Applied Signal Processing & Computer Science

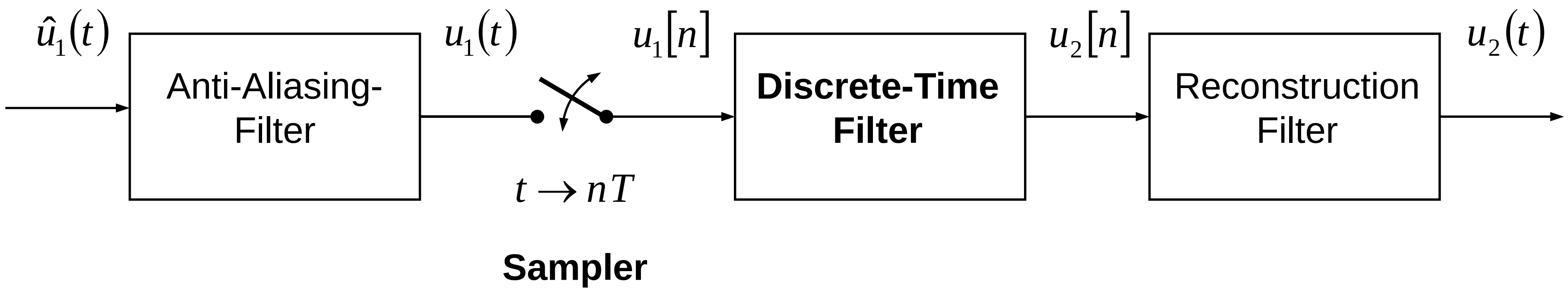
Chapter 6: Sampling and Aliasing

Xiaoxiang Zhu

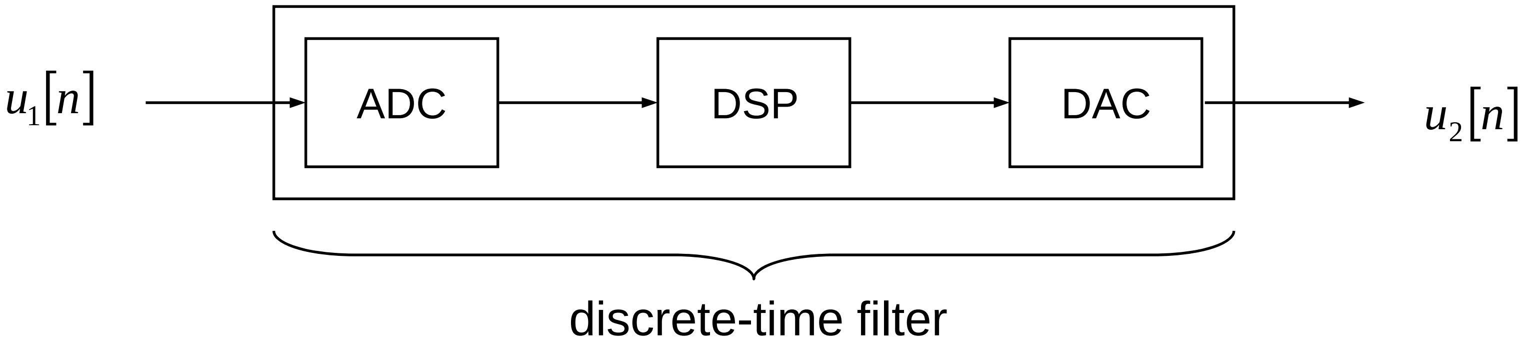
xiao.zhu@dlr.de

Remote Sensing Technology
TU München
&
Remote Sensing Technology Institute
German Aerospace Center (DLR)
Oberpfaffenhofen

Discrete-Time Signal Processing of Continuous Signals



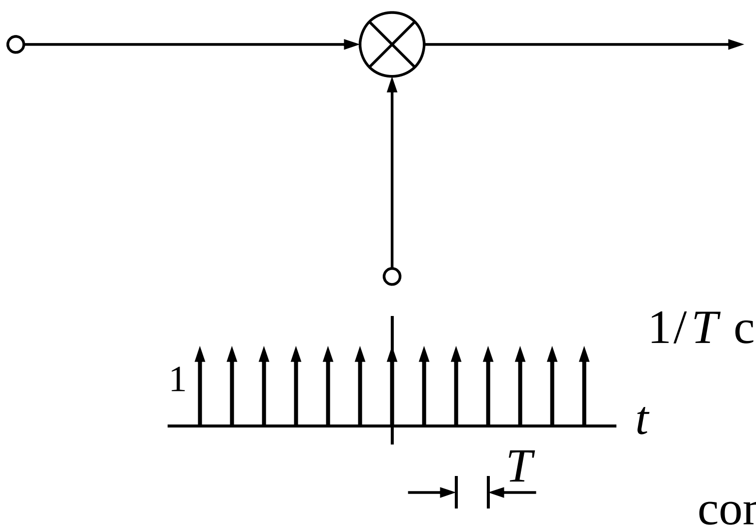
Digital Signal Processing (DSP) = discrete time <u>and</u> discrete values:



Systems Theoretical Description of Sampling

$$u(t) \longleftrightarrow U(f)$$

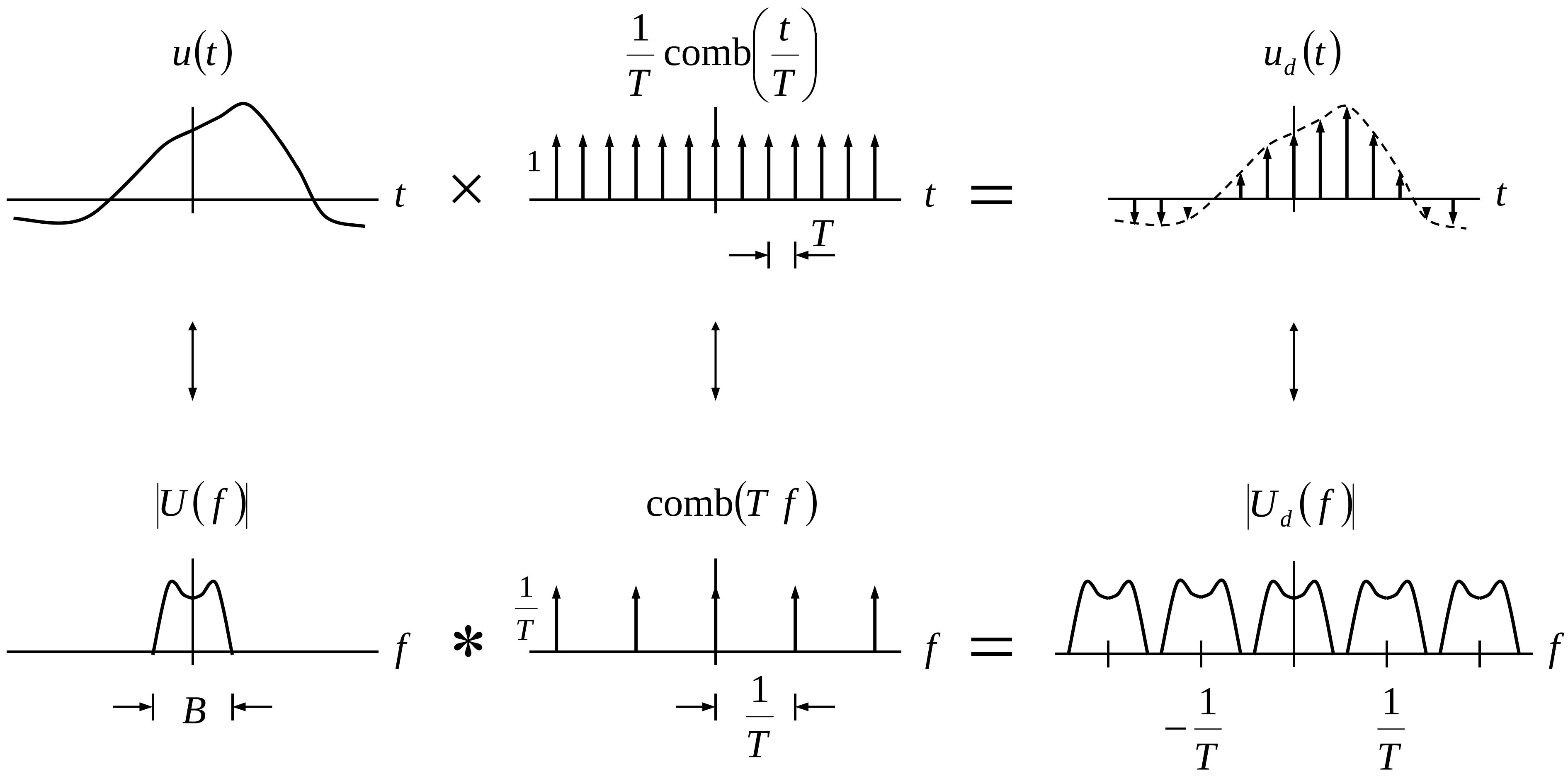
$$u_d(t) = u(t)\frac{1}{T}\operatorname{comb}\left(\frac{t}{T}\right) \longleftrightarrow U_d(f) = U(f) * \operatorname{comb}(T\,f)$$



$$1/T\,\operatorname{comb}(t/T) \longleftrightarrow \operatorname{comb}(T\,f)$$

$$U_d(f) = U(f) * \operatorname{comb}(T\,f) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} U\left(f - \frac{k}{T}\right)$$

Signal Sampling



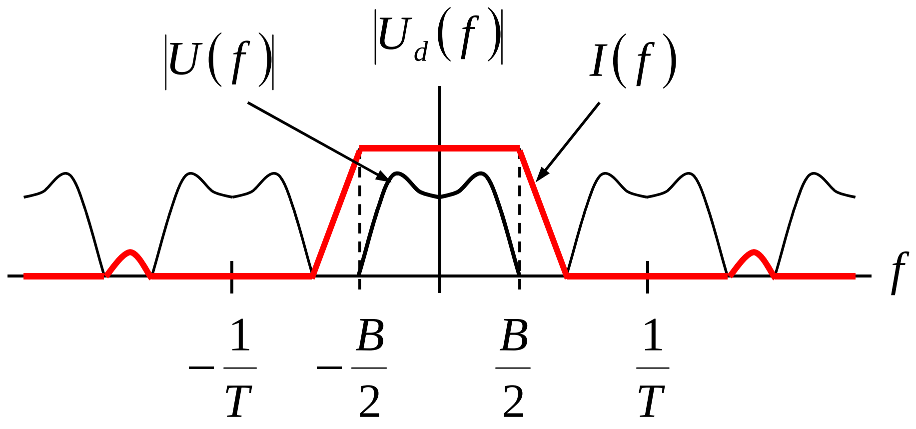
sampling of the signal = periodic replication of the spectrum

Reconstruction Filter (Interpolator)

if (mathematical) Bandwidth $B < \frac{1}{T}$, i.e. $2 f_{\max} = B < \frac{1}{T}$ (for real LP-Signals):

$\Rightarrow$ no spectral aliasing
$\Rightarrow$ lossless reconstruction by using low-pass interpolator:

$$I(f) = \begin{cases} T & |f| \leq B/2 \\ 0 & |f - n/T| \leq B/2, \quad n \neq 0 \\ \text{arbitrary} & \text{else} \end{cases}$$



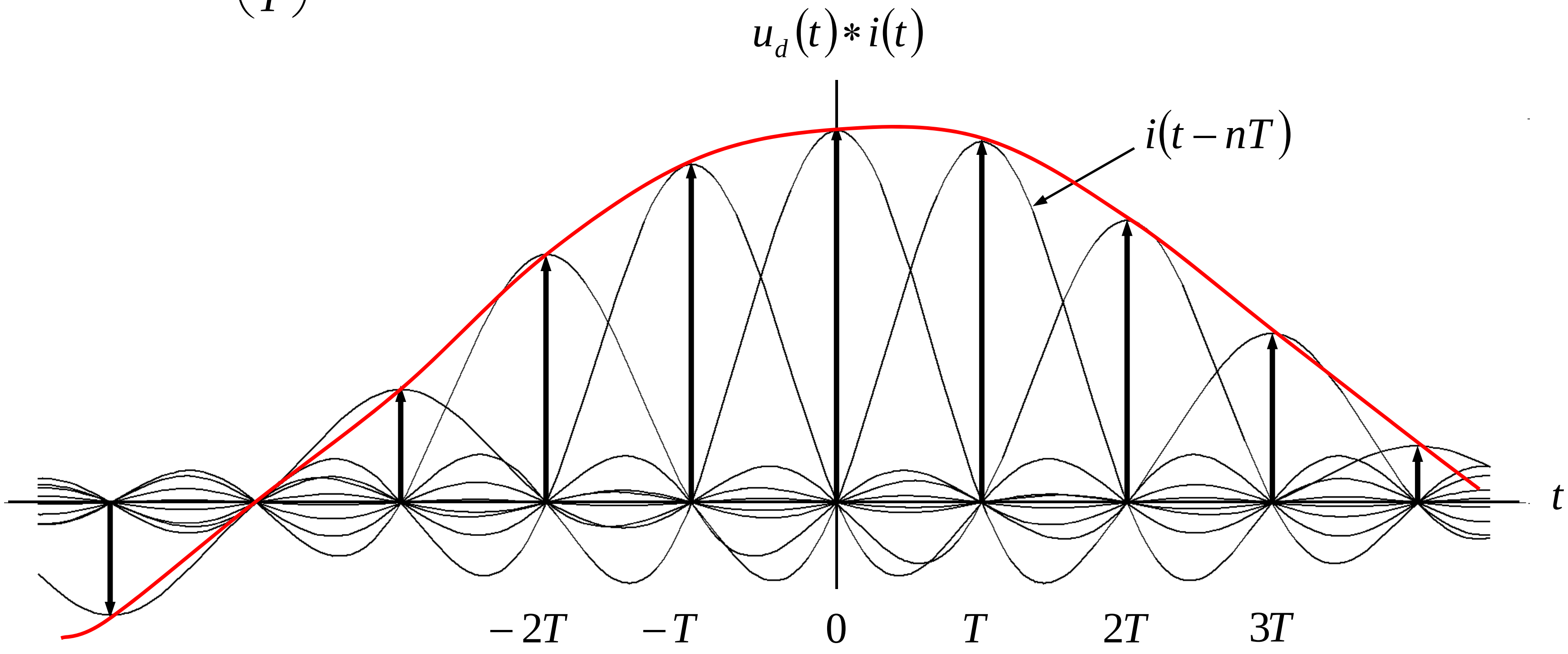
$$u(t) = u_d(t) * i(t) = \sum_{n=-\infty}^{+\infty} u[n]\, i(t - nT)$$

Example:

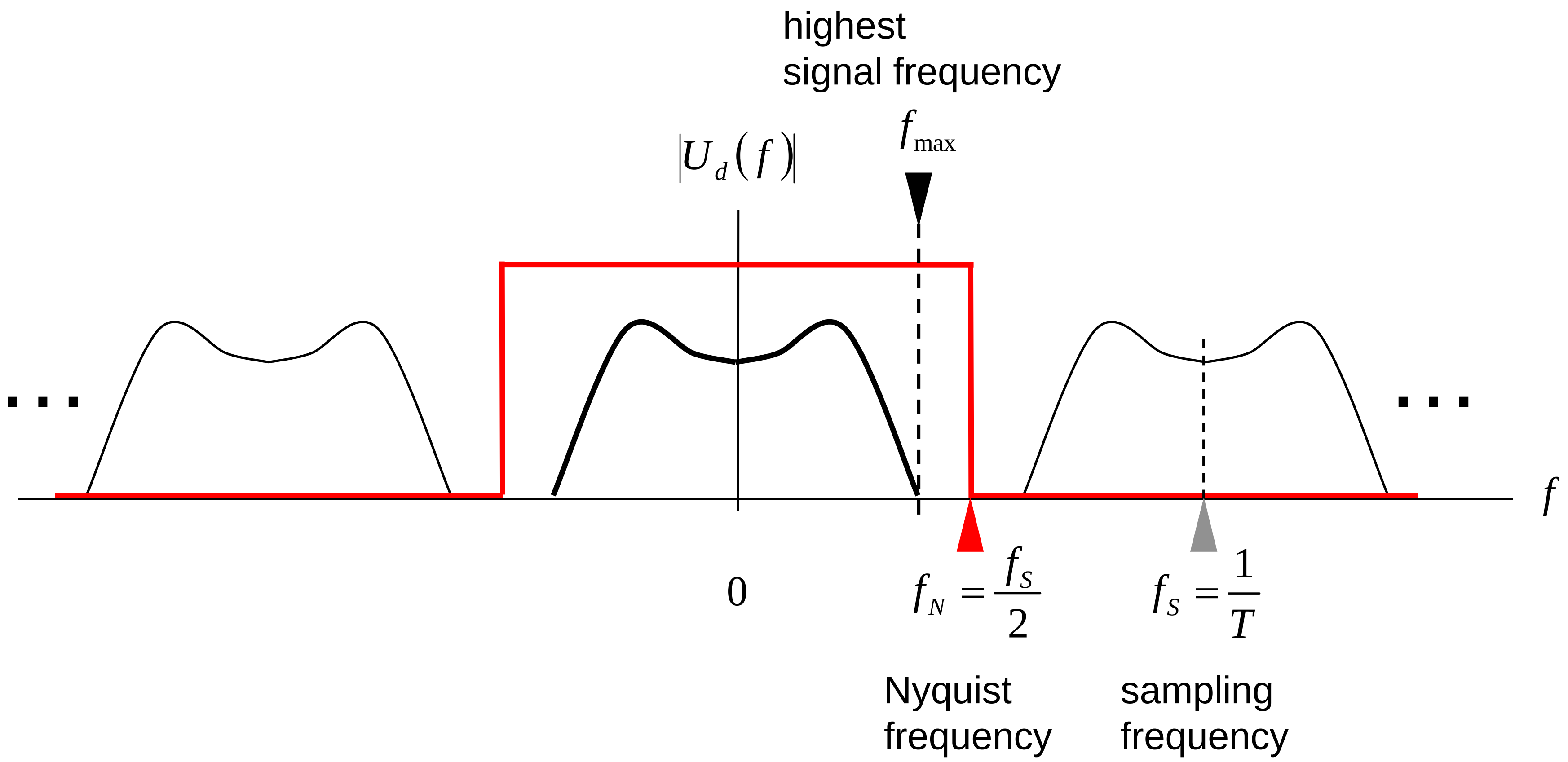
$$I(f) = T\ \mathrm{rect}(T\ f)$$ ideal low-pass

$$\updownarrow$$

$$i(t) = \mathrm{sinc}\left(\frac{t}{T}\right)$$



Nyquist-Shannon Sampling Theorem (for Real LP Signals)

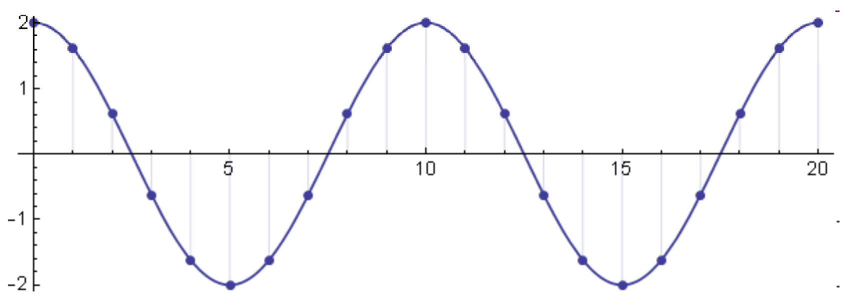


Sampling Theorem: $f_{\max} \overset{!}{<} f_N = \frac{f_S}{2}$

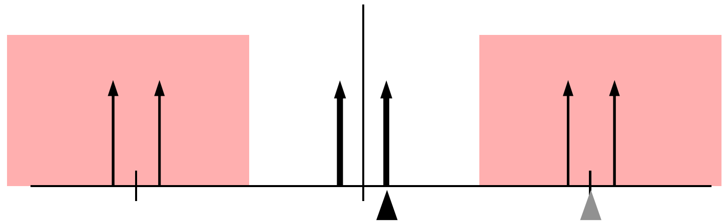
Example: Sampling of a Sinosoidal Signal

$f_S = 1{,}000\ Hz$

$$u(t) = 2\cos(2\pi f_0 t) \quad \leftrightarrow \quad U(f) = \delta(f + f_0) + \delta(f - f_0)$$

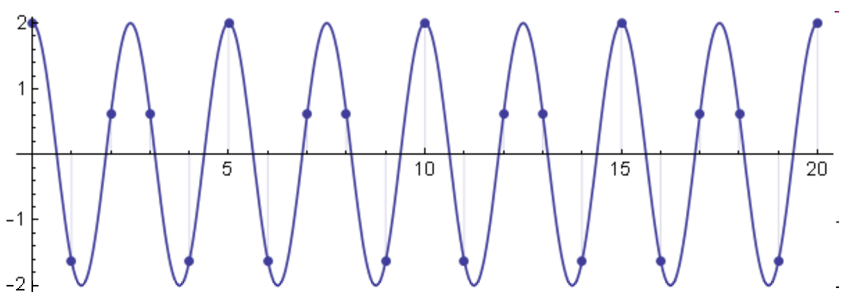


$t\,[ms]$

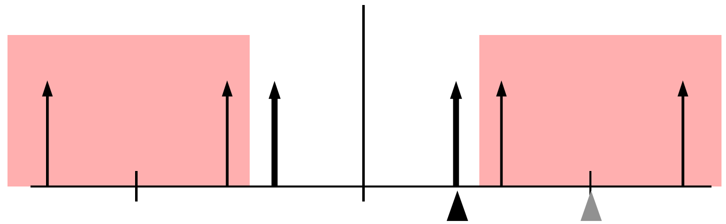


$f\,[Hz]$

$f_0 = 100\ Hz$ $\quad f_S = 1{,}000\ Hz$



$t\,[ms]$



$f\,[Hz]$

$f_0 = 400\ Hz$ $\quad f_S = 1{,}000\ Hz$

Example: Sampling of a Sinosoidal Signal (cont.)

$f_S = 1{,}000\ Hz$

$$u(t) = 2\cos(2\pi f_0 t) \quad \leftrightarrow \quad U(f) = \delta(f + f_0) + \delta(f - f_0)$$