Week 1 – Forward Contracts

MIT Sloan School of Management

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Outline

- Forward contract basics
  - Definition
  - Profit/Loss
  - A forward currency hedge and payoff diagrams
  - Review of risk-free rates and the yield curve
  - Interpretation of forward contracts as highly levered positions
- Pricing formulas
  - Financial assets: stocks, bonds, currencies
  - Commodities
- Key concepts for hedging and speculating

Forward contract basics

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Definition: Forward Contract

A **forward contract** is an agreement between two counterparties to trade a pre-specified amount of goods or securities at a pre-specified future date, T, for a pre-specified price, F_0.

Notes:

- It is free to enter into a forward contract initially: the contract is an agreement to exchange goods or securities for money in the future, and not today
  - In practice you may have to post collateral, but typically you earn a return on the collateral
  - A “pre-paid forward” is an exception. In this case the payment is made upfront.
- The pre-specified price F_0 is set to ensure that the value of the forward contract is zero for both counterparties at the inception of the contract
- **Terminology**: The counterparty who agrees to **buy (sell)** the goods or securities has **long (short)** position
- **Other features**: Forwards are traded “over-the-counter”. The contract may specify physical delivery or cash settlement.

Profit/Loss from a forward contract

Let S_t be the spot price of one unit of the good or security at t, and N be the size of the contract (# of units)

- *Definition:* The “spot price” is the current market price

The Profit/Loss (P/L) at the contract maturity T for each counterparty is

- **P/L long = N x $(S_T - F_0)$**
- **P/L short = N x $(F_0 - S_T)$**

Questions for you to think about:

- What are the details of the transactions that makes this true for physical settlement?
- What are the details of the transactions that make this true for cash settlement?
- Does it matter if the short already owns the asset?
- Does it matter whether or not the long wants to own the asset?
- How can a zero-sum contract benefit both counterparties?
- What is the P/L if you sell the contract at time $t < T$

Forward contract timing of cash flows

t=0	t = T
Long contract	Payoff = S_T-F_0
Short contract	Payoff = F_0-S_T

No initial cash flows but collateral may be required

Forward contracts are always “zero-sum” in their payoffs

Example 2.1: Hedging with a forward currency contract

A US firm has sold a piece of equipment to a German client and now it has a receivable of EUR 5 million in T = 6 months.

Let S_t = USD/EUR exchange rate at t. Assume the current rate S_0 = 1.2673

Unhedged dollar payoff at T = EUR 5 million x S_T

What is the risk?

Exchange rate risk: Euro can depreciate vs. the dollar (S_T declines)

Example 2.1 (cont.)

Hedging strategy: enter into a forward contract with a bank to exchange euros for dollars at T = 6 months at an exchange rate F, say F = 1.28, decided today

→ The firm is short the Euro forward

Dollar P/L of forward contract at T = 5 mil x $(F - S_T)$

Total payoff at T

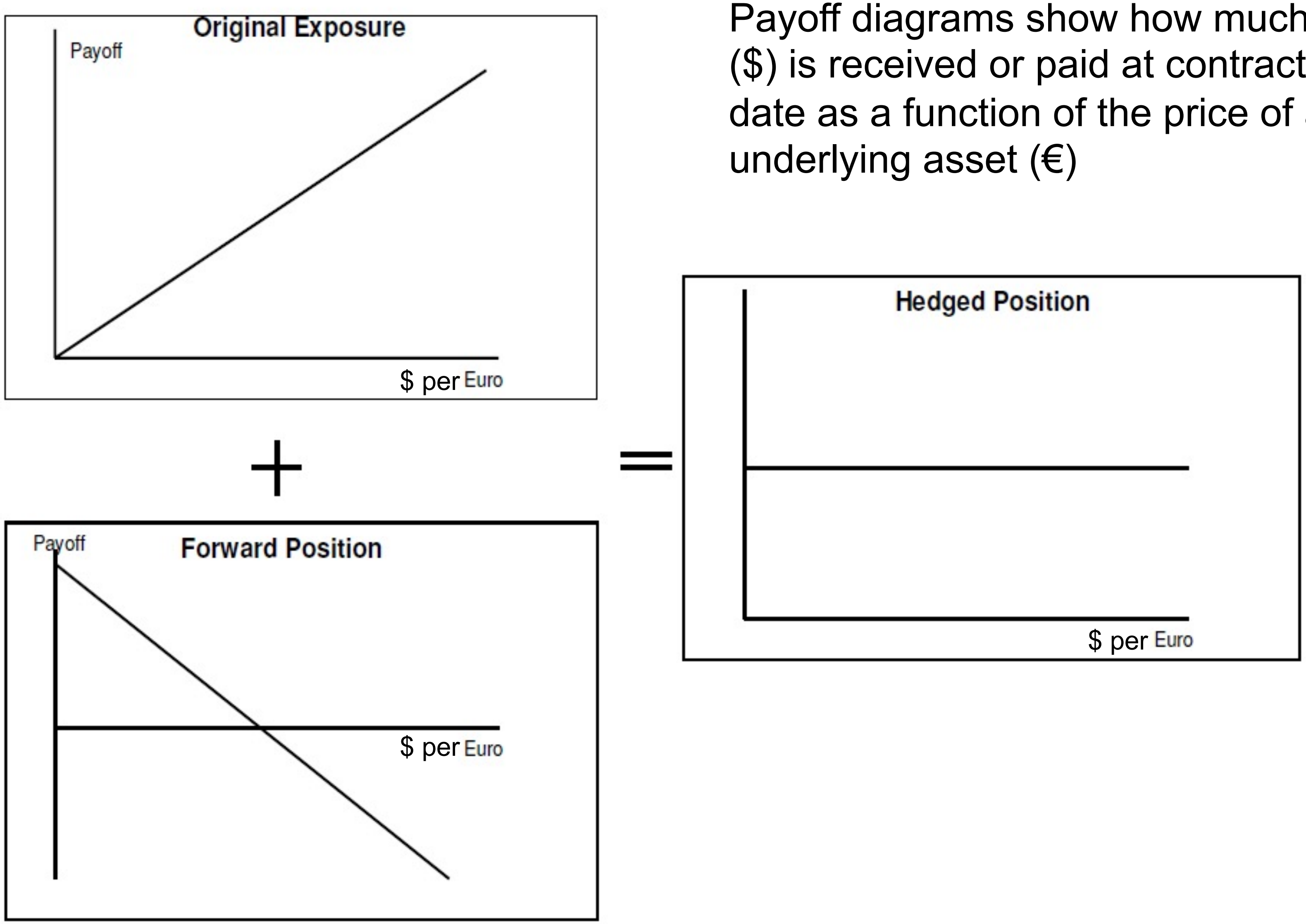
= payoff from original position T + payoff of forward contract at T

= $(N \times S_T) + N \times (F - S_T)$

= 5 mil x 1.28 = \$6.4 million

The hedge locks in the dollar payoff at T to be $N \times F$, regardless of the exchange rate movement

Payoff diagrams for hedging example

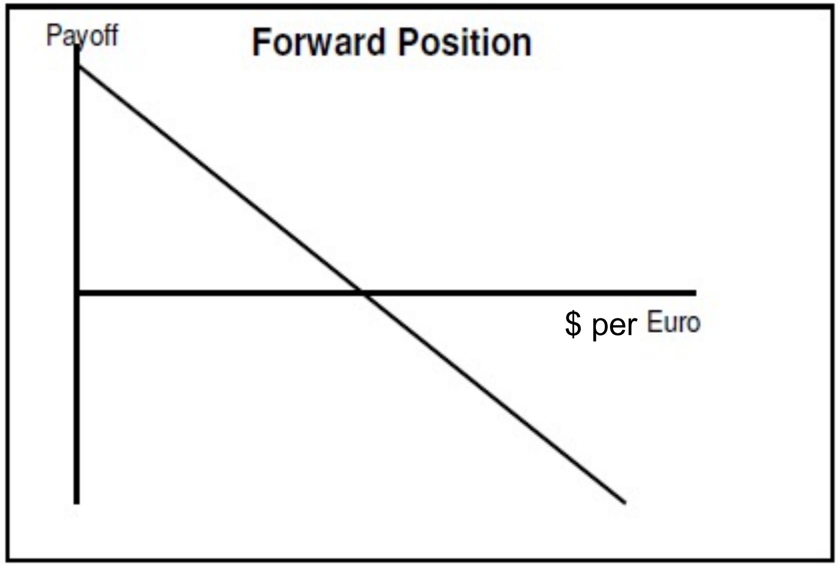


Payoff diagrams show how much money ($) is received or paid at contract expiration date as a function of the price of an underlying asset (€)

Example 2.2: Speculating with forwards

Directional bets

- Think that euro will depreciate versus dollar?
  - Short forwards or futures on dollar/euro exchange rate
  - It costs essentially nothing
  - Each tick in exchange rate movement => large profits (or losses)



Levered position: return on capital very high (or low)

Review: risk-free rates and the yield curve

The price of a bond is the present value of its promised payments

- The Treasury spot yield curve provides a reference market rate for discounting cash flows of different maturities that are free from default risk
- Rates along the Treasury yield curve make up the term structure of "risk-free" rates
- Risky or illiquid bond cash flows are discounted at rates that include a premium

Notation: The market rate to discount a cash flow arriving at time *t* back to time 0, stated on a continuous basis, is r_t

Price of a zero coupon bond with face value *Z* paid at time T is given by

$$P = e^{-r_T T} Z$$

Recall that: $\lim_{n \to \infty} (1 + r/n)^n = e^r$

Implied forward prices and forward rates

Current price of bond is maturing at T_2 is:

$$P_{B,0} = e^{-r_{T_2}T_2} Z$$

We can prove that the forward price for delivery at T_1 is:

$$F = e^{r_{T_1}T_1} P_{B,0}$$

Substituting for bond price =>

$$F = e^{-r_{T_2}T_2} Z e^{r_{T_1}T_1} = Z e^{(r_{T_1}T_1 - r_{T_2}T_2)}$$

....

The **forward price implies a "forward rate"** which is the return locked in on the bond delivered in the forward contract.

The implied forward rate $f(0,T_1,T_2)$ solves:

$$F = Z e^{-f(0,T_1,T_2)(T_2 - T_1)}$$

Note on notation: $f(0,T_1,T_2)$ denotes the forward rate between time T_1 and T_2, as of time 0.

$$\boxed{f(0,T_1,T_2) = \frac{T_2 r_{T_2} - T_1 r_{T_1}}{T_2 - T_1}}$$

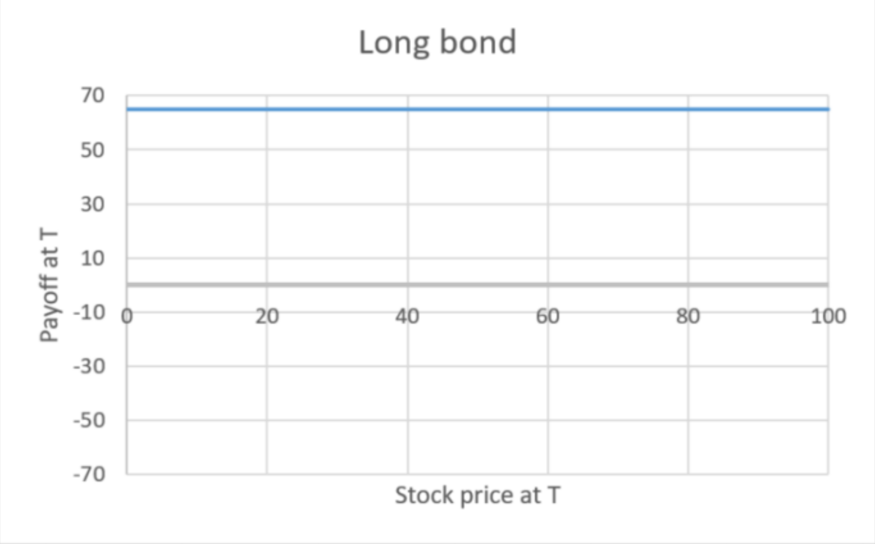
Risk-free borrowing and lending

- The standard pricing formulas for forwards that we will derive depend on the assumption that market participants can borrow and lend at a risk-free rate

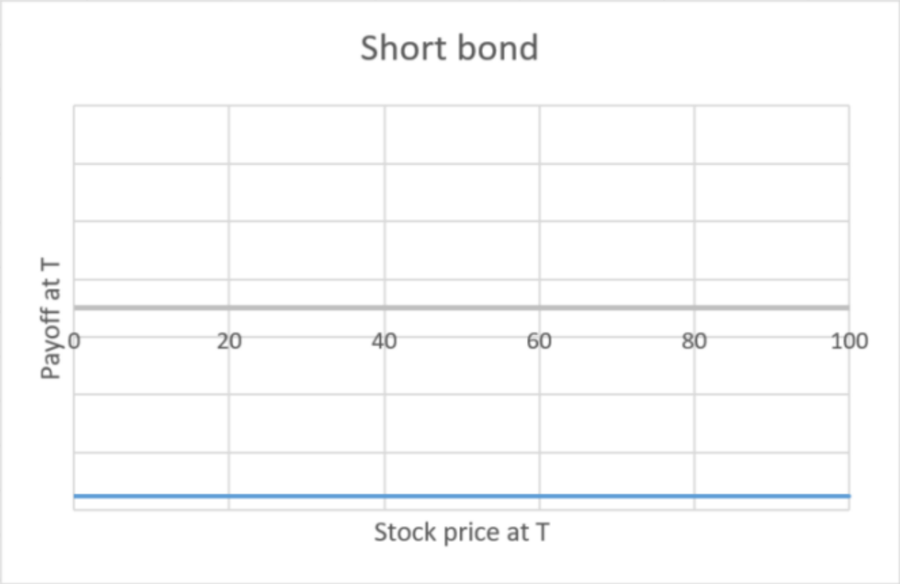
- In practice this can be accomplished by
  - Buying and selling Treasury securities
  - Collateralized borrowing and lending, e.g., using repurchase agreements (RPs or repos)

Using payoff diagrams to understand forward contracts as highly levered positions

Stock spot price T	Long bond Payoff T
0	65
10	65
20	65
30	65
40	65
50	65
60	65
70	65
80	65
90	65
100	65



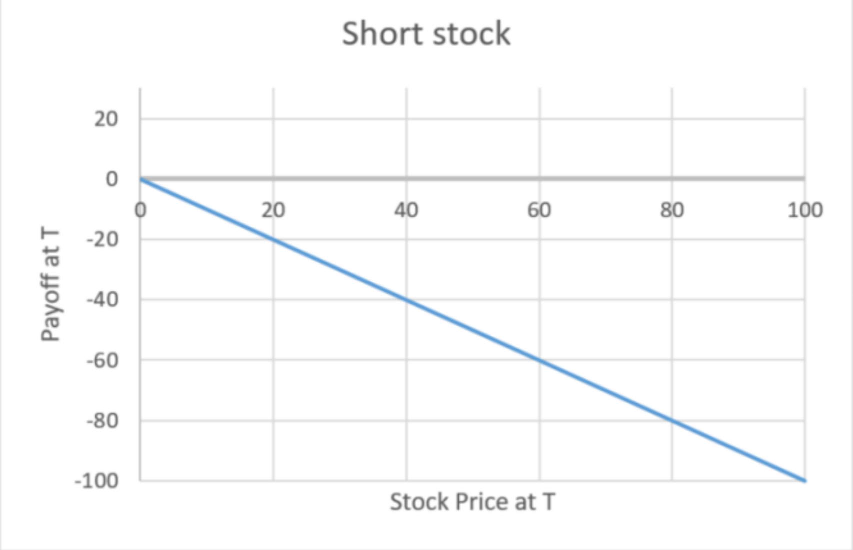
Stock spot price T	Short bond Payoff T
0	-65
10	-65
20	-65
30	-65
40	-65
50	-65
60	-65
70	-65
80	-65
90	-65
100	-65



Using pay off diagrams to understand forwards as highly levered positions

Stock spot price T	Long stock Payoff T
0	0
10	10
20	20
30	30
40	40
50	50
60	60
70	70
80	80
90	90
100	100

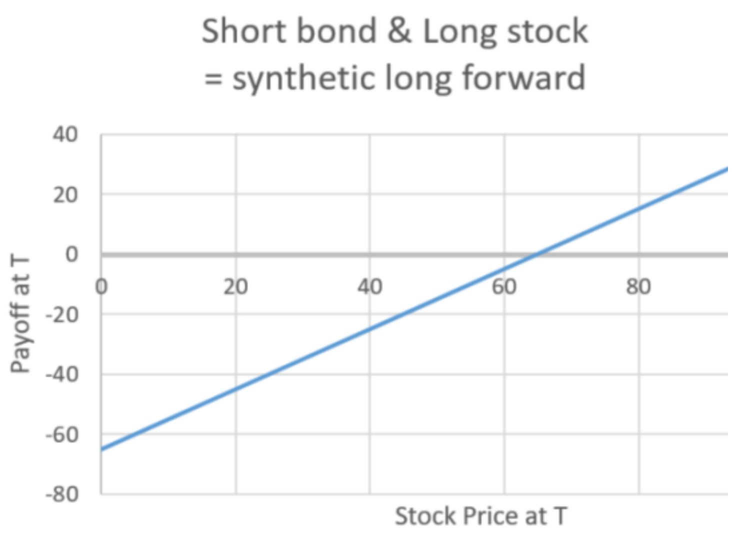
Stock spot price T	Short stock Payoff T
0	0
10	-10
20	-20
30	-30
40	-40
50	-50
60	-60
70	-70
80	-80
90	-90
100	-100



Using pay off diagrams to understand forwards as highly levered positions

Stock spot price T	Long stock forward Payoff T
0	-65
10	-55
20	-45
30	-35
40	-25
50	-15
60	-5
70	5
80	15
90	25
100	35

Stock spot price T	Short bond & Long stock Payoff T
0	-65
10	-55
20	-45
30	-35
40	-25
50	-15
60	-5
70	5
80	15
90	25
100	35



Takeaways from payoff diagram example

Long forward positions are equivalent to borrowing and going long in the underlying asset

- Allows high long risk exposure to underlying with no money down (except collateral)
- Implicit leverage allows some people to borrow who otherwise couldn’t

Forward short positions are equivalent to lending and going short the underlying

- Allows high short risk exposure to underlying with no money down (except collateral)
- Provides access to short exposure even when underlying is unavailable to borrow and short

Pricing formulas and their derivations

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Some definitions: No arbitrage

Definition*: An **arbitrage opportunity** is a trading strategy that either

(1) Yields a positive profit today, and zero cash flows in the future; or

(2) Costs nothing today and yields a positive profit in the future

- The value of many derivative securities is estimated by assuming that no arbitrage opportunities exist
- In well functioning markets, no arbitrage opportunities can persist
- If they did, arbitrageurs would take giant positions to profit from them, quickly eliminating them

Note that "no arbitrage" does not imply there are no market frictions

*This is a technical definition. The term "arbitrage" is used more loosely by traders.

Some definitions: No arbitrage

The Law of One Price:
Securities with identical payoffs must have the same price

Otherwise, an arbitrage opportunity arises..

Buy Low / Sell High

Buy the security with the low price and simultaneously sell (short) the one with a high price

- This yields an upfront profit

The arbitrageur is hedged, as future cash flows cancel out exactly

Some definitions: Short sales

Exploiting an arbitrage opportunity often requires taking offsetting long and short positions

Traders can take a short position either by

1. selling an asset they already own
2. borrowing an asset and then selling it
   - The trader must later buy the asset and return it to the lender
   - Shorting involves borrowing costs
   - Not all assets are available to short

Which forwards and futures are priced by no-arbitrage?

Forwards for **pure investment assets can be priced using** relatively simple **no-arbitrage** arguments

- For instance, in Example 2.1, we will see how the pre-specified forward exchange rate F_0 = 1.28 USD/EUR was determined

We will see that a “cash and carry” strategy can be used to derive forward prices for pure investment assets via no-arbitrage reasoning

Forwards and futures for **non-investment assets** (e.g., agricultural commodities, energy, metals) **cannot be valued simply by no-arbitrage conditions**

Inventory of pricing formulas for financial forwards

- Stocks
  - Stock with known dividend D at time $t < T$: $F_0 = [P_{S,0} - D(e^{-rt})](e^{rT})$
  - Stock with known dividend yield q: $F_0 = P_{S,0}\, e^{(r-q)T}$
    - r is the risk-free rate that matches cash flow maturity
    - $P_{S,t}$ is the stock price at time t
- Bonds
  - Bond with coupon C at time $t < T$: $F_0 = [P_{B,0} - C(e^{-rt})](e^{rT})$
    - r is the risk-free rate that matches cash flow maturity
    - $P_{B,t}$ is the bond price at time t
- Currencies
  - E.g., pay euros for dollars: $F_0 = S_0(e^{(r_\$ - r_€)T})$
    - $r_\$$ ($r_€$) is the USD (EUR) risk-free rate
    - S_t is the exchange rate (USD per EUR) at time t

Recap of example 2.1: Hedging with a forward currency contract

A US firm has sold a piece of equipment to a German client and now it has a receivable of EUR 5 million in T = 6 months.

Risk is that Euro can depreciate vs. the dollar (S_T declines)

Hedge with short forward contract in Euros

Let S_t = USD/EUR exchange rate at t. Assume the current rate S_0 = 1.2673

Unhedged dollar payoff at T = EUR 5 million x S_T

Also assume 6-month USD interest rate is 5%, & 6-month EUR interest rate is 3%

Recap example 2.1 (cont.)

Hedging strategy: enter into a forward contract with a bank to exchange euros for dollars at T = 6 months at an exchange rate F, say F = 1.28, decided today

How is the prespecified exchange rate F = 1.28 USD/EUR determined?

Practice problem: Verify that if interest rates and the spot exchange rate are as stated in this example, that the no-arbitrage forward exchange rate is 1.28.

Deriving forward prices from no-arbitrage conditions:
Cash and carry for a non-dividend paying stock

t=0	t = T
short stock forward	Receive F Deliver share of stock
buy stock for $-P_{S,0}$ borrow $P_{S,0}$	Use for delivery in forward Repay borrowing $-P_{S,0}(e^{rT})$
Net cash flow = 0	P/L = F - $P_{S,0}(e^{rT})$

Conclusion: $F \leq P_{S,0}(er^{T})$

Deriving forward prices from no-arbitrage conditions:
Reverse cash and carry for a non-dividend paying stock

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t=0	t = T
long stock forward	Pay -F receive share of stock
short stock for $P_{S,0}$ lend -$P_{S,0}$	Use stock received to cover short Receive loan repayment $P_{S,0}(e^{rT})$
Net cash flow = 0	P/L = $P_{S,0}(e^{rT}) - F$

Conclusion: $F \geq P_{S,0}(er^{T})$

Forward price with no arbitrage

If F – $P_{S,0}(e^{rT})$ > 0 then cash and carry is arbitrage opportunity

If $P_{S,0}(e^{rT})$ – F > 0 then reverse cash & carry is arbitrage opportunity

Hence, the forward price for a stock with no dividends between 0 and T must be:

$$\boldsymbol{F_0 = P_{S,0}(e^{rT})}$$

Forward prices for commodities

Some assets that are not purely financial---like oil, metals and some agricultural commodities--are storable and held as investments

Storing commodities has costs and benefits

- Storage cost “U” (lump sum payments)
- Storage cost “u” (percentage of spot price)
- Convenience yield “y” (percentage of spot price)

Forward price $F_{0,T}$ at contract initiation comes from no-arbitrage condition that again can be demonstrated using cash-and-carry and reverse cash-and-carry logic.

- Adjustments for costs and benefits are like adjustments for stock dividends and bond coupons

Forward prices for commodities

Forward price with lump-sum storage cost U

$$F_{0,T} = (S_0 + PV(U)) \times e^{rT}$$

Forward price with proportional storage cost u

$$F_{0,T} = S_0 \times e^{(r+u)T}$$

Forward price with convenience yield y

$$F_{0,T} = S_0 \times e^{(r-y)T}$$

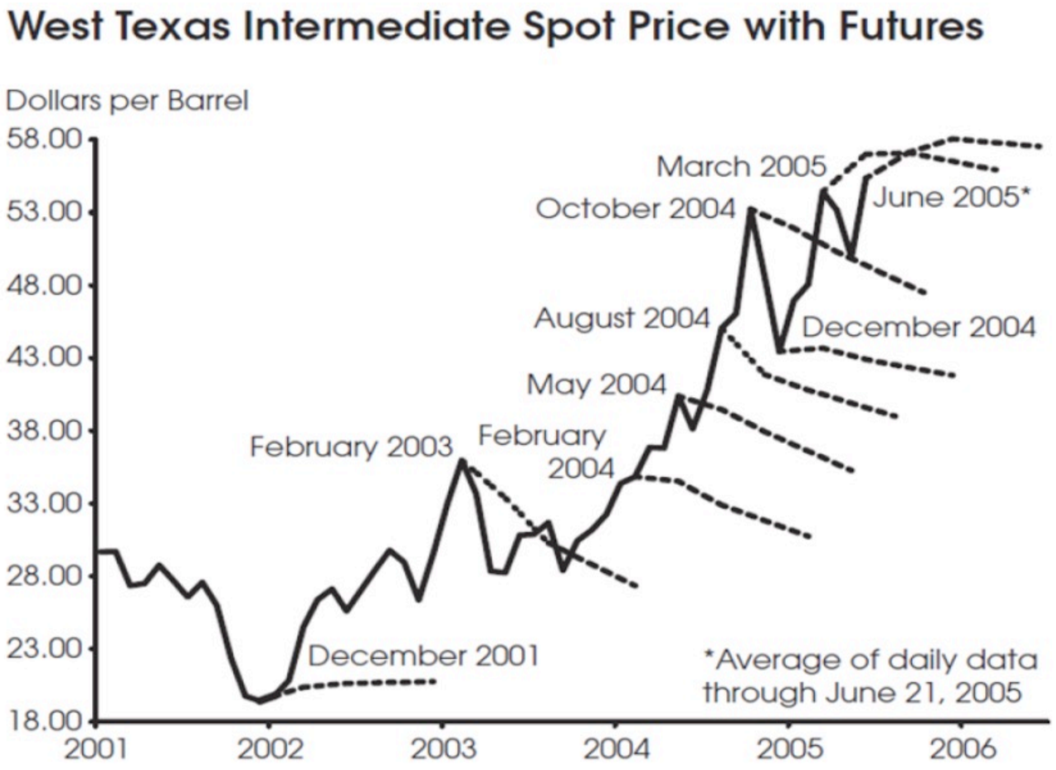
Forward price with proportional storage cost and convenience yield

$$F_{0,T} = S_0 \times e^{(r+u-y)T}$$

Describing the shape of the forward curve

Contango is a pattern of forward prices that increases with contract maturity

Backwardation is a pattern of forward prices over time that decreases with contract maturity



What about commodities that cannot be stored?

- May be no storage or very limited storage life
  - Electricity
  - Lettuce, strawberries, live concerts…
- For non-storable commodities, forward prices can have information about future spot prices because no-arbitrage conditions don’t hold
  - Approach to pricing is to model stochastic future spot prices
  - Also must infer discount rates

Key takeaways on forwards

- For stocks, bonds, currencies, metals, stored agricultural commodities, etc., there is no new information in forward prices over what can be learned from spot prices!

- Why?
  - Forward price *F* has no uncertainty associated with it, a forward transition involves a certain payment for delivery of a specific asset.
  - The forward price is tied down by no-arbitrage conditions that depend only on the underlying spot price, interest rates, and associated cash flows between 0 and T (dividends, coupons, storage costs, convenience yield)

- Implication: Don't try to predict future spot prices using forward prices!
  - Discussion question: Is the expected future price of a non-dividend paying stock higher or lower than its forward price?

Key concepts for hedging and speculating

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Valuing a forward contract over time

We have seen that the initial forward price is determined by no-arbitrage, and the initial value of the contract, f_0, is 0.

How does the value of a forward contract evolve over time?

This is determined by another no-arbitrage relationship:

- Suppose that $K = F_0$ is the original delivery price and F_t is the forward price for a contract that would be negotiated today for the same underlying and delivery date.
- By considering the difference between a contract with delivery price K and a contract with delivery price F_t we can deduce that the value of a long forward contract, $f_{t,T}$, at time t is

$$(F_t - K)e^{-r(T-t)}$$

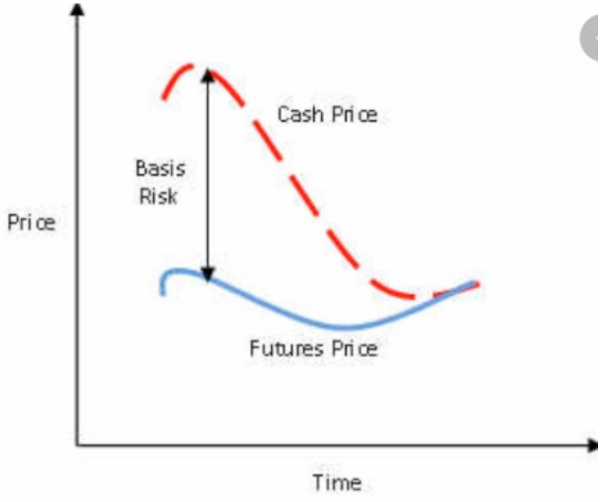
- the value of a short forward contract is

$$(K - F_t)e^{-r(T-t)}$$

Basis risk and cross hedging

Definition: **Basis** is the difference between the spot and forward price of a security or commodity.

No-arbitrage implies basis goes to zero on contract expiration date.



Definition: **Cross-hedging** involves using a contract type to hedge which differs from the security or commodity being hedged.

- Limited contract types => traders often have to cross-hedge

Hedge ratios

Definition: The **hedge ratio** is the relative number of forward contracts to units of the asset being hedged that maximizes the effectiveness of the hedge.

Goal is to have $N_s \times E(dS) = N_F \times E(dF)$ so that same total dollar expected price change

$$\Rightarrow N_s / N_F = E(dF) / E(dS)$$

- N_F = number forward units
- N_S = number spot units
- $E(.)$ = expected value
- dF = change in value of forward contract
- dS = change in value of spot security or commodity

Note: If long in spot then short in forwards, and vice versa.