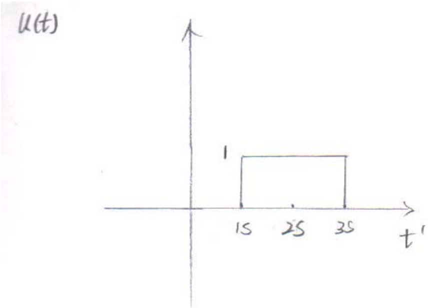
Applied Signal Processing and Computer Science

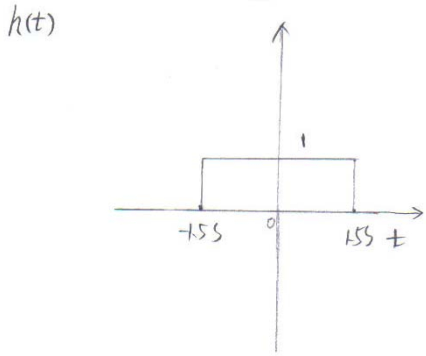
WS 10/11(Email: xiaoxiang.zhu@bv.tum.de)

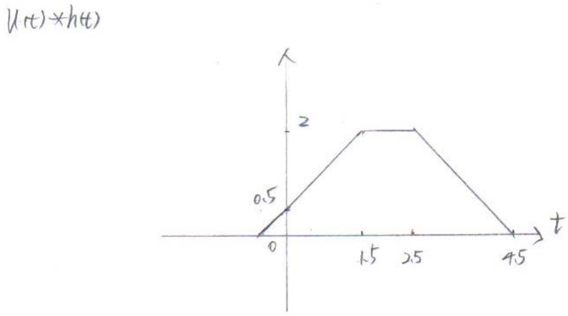
Solution 3: Convolution

1. Graphical Convolution:

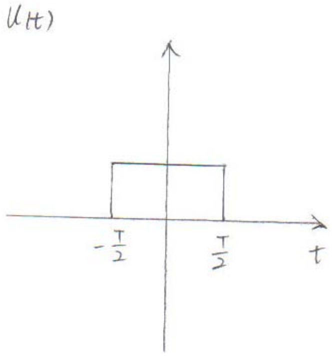
- $rect(\frac{(t-T_1)}{T_1}) * rect(\frac{t}{T_2})\ with\ T_1 = 2s\ and\ T_2 = 3s$

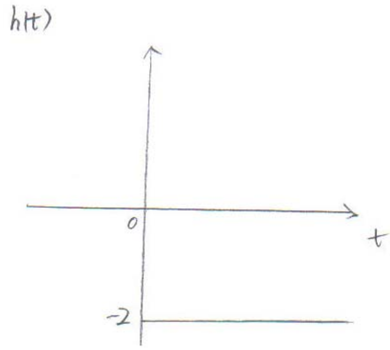


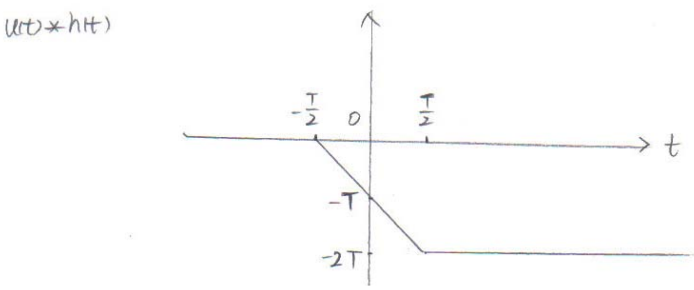




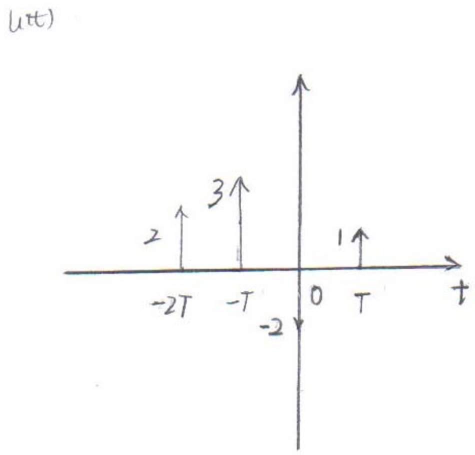
- $rect(\frac{t}{T}) * (-2\gamma(t))$

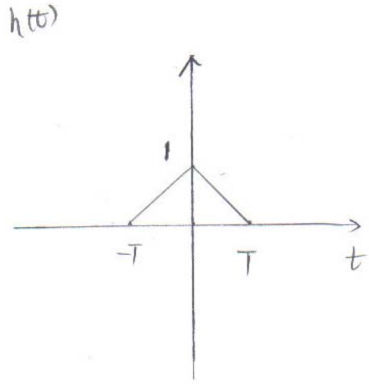


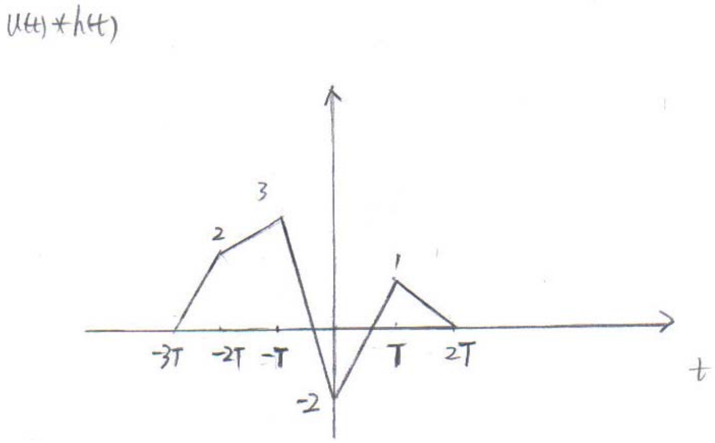




- $[2\delta(t+2T)+3\delta(t+T)-2\delta(t)+\delta(t-T)]*tri(\frac{t}{T})$







2. Analytical Convolution

2.1

- $e^{(\frac{-t^2}{a_1^2})} * e^{(\frac{-t^2}{a_2^2})}$

Hints: $\int_{-\infty}^{\infty} e^{2bx-ax^2}dx = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{a}}$

$$e^{(\frac{-t^2}{a_1^2})} * e^{(\frac{-t^2}{a_2^2})} = \int_{-\infty}^{\infty} e^{(\frac{-t'^2}{a_1^2})} e^{(\frac{-(t-t')^2}{a_2^2})} dt' = e^{(\frac{-t^2}{a_2^2})} \int_{-\infty}^{\infty} e^{(\frac{2t}{a_2^2}t' - t'^2(\frac{1}{a_1^2}+\frac{1}{a_2^2}))} dt'$$

$$b =: \frac{t}{a_2^2} \qquad a =: (\frac{1}{a_1^2}+\frac{1}{a_2^2})$$

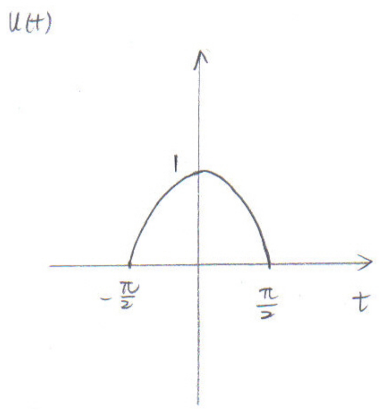
$$\Rightarrow e^{(\frac{-t^2}{a_2^2})} \int_{-\infty}^{\infty} e^{(\frac{2t}{a_2^2}t' - t'^2(\frac{1}{a_1^2}+\frac{1}{a_2^2}))} dt' = e^{(\frac{-t^2}{a_2^2})} \int_{-\infty}^{\infty} e^{2bt'-at'^2} dt' = e^{(\frac{-t^2}{a_2^2})} \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{a}}$$

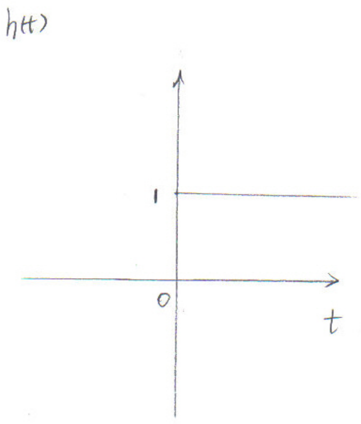
Insert a, b into the result:

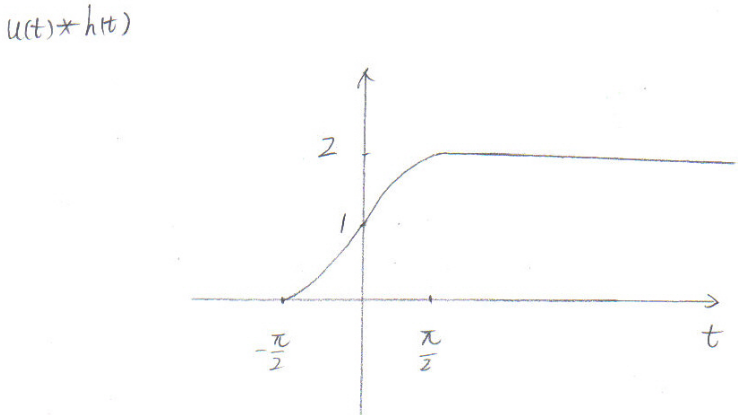
$$\Rightarrow e^{(\frac{-t^2}{a_2^2})} \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{a}} = e^{(\frac{-t^2}{a_2^2})} \sqrt{\frac{\pi}{(\frac{1}{a_1^2}+\frac{1}{a_2^2})}} e^{\frac{(\frac{t}{a_2^2})^2}{(\frac{1}{a_1^2}+\frac{1}{a_2^2})}} = \sqrt{\frac{\pi a_1^2 a_2^2}{(a_1^2+a_2^2)}} e^{-\frac{t^2}{(a_1^2+a_2^2)}}$$

2.2 $(\cos(t)\cdot rect(\frac{t}{\pi}))*\gamma(t)$

Graphical:







Analytical:

$$(\cos(t)\cdot rect(\frac{t}{\pi}))*\gamma(t) = \int_{-\infty}^{+\infty} (\cos(t^{'})\cdot rect(\frac{t^{'}}{\pi}))\cdot\gamma(t-t^{'})dt^{'}$$

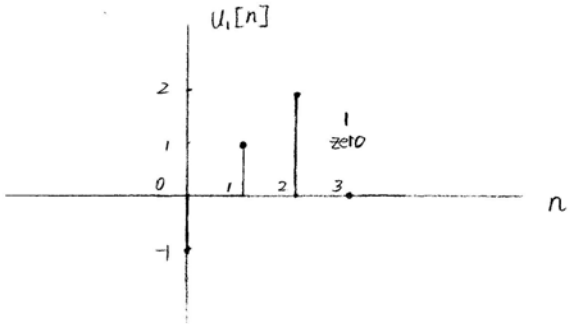
$$= \begin{cases} 0 & for \quad t < -\frac{\pi}{2} \\ \int_{-\frac{\pi}{2}}^{t} \cos(t^{'})dt^{'} & for \quad -\frac{\pi}{2} \le t \le -\frac{\pi}{2} \\ 2 & for \quad t > \frac{\pi}{2} \end{cases}$$

$$= \begin{cases} 0 & for \quad t < -\frac{\pi}{2} \\ 1+\sin(t) & for \quad -\frac{\pi}{2} \le t \le -\frac{\pi}{2} \\ 2 & for \quad t > \frac{\pi}{2} \end{cases}$$

3. Discrete-time convolution

$N_1 = 3, \quad N_h = 2, \quad N_2 = N_1 + N_h - 1 = 4$

$N = 4$



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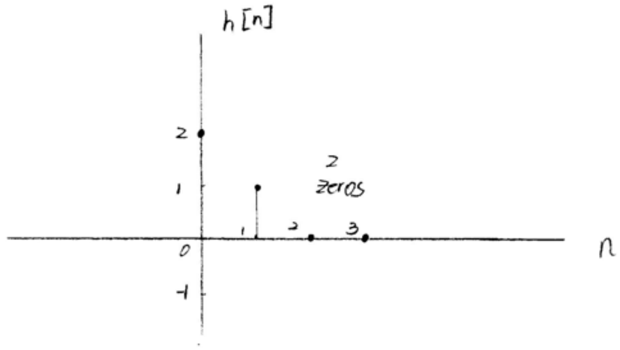

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