Recitation 19

Spring 2021

Question 1

Consider a firm that generates \$10M in earnings before interest and taxes in each year (starting in Year 1) in perpetuity with no risk. Suppose the firm takes a perpetual loan on which it is required to pay \$8M interest each year in perpetuity.
Assume that the corporate income tax rate is 25%. The risk-free rate is 2%.
What is the value of this firm?

Solutions:

We will use the APV method to find the value of this firm, which is the value of the unlevered firm plus the present value of tax shields.
The unlevered firm value is the value of its perpetual cash flows starting in year 1, discounted at the risk-free rate since the earnings have no risk. The cash flow is $(1-\tau) \times EBIT_1$. So we have:

$$\begin{aligned} V_U &= \frac{(1-\tau)EBIT_1}{r_f} \\ &= \frac{(1-25\%) \times \$10M}{2\%} \\ &= \$375M \end{aligned}$$

Now, we find the present value of tax shields. Since the firm generates \$10M in risk-free earnings, it with certainty has enough to pay interest each year. This means the debt is risk-free, so that tax shields generated are risk-free as well:

$$\begin{aligned} PVTS &= \frac{\tau \times I}{r_f} \\ &= \frac{25\% \times \$8M}{2\%} \\ &= \$100M \end{aligned}$$

So the levered firm value is:

$$\begin{aligned} V_L &= V_U + PVTS \\ V_L &= \$375M + \$100M \\ &= \$475M \end{aligned}$$

Note that roughly 21% of firm value (100/475) comes from interest tax shields.

Question 2

Consider a firm that generates \$10M in earnings before interest and taxes each year (starting in Year 1) in perpetuity. The required return on assets of this firm is 10%. Suppose that the firm's financing policy is such that it constantly maintains debt at 25% of its asset value. At this level, the debt is risk-free.
Assume that the corporate income tax rate is 25%. The risk-free rate is 2%.
What is the value of this firm?

Solutions:

We will again use the APV method to find the value of this firm. We discount the cash flows at r_A to find the unlevered value of the firm:

$$\begin{aligned} V_U &= \frac{(1-\tau)EBIT_1}{r_A} \\ &= \frac{(1-25\%) \times \$10M}{10\%} \\ &= \$75M \end{aligned}$$

Now, we find the present value of tax shields. Since the firm constantly adjusts the amount of debt such that the debt to assets ratio stays at 25%, the future debt amount and tax shields are as uncertain as the cash flows of the firm. So, we need to discount the tax shields at the asset cost of capital.
The current value of debt is $25\% \times \$75M = \$18.75M$. So the present value of tax shields is:

$$\begin{aligned} PVTS &= \frac{\tau \times r_f \times D}{r_A} \\ &= \frac{25\% \times 2\% \times \$18.75M}{10\%} \\ &= \$0.94M \end{aligned}$$

So the value of the levered firm is:

$$\begin{aligned} V_L &= V_U + PVTS \\ &= \$75M + \$0.94M \\ &= \$75.94M \end{aligned}$$

Note that only 1.2% of firm value (0.94/75.94) comes from interest tax shields.

Question 3

There is a publicly traded firm, ABC. It has 12 million shares outstanding, and its shares are traded at \$25 per share. We will assume that, as of now, this company is all equity financed and has no debt on its balance sheet. ABC is considering a share repurchase program. It plans to take out a loan of \$100 million and use these funds to repurchase some of its existing shares. Assume a tax rate of 21%.

(a) What is the current market value of this firm?

(b) What is the value of this firm and the value of equity after the loan is taken but before the shares are repurchased?

(c) What is the value of the firm and the value of equity after the shares have been repurchased?

(d) What is the share price of the stock after the recapitalization has been announced after ABC has received the loan, but before it has actually repurchased shares?

(e) How many shares can be repurchased?

(f) What happens right after the announcement of the share repurchase?

Solutions:

(a) Since the firm is all equity-financed, the firm's market value is equal to its equity value. The equity value is:

$$V_{FIRM} = V_{EQUITY} = \$25 \times 12M = \$300M$$

(b) On the assets side of the MVBS (market value balance sheet), we have the cash from the loan, the original assets, and the present value of tax shields. The firm receives \$100 million of cash and has \$300 million of original assets. The present value of the tax shields from the loan is $\tau \times \$100 = \$21M$, assuming that the loan is perpetual (never repay the principal).

On the liabilities side of the balance sheet, we have debt and equity. We know that the debt is \$100 million. So, the value of equity is the value of assets minus the value of debt:

$$\begin{aligned} V_{EQUITY} &= V_{ASSETS} - V_{DEBT} \\ &= (\$100M + \$300M + \$21M) - \$100M \\ &= \$321M \end{aligned}$$

(c) After the shares are repurchased, the cash on the asset side of the balance sheet goes to \$0. The original assets remain at \$300M, and the PVTS remains at \$21M. So, the firm has \$321M of assets on the left hand side of the MVBS. The value of the firm is \$321M.

On the liabilities side of the MVBS, we have \$100M of debt. The remaining is equity. So, the equity value of the firm after the shares have been repurchased is:

$$V_{EQUITY} = V_{ASSETS} - V_{DEBT} = \$321M - \$100M = \$221M$$

(d) We found in part b that the market value of equity is \$321 million after the loan is taken out but before the shares are repurchased. The price per share is:

$$P = \frac{V_{EQUITY}}{SHO} = \frac{\$321M}{12M} = \$26.75$$

(e) We have \$100 million to repurchase shares, each worth \$26.75. So we can repurchase $\frac{\$100M}{\$26.75} = 3.75M$ shares. There are $12 - 3.74 = 8.26$ million shares remaining.

As expected, the value of equity after the repurchase is $\$26.75 \times 8.26M = \$221M$.

(f) As soon as the announcement is made, the market will understand the tax shield implications of this recapitalization and price that into the equity. In an efficient market, the share price will jump to \$26.75 after the announcement and remain at that level through the recapitalization process.

Question 4

You are a consultant who was hired to evaluate a new product line for Kendall Enterprises. The up front investment required to launch the product line is \$10 million. The product will generate free cash flow of \$750,000 the first year, and this cash flow is expected to grow at a rate of 4% per year.
Kendall has an equity cost of capital of 11.3%, a debt cost of capital of 5%, and a tax rate of 35%. Kendall maintains a debt-to-equity ratio of 0.40.

(a) What is the NPV of the new product line (including any tax shields from leverage)?

(b) How much debt will Kendall initially take on as a result of launching this product line?

(c) How much of the product line's value is attributable to the present value of interest tax shields?

Solutions:

(a) The NPV of the new product line is the investment cost, plus the present value of cash flows from the line.

The PV of CF from the new line is:

$$PV = \frac{FCF_1}{WACC - g} = \frac{\$750K}{WACC - 4\%}$$

We need to find WACC:

$$\begin{aligned} WACC &= r_E \times \frac{E}{V} + r_D \times (1 - \tau) \times \frac{D}{V} \\ &= 11.3\% \times \frac{1}{1.4} + 5\% \times (1 - 35\%) \times \frac{0.4}{1.4} \\ &= 9\% \end{aligned}$$

So the PV of cash flows from the new line is:

$$PV = \frac{\$750K}{9\% - 4\%} = \$15M$$

The NPV of the new line is:

$$\begin{aligned} NPV &= -I + PV \\ &= -\$10M + \$15M \\ &= \$5M \end{aligned}$$

(b) This product line will add \$15M in assets to the LHS of the MVBS. We know that Kendall keeps a debt-to-equity ratio of 0.4. So, its debt-to-value ratio is $0.4/1.4 = 28.57\%$.

In order to maintain the ratio, Kendall needs to take additional debt of:

$$28.57\% \times \$15M = \$4.29M$$

The remaining \$10.71M needs to be financed with equity.

(c) We can first find the value of the project if it was unlevered, then subtract that from the actual value of the project. If the new line was financed with equity, its PV of CF would have been:

$$PV = \frac{FCF_1}{r_A - g} = \frac{\$750K}{r_A - 4\%}$$

We can find r_A:

$$\begin{aligned} r_A &= r_E \times \frac{E}{V} + r_D \times \frac{D}{V} \\ &= 11.3\% \times \frac{1}{1.4} + 5\% + \frac{0.4}{1.4} \\ &= 9.5\% \end{aligned}$$

So the present value of the project if it was equity-funded is:

$$PV = \frac{\$750K}{9.5\% - 4\%} = \$13.64M$$

So the value of the new line attributable to PVTS is $\$15M - \$13.64M = \$1.36M$.

Alternatively, we could have solved this by finding the PVTS directly from calculating interest tax shields. The initial debt is \$4.29M, so the tax shield in the first year is $4.29M \times 5\% \times 35\% = \$0.075M$. We use the r_A as the discount rate on tax shields since the firm maintains a constant debt to equity ratio, which means the tax shields have the same riskiness as the assets:

$$PVTS = \frac{\$0.075M}{9.5\% - 4\%} = \$1.36M$$

Question 5

Consider a firm that is expected to generate \$100 million in earnings before interest in Year 1. These earnings are expected to grow at a 2% rate in perpetuity. Suppose that assets of this firm have a market beta of 1.35.
The firm has a \$500 million loan, which carries an interest of 5%. Each year, the firm is planning to increase the loan amount proportionally with its earnings to maintain a constant debt-to-equity ratio in perpetuity. Assume that the cost of debt will remain at 5%.
Assume that the risk-free rate is 3% and the expected return on the market portfolio is 10%. The corporate income tax rate is 25%.

(a) What would be the value of this firm if it was 100% equity financed?

(b) What is the present value of interest tax shields?

(c) What is the value of this firm?

(d) What is the weighted average cost of capital (WACC) for this firm?

(e) Find the value of this firm using WACC and compare it to part (c).

Solutions:

(a) The unlevered value of the firm is:

$$V_U = \frac{(1-\tau) \times EBIT}{r_A - g} = \frac{(1-25\%) \times \$100M}{r_A - 2\%}$$

We need to find r_A. We can do so by using CAPM:

$$\begin{aligned} r_A &= r_f + \beta_A(E[r_M] - r_f) \\ &= 3\% + 1.35 \times (10\% - 3\%) \\ &= 12.45\% \end{aligned}$$

So,

$$\begin{aligned} V_U &= \frac{1 - 25\% \times \$100M}{12.45\% - 2\%} \\ &= \$717.70M \end{aligned}$$

(b) Since the firm value grows at a rate of 2%, the debt value has to grow at 2% as well for the firm to maintain a constant leverage ratio.

The interest tax shield in Year 1 is:

$$ITS_1 = \tau \times I_1 = 25\% \times 5\% \times \$500M = \$6.25M$$

And, since the firm maintains a constant leverage ratio, the risk of the ITS is the same as the risk of the firm's assets. So, we discount at r_A:

$$\begin{aligned} PVTS &= \frac{ITS}{r_A - g} \\ &= \frac{\$6.25M}{12.45\% - 2\%} \\ &= \$59.81M \end{aligned}$$

(c) The value of the firm is value of the unlevered firm plus the PVTS:

$$\begin{aligned} V_L &= V_U + PVTS \\ &= \$717.70M + \$59.81M \\ &= \$777.51M \end{aligned}$$

(d) By definition, WACC is:

$$WACC = r_E \times \frac{E}{V} + r_D \times (1-\tau) \times \frac{D}{V}$$

We first need to find r_E. By MM II:

$$\begin{aligned} r_E &= r_A + \frac{D}{E}(r_A - r_D) \\ &= 12.45\% + \frac{\$500M}{\$777.51M - \$500M}(12.45\% - 5\%) \\ &= 25.87\% \end{aligned}$$

So:

$$\begin{aligned} WACC &= r_E \times \frac{E}{V} + r_D(1-\tau) \times \frac{D}{V} \\ &= 25.87\% \times \frac{\$277.51M}{\$777.51M} + 5\% \times (1 - 25\%) \times \frac{\$500M}{\$777.51M} \\ &= 11.65\% \end{aligned}$$

(e) Using WACC, the firm value is:

$$\begin{aligned} V_L &= \frac{(1-\tau)EBIT}{WACC - g} \\ &= \frac{(1 - 25\%) \times \$100M}{11.65\% - 2\%} \\ &= \$777.51M \end{aligned}$$