# Recitation 18

Spring 2021

## Question 1

Boston Properties (BP) just issued a perpetual bond with $10 million face value. Suppose that BP will pay interest only on this debt. Assume that the tax rate is 25%.

(a) If the interest rate on this debt is 6%, what is the annual interest tax shield?

(b) What is the present value of interest tax shields?

(c) Suppose instead that the interest rate is 5%. What is the present value of the interest tax shield in this case?

Now, suppose that instead of issuing a perpetual bond, BP has issued a $10 million face value bond with a maturity of 15 years. Assume that the bond pays an annual coupon rate of 5% and that the bond has been issued at par.

(d) What is the present value of interest tax shields generated by this bond?

(e) What would the present value of interest tax shields be if the coupon rate was 6%?

### Solutions:

(a) The annual interest is:

$$I_t = r \times FV = 6\% \times \$10M = \$600,000$$

So the annual tax shield is:

$$\begin{aligned} ITS_t &= \tau \times I_t = 25\% \times \$600,000 \\ &= \$150,000 \end{aligned}$$

(b) The perpetual bond generates $150,000 annual interest tax shield in perpetuity, as calculated in part a. The present value is:

$$\begin{aligned} PVTS &= \frac{ITS}{r} = \frac{\$150,000}{6\%} \\ &= \$2.5M \end{aligned}$$

(c) If the interest rate is 5%, we need to adjust our annual tax shield as well as the perpetuity calculation:

$$\begin{aligned} PVTS &= \frac{\tau \times I_t}{r} = \frac{\tau \times r \times FV}{r} \\ &= \tau \times FV \\ &= 25\% \times \$10M \\ &= \$2.5M \end{aligned}$$

(d) The annual interest tax shield is:

$$ITS_t = \tau \times r \times FV = 25\% \times 5\% \times \$10M = \$125,000$$

These tax shields represent a 15-year annuity. Its present value is:

$$PVTS = \frac{ITS}{r}(1 - \frac{1}{(1+r)^{15}}) = \$1.30M$$

(e) If the interest rate $r = 6\%$, then:

$$ITS_t = \tau \times r \times FV = 25\% \times 6\% \times \$10M = \$150,000$$

The present value of the 15-year annuity is:

$$PVTS = \frac{\$150,000}{6\%}(1 - \frac{1}{(1+6\%)^{15}}) = \$1.46M$$

## Question 2

Equity Residential is considering opening a new mixed use property in Kendall Square. The property is expected to start generating cash flow in Year 1. The first cash flow is expected to be \$55 million, and this amount will grow at a rate of 2% per year from then on.
Assume that Equity Residential pays out all of its earnings to shareholders as dividends. Assume that shareholders pay no taxes on dividends.
Suppose that the cost of capital on this property is 20%. Equity Residential faces a corporate tax rate of 25%.

(a) Suppose that the property will be financed 100% with equity. What is its current market value?

(b) Equity Residential is considering borrowing \$200 million to finance this property development. This financing will either take a form of a perpetual bond or a rolling term loan. In the latter case, the company will be re-borrowing the same principal amount at maturity to cover the principal payment. Suppose that the annual interest rate will be 10%. Assume for simplicity that this interest rate will not change. What is the current market value of this property under this financing arrangement?

What is the value of equity under this arrangement? What is the current debt to equity ratio? What will the debt to equity ratio be in Year 10?

## Solutions:

(a) Since the property is financed with 100% equity, we don't have to account for any interest tax shields. The after-tax cash flow in Year 1 is:

$$\$55M \times (1 - 25\%) = \$41.25M$$

This cash flow (and therefore after-tax cash flow) will grow at 2% per year. So the current market value of the property under the 100% equity financing is:

$$V_0 = \frac{\$41.25M}{20\% - 2\%} = \$229.17M$$

(b) The value of the levered firm equals the value of the unlevered firm plus the present value of interest tax shields. We already found the value of the unlevered firm in part a, so we now need to find the present value of interest tax shields.

Since the level of debt and interest rate remain constant over time:

$$\begin{aligned} PVTS &= \frac{ITS_t}{r} = \frac{\tau \times r \times FV}{r} \\ &= \tau \times FV \\ &= 25\% \times \$200M \\ &= \$50M \end{aligned}$$

So the value of the levered firm is:

$$\begin{aligned} V_L &= V_0 + PVTS \\ &= \$229.17M + \$50M \\ &= \$279.17M \end{aligned}$$

The value of equity is

$$\begin{aligned} E &= V_L - D \\ &= \$279.17M - \$200M \\ &= \$79.17M \end{aligned}$$

So the current debt-to-equity ratio is:

$$\frac{D}{E} = \frac{\$200M}{\$79.17M} = 2.53$$

Now, we want to find the debt-to-equity ratio in year 10. To do so, we need to find the value of debt in year 10 and the value of equity in year 10. We know that the debt is perpetual, so the year 10 debt value will be \$200M.

To find the equity value in year 10, we will first find the value of the unlevered firm in Year 10:

$$V_{10}^U = \frac{FCF_{11}}{r_A - g} = \frac{FCF_1(1+g)^{10}}{r_A - g} = \frac{\$41.25M \times (1 + 2\%)^{10}}{20\% - 2\%} = \$279.35M$$

So the levered value in year 10 is $V_{10}^U + PVTS_{10} = \$279.35M + \$50M = \$329.35M$. And, the equity value is then $\$329.35M - \$200M = \$129.35M$.

So the debt to equity ratio in year 10 is:

$$\frac{D}{E} = \frac{\$200M}{\$129.35M} = 1.55$$

# Question 3

Consider a two-period setting: Year 0 and Year 1. All investors are risk-neutral, and the risk-free rate is 0%.
There is a firm whose assets in Year 1 will be valued as follows:

| State | Probability | Asset Value in Year 1 |
|---|---|---|
| Good | 1/6 | \$100 |
| Okay | 1/3 | \$50 |
| Bad | 1/2 | \$20 |

The firm has outstanding debt with face value \$50 due in Year 1.
Management of the firm has identified a new project, which requires an investment of \$6 in Year 0 and pays off \$10 in Year 1 with certainty. Management understands that the NPV of this project is \$4, but does not have cash available to finance this project.
Management considers raising external financing to fund this project. The firm has three options:

1. Raise new debt, junior to existing debt
2. Raise new equity
3. Raise new debt, senior to existing debt

You are hired as a financial advisor to the management of the firm. Determine which of these options are feasible and advise on the best financing option.

## Solutions:

Let's first determine the current value of debt and equity without the new project. We have:

| State | Probability | Asset Value in Year 1 | Debt | Equity |
|---|---|---|---|---|
| Good | 1/6 | \$100 | \$50 | \$50 |
| Okay | 1/3 | \$50 | \$50 | \$0 |
| Bad | 1/2 | \$20 | \$20 | \$0 |

The value of debt in year 0 is:

$$D_0 = \frac{1}{6} \times \$50 + \frac{1}{3} \times \$50 + \frac{1}{2} \times \$20 = \$35.00$$

The value of equity in year 0 is:

$$E_0 = \frac{1}{6} \times \$50 = \$8.33$$

We will now consider the various financing options.

**Option 1: Junior Debt**
We want to raise \$6 of junior debt. What would be the face value of this debt? Suppose it is $X$, where $X \leq 10$. The following table shows what happens to the firm in each state of the world if they take on the project:

| State | Probability | Asset Value in Year 1 | Existing Debt | Junior Debt |
|---|---|---|---|---|
| Good | 1/6 | \$110 | \$50 | \$X |
| Okay | 1/3 | \$60 | \$50 | \$X |
| Bad | 1/2 | \$30 | \$30 | \$0 |

So, we have the following pricing equation for the junior debt:

$$\$6 = \frac{1}{6}X + \frac{1}{3}X \rightarrow X = \$12$$

However, this violates our initial condition that $X \leq 10$. So, we consider the face value of junior debt where $X > 10$:

| State | Probability | Asset Value in Year 1 | Existing Debt | Junior Debt |
|---|---|---|---|---|
| Good | 1/6 | \$110 | \$50 | \$X |
| Okay | 1/3 | \$60 | \$50 | \$10 |
| Bad | 1/2 | \$30 | \$30 | \$0 |

Our pricing equation is now:

$$\$6 = \frac{1}{6}X + \frac{1}{3} \times 10 \rightarrow X = \$16$$

This is now internally consistent. So the face value of junior debt is \$16.
So, we have the following values of existing debt, new junior debt, and equity if the firm undertakes the project:

| State | Probability | Asset Value in Year 1 | Existing Debt | Junior Debt | Equity |
|---|---|---|---|---|---|
| Good | 1/6 | \$110 | \$50 | \$16 | \$44 |
| Okay | 1/3 | \$60 | \$50 | \$10 | \$0 |
| Bad | 1/2 | \$30 | \$30 | \$0 | \$0 |

Will the equity holders be in favor of the financing arrangement? The value of equity in year 0 now becomes:

$$E_0 = \frac{1}{6} \times \$44 = \$7.33$$

So equity holders will not be in favor of this financing arrangement, as it decreases the value of their claim from \$8.33 to \$7.33.
What about existing debtholders? The value of existing debt in year 0 becomes:

$$D_0 = \frac{1}{6} \times \$50 + \frac{1}{3} \times \$50 + \frac{1}{2} \times \$30 = \$40$$

So existing debtholders will be in favor of the project, as it increases the value of their claim from \$35 to \$40.

**Option 2: Issue new equity**
The following table shows the values of debt and equity if the firm issues equity to finance the project:

| State | Probability | Asset Value in Year 1 | Debt | Equity |
|---|---|---|---|---|
| Good | 1/6 | \$110 | \$50 | \$60 |
| Okay | 1/3 | \$60 | \$50 | \$10 |
| Bad | 1/2 | \$30 | \$30 | \$0 |

The value to the old equity holders in Year 0 is the value of equity with the project, minus the \$6 used to finance the project:

$$E_0 = -\$6 + \frac{1}{6} \times \$60 + \frac{1}{3} \times \$10 = \$7.33$$

So, equity holders will not agree to finance this project because doing so decreases their value from \$8.33 to \$7.33.

**Option 3: Issue senior debt**

The senior debt is risk-free, since the value of assets in all states of the world is sufficient to cover the senior debt. The value of senior debt, old debt, and equity are given in the following table:

| State | Probability | Asset Value in Year 1 | Senior Debt | Existing Debt | Equity |
|---|---|---|---|---|---|
| Good | 1/6 | \$110 | \$6 | \$50 | \$54 |
| Okay | 1/3 | \$60 | \$6 | \$50 | \$4 |
| Bad | 1/2 | \$30 | \$6 | \$24 | \$0 |

The value to equity holders in year 0 is:

$$E_0 = \frac{1}{6} \times \$54 + \frac{1}{3} \times \$4 = \$10.33$$

So the equity holders want to finance the project, since it increases their value from \$8.33 to \$10.33.

The value to existing debt holders is:

$$D_0 = \frac{1}{6} \times \$50 + \frac{1}{3} \times \$50 + \frac{1}{2} \times \$24 = \$37$$

The existing debtholders will be in favor of the project, since it increases their value from \$35 to \$37.

Overall, financing the project with either junior debt or equity is infeasible, since the equity holders will not want to pursue the project. However, if the project is financed with senior debt, it benefits both the equity holders and current debt holders, and the project will be funded. The positive NPV of the project (\$4) is split between the debt and equity holders, and each gets \$2.