15.435x Recitation Teaching Note

Constructing Binomial Trees for Dividend-Paying Stocks

Tree construction follows the logic on slides 3 to 5 of Lecture 5 with an adjustment for dividends. A dividend payment has the effect of lowering a stock's price by the amount of the dividend. A constant dividend yield, δ, reduces the average growth rate of a stock's price from μ to μ – δ (see below). The measured annual volatility of the stock price on a continuous annual basis is denoted by σ.

Then, as in the case of no dividend, the volatility over a period of length *h* is captured by the choice of the up and down multipliers:

$$u = e^{\sigma\sqrt{h}}$$

$$d = 1/u.$$

The lower average growth rate of the stock price due to the effect of the dividend yield is captured by a reduction in the probability of an up move:

$$q = \frac{e^{(\mu-\delta)h} - d}{u - d}$$

You can verify that these formulas are embedded in the spreadsheet BinomialTree.xls.

……………………………………………………..

The effect of the dividend yield on the drift can be derived by first noting that the total return over an interval of time h, inclusive of the dividend, is *vh*. Then

$$E\left[\frac{S_{t+h} - S_t + div_{t+h}}{S_t}\right] = \nu h$$

$$\frac{div_{t+h}}{S_t} \equiv \delta^* h$$

Rearranging and substituting implies:

$$E\left[\frac{S_{t+h} - S_t}{S_t}\right] = (\nu - \delta^*)h$$

Note: Here *vh* is the return over an interval *h* on a simple interest basis, and $\delta^* h$ is the dividend over an interval *h* on a simple interest basis. The variables are represented this way for ease of exposition. In the tree construction, the rates are represented on a continuously compounded basis. Hence, $\nu h = e^{\mu h} - 1$ and $\delta^* h = e^{\delta h} - 1$.