15.415x Foundations of Modern Finance

Leonid Kogan and Jiang Wang
MIT Sloan School of Management

Lecture 15: Capital Asset Pricing Model

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

Introduction

- Mean-variance portfolio theory analyzes investors' asset demand given asset returns.
  - Diversify to eliminate non-systematic risk.
  - Hold only the risk-free asset and the tangency portfolio.
- How does investors' asset demand determine the relation between assets' risk and return?

The market portfolio

- The market portfolio is the portfolio of all risky assets traded in the market.
- A total of N risky assets. Market capitalization of asset i is:

$$\mathrm{MCAP}_i = (\text{price per share})_i \times (\#\text{ of shares outstanding})_i$$

- The total market capitalization of all risky assets is:

$$\mathrm{MCAP}_M = \sum_{i=1}^{N} \mathrm{MCAP}_i$$

- The market portfolio has the following portfolio weights:

$$w_i = \frac{\mathrm{MCAP}_i}{\sum_{j=1}^{N} \mathrm{MCAP}_j} = \frac{\mathrm{MCAP}_i}{\mathrm{MCAP}_M}$$

- We denote the market portfolio by w_M.

Derivation of CAPM: assumptions

- All investors invest over the same, single period, $t = 0$ to $t = 1$.
- No other sources of income, entire wealth consists of the portfolio of financial assets.
- Investors agree on the distribution of asset returns.
- Investors have mean-variance preferences.
- There is a risk-free asset:
  - paying interest rate r_F;
  - in zero net supply.
- There are no constraints on portfolios, and no trading costs.
- Demand for assets equals supply in equilibrium.

Derivation of CAPM: implications

1. Every investor puts their money into two pots:
   - the riskless asset;
   - a single portfolio of risky assets, the tangency portfolio.
2. All investors hold the risky assets in same proportions.
   - Collectively, they hold the same risky portfolio, the tangency portfolio.
3. Impose market clearing: demand for stocks (the aggregate of investors' portfolios) must equal the supply (the aggregate stock market).
   - The tangency portfolio is the market portfolio!
   - The market portfolio must be on the CML: it has the highest possible Sharpe ratio (it is mean-variance efficient).

Example: market clearing

- In equilibrium, total asset holdings of all investors must equal the total supply of assets.
- Example: there are only three risky assets, A, B and C. Suppose that the tangency portfolio is

$$w_T = (w_A, w_B, w_C) = (0.25, 0.50, 0.25)$$

- There are only three investors in the economy, 1, 2 and 3, with total wealth of 500, 1000, 1500 dollars, respectively. Their asset holdings (in dollars) are:

Investor	Riskless	A	B	C
1	100	100	200	100
2	200	200	400	200
3	-300	450	900	450
Total	0	750	1500	750

Example: market clearing

- In equilibrium, the total dollar holding of each asset must equal its market value:

$$\text{Market Capitalization of } A = \$750$$

$$\text{Market Capitalization of B} = \$1{,}500$$

$$\text{Market Capitalization of C} = \$750$$

- The total market capitalization is

$$750 + 1{,}500 + 750 = \$3{,}000$$

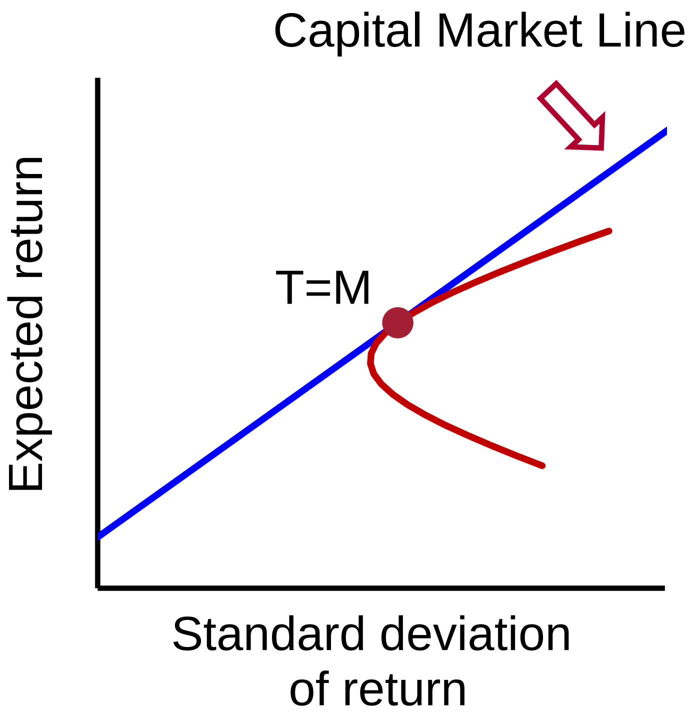
- The market portfolio is the tangency portfolio!

$$w_M = \left(\frac{750}{3{,}000}, \frac{1{,}500}{3{,}000}, \frac{750}{3{,}000}\right) = (0.25{,}0.50{,}0.25) = w_T$$

Derivation of CAPM

- The marginal contribution of asset i to the market portfolio:
  - return: $\bar{r}_i - r_F$;
  - risk: $\frac{\sigma_{iM}}{\sigma_M}$
- For the market portfolio to be optimal, the marginal return-to-risk ratio (RRR) of all risky assets must be the same:

$$RRR_i = \frac{\bar{r}_i - r_F}{\sigma_{iM}/\sigma_M} = SR_M = \frac{\bar{r}_M - r_F}{\sigma_M}$$



The CAPM

- Re-writing:

$$\frac{\bar{r}_i - r_F}{\sigma_{iM}/\sigma_M} = \frac{\bar{r}_M - r_F}{\sigma_M}$$

we have:

$$\bar{r}_i - r_F = \frac{\sigma_{iM}}{\sigma_M^2}(\bar{r}_M - r_F) = \beta_{iM}(\bar{r}_M - r_F)$$

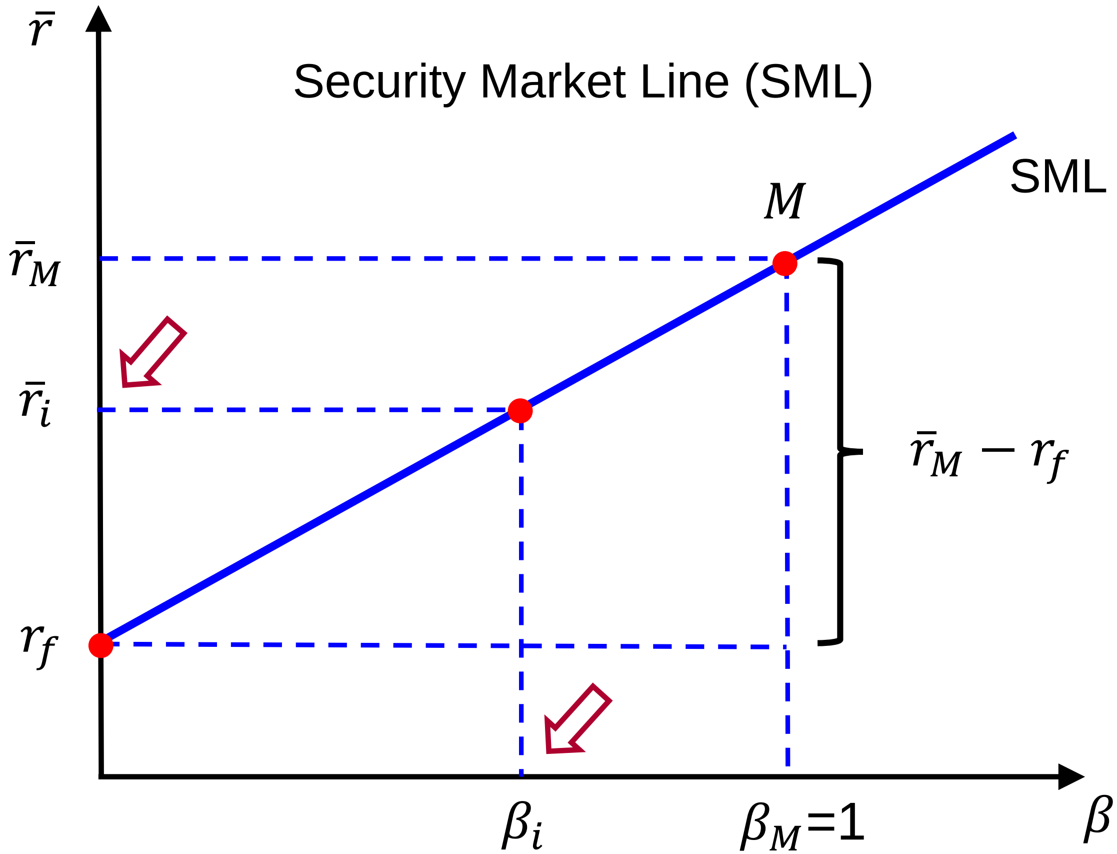
where $\beta_{iM} = \frac{\sigma_{iM}}{\sigma_M^2}$ is the beta of asset i with respect to the market portfolio.

- This is the CAPM:
  - β_{iM} is a measure of asset i's systematic risk: exposure to the market.
  - $\bar{r}_M - r_F$ gives the premium per unit of systematic risk.
- The risk premium of an asset equals its systematic risk (β_{iM}) times the price of systematic risk ($\bar{r}_M - r_F$).

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

Security Market Line (SML)



- The relation between an asset's premium and its market beta is called the Security Market Line (SML).
- Given an asset's beta, we can determine its expected return.

Example

- Suppose that CAPM holds. The expected market return is 8% and the risk-free rate is 2%.
  - What should be the expected return on a stock with $\beta = 0$?

2%: only idiosyncratic risk ⇒ risk-free rate of return

  - What should be the expected return on a stock with $\beta = 1$?

8%: same risk as the market ⇒ same expected return as the market

  - What should be the expected return on a portfolio with the market beta of 0.5?

$2\% + 0.5 \times (8\% - 2\%) = 5\%$: 50% of market risk ⇒ 50% of risk premium

  - What should be expected return on stock with $\beta = -0.6$?

$\bar{r} = 2\% + (-0.6)(8\% - 2\%) = -1.6\%$: “insurance” against market declines ⇒ negative premium

Risk and return in CAPM

- We can decompose an asset's return into three pieces:

$$\tilde{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\epsilon}_i$$

  - $\mathrm{E}[\tilde{\epsilon}_i] = 0$;
  - $\mathrm{Cov}[\tilde{r}_M, \tilde{\epsilon}_i] = 0$.
- Three characteristics of an asset:
  - Alpha: according to CAPM, alpha should be zero for all assets.
  - Beta: measures an asset's systematic risk.
  - Sigma = $\mathrm{SD}(\tilde{\epsilon}_i)$: measures non-systematic risk.

Alpha

$$\tilde{r}_i - r_F = \boxed{\alpha_i} + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\epsilon}_i$$

- According to CAPM, alpha should be zero for all assets.
- Alpha measures an asset's return in excess of its risk premium according to CAPM.
- What to conclude if we find an asset with a positive (or negative) alpha?
  - Check estimation error;
  - Past value of α may not predict its future value;
  - Positive α may be compensating for other risks;
  - Trading frictions, taxes,…

Alpha and portfolio choice

- Suppose that asset i violates the CAPM:

$$\tilde{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\epsilon}_i,$$

where idiosyncratic shocks are mutually uncorrelated.

- Consider a mean-variance optimizing investor.
  - How would the portfolio of this investor deviate from the market portfolio?
  - Compute the highest Sharpe ratio an investor can achieve in this market.

Alpha and portfolio choice

$$\tilde{r}_i = r_F + \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\epsilon}_i$$

- Construct a portfolio P, \$1 total investment:
  - Long \$1 of asset i;
  - -\$$\beta_{iM}$ units of the market portfolio;
  - \$$\beta_{iM}$ in the risk-free asset.

$$\tilde{r}_P = \tilde{r}_i - \beta_{iM}(\tilde{r}_M - r_F) = r_F + \alpha_i + \tilde{\epsilon}_i$$

- Note that return on P is uncorrelated with the market return; $SR_P = \alpha_i / \sigma_{\epsilon_i}$.
- Construct the tangency portfolio using portfolios M and P:

$$w_M = \lambda \frac{\bar{r}_M - r_F}{\sigma_M^2}, \qquad w_P = \lambda \frac{\bar{r}_P - r_F}{\sigma_P^2} = \lambda \frac{\alpha_i}{\sigma_{\epsilon_i}^2}$$

- Sharpe ratio of the tangency portfolio:

$$SR_T = \sqrt{SR_M^2 + SR_P^2}$$

Alpha and portfolio choice

Derivation of the Sharpe ratio of the tangency portfolio

$$\tilde{r}_P = r_F + \alpha_i + \tilde{\epsilon}_i$$

- Start with the weights of the tangency portfolio:

$$w_M = \lambda \frac{\bar{r}_M - r_F}{\sigma_M^2}, \qquad w_P = \lambda \frac{\bar{r}_P - r_F}{\sigma_P^2} = \lambda \frac{\alpha_i}{\sigma_{\epsilon_i}^2}$$

- Compute expected excess return and variance of the tangency portfolio:

$$\bar{r}_T - r_F = w_M(\bar{r}_M - r_F) + w_P(\bar{r}_P - r_F) = \lambda \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2}$$

$$\sigma_T^2 = w_M^2 \sigma_M^2 + w_P^2 \sigma_P^2 = \lambda^2 \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda^2 \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2} = \lambda(\bar{r}_T - r_F)$$

Alpha and portfolio choice

Derivation of the Sharpe ratio of the tangency portfolio

- Start with the weights of the tangency portfolio:

$$w_M = \lambda \frac{\bar{r}_M - r_F}{\sigma_M^2}, \qquad w_P = \lambda \frac{\bar{r}_P - r_F}{\sigma_P^2} = \lambda \frac{\alpha_i}{\sigma_{\epsilon_i}^2}$$

- Compute expected excess return and variance of the tangency portfolio:

$$\bar{r}_T - r_F = w_M(\bar{r}_M - r_F) + w_P(\bar{r}_P - r_F) = \lambda \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2}$$

$$\sigma_T^2 = w_M^2\sigma_M^2 + w_P^2\sigma_P^2 = \lambda^2 \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda^2 \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2} = \lambda(\bar{r}_T - r_F)$$

- Sharpe ratio of the tangency portfolio:

$$SR_T^2 = \frac{(\bar{r}_T - r_F)^2}{\sigma_T^2} = \frac{1}{\lambda}(\bar{r}_T - r_F)$$

$$\boxed{\sigma_T^2 = \lambda(\bar{r}_T - r_F)}$$

Alpha and portfolio choice

Derivation of the Sharpe ratio of the tangency portfolio

- Start with the weights of the tangency portfolio:

$$w_M = \lambda \frac{\bar{r}_M - r_F}{\sigma_M^2}, \qquad w_P = \lambda \frac{\bar{r}_P - r_F}{\sigma_P^2} = \lambda \frac{\alpha_i}{\sigma_{\epsilon_i}^2}$$

- Compute expected excess return and variance of the tangency portfolio:

$$\bar{r}_T - r_F = w_M(\bar{r}_M - r_F) + w_P(\bar{r}_P - r_F) = \lambda \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2}$$

$$\sigma_T^2 = w_M^2 \sigma_M^2 + w_P^2 \sigma_P^2 = \lambda^2 \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \lambda^2 \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2} = \lambda(\bar{r}_T - r_F)$$

- Sharpe ratio of the tangency portfolio:

$$SR_T^2 = \frac{(\bar{r}_T - r_F)^2}{\sigma_T^2} = \frac{1}{\lambda}(\bar{r}_T - r_F) = \frac{(\bar{r}_M - r_F)^2}{\sigma_M^2} + \frac{(\alpha_i)^2}{\sigma_{\epsilon_i}^2} = SR_M^2 + SR_P^2$$

$$SR_T = \sqrt{SR_M^2 + SR_P^2}$$

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

CAPM vs APT

- The CAPM equation is identical to the APT equation with a single factor. But the two theories have important differences.

	CAPM	APT
Preferences	Mean-variance preferences	Prefer more to less ⇒ absence of arbitrage
Factor structure in returns	Not needed	Required
Exact or approximate?	Applies exactly to all assets	Holds exactly only for well-diversified portfolios. Approximate for individual assets
Identity of systematic risk	Market return	Multiple risk factors, not identified explicitly by the theory

CAPM vs APT

- What if returns have a multi-factor structure, can the CAPM still hold?
- Yes! Under the CAPM, market return still prices all assets, including the APT factors themselves.
- The same fundamental insight in both models:

The only component of total risk relevant for pricing
is the systematic risk.

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

Applications of CAPM

- Required rates of return on IBM and Dell.
  - Use the value-weighted stock portfolio as a proxy for the Market.
  - Regress historic returns of IBM and Dell on the returns on the value-weighted portfolio. Suppose the beta estimates are:

$$\beta_{IBM,VW} = 0.8 \text{ and } \beta_{Dell,VW} = 1.3$$

  - Use historic excess returns on the value weighted portfolio of all stocks to estimated average market premium, suppose it is $\bar{r}_{VW} - r_F = 6\%$.
  - Obtain the current riskless rate. Suppose it is $r_F = 2\%$.
  - Applying CAPM:

$$\bar{r}_{IBM} = r_F + \beta_{IBM,VW}(\bar{r}_{VW} - r_F) = 2\% + 0.8 \times 6\% = 6.8\%$$

- The expected rate of return on IBM (under CAPM) is 6.8%. Similarly, the expected rate of return on Dell is 9.8%.

Advantages of using CAPM and APT

- Historical averages of returns on individual stocks are poor estimates of expected returns going forward:
  - Short samples;
  - Time-variation in risk and expected returns.
- Can estimate betas more precisely than expected returns.
- CAPM and APT allow us to use firm-level beta estimates (relatively precise) and market or factor risk premia (long sample).
- Estimates of future expected returns on an asset are based on its systematic risk, not on the average of its past returns.

Key concepts

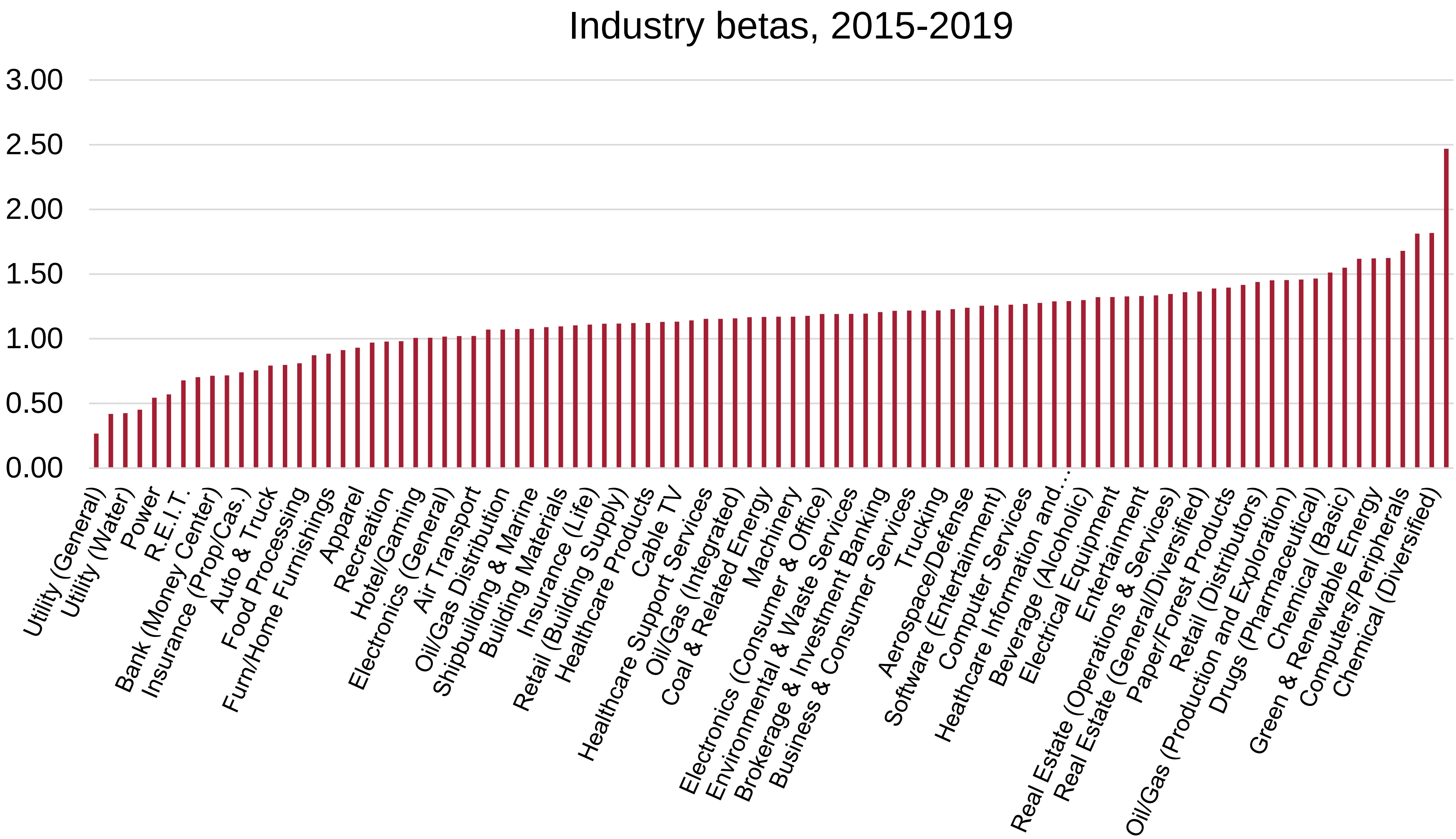
- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

Beta estimates: daily data

Regression with daily data, 1/4/19—6/5/19

Company	Beta	Total SD of return (daily, %)	SD of idiosyncratic risk (daily, %)
GENERAL MOTORS CORP	1.29	1.74	1.37
BOEING CO	1.44	1.79	1.37
BRISTOL MYERS SQUIBB CO	1.54	2.05	1.64
DELTA AIRLINES INC	1.28	1.78	1.46
HEWLETT PACKARD CO	1.56	2.43	2.10
DOW CHEMICAL CO	1.43	3.17	2.97
EXXON CORP	0.88	1.12	0.85
MERCK&CO INC	0.72	1.17	1.02
HOME DEPOT INC	0.79	1.12	0.92
MC DONALDS CORP	0.34	0.78	0.73
MICROSOFT CORP	1.33	1.37	0.85
APPLE COMPUTER INC	1.63	1.92	1.40
GOOGLE INC	1.22	1.59	1.26
WAL MART INC	0.36	0.92	0.87
JPMORGAN CHASE&CO	0.98	1.20	0.90

Industry betas



Industry betas

- Why do industry betas differ?
- Fundamental differences:
  - Cyclicality of demand,
  - Exposure to credit market risk, other aggregate shocks…
- Leverage:
  - Leverage raises equity betas relative to asset betas (enterprise betas).
  - Leverage level is a choice, related to fundamental risk.

Leverage: equity beta vs asset betas

- Firm ABC has debt and equity: debt-to-equity ratio of 0.65.
- Assume that ABC's debt has a rating of A and a beta of 0.10.
- The beta of ABC equity is 0.73.
- What is the beta of ABC's assets (it's enterprise beta)?
- The assets of the firm serve to pay all investors, and so:

$$A = E + D$$

- Thus, the firm's assets are a portfolio of its equity and debt:

$$\beta_A = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D$$
$$= \frac{1}{1+0.65}(0.73) + \frac{0.65}{1+0.65}(0.10) = 0.48$$

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

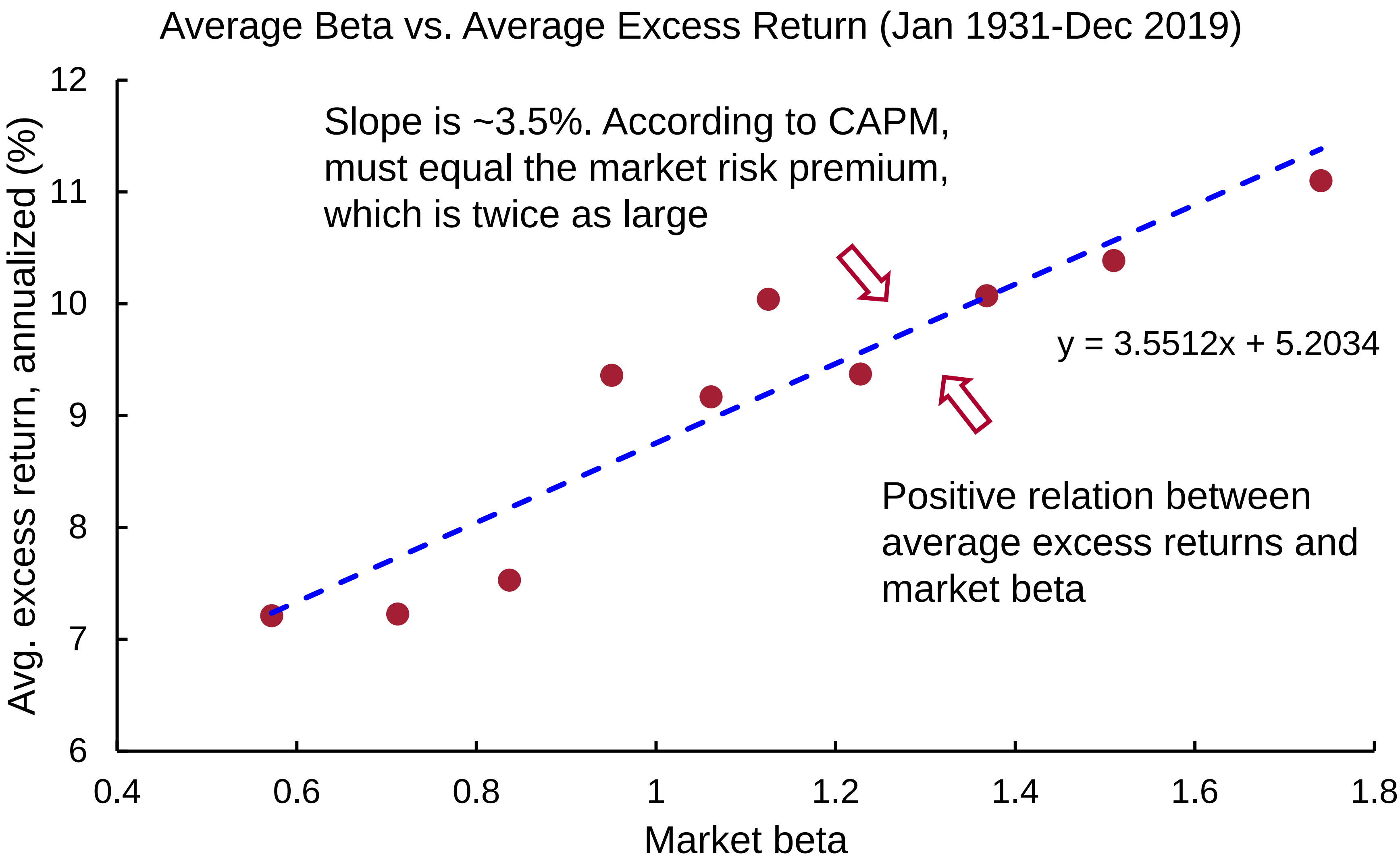
Empirical test of the CAPM: design

- Follow Black, Jensen, and Scholes (1972).
- For each security i and month t, define excess returns: $xr_t^i = r_t^i - r_{F,t}^{1M}$
  - r_t^i is the return from the start to end of month t .
  - $r_{F,t}^{1M}$ is the one-month treasury bill rate at the start of the month. This is our 1-month "risk-free" rate.
- For every (security, month) pair, estimate beta on a 60-month window, conditional on 24 non-missing month observations.
- On January 1st of every year t, create ten portfolios by sorting on betas estimates recorded in December of the previous year $t-1$.
- For each portfolio, compute value-weighted excess return xr_t^p after portfolio formation.

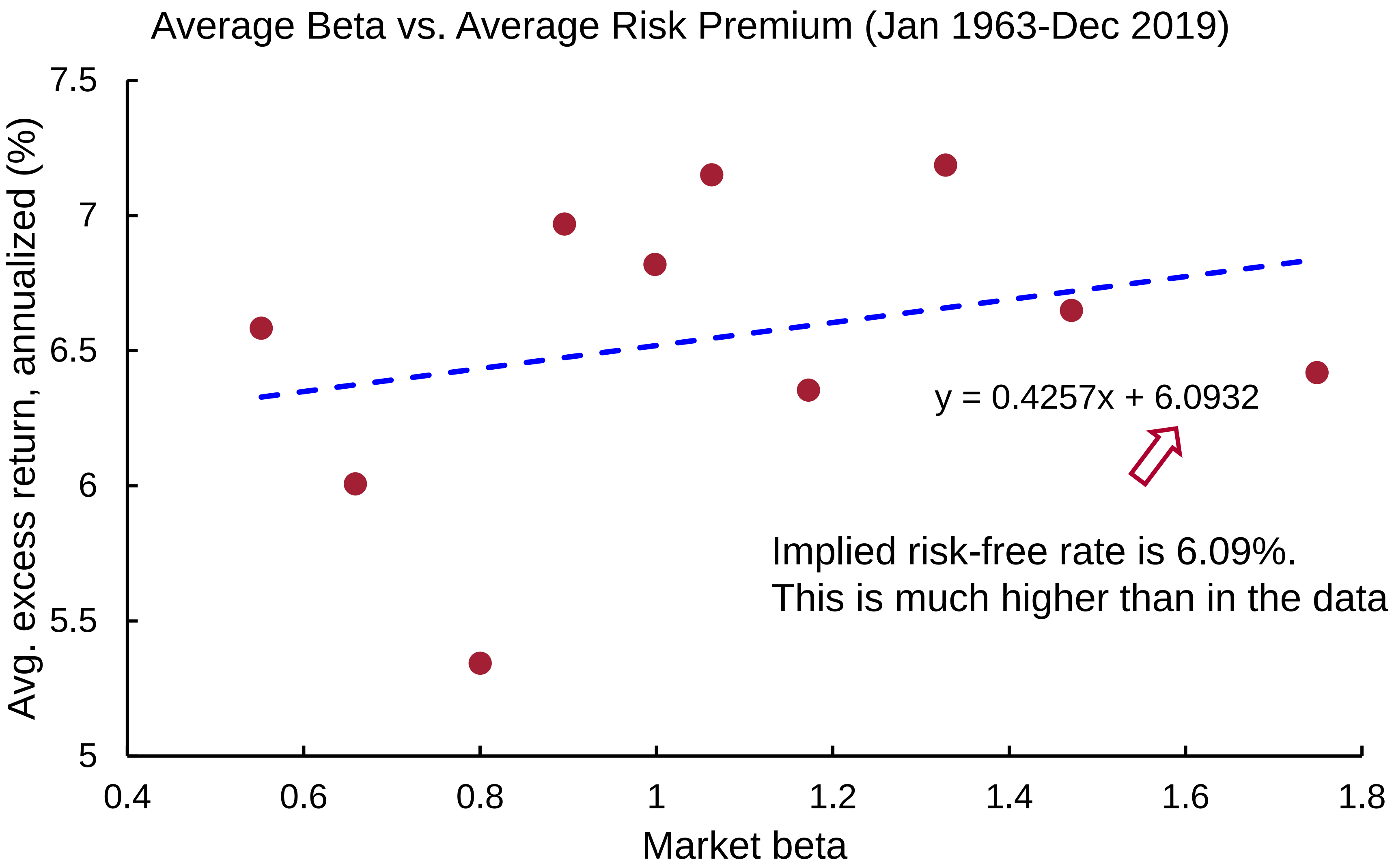
Empirical test of the CAPM: design

- For each portfolio, estimate the portfolio beta β^p by regressing portfolio excess returns xr_t^p on the value-weighted market excess returns over the full sample.
- For each portfolio, compute the time-series average of excess monthly returns, xr_t^p.

Estimated SML is too flat

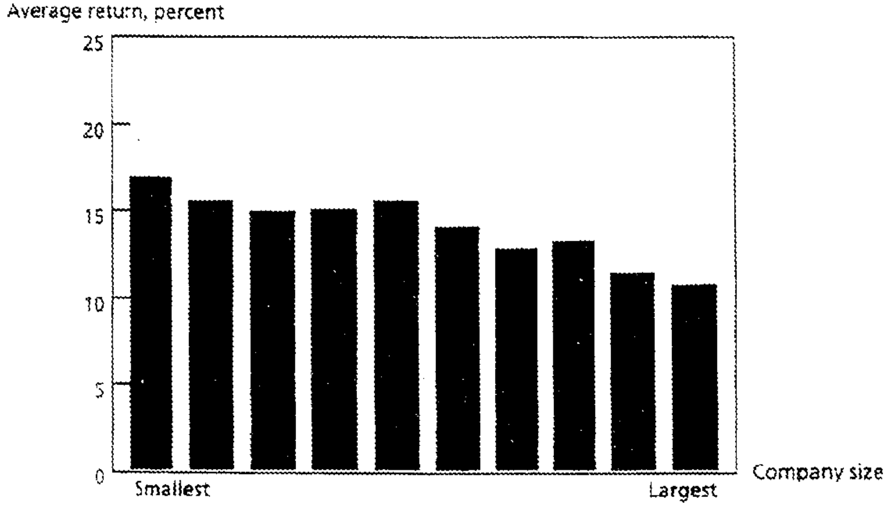


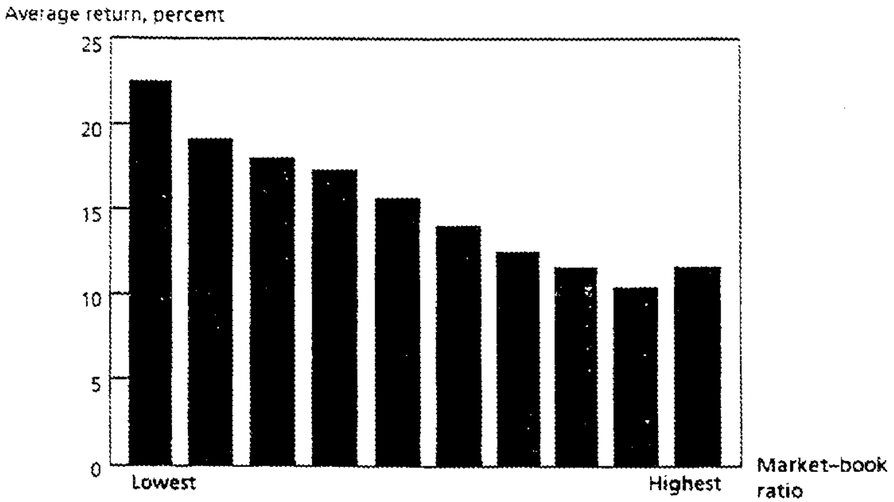
Estimated SML is really flat…



Firm size and book-to-market

- Firm characteristics seem to predict future returns.





(Source: G. Fama and K. French, 1992,
"The Cross-Section of Expected Stock Returns," *Journal of Finance*)

- Since mid-1960s:
  - Small stocks outperformed large stocks.
  - Stocks with low ratios of market-to-book value outperformed stocks with high ratios.

Interpretation

- Empirical failures of the CAPM suggest two main alternative interpretations:
  - Small stocks, high B/M stocks, low-beta stocks, etc., are mispriced;
  - The CAPM does not measure risk properly – there are missing risk factors.
- There exists a significant body of work on this topic, the question is still open.
- It is difficult to argue that CAPM alphas can be fully explained by additional risk factors, but evidence suggests that the stock market return is not the only risk factor compensated with the risk premium.
- Multi-factor models may offer a better description of the risk-return relation.

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting

CAPM and capital budgeting

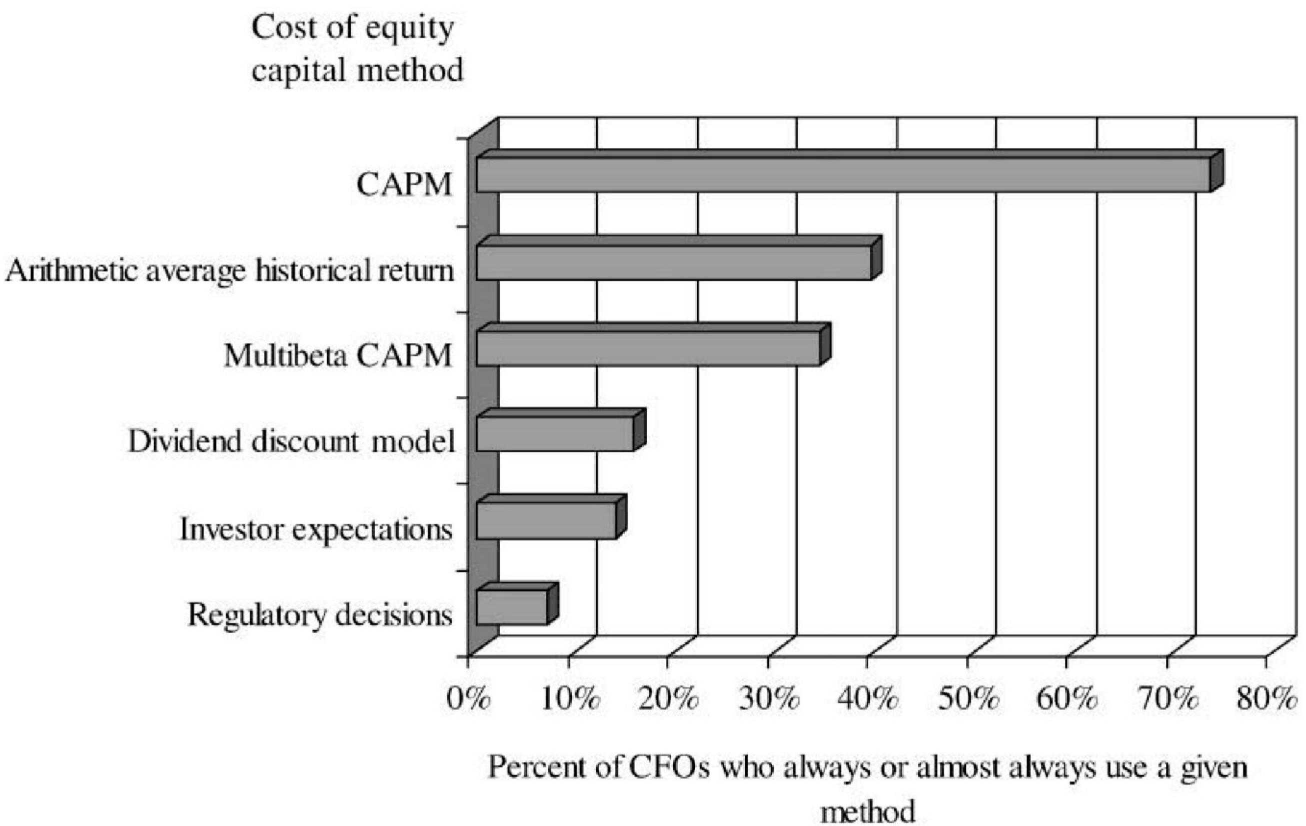
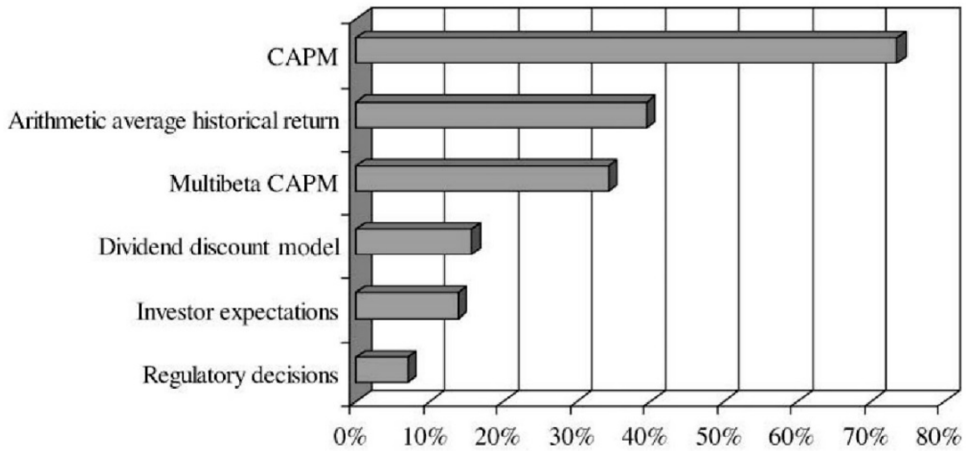


Fig. 3. Survey evidence on the popularity of different methods of calculat the cost of equity capital. We report the percentage of CFOs who always or almost always use a particular technique. CAPM represents the capital asset pricing model. The survey is based on the responses of 392 CFOs.

J. Graham and C. Harvey, "The theory and practice of corporate finance: evidence from the field," Journal of Financial Economics (2001) 187-243.

CAPM and capital budgeting



- Large firms, public firms, and low-leverage firms are more likely to use CAPM.
- Multibeta CAPM (APT). Additional risk factors:
  - Foreign exchange rate, interest rate, GDP, inflation risk, … ;
  - Large firms: FX risk most prominent;
  - Small firms: interest rate risk most prominent.

All models are wrong but some are useful

- CAPM is the leading model for capital budgeting.
- When valuing projects, forecast long-term discount rates.
- Many CAPM violations tend to be short-lived, transient.
- CAPM is not perfect, but a useful benchmark model.

Key concepts

- Derivation of CAPM
- Risk and return under CAPM
- CAPM vs APT
- Application of CAPM
- Empirical properties of CAPM betas
- Empirical tests of CAPM
- CAPM and capital budgeting