Recitation 17

Spring 2021

Question 1

Teradyne is an automatic test equipment designer and manufacturer. It is currently an all-equity firm with an expected return of 12%. It is considering a leveraged recapitalization, in which it would borrow and repurchase existing shares.

(a) Suppose Teradyne borrows to the point that its debt-equity ratio is 0.50. With this amount of debt, the debt cost of capital is 6%. What will the expected return on equity be after this transaction?

(b) Suppose instead Teradyne borrows to the point that its debt-equity ratio is 1.50. With this amount of debt, Teradyne's debt will be much risker. As a result, the debt cost of capital will be 8%. What will the expected return on equity be in this case?

(c) One of senior managers argues that option (b) is in the best interests of shareholders because it leads to higher expected return on the stock. How would you respond to this argument?

Solutions:

(a) Since Teradyne is currently an all-equity firm, its expected return on equity and assets is the same (12%). So we know r_A. We now use MM II to find r_E under the new capital structure:

$$\begin{aligned} r_E &= r_A + + \frac{D}{E}(r_A - r_D) \\ r_E &= 12\% + 0.5 \times ((12\% - 6\%) \\ &= 15\% \end{aligned}$$

(b) We use the same method as in part a, changing the debt cost of capital and debt-equity ratio:

$$\begin{aligned} r_E &= r_A + + \frac{D}{E}(r_A - r_D) \\ r_E &= 12\% + 1.5 \times ((12\% - 8\%) \\ &= 18\% \end{aligned}$$

The higher debt level increases the expected return on equity of Teradyne.

(c) Option b leads to a higher expected return on equity because the equity becomes riskier when the firm takes on a larger debt load. In other words, the return is higher because the risk is higher, and the return compensates for the risk - there is no free lunch.

Question 2

Suppose that you work as an equity analyst at an investment bank. You need to find a weighted average cost of capital for a firm, which you will use in your valuation model.
You found the following information for this firm:

- It is financed with equal amount of debt and equity, i.e. its debt-to-equity ratio is 1
- Its cost of debt is 3.5%
- The equity beta is 1.2

The economic forecasting division of your bank advises you to use a market risk premium of 6% and a risk-free rate of 2% in your model.
Suppose that the assumptions of the Modigliani-Miller theorem hold. In particular, assume that corporations do not pay corporate income taxes.
What is the weighted average cost of capital of this firm?

Solutions:

By definition, the weighted average cost of capital when there are no corporate taxes is:

$$WACC = \frac{D}{D+E} \times r_D + \frac{E}{D+E} \times r_E$$

We know that $\frac{D}{D+E} = 1$, and that $r_D = 3.5\%$. We need to find r_E to find WACC, and we'll use CAPM to do so:

$$\begin{aligned} r_E &= r_f + \beta_E \times MRP \\ r_E &= 2\% + 1.2 \times 6\% \\ &= 9.2\% \end{aligned}$$

Now we can find WACC:

$$\begin{aligned} WACC &= \frac{D}{D+E} \times r_D + \frac{E}{D+E} \times r_E \\ &= \frac{1}{2} \times 3.5\% + \frac{1}{2} \times 9.2\% \\ &= 6.35\% \end{aligned}$$

Question 3

The Brandywine Corporation has two divisions of equal market value: the brandy division and the wine division. The debt-equity ratio of Brandywine is 2/9. The company's debt is risk-free.

For the last few years, the brandy division has been using a discount rate of 11.0% in capital budgeting decisions, while the wine division has been using a discount rate of 10.0%.
The current risk-free rate is 5.0% and the expected rate of return on the market portfolio is 10.0%.

(a) What is the beta of typical projects implicit in the discount rate used by the brandy division?

(b) What is the beta of typical projects implicit in the discount rate used by the wine division?

(c) What is the equity beta of Brandywine that is consistent with the discount rates used by Brandywine's two divisions?

(d) You were hired as a consultant to advise on the use of discount rates in the brandy and wine divisions of Brandywine. As the first step, you estimated the firm's equity beta and confirmed that it equals 1.34. Next, you found a publicly traded firm, called Pure Brandy Corporation, which operates in the brandy business. You have estimated its equity beta and found that it equals 1.8. The debt-equity ratio of Pure Brandy is 1/4, and its debt beta is 0.2.

What discount rates would you recommend for projects in the brandy and in the wine divisions of Brandywine?

Solutions:

(a) We use the CAPM to find beta:

$$\begin{aligned} R_A^B &= r_f + \beta_A^B(E[r_M] - r_f) \\ 11\% &= 5\% + \beta_A^B(10\% - 5\%) \\ \beta_A^B &= 1.2 \end{aligned}$$

(b) We use CAPM, similar to part a:

$$\begin{aligned} r_A^W &= r_f + \beta_A^W(E[r_M] - r_f) \\ 10\% &= 5\% + \beta_A^W(10\% - 5\%) \\ \beta_A^B &= 1.0 \end{aligned}$$

(c) We first find the asset beta of Brandywine:

$$\begin{aligned} \beta_A^{BW} &= w_B\beta_A^B + w_W\beta_A^W \\ \beta_A^{BW} &= 0.5 \times 1.2 + 0.5 \times 1.0 \\ &= 1.1 \end{aligned}$$

We can then find the equity beta:

$$\begin{aligned} \beta_E^{BW} &= \beta_A^{BW} + \frac{D}{E}(\beta_A^{BW} - \beta_D^{BW}) \\ \beta_E^{BW} &= 1.1 + \frac{2}{9}(1.1 - 0) \\ &= 1.34 \end{aligned}$$

(d) First, note that our estimation of Brandywine's asset beta to be 1.1 remains correct, since we confirmed that Brandywine's equity beta is 1.34.

We can now find the betas for each division using the information given on Pure Brandy Corporation. We first need the asset β of Pure Brandy, since this is directly translatable to the brandy division of Brandywine. Note that since the debt-equity of Pure Brandy is 1/4, it is financed by 1/5 debt and 4/5 equity.

$$\begin{aligned}
\beta_A &= \frac{D}{D+E}\beta_D + \frac{E}{D+E}\beta_E \\
\beta_A &= \frac{1}{5} \times 0.2 + \frac{4}{5} \times 0.8 \\
&= 1.48
\end{aligned}$$

So, the brandy division beta of Brandywine is 1.48. We can now find the wine division beta:

$$\begin{aligned}
\beta_A^{BW} &= w_B \beta_A^B + w_W \beta_A^W \\
1.1 &= 0.5 \times 1.48 + 0.5 \times \beta_A^W \\
\beta_A^W &= 0.72
\end{aligned}$$

We can now suggest discount rates for the two divisions. For the brandy division, we have:

$$\begin{aligned}
r_B &= r_f + \beta_A^B (E[r_M] - r_f) \\
r_B &= 5\% + 1.48 \times (10\% - 5\%) \\
&= 12.4\%
\end{aligned}$$

For the wine division, we have:

$$\begin{aligned}
r_W &= r_f + \beta_A^W (E[r_M] - r_f) \\
r_W &= 5\% + 0.72 \times (10\% - 5\%) \\
&= 8.6\%
\end{aligned}$$

Question 4

Suppose that corporation Y is planning to acquire corporation X. Below is some current information on corporation X:

- Market value of debt is \$40 million
- Market value of equity is \$160 million
- Cost of debt capital is 7%
- Equity beta is 1.2

Below is some current information on corporation Y:

- Market value of debt is \$100 million

- Market value of equity is \$900 million
- Debt is risk-free
- Asset beta is 0.76

Below are the transaction details:

- Y will be financing the acquisition by issuing new debt
- The old debt of X will be repurchased
- The cost of debt for the combined entity will be 8%
- X will get acquired at its current market value
- No synergies

Suppose that the risk-free rate is 5%, and the expected return on the market portfolio is 15%. Assume that corporations don't pay income taxes, and that there are no other market imperfections.
What is the equity beta and the expected return on equity of firm Y after this acquisition is complete?

Solutions:

To find the equity risk of the firm, we first need to find its return on assets, which is independent of capital structure. The return on assets of the new firm is the weighted average of return on assets of firms X and Y.
The steps we will use to solve the problem are described below:

1. Find asset beta of Firm X (Note: we already know asset beta of Firm Y)
2. Find the asset beta of the new firm
3. Find equity beta and return on equity of the new firm

Step 1: Asset beta of X
We find X's asset beta using the following formula:

$$\beta_A^X = \frac{D}{D+E}\beta_D^X + \frac{E}{D+E}\beta_E^X$$

We know D, E, and β_E^X, and we need to find β_D^X. We will do so using CAPM:

$$\begin{aligned} r_D^X &= r_f + \beta_D^X(E[r_M] - r_f) \\ 7\% &= 5\% + \beta_D^X(15\% - 5\%) \\ \beta_D^X &= 0.2 \end{aligned}$$

So, the asset beta of X is:

$$\begin{aligned} \beta_A^X &= \frac{D}{D+E}\beta_D^X + \frac{E}{D+E}\beta_E^X \\ \beta_A^X &= \frac{40}{40+160} \times 0.2 + \frac{160}{160+40} \times 1.2 \\ &= 1 \end{aligned}$$

Step 2: Asset beta of combined firm
We find the new firm's asset beta as the weighted average of X and Y asset betas:

$$\begin{aligned}
\beta_A^{NEW} &= \frac{V^X}{V^X + V^Y}\beta_A^X + \frac{V^Y}{V^X + V^Y}\beta_A^Y \\
\beta_A^{NEW} &= \frac{200}{200 + 1,000} \times 1 + \frac{1,000}{200 + 1,000} \times 0.76 \\
&= 0.8
\end{aligned}$$

Step 3: Equity beta of combined firm
We find the new firm's equity beta of the combined firm using the following equation:

$$\beta_E^{NEW} = \beta_A^{NEW} + \frac{D}{E}(\beta_A^{NEW} - \beta_D^{NEW})$$

To use this equation, we first need to find the debt-to-equity ratio of the new firm, as well as the new firm's debt beta.
The total debt of the new firm is the old debt plus the new debt. Recall that Y is issuing debt to finance the purchase of X, which is \$200 million (recall that X's debt will be repurchased in the transaction). So the new firm's total debt is Y's original debt load, plus the \$200 million used to purchase X's equity and debt.

$$\text{New firm debt} = \$100\text{M} + \$200\text{M} = \$300\text{M}$$

The equity value of the firm is the firm's total value, minus the debt value:

$$\text{New firm equity} = (\$1,000\text{M} + \$200\text{M}) - \$300\text{M} = \$900\text{M}$$

So the debt to equity ratio is:

$$\frac{D}{E} = \frac{300}{900} = \frac{1}{3}$$

We now find the new firm's debt beta using CAPM and the new firm's cost of debt capital, which is given to be 8%:

$$\begin{aligned}
r_D^{NEW} &= r_f + \beta_D^{NEW}(E[r_M] - r_f) \\
8\% &= 5\% + \beta_D^{NEW}(15\% - 5\%) \\
\beta_D^{NEW} &= 0.3
\end{aligned}$$

We can now find the new firm's equity beta and expected return on equity. The equity beta is:

$$\begin{aligned}
\beta_E^{NEW} &= \beta_A^{NEW} + \frac{D}{E}(\beta_A^{NEW} - \beta_D^{NEW}) \\
\beta_E^{NEW} &= 0.8 + \frac{1}{3}(0.8 - 0.3) \\
&= 0.967
\end{aligned}$$

The expected return on equity is:

$$\begin{aligned}
r_E^{NEW} &= r_f + \beta_E^{NEW}(E[r_M] - r_f) \\
r_E^{NEW} &= 5\% + 0.967(15\% - 5\%) \\
&= 14.67\%
\end{aligned}$$

So the new firm's equity beta is 0.967, and its expected return on equity is 14.67%.

Question 5

Consider a market with two possible states a year from now: Good or Bad. In the Good state, the return on the market portfolio will be 25%, and in the Bad state it will be -25%. The probability of the Good state is 70%. The one-year risk-free rate is 5%.

Consider a corporate bond. It has a 0% coupon rate and matures one year from now. The face value of the bond is $100. In the Good state, the bond has a 2% probability of default, and the recovery ratio is 60%. In the Bad state, the bond has a 10% probability of default, and the recovery ratio is 40%.

(a) Compute the expected payoff of the bond in each of the two states one year from now.

(b) Determine the risk-neutral probabilities of the Good and Bad states.

(c) Compute the price of the corporate bond in Year 0.

(d) Compute the bond's promised yield, expected yield, the default premium, and the risk premium.

(e) Suppose that the CAPM holds. Based on one-year returns, what is the market beta of the corporate bond in this question?

Solutions:

(a) In the good (up) state, the default probability is 2%, and the recovery ratio is 60%:

$$\begin{aligned} E[CF_u] &= 0.98 \times \$100 + 0.02 \times 0.6 \times \$100 \\ &= \$99.20 \end{aligned}$$

In the bad (down) state, the default probability is 10%, and the recovery ratio is 40$:

$$\begin{aligned} E[CF_d] &= 0.9 \times \$100 + 0.1 \times 0.4 \times \$100 \\ &= \$94.00 \end{aligned}$$

(b) Since the return on market portfolio is 25% in the good state and -25% in the bad state, we have $u = 1.25$ and $d = 0.75$.

Then, the risk-neutral probabilities are:

$$\begin{aligned} q_u &= \frac{(1 + r_f) - d}{u - d} \\ &= \frac{1.05 - 0.75}{1.25 - 0.75} \\ &= 0.6 \\ q_d &= 1 - q_u \\ &= 0.4 \end{aligned}$$

(c) We use the expected payoffs in each state and risk-neutral probabilities of each state to price the bond:

$$\begin{aligned} B_0 &= \frac{q_u E[CF_u] + q_d E[CF_d]}{1 + r_f} \\ &= \$92.50 \end{aligned}$$

(d) The promised yield is:

$$y = \frac{FV}{B_0} - 1 = \frac{100}{92.50} - 1 = 8.11\%$$

The expected yield is:

$$\begin{aligned} \bar{y} &= \frac{E[CF_1]}{B_0} - 1 \\ &= \frac{p_u E[CF_u] + p_d E[CF_d]}{B_0} - 1 \\ &= \frac{0.7 \times 99.20 + 0.3 \times 94.00}{92.50} - 1 \\ &= 5.56\% \end{aligned}$$

The default premium is:

$$y - \bar{y} = 8.11\% - 5.56\% = 2.55\%$$

The risk premium is:

$$RP = \bar{y} - r_f = 5.56\% - 5\% = 0.56\%$$

(e) By CAPM:

$$E[r_i] = r_f + \beta_i(E[r_M] - r_f)$$

We can find $E[r_M]$ from the information in the problem statement, as in the good state the return is 0.25, while in the bad state it is -.25. So

$$E[r_M] = 0.7 \times 25\% + 0.3 \times (-25\%) = 10\%$$

Plugging this into CAPM for the bond, we have:

$$5.56\% = 5\% + \beta_B(10\% - 5\%) \rightarrow \beta_B = 0.1124$$

Question 6

Suppose that there are two possible states of the economy next year: good and bad. The good state occurs with probability 80%, and the bad state occurs with probability 20%. Consider a small firm whose assets in year 1 will be worth \$90 million in the good state and \$40 million in the bad state. This firm has two bonds outstanding, bond 1 and bond 2. Both bonds pay 0% coupon and mature in year 1.

The face value of bond 1 is \$40 million, and the face value of bond 2 is \$10 million. Bond 2 is junior to bond 1. This means that, in the event of default, holders of bond 2 do not receive any payment until holders of bond 1 receive their promised payment in full. We will assume that the one year risk-free rate is 5% and that corporations do not pay corporate income taxes.
Suppose that the current market value of equity is \$28.57 million. What are the current prices of bonds 1 and 2 and their promised yields? What are the expected returns on both bonds and the equity of this firm?

Solutions:

We begin by determining what the holders of each bond and the equity receive in each state of the world:

State	Assets	Bond 1	Bond 2	Equity
Good	\$90mm	\$40mm	\$10mm	\$40mm
Bad	\$40mm	\$40mm	\$0	\$0

To price the bonds, we first need to find the risk-neutral probabilities of the up and down state. We are given the current value of equity, which is \$28.57 million, and can use this to find the risk-neutral probabilities:

$$\begin{aligned}
28.57 &= \frac{q_u \times 40 + q_d \times 0}{1 + r_f} \\
28.57 &= \frac{40 q_u}{1.05} \\
q_u &= 0.75 \\
q_d &= 0.25
\end{aligned}$$

Then, the price of bond 1 is:

$$\begin{aligned}
B_1 &= \frac{q_u \times 40 + q_d \times 40}{1 + r_f} \\
&= \frac{40}{1.05} \\
&= 38.10M
\end{aligned}$$

The promised yield on bond 1 is $40/B_1 - 1 = 5\%$, which is the same as its expected return (the expected payoff of bond 1 is still 40M).

The price of bond 2 is:

$$\begin{aligned}
B_2 &= \frac{q_u \times 10 + q_d \times 0}{1 + r_f} \\
&= \frac{0.75 \times 10}{1.05} \\
&= 7.14M
\end{aligned}$$

The promised yield on bond 2 is $10/7.14 - 1 = 40\%$.

The expected return on bond 2 is $\frac{80\% \times 10 + 20\% \times 0}{7.14} - 1 = 12\%$.
Note that we can decompose the promised yield of the second bond, which is 40%, into the default premium, risk premium, and the risk-free rate of return. The default premium is $40\% - 12\% = 28\%$. The risk premium is $12\% - 5\% = 7\%$.
Finally, the expected return on equity is $\frac{80\% \times 40 + 20\% \times 0}{28.57} - 1 = 12\%$.
Note that the risk premium on equity of this firm and on bond 2 are the same. This should not be surprising because both equity and bond 2 only pay in the up state. And, in the down state, they're both worthless. In other words, bond 2 and equity of this firm are effectively identical securities.