TECHNISCHE
UNIVERSITÄT
MÜNCHEN

Remote Sensing
Technology
Prof. Dr.-Ing. R. Bamler

Practice exam

Systems Theory and Signal Processing

Date:

Duration: 90 min.

Allowed auxiliaries: non-programmable calculator, documents of the lecture and the tutorial „Systems Theory and Signal Processing“

Problem	Topic	Points possible	Points achieved
1	Graphical Convolution	13	
2	Analytical Convolution	7	
3	Fourier Transform	9	
4	Linear Time-Invariant Systems	20	
5	2-D Fourier Transform	12	
6	Radon Transform	9	
	total	70	

Last name : . . . . . Grade:

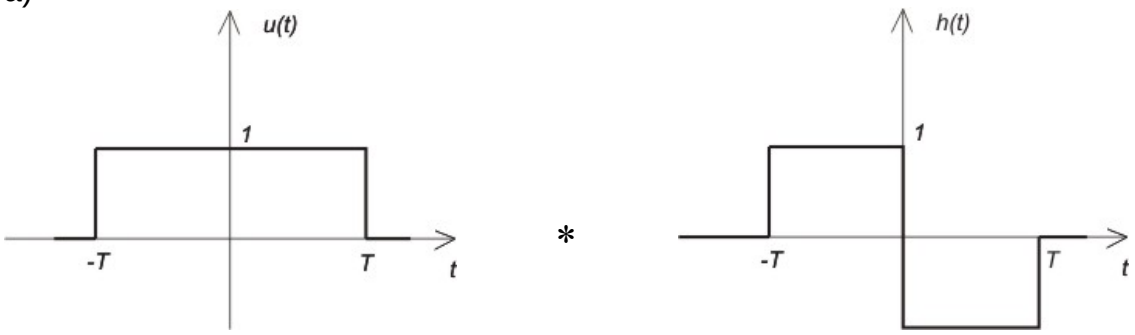
First name : . . . . . .

Student number : . . . . . .

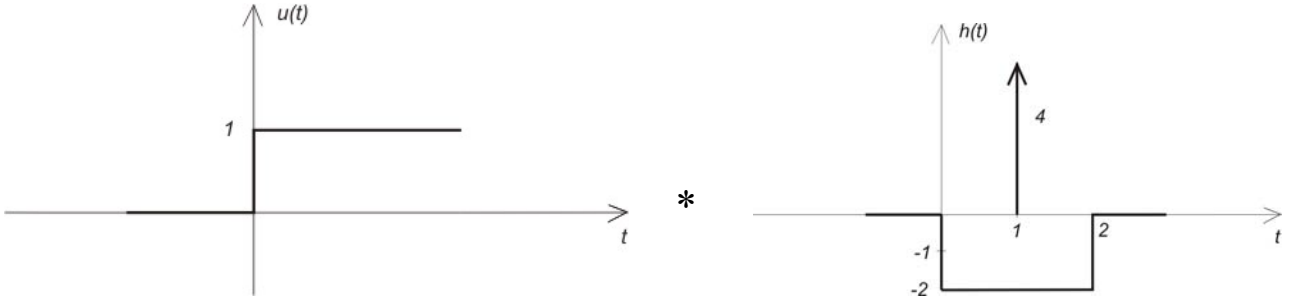
Problem 1 (13 Points): Graphical Convolution

Convolve the following signals graphically.

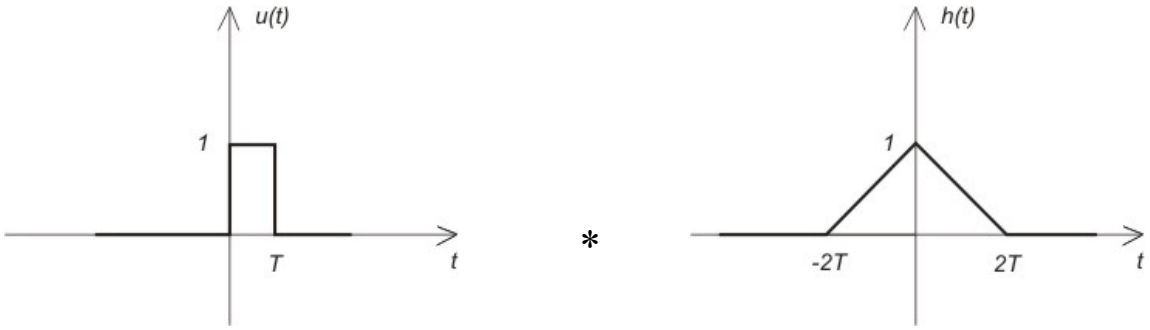
a)



b)



c)



Last name: . . . . . Points:

Problem 2 (7 Points): Analytical Convolution

Evaluate the following convolution analytically and sketch the result.

a) $y(t) = \cos\left(\frac{2\pi\ t}{T}\right) * rect\left(\frac{t - \frac{T}{2}}{\frac{T}{2}}\right)$

Problem 3 (9 Points): Fourier Transform

Evaluate the Fourier transform of the following time-domain signal $u_a(t)$ and the inverse Fourier Transform of the spectrum $U_b(f)$.

a) $u_a(t) = \gamma(t)\exp\left(-\frac{t}{T}\right)\cos\left(2\pi\ f_0 t\right)$

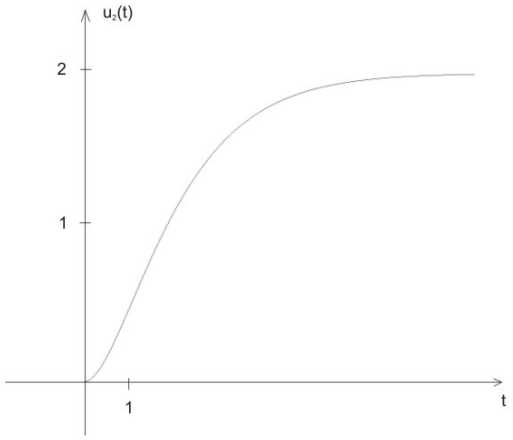
b) $U_b(f) = rect\left(\frac{f+f_0}{B}\right) - rect\left(\frac{f-f_0}{B}\right)$

Problem 4 (20 Points): Linear Time-Invariant Systems

4.1) The impulse response function $h(t)$ of a LTI system is given by $h(t) = \gamma(t)\frac{1}{a}\exp\left(-\frac{t}{a}\right)$.

Calculate the output signal $u_2(t)$ of the LTI system, if the input signal is given by $u_1(t) = \gamma(t)$.

4.2) An LTI system answers with the output signal $u_2(t) = 2\gamma(t)\left(1 - \exp(-t) - t\exp(-t)\right)$, if the input signal $u_1(t) = \gamma(t)$ is inducted.



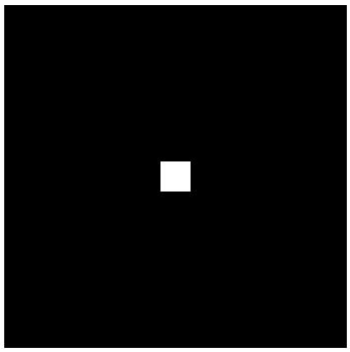
a) Calculate the system's impulse response function $h(t)$.

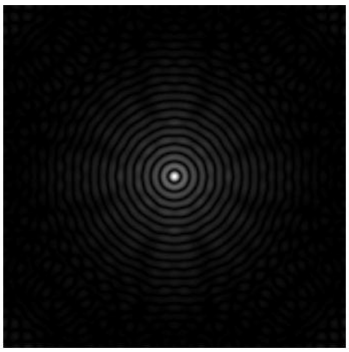
b) Evaluate its transfer function $H(f)$.

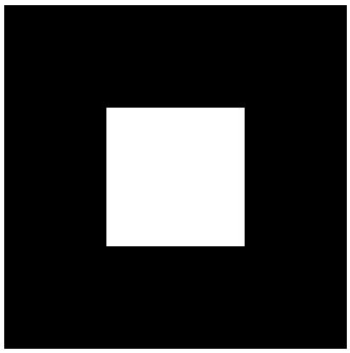
c) Evaluate the output signal for the input signal $u_1(t) = \gamma(t)\exp(-t)$.

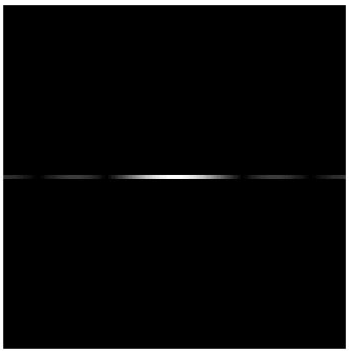
Problem 5 (12 Points): 2-D Fourier Transform

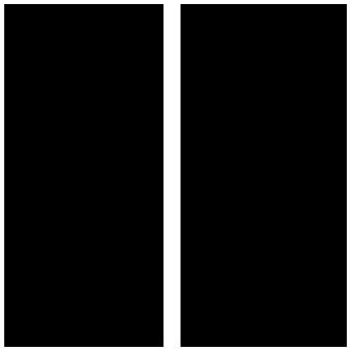
Below images of the magnitude $|u_n(x,y)|$ of six 2-D signals are shown in the spatial domain (left side). In addition, the spectra $|U_n(f_x,f_y)|$ of these signals are presented in an arbitrary order (right side). Assign the images to their respective 2-D Fourier transformed spectra. Insert the answers into the designated fields below.

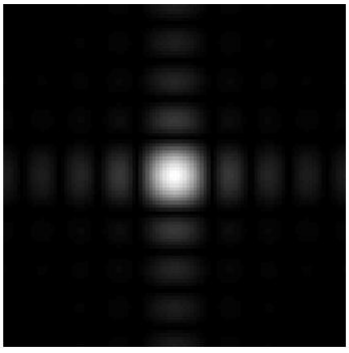
1) 

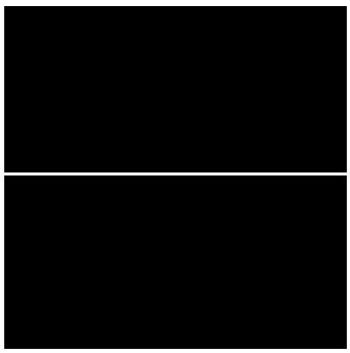
a) 

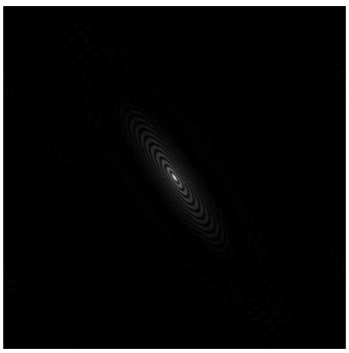
2) 

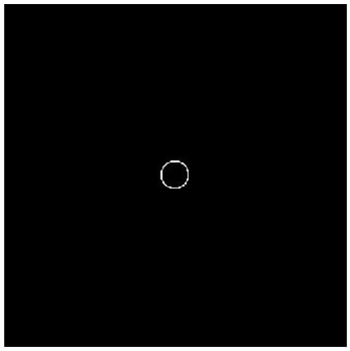
b) 

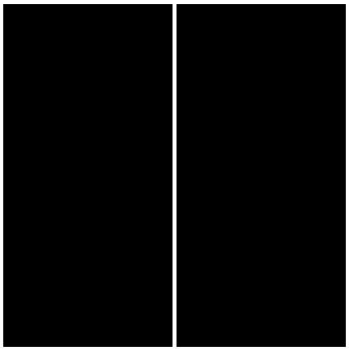
3) 

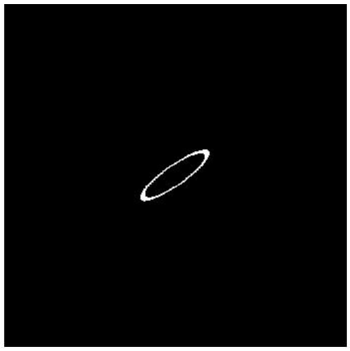
c) 

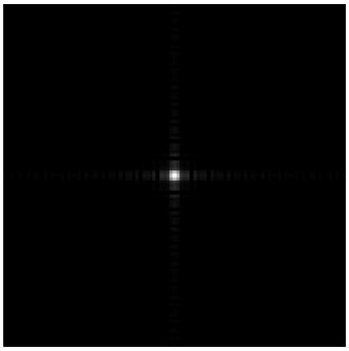
4) 

d) 

5) 

e) 

6) 

f) 

answer:

1) ↔ . . . . . .

2) ↔ . . . . . .

3) ↔ . . . . . .

4) ↔ . . . . . .

5) ↔ . . . . . .

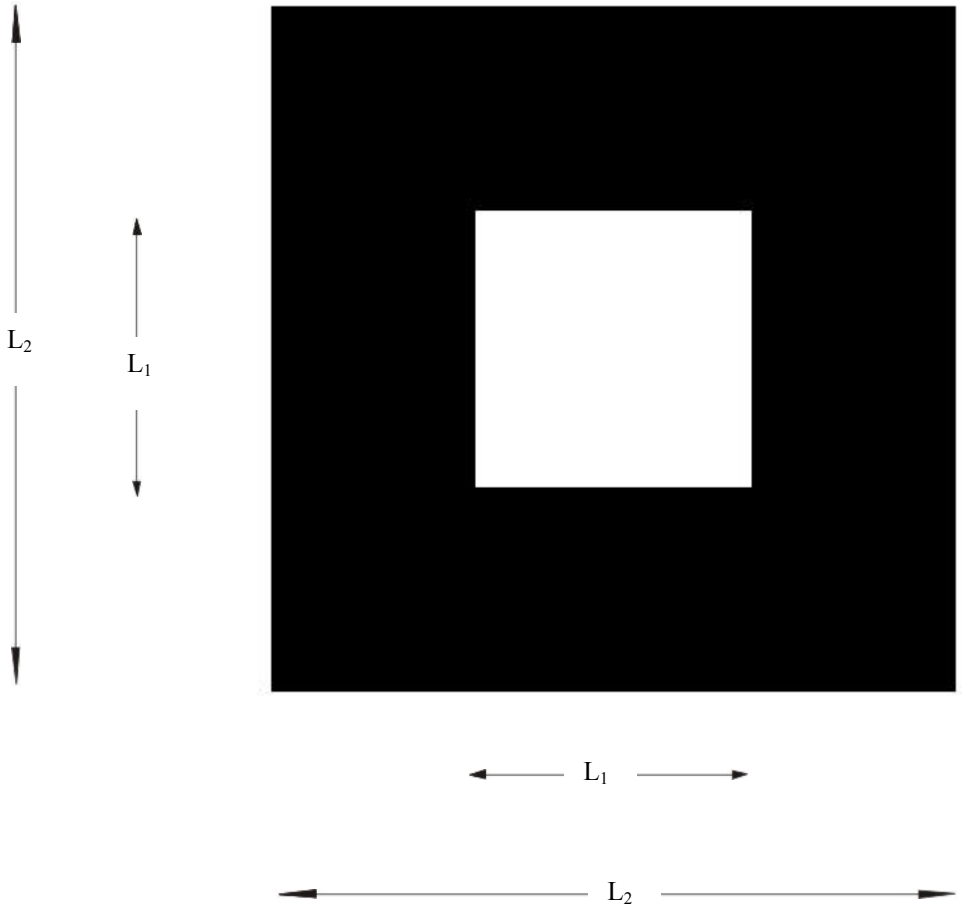
6) ↔ . . . . . .

Last name: . . . . . Points:

Problem 6 (9 Points): Radon Transform

Sketch the Radon transformed functions $u_P(R;\varphi)$, i. e. the projections, of the following signal $u_n(x,y)$ for $\varphi = 0°,\ 30°,\ 45°,\ 90°$.

$u_n(x,y)$



Last name: . . . . . Points:

Hints:

- $\int x \exp(ax) dx = \frac{\exp(ax)}{a^2}(ax-1) \qquad a \in$
- $\exp(-at) \cdot \gamma(t) \quad \leftrightarrow \quad \frac{1}{a + j2\pi \ f}$