Applied Signal Processing and Computer Science

WS10/11 (Email: xiaoxiang.zhu@mytum.de)

Solution 6: Linear time-variant systems

1. Impulse response and transfer function:

1.1

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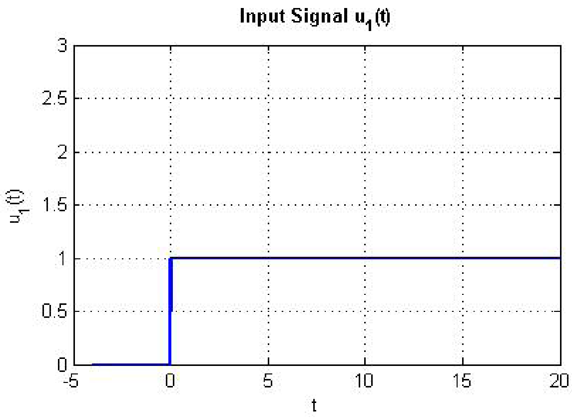

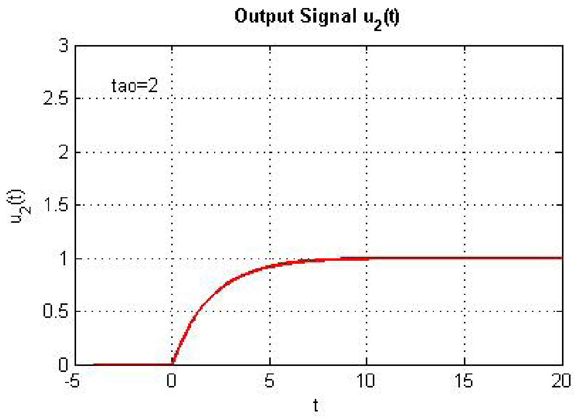

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$$u_1(t) = \gamma(t) = \delta^{-1}(t)$$

$$u_2(t) = \gamma(t)[1 - e^{-\frac{t}{\tau}}]$$

$$u_2(t) = u_1(t) * h(t) = h(t) * u_1(t) = h(t) * \delta^{-1}(t) = \gamma(t)[1 - e^{-\frac{t}{\tau}}]$$

According to:

$$u(t) * \delta^V(t) = u^V(t); \text{(properties of } \delta \text{ function)} \qquad here, V = 1 \;\; u(t) = h(t)$$

$$=> \; h^{-1}(t) = u_2(t) = \gamma(t)[1 - e^{-\frac{t}{\tau}}]$$

$$=> \; h(t) = \frac{du_2(t)}{dt} = \delta(t)[1 - e^{-\frac{t}{\tau}}] + \gamma(t)[\frac{1}{\tau} e^{-\frac{t}{\tau}}] = \gamma(t)[\frac{1}{\tau} e^{-\frac{t}{\tau}}]$$

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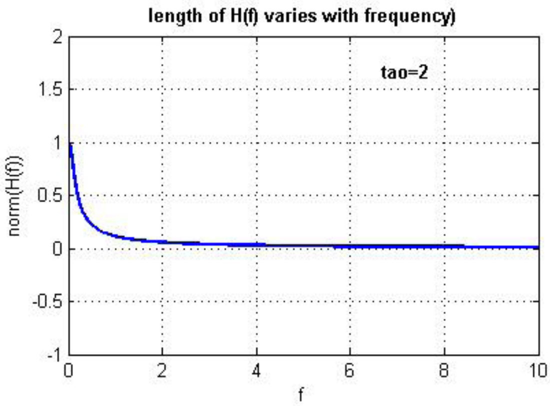
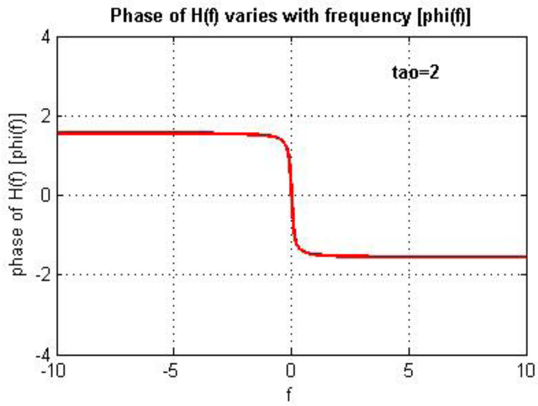
$$H(f) = \frac{1}{1 + j2\pi f \tau}$$

$$|H(f)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2 \tau^2}} \qquad \varphi(f) = a\tan(-2\pi f \tau)$$





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$$u_1(t) = \cos(2\pi f_0 t)$$

$$h(t) = \gamma(t)[\frac{1}{\tau} e^{-\frac{t}{\tau}}]$$

$$u_2(t) = u_1(t) * h(t)$$

$$U_2(f) = U(f) \cdot H(f) = \frac{1}{2}(\delta(f + f_0) + \delta(f - f_0)) \cdot H(f)$$

$$\begin{aligned} u_2(t) &= \int_{-\infty}^{\infty} U_2(f) e^{j2\pi ft} df \\ &= \frac{1}{2}[\int_{-\infty}^{\infty} \delta(f + f_0) \cdot H(f) \cdot e^{j2\pi ft} df + \int_{-\infty}^{\infty} \delta(f - f_0) \cdot H(f) \cdot e^{j2\pi ft} df] \\ &= \frac{1}{2}[H(-f_0) \cdot e^{-j2\pi f_0 t} + H(f_0) \cdot e^{j2\pi f_0 t}] \\ &= |H(f_0)|[\frac{e^{-j(2\pi f_0 t + \varphi_H(f_0))} + e^{j(2\pi f_0 t + \varphi_H(f_0))}}{2}] \\ &= |H(f_0)| \cos(2\pi f_0 t + \varphi_H(f_0)) \end{aligned}$$

1.2

➢

$$u_2(t) = u_1(t) + a u_1(t - \Delta t)$$

$$h(t) = \delta(t) + a\delta(t - \Delta t)$$

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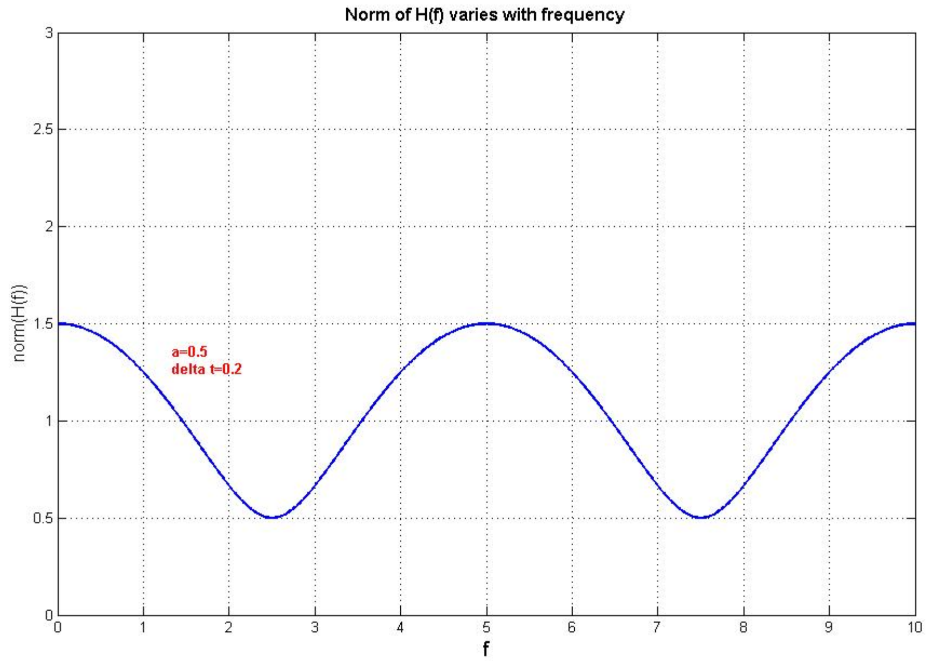
$$H(f) = 1 + a \cdot e^{-j2\pi f \Delta t}$$

$$|H(f)| = \sqrt{1 + a^2 + 2a\cos(2\pi f \Delta t)} \qquad \varphi(f) = a\tan(\frac{-a\sin(2\pi f \Delta t)}{1 + a\cos(2\pi f \Delta t)})$$

Sketch $\left|H(f)\right|$:



Sketch $\varphi_H(f)$: