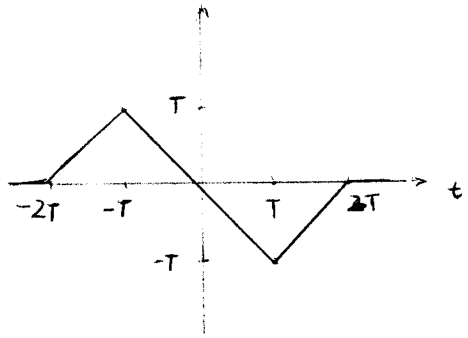
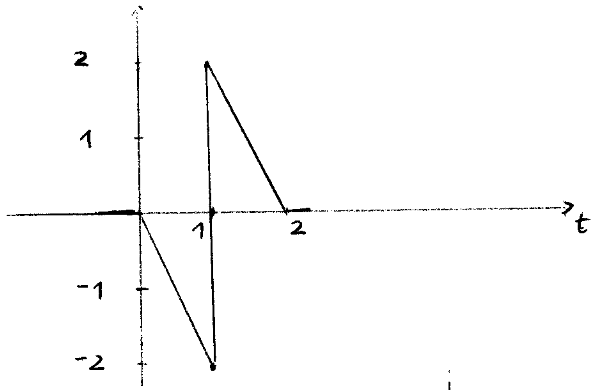
Practice Exam — Systems Theory and Signal Processing

1) Graphical Convolution:

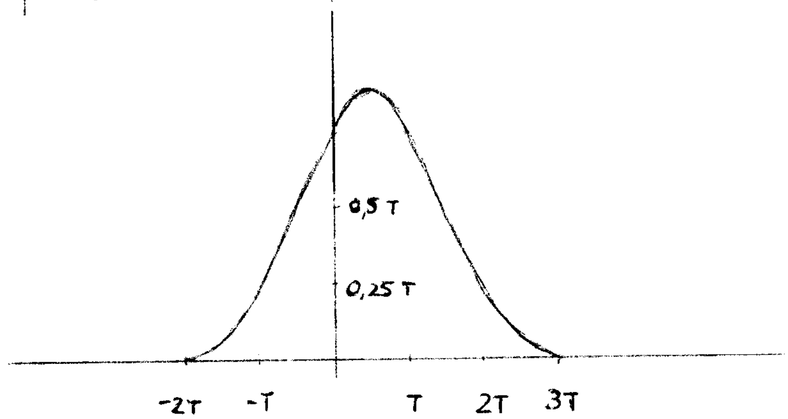
a)




b)




c)

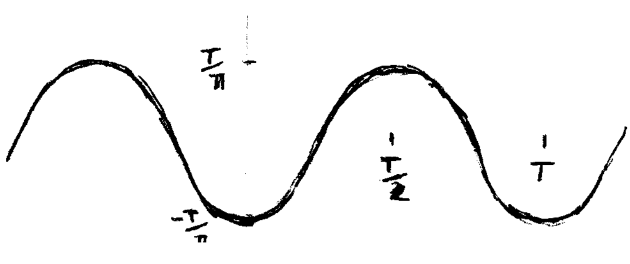



2) Analytical Convolution:

$$y(t) = \int_{-\infty}^{\infty} \cos\left(\frac{2\pi(t-t')}{T}\right) \mathrm{rect}\left(\frac{t'-\frac{T}{2}}{\frac{T}{2}}\right) dt' = \int_{\frac{T}{4}}^{\frac{3}{4}T} \cos\left(\frac{2\pi(t-t')}{T}\right) dt'$$

$$= \left[-\frac{T}{2\pi} \sin\left(\frac{2\pi(t-t')}{T}\right)\right]_{\frac{T}{4}}^{\frac{3}{4}T} = -\frac{T}{2\pi}\left[\sin\left(\frac{2\pi t}{T} - \frac{3}{2}\pi\right) - \sin\left(\frac{2\pi t}{T} - \frac{\pi}{2}\right)\right]$$

$$= -\frac{T}{2\pi}\left[\cos\left(\frac{2\pi t}{T}\right) + \cos\left(\frac{2\pi t}{T}\right)\right] = -\frac{T}{\pi} \cos\left(\frac{2\pi t}{T}\right)$$



3) Fourier Transform

a)

$$\gamma(t)\exp(-at) \longleftrightarrow \frac{1}{a+j2\pi f}$$

$$\cos(2\pi f_0 t) \longleftrightarrow \frac{1}{2}\left(\delta(f+f_0) + \delta(f-f_0)\right)$$

$$u(t)\cdot h(t) \longleftrightarrow U(f) * H(f)$$

$$\Rightarrow U_a(f) = \frac{1}{a+j2\pi f} * \frac{1}{2}\left(\delta(f+f_0) + \delta(f-f_0)\right)$$

$$= \frac{1}{2}\left[\frac{1}{a+j2\pi(f+f_0)} + \frac{1}{a+j2\pi(f-f_0)}\right] \quad \text{with} \quad a = \frac{1}{T}$$

b)

$$U_b(f) = \mathrm{rect}\left(\frac{f+f_0}{B}\right) - \mathrm{rect}\left(\frac{f-f_0}{B}\right)$$

$$= \mathrm{rect}\left(\frac{f}{B}\right) * \left[\delta(f+f_0) - \delta(f-f_0)\right]$$

$$= \mathrm{rect}\left(\frac{f}{B}\right)\frac{2}{j} * \frac{j}{2}\left[\delta(f+f_0) - \delta(f-f_0)\right]$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$u_b(t) = \frac{2}{j} B\,\mathrm{sinc}(Bt) \cdot \sin(2\pi f_0 t)$$

# 4) Linear time-invariant Systems

4.1) LTI:

$$\delta(t) \;\circ\!\!\longrightarrow\; h(t)$$

$$\gamma(t) \;\circ\!\!\longrightarrow\; u_2(t) = ?$$

$$\Rightarrow \int_{-\infty}^{t} \delta(\tau)\, d\tau = \gamma(t) \;\circ\!\!\longrightarrow\; u_2(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$u_2(t) = \int_{-\infty}^{t} \gamma(\tau)\, \frac{1}{a} \exp\left(-\frac{\tau}{a}\right) d\tau = \frac{1}{a} \int_{0}^{t} \exp\left(-\frac{\tau}{a}\right) d\tau = \ldots =$$

$$= 1 - \exp\left(-\frac{t}{a}\right)$$

4.2) a)

$$\delta(t) = \frac{d}{dt}\gamma(t) \;\circ\!\!\longrightarrow\; h(t) = \frac{d}{dt} u_2(t)$$

$\underline{t>0}$:

$$h(t) = \frac{d}{dt}\left[2\left(1 - \exp(-t) - t\exp(-t)\right)\right] = 2t\exp(-t)$$

$\underline{\forall t}$:

$$h(t) = 2\gamma(t)\, t\exp(-t)$$

b)

$$H(f) = \int_{-\infty}^{\infty} 2\gamma(t)\, t\exp(-t) \cdot \exp(-j2\pi f t)\, dt = 2\int_{0}^{\infty} t\exp(-t(1+j2\pi f))\, dt$$

$$= \frac{2}{(1+j2\pi f)^2}$$

c)

$$u_2(t) = u_1(t) * h(t) = \int_{-\infty}^{\infty} u_1(t-t') \cdot h(t')\, dt' = 2\int_{0}^{t} \exp(-t)\, t'\, dt'$$

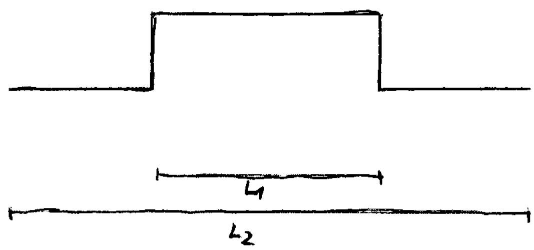
$$= \exp(-t) \cdot t^2$$

# 5) 2-D Fourier Transform

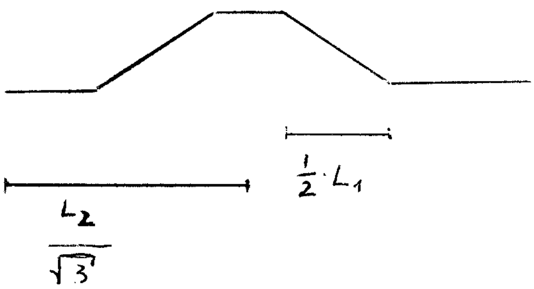
1) $\leftrightarrow$ c)
2) $\leftrightarrow$ f)
3) $\leftrightarrow$ b)
4) $\leftrightarrow$ e)
5) $\leftrightarrow$ a)
6) $\leftrightarrow$ d)

# 6) Radon Transform

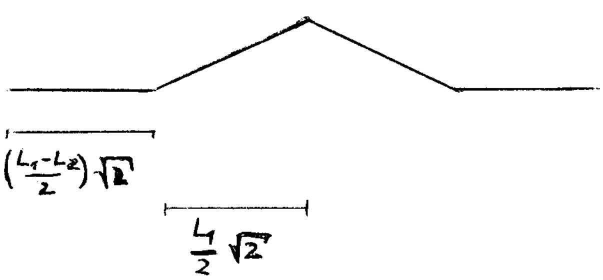
$\varphi = 0°$ : $u_p(R, 0°)$ :




$\varphi = 30°$




$\varphi = 45°$ :




$\varphi = 90°$ $\qquad u_p(R, 0°) = u_p(R, 90°)$