# Recitation 15

Spring 2021

## Question 1

Suppose that the expected return on the market portfolio is 14% and the standard deviation of its returns is 25%. The risk-free rate is 6%.
Consider a portfolio with expected return of 16% and assume that it is located on the Capital Market Line (CML).

(a) What is the beta of this portfolio?

(b) What is the standard deviation of this portfolio?

(c) What is its correlation with the market?

### Solutions:

(a) According to the capital asset pricing model (CAPM):

$$\begin{aligned} E[r_P] &= r_f + \beta_P(E[r_M] - r_f) \\ 16\% &= 6\% + \beta_P(14\% - 6\%) \\ \beta_P &= 1.25 \end{aligned}$$

(b) We can replicate this portfolio with the market portfolio and the risk-free asset to determine the portfolio's standard deviation. *Since the portfolio is located on the CML,* the expected return and the standard deviation of portfolio P and the replicating portfolio R should be the same.

Say we invest $w$ in the risk-free asset and $1 - w$ in the market portfolio. We want this portfolio to have an expected return of 16%. So:

$$\begin{aligned} 16\% &= w \times 6\% + (1 - w) \times 14\% \\ w &= -0.25 \end{aligned}$$

Since we know the weights on the risk-free asset ($-0.25$) and the market portfolio (1.25), we can compute the variance (and standard deviation) of the replicating portfolio $R$:

$$\begin{aligned} Var(r_R) &= Var(wr_f + (1 - w)r_M) = (1 - w)^2 Var(r_M) \\ \sigma_R &= (1 - w)\sigma_M = 1.25 \times 25\% = 31.25\% \end{aligned}$$

The standard deviation of the replicating portfolio is the same as the standard deviation of the portfolio in question: $\sigma_P = 31.25\%$.

(c) We already know the portfolio $\beta_P$ and $\sigma_P$. We find the correlation of the portfolio with the market using the formula for Beta:

$$\beta_P = \frac{Cov(r_P, r_M)}{Var(r_M)} = \frac{\rho_{P,M}\sigma_M\sigma_P}{\sigma_M^2} = \frac{\rho_{P,M}\sigma_P}{\sigma_M}$$

$$\begin{aligned}\rho_{P,M} &= \frac{\beta_P\sigma_M}{\sigma_P} \\ &= \frac{1.25 \times 25\%}{31.25\%} \\ &= 1\end{aligned}$$

## Question 2

Consider the following situations in the market. Are they consistent with the capital asset pricing model?

(a) There are two stocks, A and B, with the following expected returns and betas:

| Stock | Expected Return | Beta |
|---|---|---|
| A | 25% | 0.8 |
| B | 15% | 1.2 |

(b) The expected returns and standard deviation of stock A and the market portfolio are as follows:

| Stock | Expected Return | Standard Deviation |
|---|---|---|
| A | 25% | 30% |
| Market | 15% | 30% |

(c) There are three assets, with the following expected returns and standard deviations:

| Asset | Expected Return | Standard Deviation |
|---|---|---|
| Risk-free | 10% | 0% |
| Market | 18% | 24% |
| A | 16% | 12% |

(d) There are three assets, with the following expected returns and betas:

| Asset | Expected Return | Beta |
|---|---|---|
| Risk-free | 5% | 0 |
| Market | 15% | 1 |
| A | 20% | 1.5 |

(d) Suppose there are two funds, with the following expected returns and standard deviations:

| Asset | Expected Return | Standard Deviation |
|---|---|---|
| Real Estate Fund | 30% | 35% |
| Market | 40% | 25% |

## Solutions:

(a) We see from the CAPM model: $E[r_i] = r_f + \beta_i(E[r_M] - r_f)$ that a higher beta should imply a higher expected return ($r_f$ and $E[r_M]$ don't change across stocks). However, this is not the case for the stocks in this situation, so CAPM does not hold here.

(b) As shown below, stock A lies outside of the CML and outside of the efficient frontier. Therefore, the market portfolio must be inefficient, and CAPM does not hold.

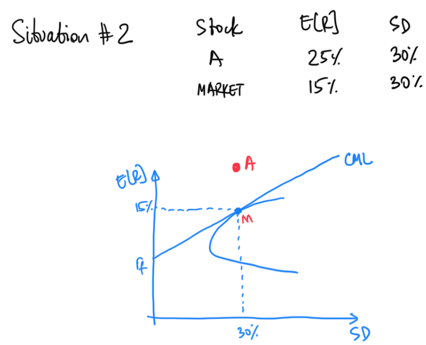


Figure 1: Stock A, the CML, and the efficient frontier (2b)

(c) As shown in the below diagram, stock A lies outside the capital market line and outside the efficient frontier.

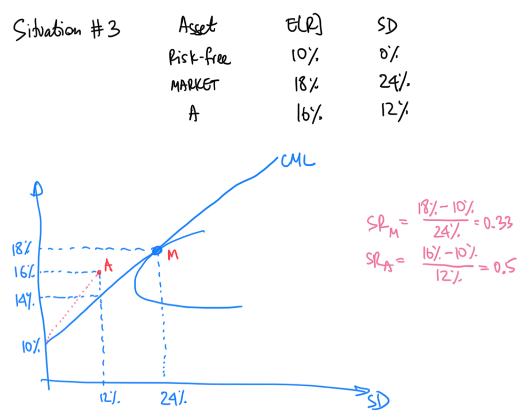


Figure 2: Stock A, the CML, and the efficient frontier (2c)

This can be shown numerically by the Sharpe ratios of the market portfolio and of stock A, as shown in the above diagram as well. We see that the Sharpe ratio for stock A is 0.5, which is greater than the market portfolio Sharpe ratio of 0.33. Since CAPM tells us that the market portfolio is efficient, CAPM does not hold in this situation.

(d) We can fit these expected returns and Betas to the CAPM model to see if CAPM holds. The following must hold for stock $A$:

$$
\begin{aligned}
E[r_A] &= r_f + \beta_A(E[r_M] - r_f) \\
20\% &= 5\% + 1.5(15\% - 5\%) \\
20\% &= 20\%
\end{aligned}
$$

Since the model holds, CAPM holds for this situation. We can also visualize this as in the below diagram, which shows that stock A lies on the security market line, as required by CAPM.

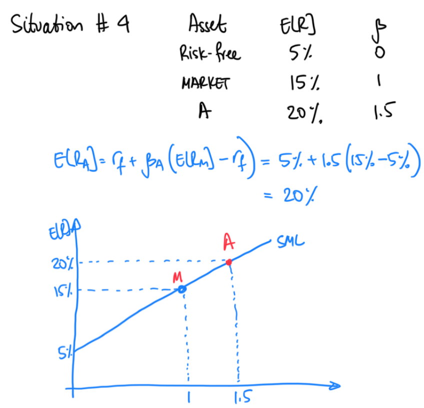


Figure 3: Stock A, the market portfolio, and the SML (2d)

(e) This situation is consistent with CAPM. In CAPM, investors are compensated only for systematic risk, which is measured by beta. On the other hand, standard deviation measures total risk, including idiosyncratic risk, which investors are not compensated for in CAPM - according to the model, investors can diversify away this type of risk. Therefore, the real estate fund having a lower expected return and higher standard deviation than the private equity fund does not mean that CAPM is violated - there may be assets in the market with low return and high standard deviation.

# Question 3

Suppose an endowment fund is currently investing 20% of its portfolio in a risk-free asset and investing the remaining 80% with a private equity fund, Eternal Growth LLC. The endowment fund analyzed holdings of Eternal Growth's portfolio and found that its expected returns over the next year will be 10%, with a standard deviation of 25%.

Research conducted by the endowment fund indicates that the market portfolio will have expected return of 10% and standard deviation of 20% during the next year. The correlation of returns between Eternal Growth's portfolio and the market portfolio is 0.8.
Suppose that the risk-free asset will yield 5% return over the next year.

(a) Compute the CAPM beta of Eternal Growth's portfolio.

(b) Compute the CAPM beta and alpha of the endowment fund's portfolio.

(c) Suppose that you were retained as an advisor to the endowment fund. Would you recommend divesting part of the holdings in Eternal Growth and reallocating them to the market portfolio?

## Solutions:

(a) CAPM beta of Eternal Growth LLC.

Recall that:

$$\beta_{EG} = \frac{\rho_{EG,M} \times \sigma_{EG}}{\sigma_M} = \frac{0.8 times 0.25}{0.2} = 1$$

So the CAPM beta of Eternal Growth's portfolio is 1.

(b) CAPM beta and alpha of the endowment fund.

The endowment fund invests 20% into the risk-free asset and 80% into Eternal Growth. So,

$$\beta_{EF} = 20\% \beta_{r_f} + 80\% \beta_{EG} = 80\% \times 1 = 0.8$$

To compute the alpha, we first find the endowment fund's expected return:

$$\bar{r}_{EF} = 20\% r_f + 80\% \bar{r}_{EG} = 20\% \times 5\% + 80\% \times 10\% = 9\%$$

Then, the CAPM alpha is the difference between the expected return that the endowment fund expects to generate (9%) and the expected return under the CAPM. The expected return of the endowment fund under CAPM is:

$$E[r_{EF}] = r_f + \beta_{EF}(E[r_M] - r_f) = 5\% + 0.8 \times (10\% - 5\%) = 9\%$$

$E[r_{EF}]$ under CAPM is the same as the endowment fund's expected return. So the CAPM alpha is 0.

(c) We can compare the Sharpe ratio of the endowment fund to the Sharpe ratio of the market portfolio. The endowment fund Sharpe ratio is given below:

$$SR_{EF} = \frac{\bar{r}_{EF} - r_f}{\sigma_{EF}} = \frac{9\% - 5\%}{20\%} = 0.2$$

Note that the standard deviation is computed as $\sigma_{EF} = 0.8\sigma_{EG} = 0.8 \times 25\% = 20\%$.

The Sharpe ratio of the market portfolio is given below:

$$SR_M = \frac{\bar{r}_M - r_f}{\sigma_M} = \frac{10\% - 5\%}{20\%} = 0.25$$

Since the market portfolio Sharpe ratio is greater than the endowment fund Sharpe ratio, the endowment fund is not efficient. Its Sharpe ratio can be improved by reallocating assets from the Eternal Growth fund to the market portfolio.

We can also visualize this by plotting the endowment fund, eternal growth fund, market portfolio, efficient frontier, and CML:

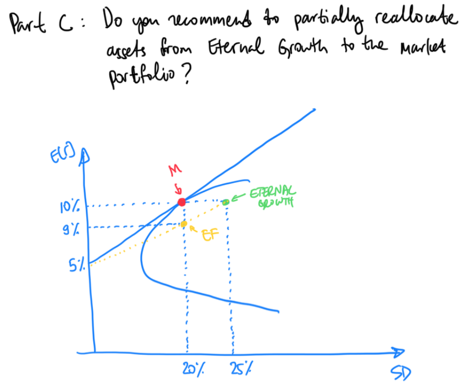


Figure 4: Endowment fund, eternal growth, the CML, and the efficient frontier (3c)

## Question 4

Given the data in the spreadsheet, estimate the CAPM betas for the given firms. Then, assuming the future monthly expected return on the market is 0.8%, use the estimated betas to find future expected returns.

### Solutions:

We are given 180 monthly observations for each firm. Returns are given in column C. We're also given the risk free rate in column D, and we're given the market return in column E.

To estimate the CAPM model, we need to run the regression of a stock's excess return on the excess market return. We first need to compute the excess return for each of the stocks by subtracting the risk-free rate from the return. To compute excess market return, we subtract the risk-free rate from the market return.

Next, we need to run the linear regression using the LINEST function in Excel. Its inputs are the the excess return on the stock (the left-hand side variable), the excess market return (right-hand side / known variable), and whether the regression is run with a constant or not. We want to have a constant. Note that a fourth regression parameter can be included in the function for additional regression statistics. And, since the function is an array function, we need to use Shift-Control-Enter to estimate both beta and alpha in the CAPM equation.

The resulting betas and alphas are given below:

| | $\beta_{MKT}$ | α |
|---|---|---|
| Kinross Gold Corporation | 0.54 | 0.45% |
| AngloGold Ashanti | 0.32 | 0.53% |
| Barrick Gold Corporation | 0.39 | 0.43% |
| Newmont Corporation | 0.38 | 0.36% |
| | | |
| Apple Inc. | 1.07 | 2.05% |
| Microsoft Corporation | 0.89 | 0.72% |
| Oracle Corporation | 0.97 | 0.31% |
| Intel Corporation | 0.91 | 0.33% |

As shown above, the CAPM betas for the gold mining stocks vary from 0.32 to 0.54, and the CAPM betas for technology stocks vary from 0.89 to 1.07. These results show that gold mining stocks have much lower return sensitivity with respect to the market when compared to technology stocks.

We can now use the estimated betas to find future expected returns using the capital asset pricing model. We assume that the future monthly expected return on the market is 0.8%. The effective annual rate is $1.008^{12} - 1$, which is about 10%. According to CAPM, the expected excess return on a stock is given by beta times the expected excess return on the market. Implementing this gives the following expected returns (plotted alongside the historical average monthly excess return, historical standard deviation, and 95% confidence interval):

| | $\beta_{MKT}$ | α | Expected excess return | Historical average monthly excess returns, 2005-2019 | Historical standard deviation | 95% CI | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|---|---|
| Kinross Gold Corporatior | 0.54 | 0.45% | 0.43% | 0.84% | 15.06% | 2.20% | -1.36% | 3.04% |
| AngloGold Ashanti | 0.32 | 0.53% | 0.26% | 0.77% | 14.08% | 2.06% | -1.29% | 2.82% |
| Barrick Gold Corporatior | 0.39 | 0.43% | 0.31% | 0.72% | 12.49% | 1.82% | -1.11% | 2.54% |
| Newmont Corporation | 0.38 | 0.36% | 0.31% | 0.64% | 10.43% | 1.52% | -0.88% | 2.17% |
| | | | | | | | | |
| Apple Inc. | 1.07 | 2.05% | 0.86% | 2.84% | 9.23% | 1.35% | 1.49% | 4.19% |
| Microsoft Corporation | 0.89 | 0.72% | 0.72% | 1.38% | 6.54% | 0.96% | 0.42% | 2.33% |
| Oracle Corporation | 0.97 | 0.31% | 0.78% | 1.03% | 6.45% | 0.94% | 0.09% | 1.97% |
| Intel Corporation | 0.91 | 0.33% | 0.73% | 1.00% | 6.92% | 1.01% | -0.01% | 2.01% |

Note that for all eight of our stocks, the estimation from the asset pricing model is lower than the historical average return. And as illustrated by the confidence intervals in the table, it is very difficult to infer average returns from the historical data - this imprecision is exactly why we use asset pricing models to forecast future returns.

## Question 5

Suppose company XYZ is a publicly traded firm. Its current share price is \$22.5. The total number of outstanding shares is 20 million.
XYZ has 525,000 bonds outstanding, each with a face value of \$100, maturing one year from now. The bonds do not pay a coupon and are considered risk-free.
XYZ's stock has a market beta of 0.8. The expected market risk premium is 6%, and the risk-free rate is 5%.

(a) What is the current market value of the firm?

(b) What is the market beta of the firm?

(c) Suppose that the beta of the firm's assets and the expected interest rates are constant over the next two years. What is the expected price of the stock in two years if it does not pay dividends? Assume that the performance of the company in Year 1 does not predict its performance in Year 2.

(d) If the firm issues 551,250 2-year zero-coupon bonds with face value \$100 to buy back its stock, will it change the equity beta of the firm? If yes, compute the new equity beta. Suppose that investors consider these new bonds risk-free.

## Solutions:

(a) Current market value of the firm.

The current market value of equity is:

$$E = \$22.5 \times 20,000,000 = \$450,000,000$$

The current market value of debt is:

$$D = \frac{1}{1 + 5\%} \times \$100 \times 525,000 = \$50,000,000$$

So the market value of the firm's assets is:

$$A = E + D = \$450,000,000 + \$50,000,000 = \$500,000,000$$

The current market value of the firm is \$500 million.

(b) Market beta of the firm's assets.

Recall that:

1. The firm's assets are a portfolio of its equity and debt
2. The beta of a portfolio is a weighted average of betas of assets in the portfolio

So, we have:

$$\beta_A = \frac{E}{E + D}\beta_E + \frac{D}{E + D}\beta_D$$

Since the debt is risk-free, we have $\beta_D = 0$. And, we are given the equity beta $\beta_E = 0.8$. So,

$$\beta_A = \frac{E}{E+D}\beta_E = \frac{\$450\text{mm}}{\$500\text{mm}} \times 0.8 = 0.72$$

The market beta of the firm's assets is 0.72.

(c) Expected stock price in Year 2.

Recall the dividend discount model:

$$P_0 = \frac{D_1}{1+r_1} + \frac{D_2 + P_2}{(1+r_1)(1+r_2)}$$

Since the stock does not pay dividends, we have:

$$P_2 = P_0(1+r_1)(1+r_2)$$

Taking the expectation of both sides gives us:

$$E[P_2] = P_0 \times E[(1+r_1)(1+r_2)]$$

Next, note that we assume the stock performance in year 1 is independent of its performance in year 2. This allows us to separate the $(1+r_1)$ and $(1+r_2)$ when taking the expectation of the product:

$$E[P_2] = P_0 \times E[(1+r_1)] \times E[(1+r_2)]$$
$$E[P_2] = P_0 \times (1+E[r_1]) \times (1+E[r_2])$$
$$E[P_2] = P_0(1+\bar{r}_1)(1+\bar{r}_2)$$

To determine the expected stock price in year 2, we need to determine the expected returns in year 1 and year 2. We will use the capital asset pricing model to do this. Note that in year 2, the equity beta is 0.72 (equal to the asset beta found in part b) since the debt matures at the end of year 1, making the firm all equity-financed. We have:

$$E[r_1] = r_f + \beta_E(E[r_M] - r_f) = 5\% + 0.8 \times 6\% = 9.8\%$$
$$E[r_2] = r_f + \beta_E(E[r_M] - r_f) = 5\% + 0.72 \times 6\% = 9.32\%$$

We can plug these into our previous equation to find the expected share price in Year 2:

$$E[P_2] = \$22.5 \times (1+9.8\%) \times (1+9.32\%) = \$27.01$$

So the expected share price in 2 years is \$27.01.

(d) Find the new equity beta if the firm issues 551,250 2-year zero-coupon bonds with \$100 face value.

Again, the firm's assets are a portfolio of its equity and debt:

$$\beta_A = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D$$

Since both the old and new tranches of debt are risk-free, $\beta_D = 0$. So,

$$\beta_A = \frac{E}{E+D}\beta_E$$

Rearranging give us:

$$\beta_E = \frac{E+D}{E}\beta_A = \frac{A}{E}\beta_A$$

The value of the firm's assets does not change as a result of this new debt issuance. To find the new equity value, we need to find the new value of debt:

$$D_{\text{new}} = \frac{1}{(1+5\%)^2} \times 551,250 \times \$100 = \$50\text{million}$$

The total value of debt $D = D_{\text{old}} + D_{\text{new}} = \$50\text{mm} + \$50\text{mm} = \$100$ million. The new value of equity after the bond issuance and share buy-back is $E = A - D = \$500\text{mm} - \$100\text{mm}$. So the new equity beta is:

$$\beta_E = \frac{A}{E} \times \beta_A = \frac{\$500\text{mm}}{\$400\text{mm}} \times 0.72 = 0.9$$

So issuing the new tranche of debt increases the equity beta of this firm from 0.8 to 0.9.