15.415x Foundations of Modern Finance

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Lecture 13: Options, Part 2

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model

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Binomial option pricing model

- Consider the binomial model in a general form.
- The stock prices follows a binomial process:

$$
S_0 \begin{cases} uS_0 \\ dS_0 \end{cases}
$$

- Interest rate is r: assume $u > 1 + r > d$ to avoid arbitrage between the stock and the risk-free asset.
- Consider a European call option on the stock with a strike of *K*. Its payoff is

$$
C_0 \begin{cases} C_u = \max[0, uS_0 - K] \\ C_d = \max[0, dS_0 - K] \end{cases}
$$

- We price the call by replication.

Binomial option pricing model

- Form a replicating portfolio with the stock and bond:
  - δ shares of the stock,
  - b dollars in the riskless bond.

  such that:

$$\delta u S_0 + b(1+r) = C_u$$

$$\delta d S_0 + b(1+r) = C_d$$

- Unique solution:

$$\delta = \frac{C_u - C_d}{(u-d)S_0}, \qquad b = \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)}$$

- We then have:

$$C_0 = \delta S_0 + b = \frac{C_u - C_d}{u-d} + \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)}$$

Risk neutral probability

- Define:

$$q_u = \frac{(1+r)-d}{u-d}, \qquad q_d = \frac{u-(1+r)}{u-d}$$

- Since $0 < q_u, q_d < 1$ and $q_u + q_d = 1$, we can interpret $q_u = q$ and $q_d = 1 - q$ as probabilities for the up- and down-states.

- We can then write:

$$C_0 = \frac{q_u C_u + q_d C_d}{1+r} = \frac{\mathrm{E}^Q[C_T]}{1+r}$$

where $\mathrm{E}^Q[\cdot]$ is the expectation under probability $Q = (q, 1-q)$, which is called the risk-neutral probability.

Why do risk neutral probabilities work?

- Start with the knowledge that the payoff of any option can be replicated by trading in the stock and the bond.
- Change probabilities so that expected stock return is equal to the risk-free rate at each node. Call the new probabilities Q-probabilities.
- Then, expected return on the replicating portfolio under the Q-probabilities is the weighted average of stock and bond expected returns, so it equals the risk-free rate.
- Apply the DCF formula to the terminal value of the replicating portfolio, which equals the option payoff.
  - Discount rate in the DCF formula is the expected return on the replicating portfolio, which is the risk-free rate under the Q-probabilities.
  - Conclusion: option price at time $t = 0$ equals the expected payoff, under the Q-probabilities, discounted at the risk-free rate.

Risk-neutral valuation applies to all assets

- With the risk neutral probability, we can price any asset easily.
- Consider the example from "Options, Part 1:"
  - Parameters are $S = 50$ and $u = 1.5, d = 0.5, r = 1.1$. Then,
  $$q = \frac{1.1 - 0.5}{1.5 - 0.5} = 0.6$$
  - The stock price is:
  $$S_0 = \frac{(0.6)(75) + (0.4)(25)}{1 + 0.1} = 50$$
  - The bond price is:
  $$B = \frac{(0.6)(1.1) + (0.4)(1.1)}{1 + 0.1} = 1$$
  - The price of a call option on the stock with a strike of \$50 is:
  $$C_0 = \frac{(0.6)(25) + (0.4)(0)}{1 + 0.1} = 13.64$$

Multiple periods

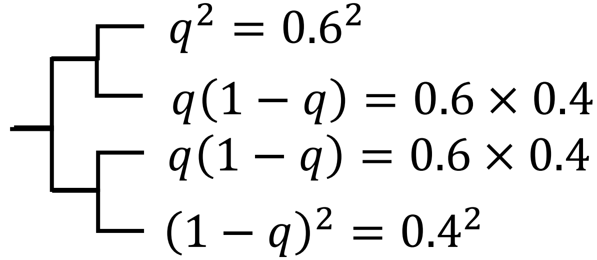
- A two-period call on the stock with a strike $K = 50$:

$$C_0 = \frac{\mathrm{E}^Q[C_2]}{(1+r)^2}$$

$$= \frac{\begin{pmatrix}(0.6)^2(62.5) + (0.6)(0.4)(0) \\ +(0.4)(0.6)(0) + (0.4)^2(0)\end{pmatrix}}{(1+0.1)^2}$$

$$= \frac{22.5}{1.1^2} = 18.60$$

Q-probabilities of time-2 states



- A put on the stock with a strike $K = 50$:

$$P_0 = \frac{(0.6)^2(0) + 2(0.6)(0.4)(12.5) + (0.4)^2(37.5)}{(1+0.1)^2}$$

$$= \frac{12.0}{1.1^2} = 9.92$$

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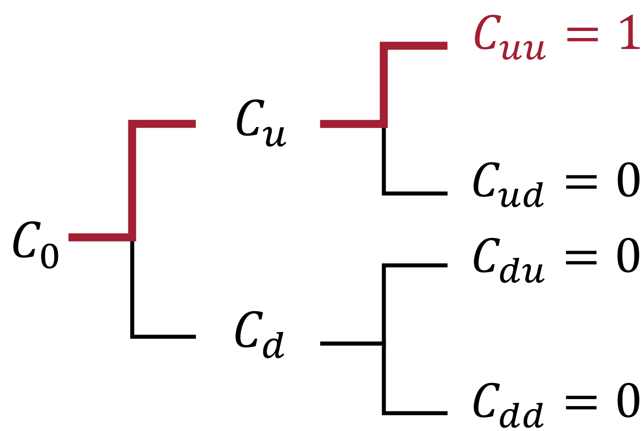
State prices

- We can consider the following "digital option": it pays off \$1 only in a given future state.

- A digital option that pays \$1 at $t = 2$ only if stock price goes up in both periods:



- Denote the price of this option by ϕ_{uu}. Similarly, we have ϕ_{ud}, ϕ_{du}, ϕ_{dd}.

- ϕ_{uu}, ϕ_{ud}, ϕ_{du}, and ϕ_{dd} are the (Arrow-Debreu) state prices.

- Each gives the price of a "state-contingent claim", which pays off one unit only in a given state.

State prices and risk-neutral probabilities

- The price of a state-contingent claim is equal to the probability of the state with the payoff of 1, discounted back to time 0 at the risk-free rate.

- State prices are proportional to risk-neutral probabilities, also reflect time value of money:

$$\phi_u = \frac{q}{1+r}, \quad \phi_d = \frac{1-q}{1+r}$$

$$\phi_{uu} = \frac{q^2}{(1+r)^2}, \quad \phi_{ud} = \frac{q(1-q)}{(1+r)^2}, \quad \phi_{du} = \frac{(1-q)q}{(1+r)^2}, \quad \phi_{dd} = \frac{(1-q)^2}{(1+r)^2}$$

- With state prices, can price any state-contingent payoff as a portfolio of state-contingent claims: mathematically equivalent to the risk-neutral valuation formula.

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Exotic options: risk-neutral pricing

- Payoff of exotic options is path-dependent.
- For example, payoff of an Up-and-Out Put option depends on the maximum stock price observed over the life of the option contract.
- Original pricing by replication is not practical: the tree does not recombine, and the number of distinct nodes on the binomial tree grows exponentially with the number of time periods.
  - Without path dependence, the tree *recombines*: “ud” node = “du” node, etc. Combine all paths leading to the same node into a single state.
- Can use risk-neutral pricing for exotic options.
- Estimate the option price by Monte Carlo simulation: sample from the set of terminal nodes according to their risk-neutral probabilities.
- Replicating portfolio can be computed at any node once the option prices are known.

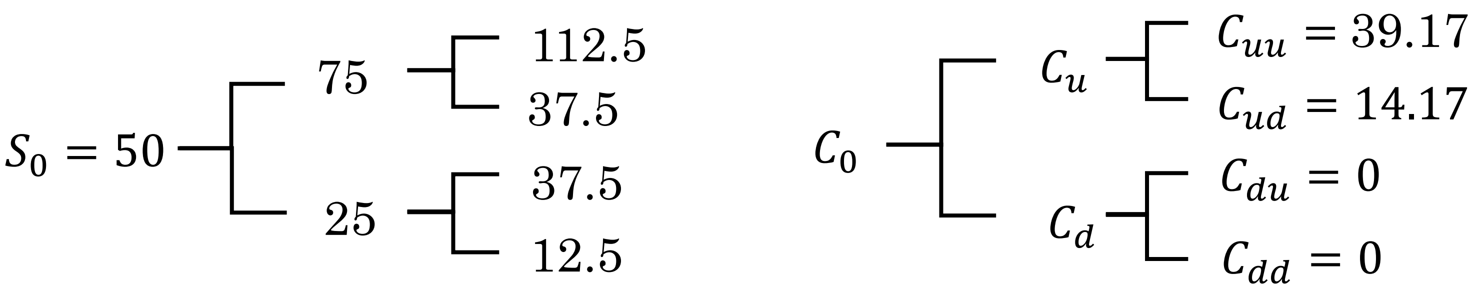
Example: Asian option

- Two-period ($T = 2$) Asian call option with a strike of \$40. Its payoff is:

$$C_2 = \max[0, \bar{S}_2 - 40]$$

  where $\bar{S}_2$ is the average price between $t = 0$ and 2.

- Then



- The price of the call is therefore ($q = 0.6$):

$$C_0 = \frac{(0.6)^2(39.17) + (0.6)(0.4)(14.17) + (0.4)(0.6)(0) + (0.4)^2(0)}{(1+0.1)^2}$$

$$= \frac{17.50}{1.1^2} = 14.46$$

Example: Asian option

- Compute the replicating portfolio as needed, for each visited node.

- For example, to compute the replicating portfolio at node "u" at $t = 1$, need to know only the prices of the option in nodes "uu," "ud," and "u."

  - Buy δ shares of stock, and invest b at the risk-free rate, where

$$
S_0 = 50 \begin{cases} 75 \begin{cases} 112.5 \\ 37.5 \end{cases} \\ 25 \begin{cases} 37.5 \\ 12.5 \end{cases} \end{cases}
\qquad\qquad
C_0 \begin{cases} C_u \begin{cases} C_{uu} = 39.17 \\ C_{ud} = 14.17 \end{cases} \\ C_d \begin{cases} C_{du} = 0 \\ C_{dd} = 0 \end{cases} \end{cases}
$$

$$\delta = \frac{39.17 - 14.17}{112.5 - 37.5} = 0.333$$

$$b = C_u - \delta u S_0 = \frac{0.6 \times 39.17 + 0.4 \times 14.17}{1.1} - 0.333 \times 75 = 1.52$$

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American options: pricing

- The holder of an American option may decide to exercise at any point before maturity.
- Option value P_t satisfies:

$$P_t = \max\left(\text{Payoff}_t, \underbrace{\frac{1}{1+r_f}\mathrm{E}_t^Q[P_{t+1}]}_{\text{Continuation value}}\right)$$

American put option

		t=0	t=1	t=2	Option Payoff
Strike	100				
u	1.2	**Stock Tree**		144.00	0.00
d	0.8		120.00	96.00	4.00
r_f	0.1	100.00			
q	0.75		80.00	96.00	4.00
				64.00	36.00
Physical probability		**Option Tree**			
p	0.5			0.00	
			0.91	4.00	
		5.17			
			$\boxed{20.00}$	4.00	
		No-exercise value	*10.91*	36.00	

American options: dynamic replication

- Replicate the option using the same algorithm as for European options: at $t = 0$, compute the option's delta from option prices and stock prices:

$$\delta = \frac{C_u - C_d}{(u-d)S_0} = \frac{0.91 - 20.00}{120.00 - 80.00}$$

$$= -0.48$$

$$b = \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)} = 52.89$$

t=0	t=1	t=2
Stock Tree		144.00
	120.00	96.00
100.00		
	80.00	96.00
		64.00
Option Tree		
		0.00
	0.91	4.00
5.17		
	20.00	4.00
No-exercise value	*10.91*	36.00
Delta		
	-0.08	
$\boxed{-0.48}$		
	-1.00	
Risk-free investment		
	10.91	
52.89		
	100.00	

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Implementing binomial model

- As we reduce the length of the time step, holding the maturity fixed, the binomial distribution of log returns converges to Normal distribution.

- Key model parameters u, and d need to be chosen to reflect the distribution of the stock return.

- One possible choice is:

$$u = \exp\left(\sigma\sqrt{\frac{T}{n}}\right), \qquad d = 1/u, \qquad p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\frac{T}{n}}$$

where μ and σ describe the first two moments of stock returns:

$$\mathrm{E}_0\left[\frac{S_T}{S_0}\right] = \exp(\mu T), \qquad \mathrm{Var}_0\left[\ln\frac{S_T}{S_0}\right] = \sigma^2 T$$

- We refer to σ as the stock's volatility.

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Black-Scholes-Merton formula

- If we let the period length get smaller and smaller, in the limit we obtain the option pricing formula:

$$\boxed{C_0 = C(S_0, K, T, r, \sigma) = S_0 N(x) - Ke^{-rT} N(x - \sigma\sqrt{T})}$$

  - x is defined by:

$$x = \frac{\ln\left(\frac{S_0}{Ke^{-rT}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

  - $N(\cdot)$ is the normal cumulative distribution function;

- T is time to option maturity, in units of a year;
- r is the continuously-compounded annual riskless interest rate;
- σ is the volatility of annual returns on the underlying asset;
- S_t is log-normally distributed (i.e., $\ln S$ is normally distributed).

Black-Scholes-Merton formula

- An interpretation of the Black-Scholes-Merton formula:

$$C_0 = C(S_0, K, T, r, \sigma) = S_0 N(x) - Ke^{-rT} N(x - \sigma\sqrt{T})$$

  - The call is equivalent to a levered long position in the stock;
  - $S_0 N(x)$ is the amount invested in the stock;
  - $Ke^{-rT} N(x - \sigma\sqrt{T})$ is the dollar amount borrowed;
  - The option's delta is $N(x) = \frac{\partial C}{\partial S}$. It is the limit of the binomial formula as the time step converges to zero, and single-period stock price movements become infinitesimal:

$$\frac{C_u - C_d}{uS_0 - dS_0} \to \frac{\partial C_0}{\partial S_0}$$

Option prices and underlying volatility

- BSM model:
  - The stock price follows a geometric Brownian motion: lognormal, independently and identically distributed (IID) returns.
  - The interest rate is constant.
- The BSM Call and Put prices increase with stock return volatility σ.

Option prices and underlying volatility

- Option value increases with the volatility of underlying asset.
- A simple example: two firms, A and B, with the same current price of \$100.
- B has higher volatility of future prices.
- Consider call options written on A and B, respectively, with the same exercise price \$100.

	Good state	Bad state
	Probability $= p$	Probability $= 1 - p$
Stock A	120	80
Stock B	150	50
Call on A	20	0
Call on B	50	0

- Clearly, call on stock B should be more valuable.
- Put value also increases with underlying volatility (by Put-Call parity).

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Option Greeks

- Option Greeks measure sensitivity of option prices to small changes in various inputs: the underlying price and model parameters:
  - Delta: $\delta = \frac{\partial C}{\partial S}$
  - Omega: $\Omega = \frac{\partial C}{\partial S}\frac{S}{C}$
  - Gamma: $\Gamma = \frac{\partial \delta}{\partial S} = \frac{\partial^2 C}{\partial S^2}$
  - Theta: $\Theta = \frac{\partial C}{\partial T}$
  - Vega: $\nu = \frac{\partial C}{\partial \sigma}$

"Vega" is not a Greek letter,
it was invented in the context of option pricing

Option Greeks: an empirical example

US Budget Impasse Threatened Default in August 2011:
Stocks plummeted, calls dropped sharply, puts surged

S&P futures, Sep 2011 contract, call options

	Strike	8/1/2011	8/2/2011	8/3/2011	8/4/2011	8/5/2011	8/8/2011	8/9/2011	8/10/2011	8/11/2011	8/12/2011
	SPU1 Index	**1279.7**	**1247.3**	**1254.5**	**1198.7**	**1197.8**	**1111.3**	**1171.7**	**1123.5**	**1168.5**	**1176.8**
	1000	280.90	249.00	255.90	204.60	204.50	137.40	180.50	143.00	180.20	185.90
	1050	232.00	200.80	207.20	158.80	158.90	99.30	136.40	102.80	136.70	141.50
	1100	184.00	153.80	159.70	116.10	116.10	65.70	95.50	67.40	96.30	100.00
	1150	137.80	109.70	114.70	77.30	78.20	38.20	59.10	37.80	60.80	63.10
Strike price	1200	94.30	69.50	73.50	44.10	43.70	17.80	29.40	16.60	31.90	33.20
	1250	55.80	36.50	39.00	20.10	18.80	6.20	10.70	5.20	12.60	13.00
	1300	25.80	13.60	14.70	6.20	5.30	2.05	2.50	1.20	3.50	3.50
	1350	7.40	3.00	3.15	1.40	1.05	0.60	0.70	0.45	0.70	0.75
	1400	1.20	0.55	0.65	0.45	0.40	0.20	0.30	0.15	0.25	0.20
	1450	0.25	0.10	0.15	0.10	0.05	0.05	0.10	0.05	0.05	0.05

S&P futures, Sep 2011 contract, put options

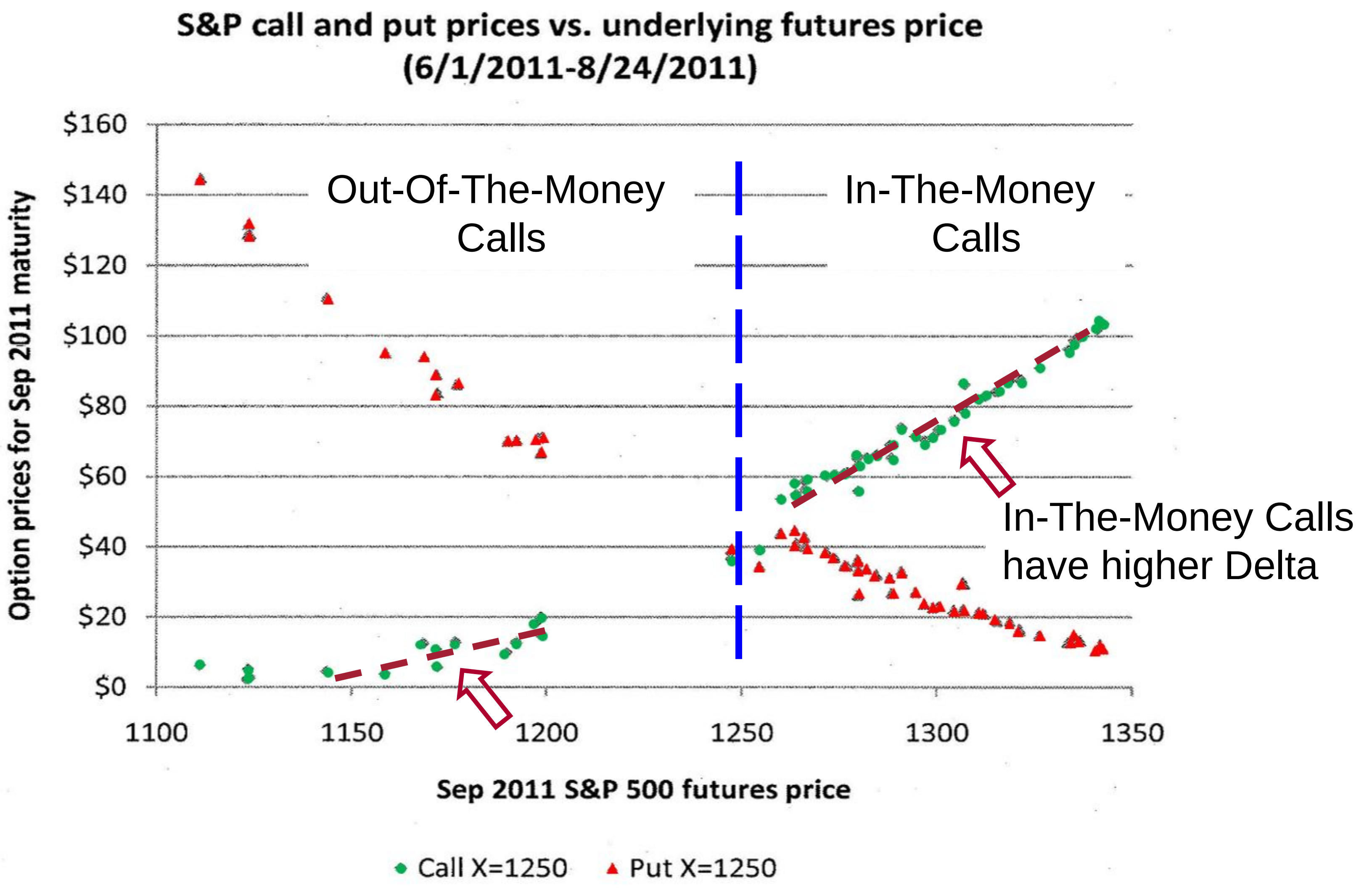
	Strike	8/1/2011	8/2/2011	8/3/2011	8/4/2011	8/5/2011	8/8/2011	8/9/2011	8/10/2011	8/11/2011	8/12/2011
	SPU1 Index	**1279.7**	**1247.3**	**1254.5**	**1198.7**	**1197.8**	**1111.3**	**1171.7**	**1123.5**	**1168.5**	**1176.8**
	1000	1.50	1.95	1.60	6.10	6.90	26.20	9.00	19.60	11.90	9.30
	1050	2.55	3.65	2.90	10.30	11.30	38.10	14.80	29.40	18.30	14.80
	1100	4.45	6.70	5.40	17.50	18.40	54.40	23.90	43.90	27.90	23.30
	1150	8.20	12.50	10.30	28.70	30.50	76.90	37.40	64.30	42.30	36.30
Strike price	1200	14.70	22.30	19.10	45.40	45.90	106.40	57.70	93.00	63.40	56.40
	1250	26.10	39.20	34.50	71.30	70.90	144.80	88.90	131.60	94.00	86.10
	1300	46.10	66.20	60.10	107.40	107.40	190.60	130.70	177.60	134.90	126.60
	1350	77.60	105.60	98.60	152.60	153.10	239.10	178.90	226.80	182.10	173.80
	1400	121.40	153.10	146.00	201.60	202.50	288.80	228.50	276.50	231.60	223.30
	1450	170.40	202.70	195.50	251.30	252.30	338.70	278.30	326.50	281.50	273.20

Call options and Greeks

Call Option Price Changes ($ and %) Depend on Strike Price
In the money: big $ moves, modest % moves
Out of the money: smaller $ moves, bigger % moves

SP500 Call Options			$\delta \approx \left(\frac{\Delta C}{\Delta S}\right)$		$\Omega \approx \left(\frac{\Delta C}{\Delta S}\frac{S}{C}\right)$	
SP500	8/1/2011	8/10/2011	Change	Chg/ChgU	%Change	Elasticity
Underlying	1279.70	1123.50	-156.20	1.00	-12.2	1.0
1000	280.90	143.00	-137.90	0.88	-49.1	4.0
1050	232.00	102.80	-129.20	0.83	-55.7	4.6
1100	184.00	67.40	-116.60	0.75	-63.4	5.2
1150	137.80	37.80	-100.00	0.64	-72.6	5.9
1200	94.30	16.60	-77.70	0.50	-82.4	6.8
1250	55.80	5.20	-50.60	0.32	-90.7	7.4
1300	25.80	1.20	-24.60	0.16	-95.3	7.8
1350	7.40	0.45	-6.95	0.04	-93.9	7.7
1400	1.20	0.15	-1.05	0.01	-87.5	7.2
1450	0.25	0.05	-0.20	0.00	-80.0	6.6

Option Greeks: an empirical example



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Implementing the BSM model

- To implement the BSM formula, we need to estimate volatility σ.
- Two possible estimates.
  - Historic volatility: develop statistical estimates using past returns on the underlying asset.
    - E.g., use daily returns over a given period to estimate daily volatility (standard deviation);
    - Annualize by multiplying daily volatility by $\sqrt{252}$.

≈ 252 trading days in a calendar year

  - Implied volatility: price options relative to each other.
    - Use the market prices of another options;
    - Assume that they are given by the BSM formula and solve for σ, which gives the implied volatility.

Example: implied volatility

- Need to price a call on a stock with a strike price of \$110 and a maturity of 1 year.

- Suppose that the current stock price is \$100 and the one-year interest rate 6.18%.

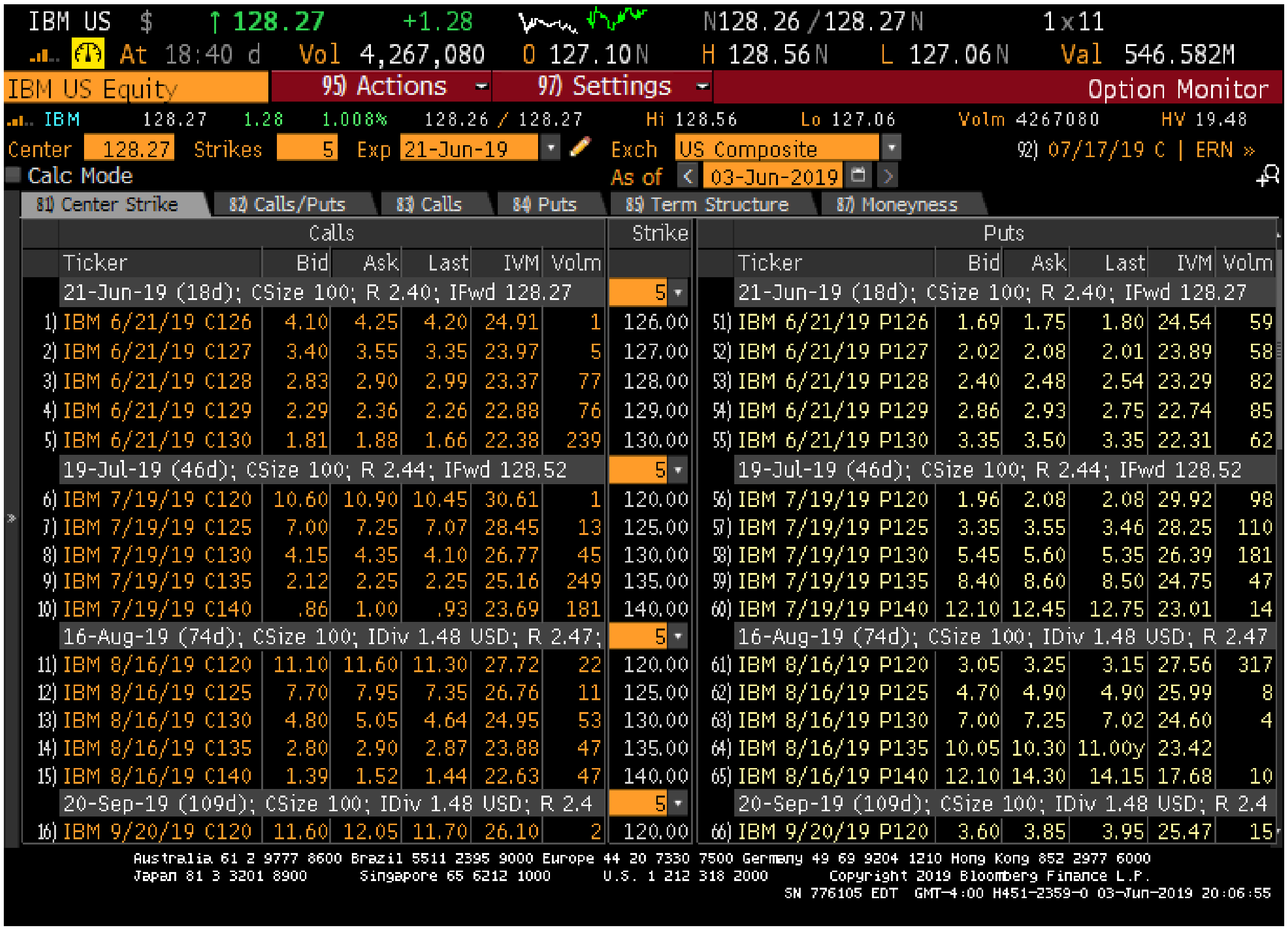
- Suppose that another call with a strike price of \$120 is trading at a market price of \$3.16. The volatility that makes the BSM price of this call equal to its market price is $\sigma = 19\%$. This is the implied volatility.

- We can then use 19% in the BSM formula to obtain the price of the first call.

- Potential problem: Implied volatility may be different for options with different strikes and maturities (smile and smirk patterns in implied volatility).

Implied volatility

Call/Put options on IBM, as of 06/03/2019



IBM US $ ↑128.27 +1.28 N128.26 / 128.27 N 1 x 11

At 18:40 d Vol 4,267,080 O 127.10 N H 128.56 N L 127.06 N Val 546.582M

IBM US Equity 95) Actions 97) Settings Option Monitor

IBM 128.27 1.28 1.008% 128.26 / 128.27 Hi 128.56 Lo 127.06 Volm 4267080 HV 19.48

Center 128.27 Strikes 5 Exp 21-Jun-19 Exch US Composite 92) 07/17/19 C | ERN »

Calc Mode As of 03-Jun-2019

81) Center Strike 82) Calls/Puts 83) Calls 84) Puts 85) Term Structure 87) Moneyness

Calls						Strike	Puts					
Ticker	Bid	Ask	Last	IVM	Volm		Ticker	Bid	Ask	Last	IVM	Volm
21-Jun-19 (18d); CSize 100; R 2.40; IFwd 128.27						5	21-Jun-19 (18d); CSize 100; R 2.40; IFwd 128.27					
1) IBM 6/21/19 C126	4.10	4.25	4.20	24.91	1	126.00	51) IBM 6/21/19 P126	1.69	1.75	1.80	24.54	59
2) IBM 6/21/19 C127	3.40	3.55	3.35	23.97	5	127.00	52) IBM 6/21/19 P127	2.02	2.08	2.01	23.89	58
3) IBM 6/21/19 C128	2.83	2.90	2.99	23.37	77	128.00	53) IBM 6/21/19 P128	2.40	2.48	2.54	23.29	82
4) IBM 6/21/19 C129	2.29	2.36	2.26	22.88	76	129.00	54) IBM 6/21/19 P129	2.86	2.93	2.75	22.74	85
5) IBM 6/21/19 C130	1.81	1.88	1.66	22.38	239	130.00	55) IBM 6/21/19 P130	3.35	3.50	3.35	22.31	62
19-Jul-19 (46d); CSize 100; R 2.44; IFwd 128.52						5	19-Jul-19 (46d); CSize 100; R 2.44; IFwd 128.52					
6) IBM 7/19/19 C120	10.60	10.90	10.45	30.61	1	120.00	56) IBM 7/19/19 P120	1.96	2.08	2.08	29.92	98
7) IBM 7/19/19 C125	7.00	7.25	7.07	28.45	13	125.00	57) IBM 7/19/19 P125	3.35	3.55	3.46	28.25	110
8) IBM 7/19/19 C130	4.15	4.35	4.10	26.77	45	130.00	58) IBM 7/19/19 P130	5.45	5.60	5.35	26.39	181
9) IBM 7/19/19 C135	2.12	2.25	2.25	25.16	249	135.00	59) IBM 7/19/19 P135	8.40	8.60	8.50	24.75	47
10) IBM 7/19/19 C140	.86	1.00	.93	23.69	181	140.00	60) IBM 7/19/19 P140	12.10	12.45	12.75	23.01	14
16-Aug-19 (74d); CSize 100; IDiv 1.48 USD; R 2.47;						5	16-Aug-19 (74d); CSize 100; IDiv 1.48 USD; R 2.47					
11) IBM 8/16/19 C120	11.10	11.60	11.30	27.72	22	120.00	61) IBM 8/16/19 P120	3.05	3.25	3.15	27.56	317
12) IBM 8/16/19 C125	7.70	7.95	7.35	26.76	11	125.00	62) IBM 8/16/19 P125	4.70	4.90	4.90	25.99	8
13) IBM 8/16/19 C130	4.80	5.05	4.64	24.95	53	130.00	63) IBM 8/16/19 P130	7.00	7.25	7.02	24.60	4
14) IBM 8/16/19 C135	2.80	2.90	2.87	23.88	47	135.00	64) IBM 8/16/19 P135	10.05	10.30	11.00y	23.42	
15) IBM 8/16/19 C140	1.39	1.52	1.44	22.63	47	140.00	65) IBM 8/16/19 P140	12.10	14.30	14.15	17.68	10
20-Sep-19 (109d); CSize 100; IDiv 1.48 USD; R 2.4						5	20-Sep-19 (109d); CSize 100; IDiv 1.48 USD; R 2.4					
16) IBM 9/20/19 C120	11.60	12.05	11.70	26.10	2	120.00	66) IBM 9/20/19 P120	3.60	3.85	3.95	25.47	15

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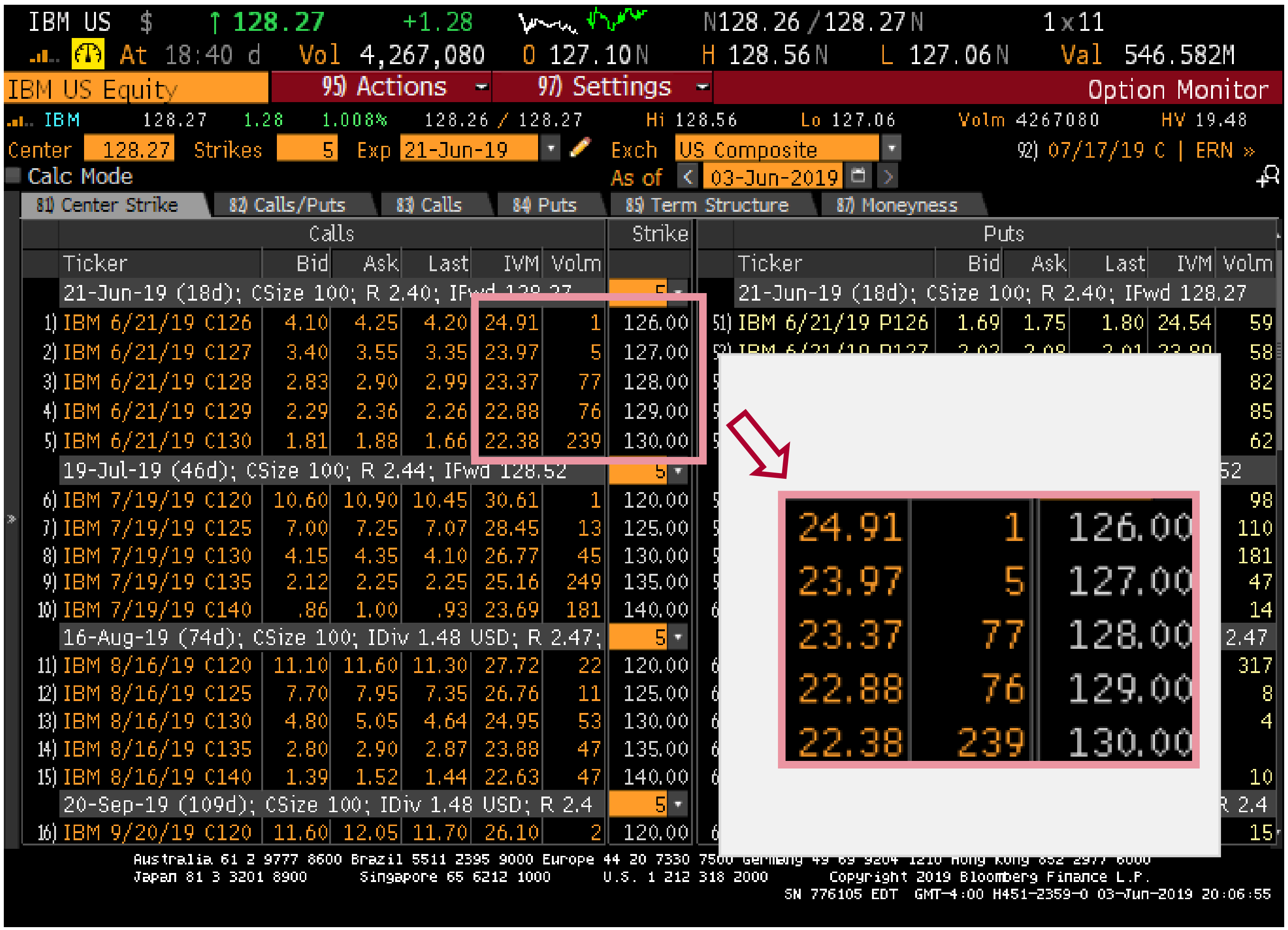
Implied volatility differs across strikes



Implied volatility differs across maturities

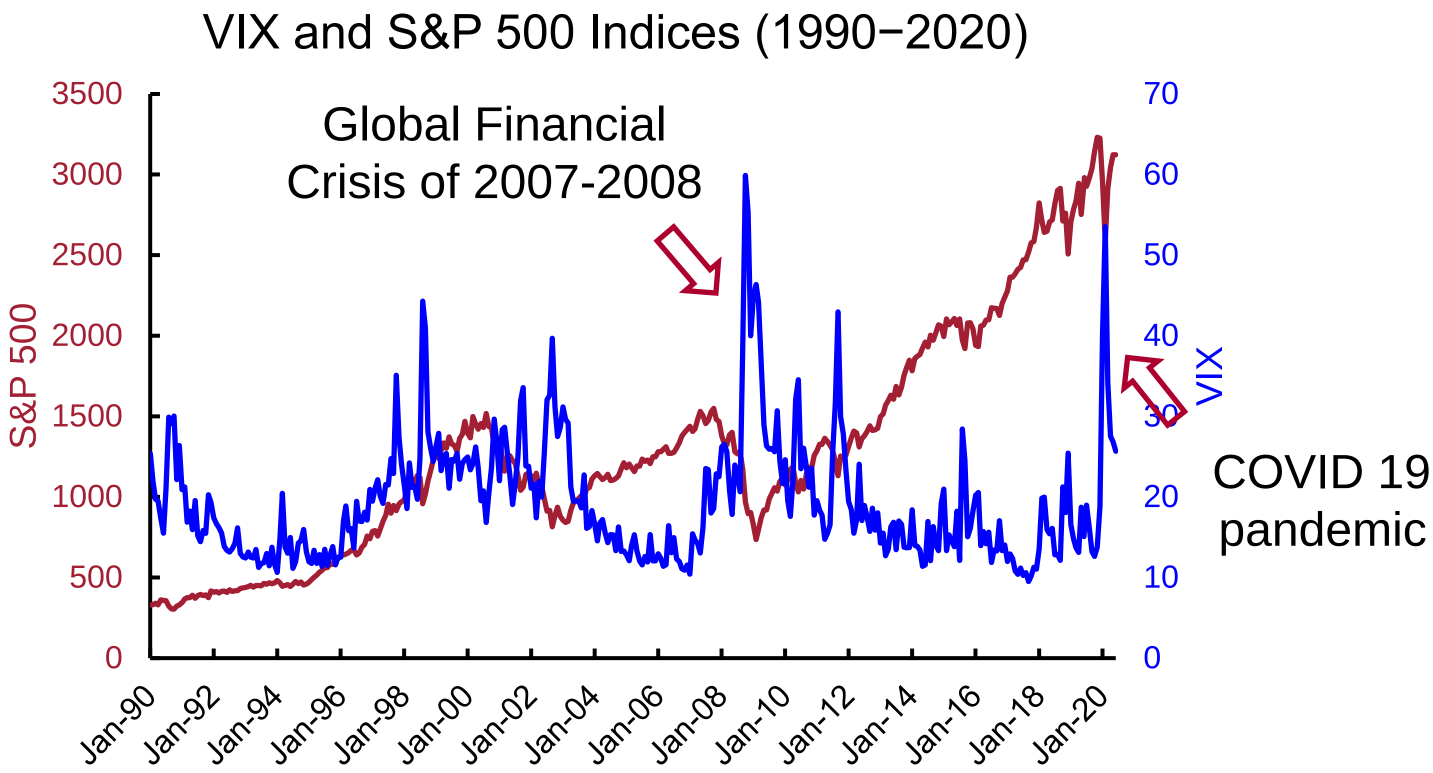
	Calls						Strike
	Ticker	Bid	Ask	Last	IVM	Volm	
	21-Jun-19 (18d); CSize 100; R 2.40; IFwd 128.27						5
1)	IBM 6/21/19 C126	4.10	4.25	4.20	24.91	1	126.00
2)	IBM 6/21/19 C127	3.40	3.55	3.35	23.97	5	127.00
3)	IBM 6/21/19 C128	2.83	2.90	2.99	23.37	77	128.00
4)	IBM 6/21/19 C129	2.29	2.36	2.26	22.88	76	129.00
5)	IBM 6/21/19 C130	1.81	1.88	1.66	22.38	239	130.00
	19-Jul-19 (46d); CSize 100; R 2.44; IFwd 128.52						5
6)	IBM 7/19/19 C120	10.60	10.90	10.45	30.61	1	120.00
7)	IBM 7/19/19 C125	7.00	7.25	7.07	28.45	13	125.00
8)	IBM 7/19/19 C130	4.15	4.35	4.10	26.77	45	130.00
9)	IBM 7/19/19 C135	2.12	2.25	2.25	25.16	249	135.00
10)	IBM 7/19/19 C140	.86	1.00	.93	23.69	181	140.00
	16-Aug-19 (74d); CSize 100; IDiv 1.48 USD; R 2.47;						5
11)	IBM 8/16/19 C120	11.10	11.60	11.30	27.72	22	120.00
12)	IBM 8/16/19 C125	7.70	7.95	7.35	26.76	11	125.00
13)	IBM 8/16/19 C130	4.80	5.05	4.64	24.95	53	130.00
14)	IBM 8/16/19 C135	2.80	2.90	2.87	23.88	47	135.00
15)	IBM 8/16/19 C140	1.39	1.52	1.44	22.63	47	140.00

Implications of implied volatility smile/smirk

- The fact that implied volatilities depend on the strike price of the option is a violation of the Black-Scholes-Merton model.
  - Under the Black-Scholes-Merton model, implied volatility must equal physical volatility.
- To improve performance of the Black-Scholes-Merton model, it is common to extend the model by adding
  - Stochastic return volatility;
  - Jumps in underlying price (discontinuous return changes).
- Black-Scholes-Merton implied volatilities are commonly used to quote prices of options.
  - For that, the model itself does not need to be valid. The implied volatility is always well defined.

Implied volatility: VIX

- VIX is a composite summary of implied vols across call and put options with different strike prices.
- VIX is an indicator of future stock market volatility over the next 30 days.



Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model