# Recitation 16

Spring 2021

## Question 1

Suppose there is a private shoe manufacturer called XYZ. XYZ is going public (doing an initial public offering) and has hired an investment bank to assess the firm's fair market value. One of the key ingredients to the fair market value is the firm's business risk.

The investment bank decides to use a comparable companies analysis, using publicly traded shoe manufacturers to assess the business risk of such a business. Suppose the investment bank was not able to find any "pure plays" (companies that only manufacture shoes), but found two companies that manufacture both shoes and apparel, firms A and B. According to the investment bank's analysis, the firms betas (business risk) and percentage of business in shoes vs. apparel are given below:

| Firm | Firm beta | Shoe | Apparel |
|---|---|---|---|
| A | 0.9 | 40% | 60% |
| B | 0.7 | 70% | 30% |

Assume both firms are fully equity-financed (have no debt). Based on this information, assess the business risk of the shoe manufacturer XYZ.

### Solutions:

Let $\beta_S$ be the beta of a pure shoe manufacturing business, and $\beta_A$ be the beta of a pure apparel manufacturing business. We know that $\beta_S$ represents the business risk for firm XYZ. Since we know the market weights on shoes vs. apparel for firms A and B, we can represent firm A and firm B as a portfolio of the pure-play shoe and pure-play apparel manufacturing businesses in order to solve for $\beta_S$ and $\beta_A$.

$$\text{Firm A: } 0.9 = 40\%\beta_S + 60\%\beta_A$$
$$\text{Firm B: } 0.7 = 70\%\beta_S + 30\%\beta_A$$

Solving this system of two equations with two unknowns yields $\beta_S = 0.5$ and $\beta_A = 1.17$. Therefore, the beta for firm XYZ, a pure-play shoe manufacturing business, is $\beta_S = 0.5$.

## Question 2

A private equity fund is considering buying a gold mining company, Kinross Gold. The firm is expected to generate an after-tax cash flow of \$540mm next year (Year 1). This cash flow is expected to grow at 2.5% in perpetuity. The private equity fund needs to determine the cost of capital for Kinross Gold in order to estimate its value.
Suppose that asset returns are characterized by a two-factor model:

$$r_i = E[r_i] + \beta_{i,1} f_1 + \beta_{i,2} f_2 + \epsilon_i$$

The first risk factor is the market risk, and the second risk factor is the price of gold.
You have estimated factor loadings for Kinross Gold and found that its loading on the market risk is 0.3, and its loading on the gold price risk is 1.92. In order to estimate Kinross Gold's cost of capital, you have obtained the following information on three well-diversified portfolios, A, B, and C:

| Portfolio | $E[r_i]$ | $\beta_{i,1}$ | $\beta_{i,2}$ |
|---|---|---|---|
| A | 4% | 0.0 | 1.0 |
| B | 6% | 0.5 | 0.5 |
| C | 4% | 0.5 | -0.5 |

What price should the private equity fund be willing to pay in order to buy Kinross Gold? Assume that Kinross Gold is all equity-financed (has no debt on its balance sheet).

### Solutions:

The value of Kinross Gold can be found using the Gordon Growth model:

$$V_0 = \frac{FCF_1}{r - g}$$
$$V_0 = \frac{\$540\text{mm}}{r - 2.5\%}$$

To find the value of the firm, we need to find $r$, its **cost of capital**.
Recall the APT pricing equation:

$$E[r_p] = r_f + \lambda_1 \beta_{P,1} + \lambda_2 \beta_{P,2}$$

We can write down this pricing equation for all three portfolios.

$$\begin{aligned}
&\text{A: } 4\% = r_f + \lambda_1 \times 0 + \lambda_2 \times 1 \\
&\text{B: } 6\% = r_f + \lambda_1 \times 0.5 + \lambda_2 \times 0.5 \\
&\text{C: } 4\% = r_f + \lambda_1 \times 0.5 + \lambda_2 \times (-0.5)
\end{aligned}$$

This a system of three equations with three unknowns. Solving yields $r_f = 2\%$, $\lambda_1 = 6\%$, $\lambda_2 = 2\%$.
So, the expected return on Kinross Gold is:

$$E[r] = 2\% + 6\% \times 0.30 + 2\% \times 1.92 = 7.64\%$$

Substituting $r = 7.64\%$ yields the following value of Kinross Gold:

$$V_0 = \frac{\$540\text{mm}}{7.64\% - 2.5\%} = \$10.51\text{bn}$$

## Question 3

Consider an asset with a price linked to the price of crude oil. Suppose the current price of crude oil is \$55 per barrel. Assume the risk-free rate $r_f = 2\%$.

(a) Suppose the asset in question pays in Year 5 the realized price of oil in Year 1. What is the asset's current value?

(b) Suppose the asset in question pays in Year 5 the realized price of oil in Year 2. What is the asset's current value?

(c) Suppose the asset has a Year 5 payoff of $\frac{P_1+P_2+P_5}{3}$, where $P_i$ is the realized price of oil in year $i$. What is the asset's current price?

### Solutions:

(a) We can start by thinking about what the price of the asset would be in year 1. Between now (year 0) and the end of year 1, the price is uncertain. However, between year 1 and year 5, the asset payoff is certain, so the asset is therefore risk free. Let $P_1$ be the price of oil in year 1, which is what the asset will pay off in year 5. At the end of year 1, $P_1$ is known. So, the value of the asset in year 1 is:

$$\text{Value}_1 = \frac{P_1}{(1+r_f)^4}$$

Now, we consider the value of the asset at year 0. At year 0, we don't know what the price of oil will be at the end of year 1. Let $\bar{r}_0$ be the annual expected return on oil. We will assume this stays constant over all time horizons (the term structure of these rates are flat). The value of the asset in year 0 is then the expected value of the asset in year 1, discounted at $\bar{r}_0$ to reflect the riskiness of the asset between year 0 and year 1:

$$\begin{aligned}
\text{Value}_0 &= \frac{E[\text{Value}_1]}{(1+\bar{r}_0)} \\
&= \frac{E[P_1/(1+r_f)^4]}{(1+\bar{r}_0)} \\
&= \frac{E[P_1]}{(1+\bar{r}_0)(1+r_f)^4} \\
&= \frac{P_0(1+\bar{r}_0)}{(1+\bar{r}_0)(1+r_f)^4} \\
&= \frac{P_0}{(1+r_f)^4}
\end{aligned}$$

Since $r_f = 2\%$ and $P_0 = \$55$, the value of the asset at Year 0 is $\frac{\$55}{(1+.02)^4} = \$50.81$.

(b) We start by thinking about the price of the asset in year 2, once there is no uncertainty around the asset's year 5 payoff. The value of the asset in year 2 is:

$$\text{Value}_2 = \frac{P_2}{(1+r_f)^3}$$

We now consider year 0. Again, we assume the annual expected return on oil $\bar{r}_0$ stays constant over all time horizons. The value in year 0 is the expected value of the asset in year 2, discounted to year 0 at $\bar{r}_0$ to reflect the riskiness of the asset between Year 0 and 2:

$$\begin{aligned}
\text{Value}_0 &= \frac{E[\text{Value}_2]}{(1+\bar{r}_0)^2} \\
&= \frac{E[P_2/(1+r_f)^3]}{(1+\bar{r}_0)^2} \\
&= \frac{E[P_2]}{(1+\bar{r}_0)^2(1+r_f)^3} \\
&= \frac{P_0(1+\bar{r}_0)^2}{(1+\bar{r}_0)^2(1+r_f)^3} \\
&= \frac{P_0}{(1+r_f)^3}
\end{aligned}$$

So, the value of the asset in year 0 is $\frac{\$55}{1.02^3} = \$51.83$.

(c) We can construct this asset's payoff with a portfolio of assets paying $P_1$ in Year 5, $P_2$ in Year 5, and $P_5$ in Year 5, each with a 1/3 weight. We have the values of the first two assets from part a and b, so we must now find the value of the asset that in year 5 pays $P_5$.

The value of such an asset in year 5 is simply the price of crude oil in year 5, $P_5$. Then, the value at year 0 is:

$$\begin{aligned}
\text{Value}_0 &= \frac{E[\text{Value}_5]}{(1+\bar{r}_0)^5} \\
&= \frac{E[P_5]}{(1+\bar{r}_0)^5} \\
&= \frac{P_0(1+\bar{r}_0)^5}{(1+\bar{r}_0)^5} \\
&= P_0 \\
&= \$55
\end{aligned}$$

So the value of the asset with a payoff of $\frac{P_1+P_2+P_5}{3}$ in year 5 is:

$$\frac{1}{3} \times \$50.81 + \frac{1}{3} \times \$51.83 + \frac{1}{3} \times \$55 = \$52.55$$

# Question 4

Consider a firm which operates as a restaurant. The restaurant is expected to generate an after-tax cash flow of one million dollars next year (Year 1). This cash flow is expected to grow at a 2% annual rate in perpetuity.
Next year (Year 1), the firm will have an opportunity to open a new restaurant. If this restaurant is opened, it will start generating exactly the same cash flow as the first restaurant in Year 2.

*Note: the first restaurant will generate 1.02 million in year 2. If the second restaurant is built, it will generate 1.02 million in cash flow that year as well.*
There is uncertainty with respect to the investment costs needed to open the new restaurant next year. Depending on the state of the economy and availability of retail space, these costs can be either \$5mm (20% probability) or \$15mm (80% probability).
Assume that the appropriate discount for the restaurant business is 12%. What is the present value of this growth opportunity?

## Solutions:

We start by determining the optimal decision in Year 1 - should the firm open the second restaurant?

**Case 1: opening costs are \$5mm**. We are in Year 1. Should we open the restaurant? If we open the new restaurant, the firm value will be

$$\begin{aligned} V_1 &= -\$5\text{mm} + \frac{\$1.02\text{mm} \times 2}{r-g} \\ &= -\$5\text{mm} + \frac{\$1.02\text{mm} \times 2}{12\% - 2\%} \\ &= \$15.4\text{mm} \end{aligned}$$

If we don't open the new restaurant, the firm value will be

$$\begin{aligned} V_1 &= \frac{\$1.02\text{mm}}{r-g} \\ &= \frac{\$1.02\text{mm}}{12\% - 2\%} \\ &= \$10.2\text{mm} \end{aligned}$$

So, if opening costs are \$5mm, the optimal decision is to open the new restaurant. In this scenario, the firm value in year 1 is \$15.4mm.

**Case 2: opening costs are \$15mm**. We are in Year 1. Should we open the restaurant? If we open the new restaurant, the firm value will be

$$\begin{aligned} V_1 &= -\$15\text{mm} + \frac{\$1.02\text{mm} \times 2}{r-g} \\ &= -\$15\text{mm} + \frac{\$1.02\text{mm} \times 2}{12\% - 2\%} \\ &= \$5.4\text{mm} \end{aligned}$$

If we don't open the new restaurant, the firm value will be

$$\begin{aligned} V_1 &= \frac{\$1.02\text{mm}}{r-g} \\ &= \frac{\$1.02\text{mm}}{12\% - 2\%} \\ &= \$10.2\text{mm} \end{aligned}$$

So, if opening costs are \$15mm, the optimal decision is to not open the new restaurant. In this scenario, the firm value in year 1 is \$10.2mm.

We now consider the value of the firm in year 0:

$$V_0 = \frac{FCF_1}{(1+r)} + \frac{V_1}{(1+r)}$$

Weighting the value of the firm under optimal deciion making in year 1, we have:

$$\begin{aligned} V_0 &= \frac{\$1\text{mm}}{(1+12\%)} + \frac{20\% \times \$15.4\text{mm} + 80\% \times \$10.2\text{mm}}{(1+12\%)} \\ &= \$10.93\text{mm} \end{aligned}$$

The value of the growth option is the firm value with the growth option minus the firm value without the growth option. We have the value of the firm with the growth option, so we now must compute the value without the growth option:

$$\text{No growth option } V_0 = \frac{\$1\text{mm}}{(r-g)} = \frac{\$1\text{mm}}{12\% - 2\%} = \$10\text{mm}$$

So the value of the growth option is:

$$PV(GO) = \$10.93\text{mm} - \$10\text{mm} = \$0.93\text{mm}$$

# Question 5

Kinross Gold is considering acquiring a gold mine in Russia. The estimated reserves are one million ounces. It takes one year to extract gold, and the estimated extraction costs are \$1,000 per oz.
The current price of gold is \$1,200 / oz. It will either increase to \$1,600 per oz or decrease to \$900 per oz next year, and will stay at that level forever.
The risk-free rate is 2%. The gold price risk is not diversifiable.

(a) What is the value of this mine of extraction starts today?

(b) If Kinross Gold can wait for a year to start the extraction process and they can also abandon this mine, what is its current value?

(c) What is the value of the abandonment option?

## Solutions:

(a) Since the gold price risk is not diversifiable, we can't discount at the risk-free rate. However, we are not given the discount rate. To solve this problem, we will use the prices of traded assets to obtain state prices, then use state prices to find the value of the mine.

    The current price of gold is \$1,200 per oz. In the up state, the price goes to \$1,600. So $\$1,600 = u \times \$1,200 \rightarrow u = 1.33$. In the down state, the price goes to \$900. Similarly, we have $d = 900/1200 = 0.75$.

We can then find the risk-neutral probabilities of the up and down states using $u$ and $d$:

$$q_u = \frac{(1+r_f)-d}{u-d} = \frac{1.02-0.75}{1.33-0.75} = 0.463$$
$$q_d = \frac{u-(1+r_f)}{u-d} = \frac{1.33-1.02}{1.33-0.75} = 0.537$$

The state prices then follow from the risk-neutral probabilities:

$$\phi_u = \frac{q_u}{(1+r_f)} = 0.454$$
$$\phi_d = \frac{q_d}{(1+r_f)} = 0.527$$

With the state prices, we can find the present value of revenue if we start extraction today. Recall that it takes one year to finish the extraction. The expected revenue per oz of gold is $\$1,600\phi_u + \$900\phi_d = \$1,200$. This is unsurprising, since the expected value should equal the current price of gold.

The cash flow per ounce of gold is $\$1,200 - \$1,000 = \$200$ per oz. So the value of the mine is \$200 per oz $\times$ 1mm oz = \$200mm.

(b) Here, we can delay extraction for one year and abandon the mine if needed. In the up state, we have a a positive cash flow per oz of gold ($\$1,600 - \$1,000 > 0$), so we choose to extract. In the down state, we abandon the mine since the cash flow per oz of gold is negative.

The following diagram demonstrates the timeline under this scenario:


The value in the up state, where you pay \$1,000 per oz in year 1 and receive \$1,600 per oz with certainty starting in year 2, is $-\$1,000 + \frac{1600}{1.02} = \$568.63$. So, the current value of this mine per oz is:

$$\$568.63 \times \phi_u + \$0 \times \phi_d = \$258.03$$

So the value of the mine is \$258.03 per oz $\times$ 1mm oz = \$258.03mm.

(c) The value of the abandonment option is the value of the mine with the option minus the value of the mine without the option. We have the value of the mine with the option, which we found in the previous part - it is \$258.03mm. Now, we need to find the value without the abandonment option.

To isolate the value of the abandonment option, we must separate its value from the value of the option to delay production by a year. We do this by considering the case where the firm can delay production but can't abandon the mine:

In the up state, we still have a value of \$258.03 per oz.

In the down state, rather than 0, we now have a value of $-\$1,000 + \frac{\$900}{1.02} = -\$117.65$.

So the value per oz without the abandonment option and delaying production by a year is:

$$\$258.93 \times \phi_u - \$117.65 \times \phi_d = \$196.08$$

So the value of the mine if we delay production by a year and don't have the abandonment option is \$196.08mm. Recall from part a that the value of the mine without the abandonment option and starting extraction now is \$200mm, which is greater than \$196.08mm. It is therefore optimal to start producing now if the abandonment option is not available (the firm will not exercise the option to delay production), so the firm value without the abandonment option is \$200mm.

We have that:

- The value of the mine with the abandonment option is \$258.93mm
- The value of the mine without the abandonment option is \$200mm, since the firm will choose to start producing now if it can't abandon the mine

So the value of the abandonment option is then $\$258.93\text{mm} - \$200\text{mm} = \$58.03\text{mm}$.