15.435x: Recitation 9

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Outline

Overview

Problem 1

Consider two 7% (annual) coupon corporate bonds, each with one year until maturity. Both are expected to default with a 20% probability. Investors demand a risk-adjusted expected return of 4.5% on both bonds. The only difference is that, in the event of default, the expected recovery rate on the first bond is 75%, and the expected recovery rate on the second bond is 25%.

Use this information to estimate the difference in the quoted yield to maturity between the two bonds.

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Use this information to estimate the difference in the quoted yield to maturity between the two bonds.

Solution: Let P_1 be the value of the first bond with expected recovery rate of 75%, and P_2 be the value of the second bond with expected recovery rate of 25%. Assume that both bonds have a face value of 100.

Problem 1

Solution (Cont.): Since there's one year until maturity, the holder of each bond will receive its face value of 100 plus a terminal coupon of 7 if the bond doesn't default, which occurs 100% - 20% $=$ 80% of the time. If the first bond defaults, then the holder of the bond will receive $75\% \times 107 = 80.25$; if the second bond defaults, then the holder will receive $25\% \times 107 = 26.75$.

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Thus, we can solve for P_1 as:

$$P_1 = \frac{0.8 \times 107}{1.045} + \frac{0.2 \times 0.75 \times 107}{1.045} = 97.273.$$

Similarly, P_2 is equal to:

$$P_2 = \frac{0.8 \times 107}{1.045} + \frac{0.2 \times 0.25 \times 107}{1.045} = 87.033.$$

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Solution (Cont.): Finally, we can find the yield to maturity of each bond as:

$$\frac{107}{1+y_1} = 97.273 \Rightarrow y_1 = 10\%$$
$$\frac{107}{1+y_2} = 87.033 \Rightarrow y_2 = 22.94\%$$

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The difference between these two yields is 12.94%. Intuitively, the higher yield on the second bond reflects its greater credit risk.

Overview

Problem 2

Your firm owns 1,000 shares of ABC stock and is now seeking a loan using the stock as collateral. The current market price of the stock is \$80 per share. You have just been offered a loan of $\$80,000$, the full current value of the stock, at an interest rate of 25% per period. At the end of each of the next two periods, your firm will owe an interest payment of $\$20,000$; the first payment is due one period from now. The principal is due three periods from now, so at that time your firm will owe a total payment of $\$100,000$.

Problem 2

The 1,000 shares of stock will serve as the sole collateral for the loan. If your firm defaults on any of the payments, the lender will immediately take possession of the stock but will have no further claim on any of your firm's other assets. A default on this special arrangement will have no effect whatsoever on the other activities of your firm.

The ABC stock pays no dividends, and over each period its value will either increase by 50% or decrease by 50%, with the risk-neutral probability of an "up" move equal to 60%. The interest rate on default-free loans is 10% per period. Is this an attractive opportunity for your firm?

Problem 2

Solution: A default-free bond having the same promised payments as the proposed loan would have a current value of

$$\frac{20,000}{1.1} + \frac{20,000}{1.1^2} + \frac{100,000}{1.1^3} = \$109,842,$$

which is substantially greater than the $80,000 your firm would receive for undertaking the same promised payments. However, your firm would also have a valuable option to cancel the loan at any time by surrendering the underlying collateral. Defaulting in this way would be in the firm's interest whenever the value of the remaining payments exceeds the value of the collateral.

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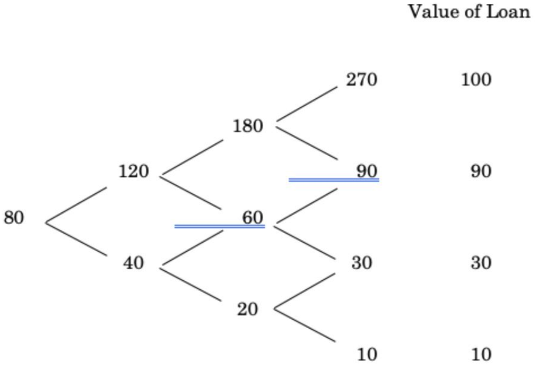
Thus, the proper value of the remaining payments must reflect the fact that even if the current payment is made, default may still occur on one or more of the future payments. The current value of the remaining payments will thus depend on the value of the collateral.

Problem 2

Solution (Cont.): The following diagram shows the possible prices for the underlying collateral, 1,000 shares of ABC stock, over the next three periods along with the corresponding possible values of the loan on its maturity date. The last three values reflect the fact that default would occur in those circumstances. All values are in thousands of dollars.



Problem 2

Solution (Cont.): As always, we work backwards through the binomial tree. Let $V(S, n)$ be the value of the current and future payments on the loan (in thousands of dollars) when the stock price is S and there are n periods remaining. Beginning with the $n = 1$ case, we have that:

$$V(180, 1) = \min\{180, 20 + [(0.6 \times 100 + 0.4 \times 90)/1.1]\} = 107.27$$
$$V(60, 1) = \min\{60, 20 + [(0.6 \times 90 + 0.4 \times 30)/1.1]\} = 60$$
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Thus, the optimal choice for the firm is to default on the loan at the nodes where $S = 60$ or 20 at $n = 1$.

Problem 2

Solution (Cont.): Now, for the $n = 2$ case, we have that:

$$V(120, 2) = \min\{120, 20 + [(0.6 \times 107.27 + 0.4 \times 60)/1.1]\} = 100.33$$
$$V(40, 2) = \min\{40, 20 + [0.6 \times 60 + 0.4 \times 20)/1.1]\} = 40$$

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Finally, at the initial node when $n = 3$ and $S = 80$, we have that:

$$V(80, 3) = \min\{80, (0.6 \times 100.33 + 0.4 \times 40)/1.1\} = 69.27$$

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Overall, the loan is an attractive opportunity for your firm. You have been offered $\$80,000$ for a series of contingent future payments having a current fair value of only $\$69,270$!

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Problem 3

Consider a risky 1-year zero-coupon bond priced to yield 6.375% on a bond equivalent basis (b.e.b.). (Recall from the Week 3 recitation that a **bond equivalent yield** is the single interest rate that equates the price of a bond to the present value of its future cash flows, *assuming semiannual compounding*.) At the same time, the 1-year spot yield on a Treasury bill is 2.3% on a b.e.b.

A credit default swap (CDS) is available on the risky bond. The upfront premium for the CDS is $4,500 per $100,000 face value of the risky bond.

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(a) Does there appear to be an arbitrage opportunity between the cash and CDS markets? Explain with a calculation.

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As we saw in the Week 9 lecture, the difference between the risk-free bond price and the risky bond price is equal to the fair price of the CDS premium, which in this case is $\$97,739.08 - \$93,917.35 = \$3,821.73$. Thus, there appears to be an arbitrage opportunity, as the CDS is selling for $\$4,500$ and is relatively overpriced.

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(b) Describe the transactions you would have to simultaneously make in the cash and CDS markets to exploit the arbitrage opportunity.

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Notice that there are zero future cash flows paid or received from this strategy. Why?

If the risky bond defaults, then the CDS payment is covered by the difference between the face value of the Treasury bill and the recovery value of the risky bond. If the risky bond doesn't default, then the CDS pays nothing, and the face values cancel out on the long and short bond positions.

Overview

Problem 4

Your boss asks you for a quick price estimate for a credit guarantee on a risky bond of a publicly traded firm. You have the following information:

- The Treasury yield curve is flat at 4% (on a continuous basis).
- The bond to be guaranteed is zero-coupon and has a face value of \$50 million. It matures in 3.25 years. It is the only debt of the firm. The guarantee ensures payment to the holder of the full face value at maturity.
- The firm has 5.1 million shares of equity outstanding priced at \$10.69 per share. The equity volatility is 25% per year.
- Using the Merton model for valuing risky debt, you conclude that the market value of assets is \$95 million and the asset volatility is 12% per year.

Based on this information, what is your best estimate of the value of the credit guarantee?

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From the information provided, we can compute the market value of the firm's equity to be $E = \$10.69 \times 5.1 \text{ million } = \$54.519$ million.

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From the information provided, we can compute the market value of the firm's equity to be $E = \$10.69 \times 5.1$ million $= \$54.519$ million.

Since the market value of the firm's assets is \$95 million according to the Merton model, we can infer that $D = A - E = 95 - 54.519 = \$40.481$ million.

Problem 4

Solution (Cont.): As we saw in the Week 9 lecture, the value of a credit guarantee on a risky bond is equal to the difference between the hypothetical price of the promised payments of the bond if they were risk-free and the price of the risky bond.

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If the firm's debt were risk-free, then its price would be equal to $50 * \exp(-0.04 * 3.25) = \$43.905$ million. Thus, the credit guarantee is estimated to be worth $43.905 - 40.481 = \$3.424$ million.