Applied Signal Processing and Computer Science

WS 11/12 (email: xiao.zhu@dlr.de)

Solution 1: Complex Numbers

1. Arithmetic of Complex Numbers:

1.1 Evaluate the following complex numbers:

- $n = 2,3,4,...,9 \quad \rightarrow j^n = -1,-j,1,j,-1,-j,1,j$
- $j^{4n} = 1,\ j^{4n+1} = j,\ j^{4n+2} = -1,\ j^{4n+3} = -j$

1.2 Convert the following numbers into a $a + jb$ representation:

- $2j$
- $-2j$
- $2j$

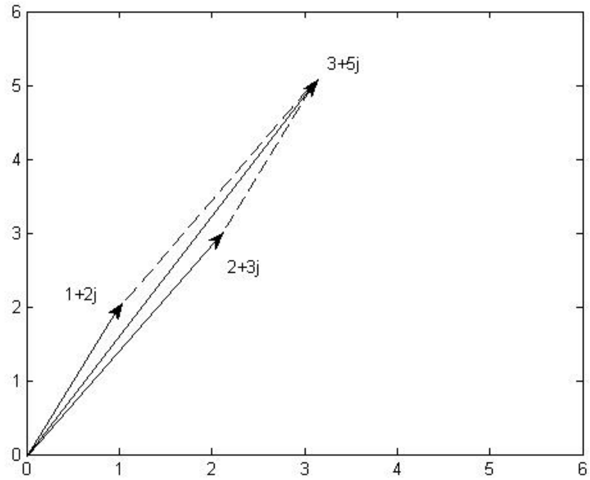
1.3 Evaluate:

- -12
- $10 + 5j$
- $56 - 58j$

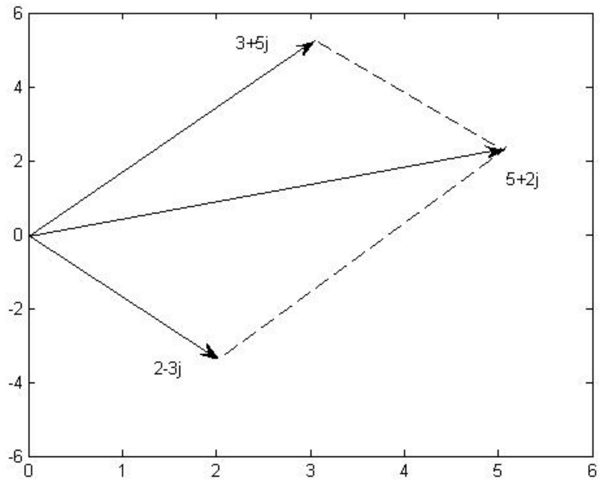
2. The complex plane

2.1 Represent the following numbers in the complex plane by using vector sums, and write the respective result as a complex number:

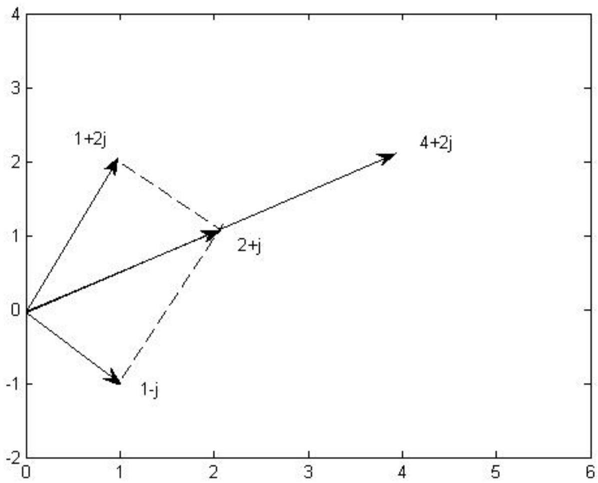
- $(2+3j)+(1+2j)$



➢ $(2-3j)+(3+5j)$



➢ $(1+2j)+(2+j)+(1-j)$



2.2

$z_1 + z_2 = 1.8 + 2.4j$ $\quad |z_1 + z_2| = 3$; $\quad z_1 \cdot z_2 = -0.56 + 1.92j$ $\quad |z_1 \cdot z_2| = 2$

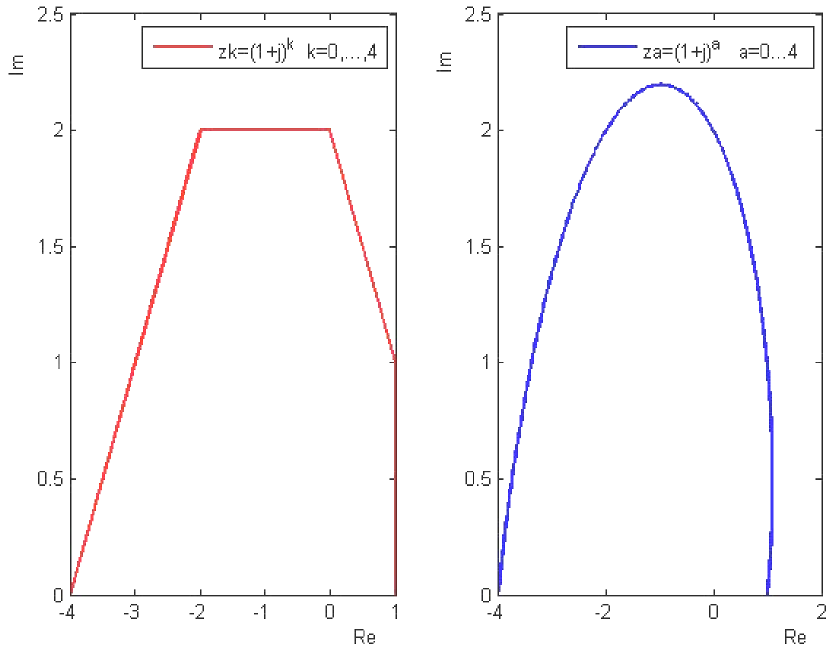
2.3 Convert the following numbers into an Euler representation (magnitude and phase)

- $\sqrt{2}\, e^{j\frac{\pi}{4}}$
- $2\sqrt{3}\, e^{j\frac{\pi}{6}}$
- $z_1 \cdot z_2 = 4\, e^{j\frac{\pi}{6}} \quad \dfrac{z_1}{z_2} = e^{j\frac{3\pi}{2}}$
- $z_1 \cdot z_2 = e^{0j} \quad z_1 \cdot z_2^{\,*} = e^{j\frac{\pi}{2}}$

2.4 Demonstrate the validity of the following Euler laws for all $\varphi \in \Re$:

- $e^{j\varphi} = \cos\varphi + j\sin\varphi \;\; => \;\; (e^{j\varphi})^* = \cos\varphi - j\sin\varphi = e^{j\,a\tan(-\frac{\sin\varphi}{\cos\varphi})} = e^{-j\varphi}$
- $\dfrac{1}{2}(e^{j\varphi} + e^{-j\varphi}) = \dfrac{1}{2}(\cos\varphi + j\sin\varphi + \cos\varphi - j\sin\varphi) = \cos\varphi$
- $\dfrac{1}{2j}(e^{j\varphi} - e^{-j\varphi}) = \dfrac{1}{2j}(\cos\varphi + j\sin\varphi - \cos\varphi + j\sin\varphi) = \dfrac{1}{2j} * 2j\sin\varphi = \sin\varphi$

2.5



3. Complex Harmonic Oscillation

3.1

$$\mathrm{Re}\{u(t)\} = A\sin(2\pi\, ft + \pi/3) = A\cos(2\pi\, ft - \pi/6)$$

$$u(t) = A(\cos(2\pi\, ft - \pi/6) + j\sin(2\pi\, ft - \pi/6))$$

$$\mathrm{Im}\{u(t)\} = \sin(2\pi\, ft - \pi/6)$$

3.2

$$A = \sqrt{5} \qquad \varphi = a\tan(-\frac{1}{2})$$