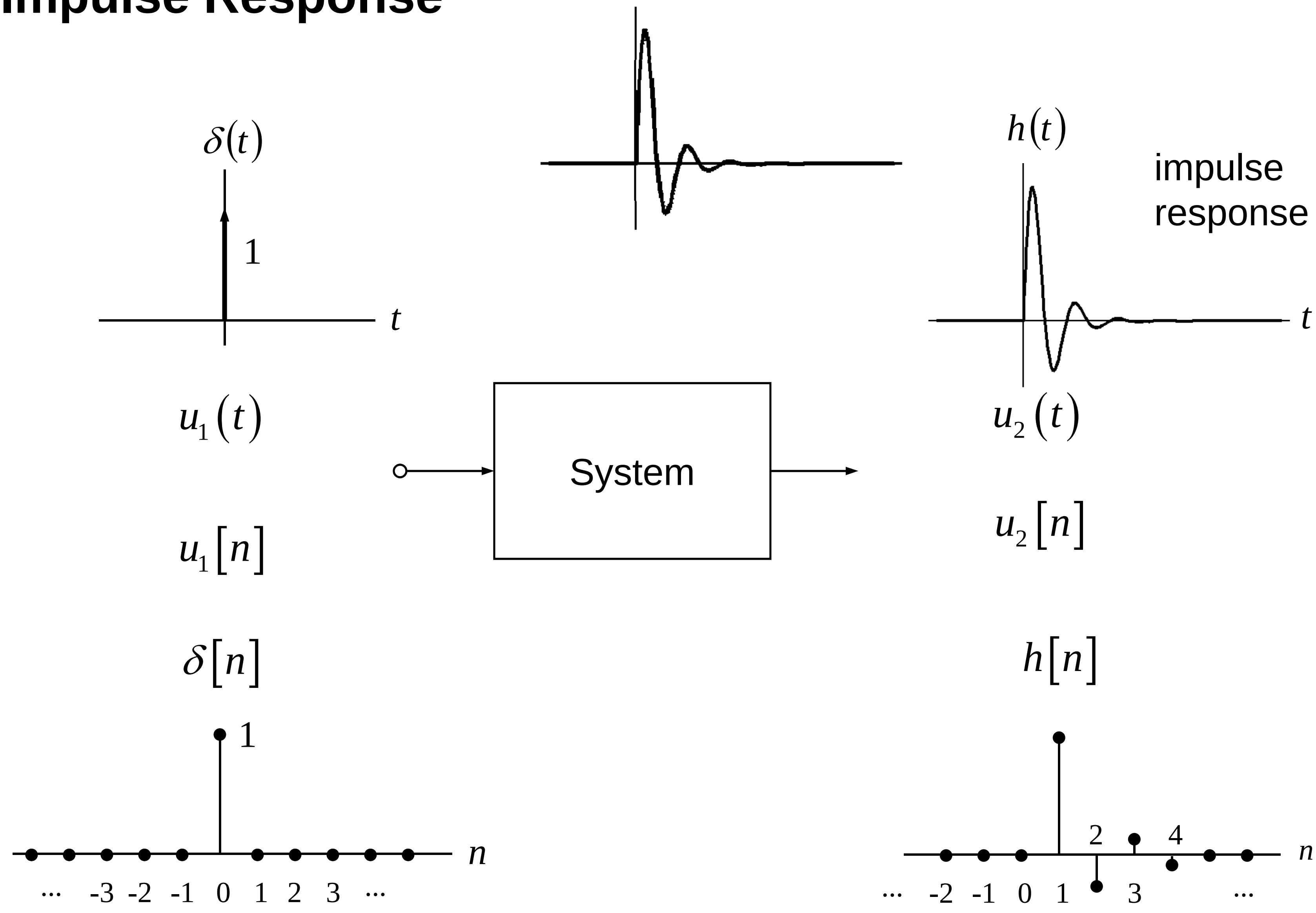
Discrete-Time Systems: Bilinear Interpolator

The output is the mean of a pair of upsampled adjacent values



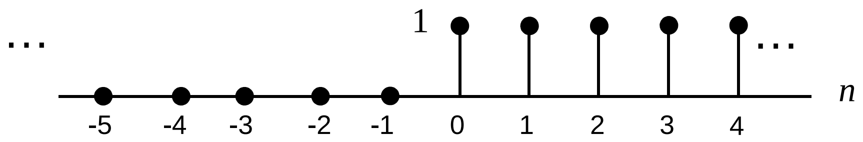


Impulse Response

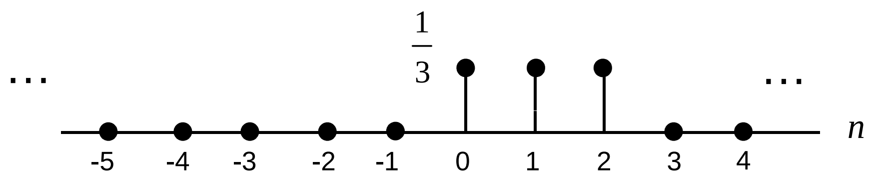


Impulse Response

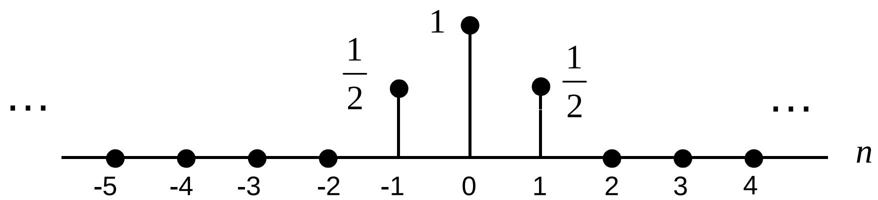
- Accumulator: $h[n] = [\cdots \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad \cdots]$



- Moving Average Filter: $h[n] = [\cdots \quad 0 \quad 0 \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad 0 \quad 0 \quad \cdots]$



- Linear Interpolator: $h[n] = [\cdots \quad 0 \quad 0 \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad 0 \quad 0 \quad \cdots]$

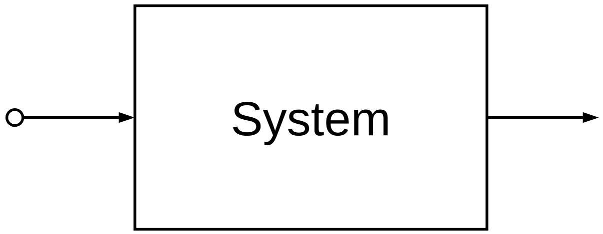


Linear Time-Invariant (LTI) System

- Linearity
  - Superposition of each impulse

$$u_{1,1}[n] \qquad\qquad u_{2,1}[n]$$

$$u_{1,2}[n] \qquad\qquad u_{2,2}[n]$$



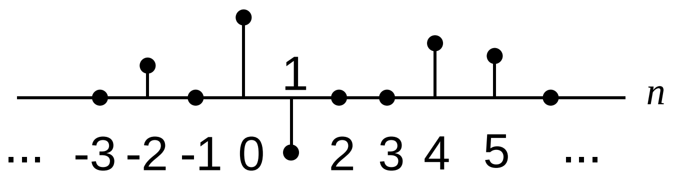
$$u_1[n] = \alpha u_{1,1}[n] + \beta u_{1,2}[n] \qquad\qquad u_2[n] = \alpha u_{2,1}[n] + \beta u_{2,2}[n]$$

- Time-Invariant
  - Output is **independent** of the time when input is applied
  - $u_1[n] = u_1'[n-n_0] \Rightarrow u_2[n] = u_2'[n-n_0]$

Input-Output Relationship

- Input: Superposition of many shifted **Impulses**

- Output: Superposition of many shifted **Impulse Responses**



$$u_1[n] = 0.5\delta[n+2] + 1.5\delta[n] - \delta[n-1] + \delta[n-4] + 0.7\delta[n-5]$$

$$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$$

$$u_2[n] = 0.5h[n+2] + 1.5h[n] - h[n-1] + h[n-4] + 0.7h[n-5]$$

$$u_2[n] = \sum_{k=-\infty}^{\infty} u_1[k]\, h[n-k], \text{ or} \quad u_2[n] = u_1[n] * h[n]$$

Convolution

= linear time-invariant operation

Definition:

$$u(t) * h(t) = \int_{-\infty}^{+\infty} u(t')\, h(t-t')\, dt'$$

$$= \int_{-\infty}^{+\infty} u(t-t')\, h(t')\, dt' = h(t) * u(t)$$

Interpretation 1: $h(t)$ as Integration Kernel

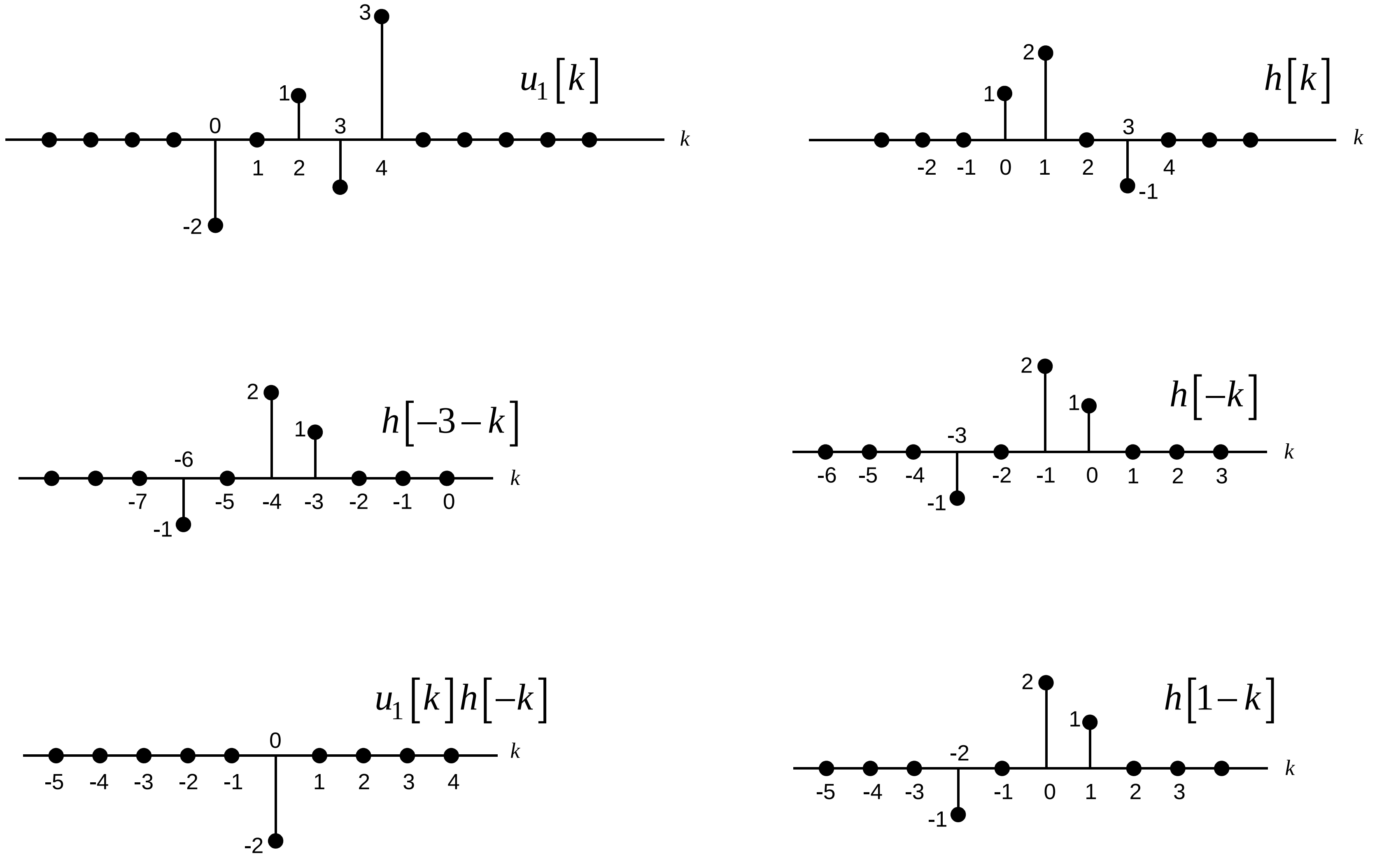
1 $\quad u(t) \rightarrow u(t')$

2 $\quad h(t) \rightarrow h(-t')$ Reflection at ordinate

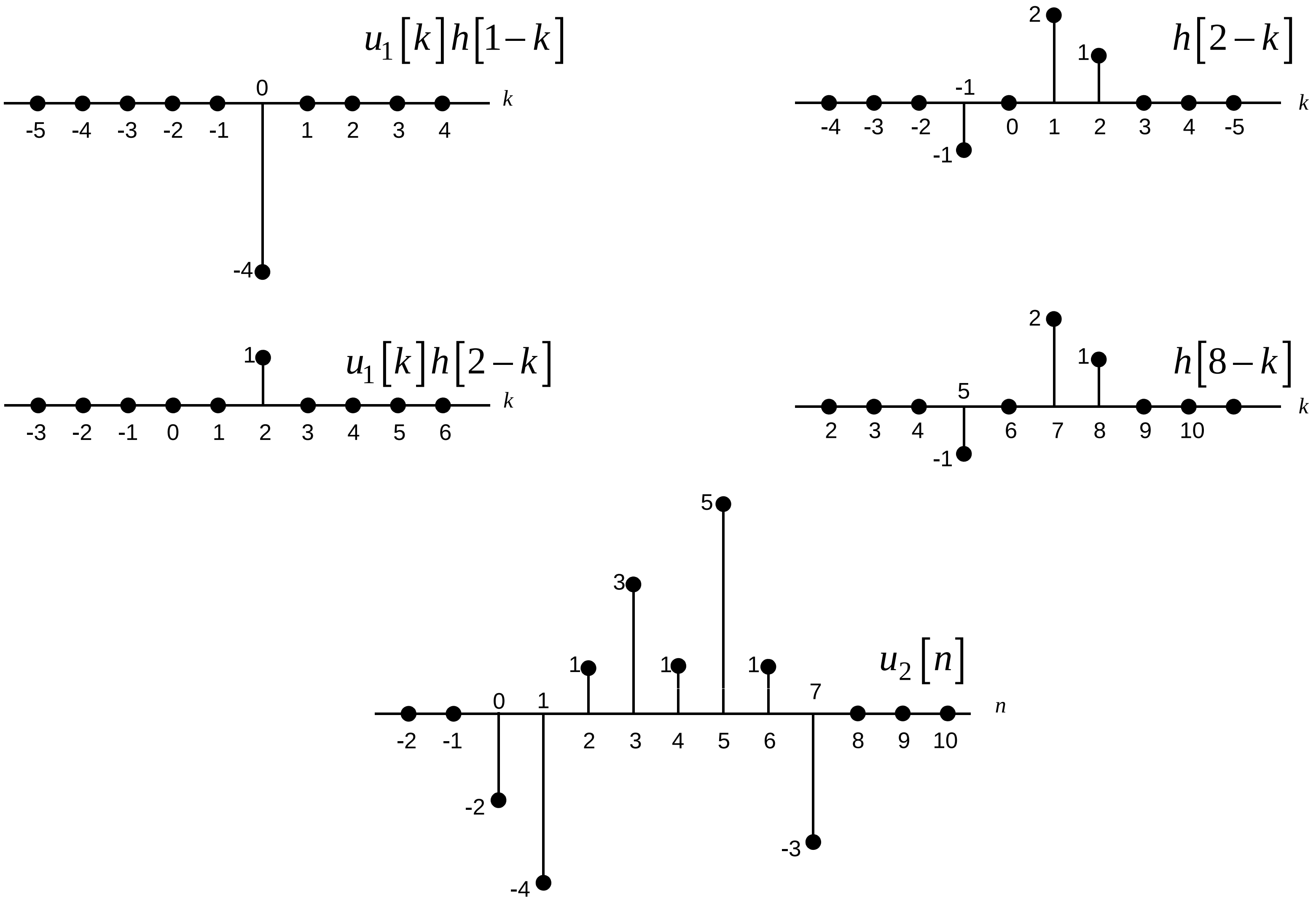
3 $\quad h(-t') \rightarrow h(t-t')$ Shift to position t

4 $\quad \int_{-\infty}^{+\infty} u(t')h(t-t')dt'$ Multiplication and integration of the product

Graphic Convolution Sum Computation (cont.)



Graphic Convolution Sum Computation



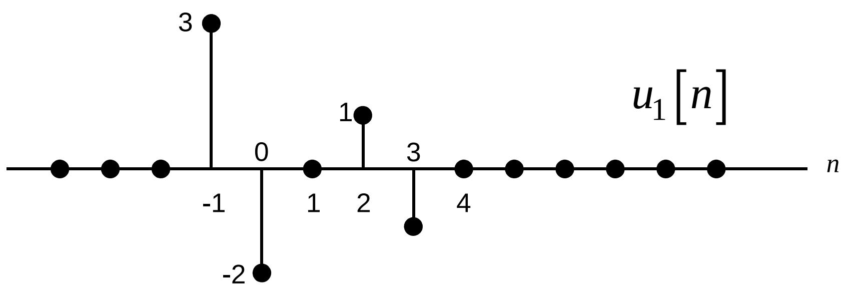
Tabular Method of Convolution Sum Computation

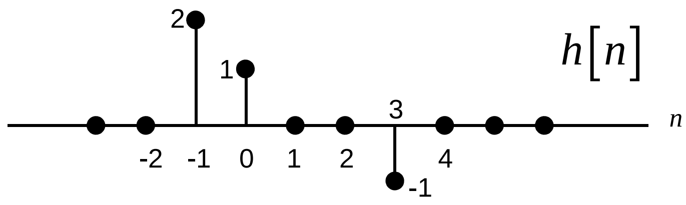
n:	0	1	2	3	4	5
$u_1[n]$:	$u_1[0]$	$u_1[1]$	$u_1[2]$	$u_1[3]$		
$h[n]$:	$h[0]$	$h[1]$	$h[2]$	-		
	$u_1[0]h[0]$	$u_1[1]h[0]$	$u_1[2]h[0]$	$u_1[3]h[0]$		
		$u_1[0]h[1]$	$u_1[1]h[1]$	$u_1[2]h[1]$	$u_1[3]h[1]$	
			$u_1[0]h[2]$	$u_1[1]h[2]$	$u_1[2]h[2]$	$u_1[3]h[2]$
$u_2[n]$:	$u_2[0]$	$u_2[1]$	$u_2[2]$	$u_2[3]$	$u_2[4]$	$u_2[5]$

Tabular Method of Convolution Sum Computation

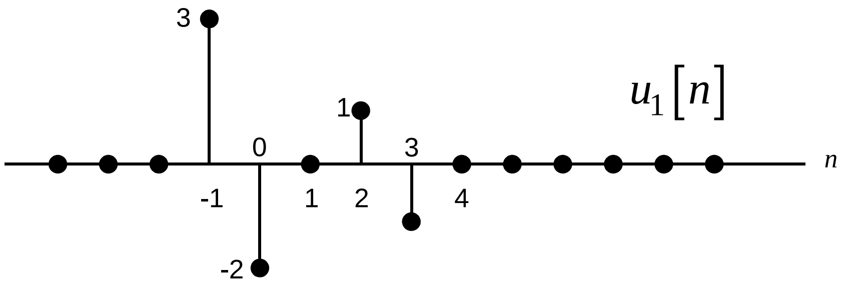
n:	0	1	2	3	4	5	6	7
$u_1[n]$:	-2	0	1	-1	3			
$h[n]$:	1	2	0	-1	-			
	-2	0	1	-1	3			
	-	-4	0	2	-2	6		
	-	-	0	0	0	0	0	-
	-	-	-	2	0	-1	1	-3
$u_2[n]$:	-2	-4	1	3	1	5	1	-3

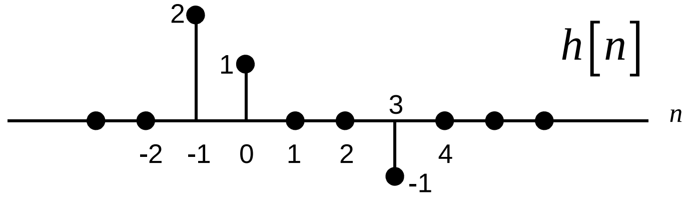
Exercise Graphic Convolution



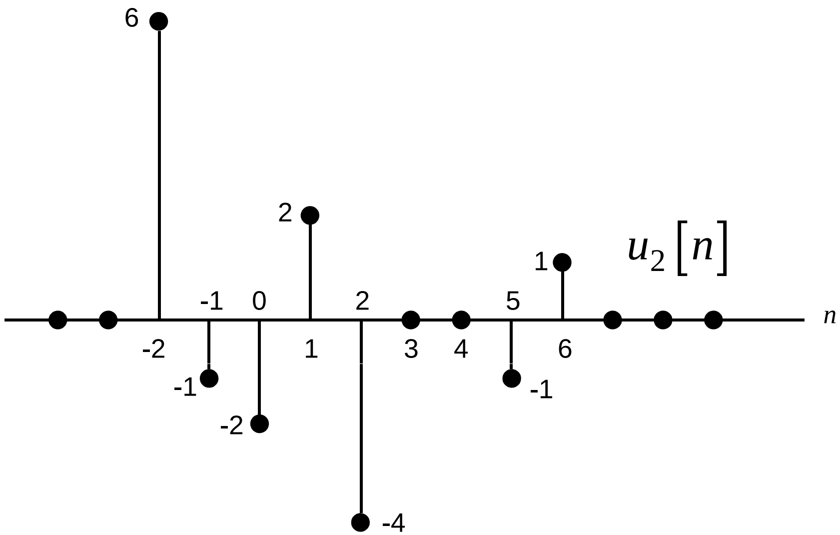


Exercise Graphic Convolution





Solution:



Exercise Convolution Tabular Method

$n:$	0	1	2	3	4	5	6	7
$u_1[n]:$	1	0	-2	5	-			
$h[n]:$	2	-2	0	3	-			
	_	_	_	_	_	_	_	_
	_	_	_	_	_	_	_	_
	_	_	_	_	_	_	_	_
	_	_	_	_	_	_	_	_
$u_2[n]:$	_	_	_	_	_	_	_	_

Exercise Convolution Tabular Method

$n:$	0	1	2	3	4	5	6	7
$u_1[n]:$	1	0	-2	5	-			
$h[n]:$	2	-2	0	3	-			
	2	0	–4	10	–	–	–	–
	–	–2	0	4	–10	–	–	–
	–	–	0	0	0	0	–	–
	–	–	–	3	0	–6	15	–
$u_2[n]:$	2	-2	-4	17	-10	-6	15	-

Correlation of Signals

"The Twin Brother of Convolution"

Definition:

$$u(t) \otimes h(t) = \int_{-\infty}^{+\infty} u(t)\, h(t - t')\, dt$$

To compare the **similarity** of two signals.