# Week 2 – Futures and Swaps

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# Outline

- Futures markets: institutions and some popular contracts
- Plain vanilla interest rate swaps
  - Structure, terminology, and pricing
  - Application: hedging interest rate risk for a bank
- Other popular swap products
  - Currency, commodity, and total-rate-of-return swaps

# Futures: institutions and some popular contracts

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# Futures contracts

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## A specific type of forward contract

- Locks in a pre-specified buy or sell price for a transaction on a pre-specified future date

## Key features include:

- The contracts are traded on an exchange (e.g. CME, ICE), which is a central counterparty (CCP) that clears all trades
- The contracts are standardized (assets, size, maturity, delivery)
  - Facilitates liquidity but creates basis risk
- Profits and losses are marked to market (daily settlement)
  - Margin account serves as collateral and absorbs gains and losses.
  - Reduces credit risk

Notes:

- In practice, most contracts are closed out before maturity.
- A futures position can be closed out on any day by entering into an offsetting contract.
  - E.g., Have long position in 10 April corn contracts. Close out by taking short position in 10 April corn contracts.

# Example 2.3: S&P e-mini contract

https://www.cmegroup.com/trading/equity-index/us-index/e-mini-sandp500.html

## E-MINI S&P 500 FUTURES - QUOTES

Globex

GLOBEX FUTURES | GLOBEX OPTIONS

AUTO REF

Market data is delayed by at least 10 minutes.

All market data contained within the CME Group website should be considered as a reference only and should not be used as validation against, nor as a complement to data feeds. Settlement prices on instruments without open interest or volume are provided for web users only and are not published on Market Data Platform (MDP). The based on market activity.

| MONTH | OPTIONS | CHARTS | LAST | CHANGE | PRIOR SETTLE | OPEN | HIGH | LOW | VOLUME |
|---|---|---|---|---|---|---|---|---|---|
| MAR 2021 | OPT | | 3900.25 | -27.50 | 3927.75 | 3929.00 | 3932.25 | 3896.50 | 821,798 |
| JUN 2021 | OPT | | 3889.25 | -28.00 | 3917.25 | 3920.50 | 3921.75 | 3887.00 | 1,038 |
| SEP 2021 | OPT | | 3894.25 | -12.75 | 3907.00 | 3900.00 | 3900.00 | 3878.00 | 74 |
| DEC 2021 | OPT | | - | - | 3893.50 | - | - | - | 0 |
| MAR 2022 | OPT | | - | - | 3879.75 | - | - | - | 0 |

# Example 2.4: Bitcoin futures

https://www.investopedia.com/news/bitcoin-futures-cboe-vs-cme-whats-difference/

https://www.cmegroup.com/trading/bitcoin-futures.html

Interesting case study in introducing a new contract

On plus side

- Appeals to many traders
- Potential institutional demand
- Provides liquid exposure to asset class
- Can't short bitcoins but can short bitcoin futures
- Lots of publicity from introduction

Negatives

- Unreliable reference prices
- High risk product, esp. for small investors
- Reputational risk
- Volume could be low

**Bitcoin jumps to a new high above $51,700, extending its year-to-date rally to 78%**

Harry Robertson

Feb. 17, 2021, 01:18 PM

SHARE

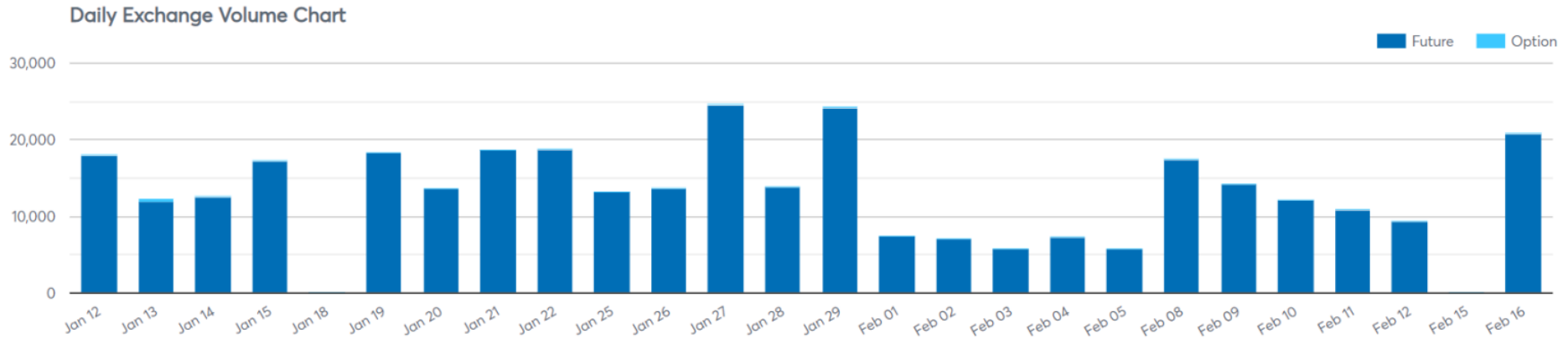
Chesnot/Getty Images

- **The bitcoin price hit a new high above $51,700 after breaking the $50,000 mark on Tuesday.**
- **Yet JPMorgan said the rally looks unsustainable unless bitcoin's volatility falls.**
- **Bitcoin's market capitalization has skyrocketed to close to $1 trillion.**

# Example 2.4: Bitcoin futures

Trade Date: Tuesday, 16 Feb 2021 (Final)

DOWNLOAD DATA




| MONTH | VOLUME | | | | | | | | | DELIVERIES | OPEN INTEREST | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | VENUE DETAIL | | | | TRADE TYPE DETAIL | | | | | | AT CLOSE | CHANGE |
| | GLOBEX | OPEN OUTCRY | PNT / CLEARPORT | TOTAL VOLUME | BLOCK TRADES | EFP | EFR | EFS | TAS | | | |
| FEB 21 | 15,414 | 0 | 0 | 15,414 | 0 | 0 | 0 | 0 | 0 | 0 | 7,188 | -276 |
| MAR 21 | 3,530 | 0 | 0 | 3,530 | 0 | 0 | 0 | 0 | 0 | 0 | 3,176 | 274 |
| APR 21 | 1,384 | 0 | 0 | 1,384 | 0 | 0 | 0 | 0 | 0 | 0 | 704 | 87 |
| MAY 21 | 514 | 0 | 0 | 514 | 0 | 0 | 0 | 0 | 0 | 0 | 300 | 40 |
| | 44 | 0 | 0 | 44 | 0 | 0 | 0 | 0 | 0 | 0 | 50 | -5 |

//www.cmegroup.com

# Margins and marking to market

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Futures traders have to maintain a minimum balance in a margin account

- Daily settlement of gains and losses and minimum margin greatly reduce default risk
- Margin may be cash or securities

## Example 2.5 Tracking a margin account over time

An investor takes a long position in two December gold futures contracts on June 5.

The contract size is 100 oz.

The futures price is $1250 per ounce.

The initial margin requirement is $6,000/contract ($12,000 in total).

Maintenance margin is $4,500/contract ($9,000 in total).

# Example 2.5 (cont.)

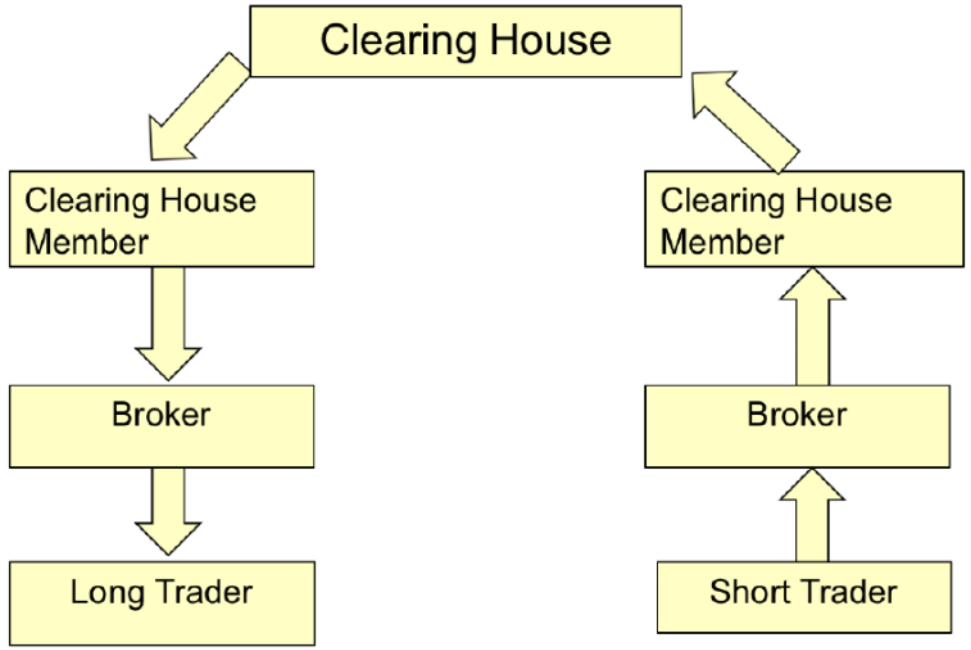
| Day | Trade Price ($) | Settle Price ($) | Daily Gain ($) | Cumul. Gain ($) | Margin Balance ($) | Margin Call ($) |
|---|---|---|---|---|---|---|
| 1 | 1,250.00 | | | | 12,000 | |
| 1 | | 1,241.00 | -1,800 | -1,800 | 10,200 | |
| 2 | | 1,238.30 | -540 | -2,340 | 9,660 | |
| ..... | | ..... | ..... | ..... | ..... | |
| 6 | | 1,236.20 | -780 | -2,760 | 9,240 | |
| 7 | | 1,229.90 | -1,260 | -4,020 | 7,980 | 4,020 |
| 8 | | 1,230.80 | 180 | -3,840 | 12,180 | |
| ..... | | ..... | ..... | ..... | ..... | |
| 16 | 1,226.90 | | 780 | -4,620 | 15,180 | |

On day 7, the margin balance falls below the maintenance margin, so the investors has to deposit into the account to restore the initial margin requirement.

# Clearing houses

End-users trade futures via brokers that submit orders with clearing members of the exchange

Cash flow when futures price increases looks like:




Flow direction reverses when price decreases.

Structure has implications for stability and safety; CCPs are subject to regulation.

# Do forward prices equal futures prices?

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## Theoretically they are the same when interest rates are constant

- E.g., if futures price is less than the forward price for the same underlying and delivery date, then long futures, short forward, arbitrage profit at delivery date

## Small differences can arise because of margin account dynamics

- Margin accounts bear interest
- If long (short) margin account tends to be bigger when rates are higher, that is an advantage to a long (short) position in futures relative to uncollateralized forwards.
- If long (short) is better off with futures, then futures price is higher (lower) than forward price.
- Difference was of greater importance when interest rates were higher and forwards were not collateralized

# Swaps: products, pricing and risk management applications

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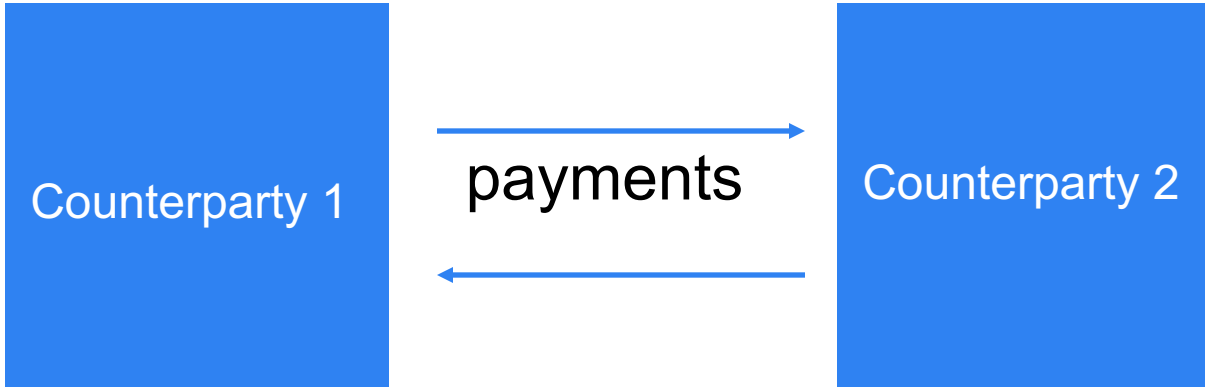
# Swap basics

A swap is a contract calling for an exchange of payments, on one or more future dates, determined by the difference in two reference prices or interest rates

A single-payment swap is equivalent to a cash-settled forward contract

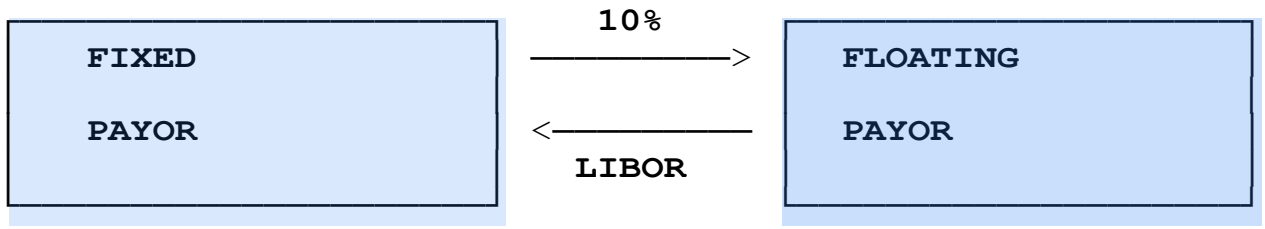
A swap provides a means to hedge or speculate on a *stream* of risky cash flows



# Structure of a "Plain Vanilla" Interest Rate Swap

In the most common type of interest rate swap (fixed for floating), fixed interest rate payments are exchanged for floating interest rate payments at regular intervals over the life of the contract

No principal is exchanged




*LIBOR is the London Interbank Offer Rate. For many years it was the most common reference floating rate for swaps. It is being replaced by other reference rates, e.g., SOFR.*

- An interest rate swap can also be described as a package of forward rate agreements (FRAs)
  - A forward rate agreement is a one-time exchange based on a fixed interest rate and a floating one
  - No-arbitrage => swaps must be priced consistently with FRAs, futures and the cash bond market

# Terminology for interest rate swaps

## Notional Principal

- Amount of principal upon which the interest payments are based. This principal is never exchanged.

## Counterparties

- The two participants in the swap.

## Fixed Rate Payor

- The counterparty who pays a fixed rate, and receives a floating rate in the swap.
  *The fixed rate payor is said to have "bought the swap" or is long in the swap.*

## Floating Rate Payor

- The counterparty who pays a floating rate, and receives a fixed rate in the swap.
  *The floating rate payor is said to have "sold the swap" or is short in the swap.*

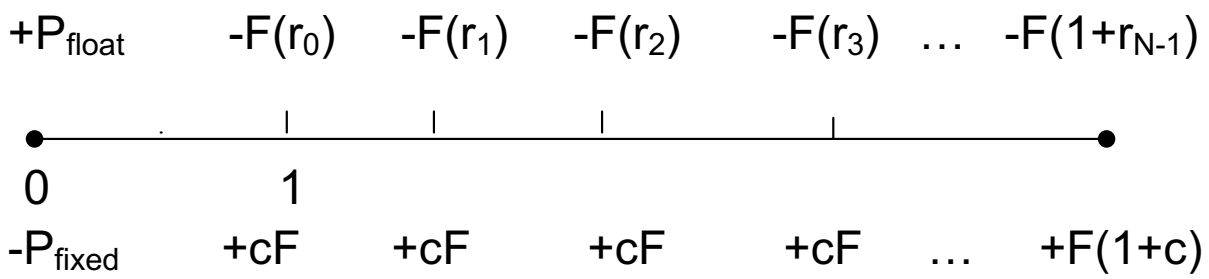
***Note**: All interest rates here are stated on a simple per period basis. For example, if "r" represents the rate earned over 1 period, investing $100 returns $100(1+r) at the end of one period. A common swap payment frequency is semi-annual.*

# Interest rate swap pricing

For floating rate payor, swap is initially equivalent to going long in a fixed rate bond priced at par, and going short in a floating rate bond priced at par

For the fixed rate payor, the equivalent cash position is the opposite

**In general, we price a swap by finding the difference between the present value of the fixed and floating rate payments.**




- Note: the floating payments $r_1$, $r_2$, …, are stochastic from time 0 perspective.
- The fixed rate on the swap is “c”
- The principal value of the swap is “F”

# Swap Pricing: Valuing a Floating Rate Bond

**<u>Fact:</u> Floating rate bonds always are priced at par at reset dates.**

- *Assumes no credit spread and no change in risk over time*

**Proof**: Let $r_i$ be the one-period reset rate realized at time i.

We find the price at time 0 by working backwards.

At time N-1 there is one remaining payment of principal and interest, equal to $F(1 + r_{N-1})$. Its value at time N-1, $P_{N-1}$, is $F(1 + r_{N-1})/ (1 + r_{N-1}) = F$.

Stepping back to time N-2, $P_{N-2} = ( F(r_{N-2}) + P_{N-1})/ (1 + r_{N-2}) =$

$$F(1 + r_{N-2})/ (1 + r_{N-2}) = F.$$

Continuing in this way, it is clear that the price equals the face value on all reset dates, including at time 0

# Swap Pricing: A No-Arbitrage Condition

At swap initiation, the present value of the fixed and floating rate payments must be equal

- That is because entering into a swap is free, and a voluntary exchange has to be fair to both sides

Because we know that the present value of the floating payments equals the face value of the floating rate bond, the present value of the fixed rate payments also must equal the face value of the fixed rate bond

Thus, **the fixed rate on the swap is determined by setting the present value of the future fixed rate payments equal to par**

# Swap Pricing: Implementation

Imagine that you have derived a spot yield curve $Y_1, Y_2, \ldots, Y_N$ that is appropriate for discounting the fixed rate swap payments

Then the coupon rate on the swap solves:

$$F = \frac{cF}{(1+Y_1)^1} + \frac{cF}{(1+Y_2)^2} + \ldots + \frac{F(1+c)}{(1+Y_N)^N}$$

Rearranging implies the fixed coupon rate is:

$$c = \frac{1 - \frac{1}{(1+Y_N)^N}}{\frac{1}{(1+Y_1)^1} + \frac{1}{(1+Y_2)^2} + \ldots + \frac{1}{(1+Y_N)^N}}$$

**Key insights:**

- Swaps are priced to be consistent with the yield curve and hence with implied forward rates, FRAs, and other interest rate forwards and futures
- Over time the value of the swap changes with market interest rates. Like forward contracts, it is zero sum across the two counterparties.

# Example 3.1: Hedging bank balance sheet risk

It is December 2025. Southwest savings bank is expanding its holdings:

- It funds $1mm of new 10-year mortgages using 3-month time deposits
- The current mortgage rate is 10% per year, fixed.
- The current 3-month rate on time deposits is 8%.

A profit of 2% is locked in over the first three months

After that, the bank bears the risk that interest rates might rise. This risk can be **hedged** with **futures contracts**, or more effectively, with an **interest rate swap**

**Balance Sheet**

| **Assets** | **Liabilities** |
|---|---|
| 10 year Mortgages | 3 month time deposits |
| | Equity |

# Example 3.1: (*continued*)

## Hedging with 3-month Futures

### 3-month Futures Quotations

| | Quoted Price | Effective Rate (360 day basis) | Open Interest |
|---|---|---|---|
| March 2025 | 92.11 | 7.89% | 282,867 |
| June 2025 | 92.05 | 7.95% | 158,974 |
| Sept 2025 | 91.83 | 8.17% | 102,620 |
| Dec 2025 | 91.84 | 8.16% | 65,656 |
| March 2026 | 91.56 | 8.44% | 33,065 |
| June 2026 | 91.50 | 8.50% | 27,220 |
| Sept 2026 | 91.51 | 8.49% | 24,899 |
| Dec 2026 | 91.49 | 8.51% | 15,108 |
| March 2027 | 91.45 | 8.55% | 12,763 |
| June 2027 | 91.48 | 8.52% | 21,028 |
| Sept 2027 | 91.46 | 8.56% | 10,832 |
| Dec 2027 | 91.45 | 8.55% | 2,532 |

Shorting one of each of these contracts locks in a **borrowing** rate on $1 million of funds as listed above over the next three years. This is called "shorting a strip of futures."

*What are the drawbacks of this hedging strategy?*

# Example 3.1: (*continued*)

## Hedging with an interest rate swap

- Imagine that Southwest  can enter into an interest rate swap with the following terms:
  - **Maturity** = 10 years
  - **Fixed rate payor** = Southwest S&L.
  - **Fixed Rate** = 8.65%.
  - **Floating Rate** = LIBOR
  - **Payment Frequency** = Semiannual for both fixed and floating.
- Now Southwest can use the fixed (10%) payments from the mortgages to meet their obligations in the swap.
- The floating rate payments received in the swap will be used to pay interest on the deposits backing the mortgages.

# Example 3.1: (*continued*)

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**The advantages over the strip of futures contracts include:**

- only one contract
- covers the entire 10 years
- avoids illiquid contracts for long-dated futures
- the timing is more flexible

**But this swap is not a perfect hedge for Southwest:**

- Mortgages are usually amortized over their lifetime, so that the principal balance is declining
- The frequency of the mortgage payments (often monthly) does not match the semiannual frequency of the fixed payments on a plain vanilla swap
- The three month rate paid to depositors does not match the six month LIBOR rate received in the swap
- Mortgages can usually be prepaid

# Customized Swap Contracts

Those features of mortgages, which make a plain vanilla swap a less-than-perfect hedge for Southwest, is an example of why there is a demand for more specialized swap products such as:

- **Amortizing Swaps**
- **Basis Swaps**
- **Swaptions**

Specialized swaps tend to be more expensive than a plain vanilla swap and a counterparty may be harder to locate

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# Currency, commodity and total-rate-of-return swaps

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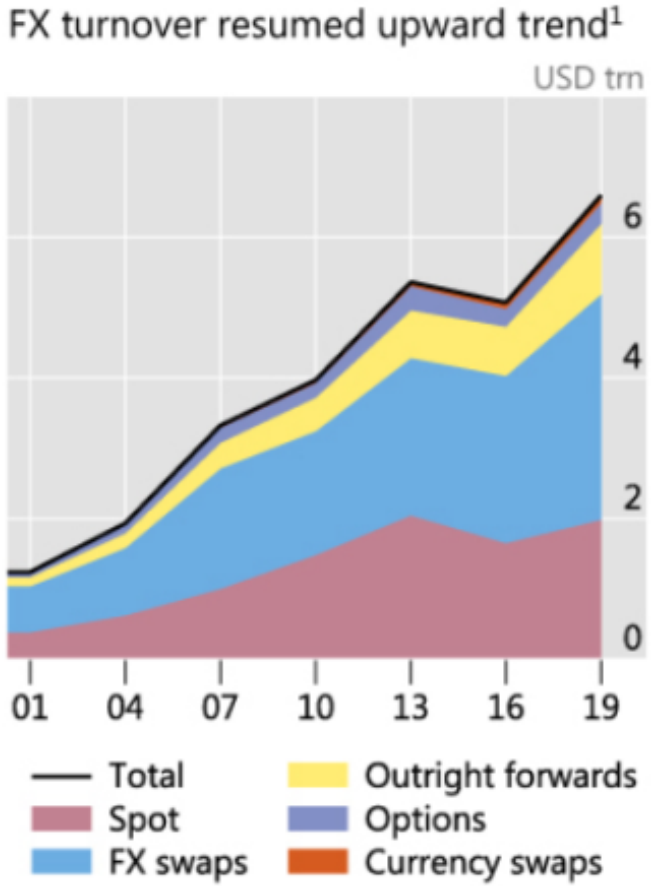
# Currency Swaps

A **currency swap** is an agreement to periodically exchange a payment in one currency for a payment in a second currency

- Payments can be fixed for fixed, fixed for floating, or floating for floating
- Fixed for fixed is like a portfolio of forward currency contracts



# Currency swap example

Back to our example of hedging currency risk, assume that the US exporter is due to receive the 5 mil euros in 5 equal installments, every 6 months for 2.5 years.

The US company can enter into five forward (or futures) contracts to hedge each installment as a stand-alone cash flow.

Recall that $S_0$ = 1.2673, $r_{\$}$ = 5%, and $r_{€}$ = 3%  (flat term structure in both countries)

Using $F = S_0 e^{(r_{\$} - r_{€})T}$ implies the forward rate schedule:

| Maturity | .5 | 1 | 1.5 | 2 | 2.5 |
|---|---|---|---|---|---|
| Forward rate | 1.28 | 1.2929 | 1.3059 | 1.3190 | 1.3323 |

# Currency swap example (cont.)

Alternatively, the US firm can enter into a currency swap

For instance, the swap contract between the US firm and a bank may be specified as:

- US firm pays bank 1 mil euros on T = 0.5, 1, . . . , 2.5
- Bank pays US firm 1 mil ×K (where K is the swap rate, say K = 1.306) dollars on the same dates.

What is the net $ cash flow for the U.S. firm from the swap at any payment date?

- At every $T$, the firm receives 1 mil × $K$ dollars, and must pay 1 mil euros ×$S_T$ dollars/euro (cash settled)
- **Net amount received in swap is $1 mil x ($K$ – $S_T$)**

U.S. firm also sells Euros received for dollars at current spot rate $S_T$. Net $ cash flow = 1 mil x $K$

# Currency swap example (cont.)

How is the swap rate $K = 1.306$ determined?

The swap rate $K$ is chosen at time 0 so that the value of the swap is equal to zero, i.e., no exchange of money at inception but only in the future.

- It is determined by a no-arbitrage condition between the forward and swap markets

Strategy: Construct a portfolio with swap & set of reverse forward contracts

- Payoff at every $T$ from swap + forward

$$= 1\ mil \times (K - S_T + S_T - F_T)$$

$$= 1\ mil \times (K - F_T)$$

- Present value of cash flows is 1 mil multiplied by:

$$e^{-r_{\$}.5}(K - F_{.5}) + \ldots + e^{-r_{\$}2.5}(K - F_{2.5})$$

- Set $K$ so that present value of portfolio equals zero, and hence swap value is zero (Why?)

$$K = w_{.5}F_{.5} + \ldots + w_{2.5}F_{2.5}$$

$$w_T = \frac{e^{-r_{\$}T}}{e^{-r_{\$}.5} + \ldots + e^{-r_{\$}2.5}}$$

# Currency swap example (cont.)

We obtain an alternative (equivalent) formulation by substituting the forward prices

$$F = S_0 e^{(r_\$ - r_\varepsilon)T}$$

Then we have a currency swap rate

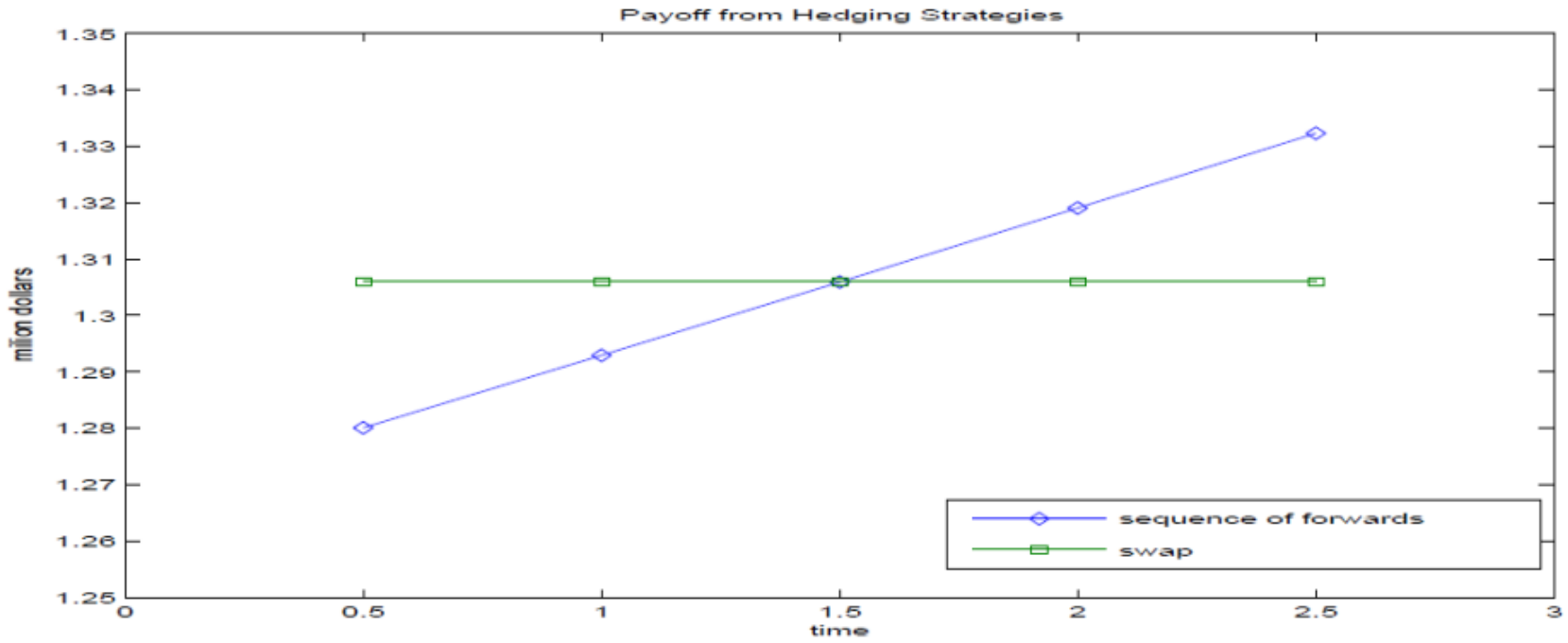
$$K = S_0 \frac{e^{-r_\varepsilon .5} + e^{-r_\varepsilon 1} + \ldots e^{-r_\varepsilon 2.5}}{e^{-r_\$ .5} + e^{-r_\$ 1} + \ldots e^{-r_\$ 2.5}}$$

The currency swap rate equals the current exchange rate multiplied by the ratio of the relative risk-free borrowing costs in the two currencies

# Hedging with swaps versus forwards

The payoff profile from the sequence of forwards and one swap is different:

- The sequence of forwards implies the US firm gets less money early on, and more later on (from $1.28 mil to $1.3323 mil)
- The swap implies the firm gets a constant amount $1.306 mil every payment




Both strategies perfectly hedge the exposure, as the exchange rate risk is eliminated and both payoff profiles are known at 0. And both have the same present value.

***Why then use one rather than the other?***

# Commodity swaps

A **commodity swap** is an agreement to periodically exchange a pre-specified fixed payment for a payment linked to the market price of a commodity

- Usually contract calls for cash-settlement but in principle could require physical deliveries

Example of commodity swap

- A company needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.
- The forward prices for deliver in 1 year and 2 years are $110 and$111/barrel.
- The 1- and 2-year zero-coupon bond yields are 6% and 6.5%

# Commodity swap example (cont.)

Company can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years.

The PV of this cost per barrel is

$$\frac{110}{1.06}+\frac{111}{(1.065)^2}=201.638$$

Any payments that have a present value of $201.638 are consistent with the forward prices.

Typically a swap will call for equal payments each year.

For example, the payment per barrel, x, will have to be $110.483 to satisfy:

$$\frac{x}{1.06}+\frac{x}{(1.065)^2}=201.638$$

Then the no-arbitrage 2-year swap price per barrel is $110.483

# Commodity swap example (cont.)

This example illustrates that swaps are equivalent to forward contracts coupled with borrowing and lending money.

Consider the swap price of $110.483/barrel versus the forward prices.

- Relative to the forward curve price of $110 in 1 year and $111 in 2 years, we are overpaying by $0.483 in the first year, and we are underpaying by $0.517 in the second year.

Thus, by entering into the swap, we are lending the counterparty money for 1 year. The implied interest rate on this loan is

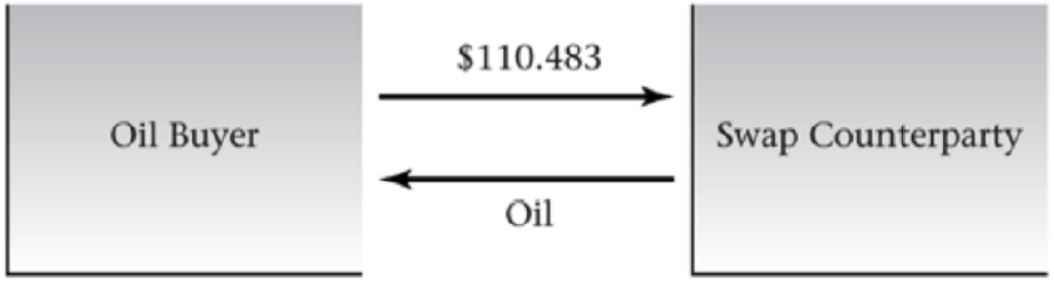
$$\frac{.517}{.483} - 1 = 7\%$$

Given the 1 and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the implied forward yield between years 1 and 2.

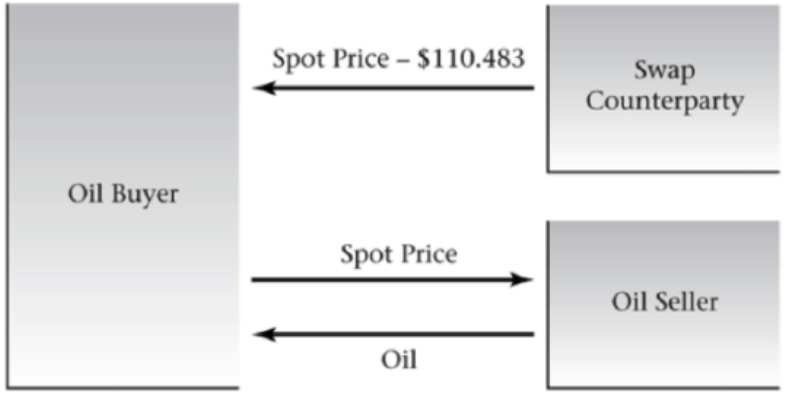
The deal, which is fairly priced, has an embedded borrowing and lending rates equal to the implied forward rates in the yield curve.

# Physical vs. financial settlement of swaps

Physical settlement of the swap




Financial settlement of the swap, plus cash transaction




- Oil buyer pays the swap counterparty the difference between $110.483 and the spot price, and the oil buyer then buys oil at the spot price.
- Whatever the market price of oil, the net cost to the buyer is the swap price, $110.483

# Total Rate of Return (TROR) Swaps

- An exchange of an interest payment for the total return on a reference asset, paid periodically over the life of the TROR contract.

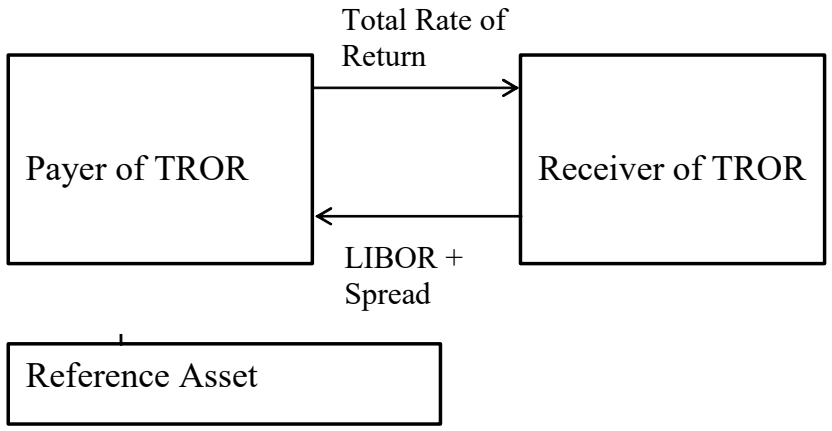
**Total Return = Cash Flows + (Change in Market Value)**

**Fixed maturity date**.

- Need not match reference asset maturity

**Reference Asset:**

- Bond, Loan (e.g., emerging market, sovereign, bank debt, mortgage-backed securities, or corporate loan), Index, Equity, Commodity
- Payor need not own reference asset, but if it is owned, it hedges the cash flows for the payor.



# Comprehension check

Total rate of return swap with notional principal $10 million. Sold at LIBOR flat.

Reference asset earns -10% over the period (interest and capital gain/loss)

LIBOR is 4.5% over the period.

What is the net cash flow on the swap on the payment date?

# TROR Swap – Pricing Basics

From the TROR **payor's perspective**

- on each leg the swap is equivalent to a **short position in the reference asset and a long position in a floating rate bond.**
- This abstracts from transactions costs and counter-party risk

From the TROR **receiver's perspective**

- the swap is equivalent to a **long position in the reference asset and a short position in a floating rate bond.**

The net swap value is zero at payment dates, after the total net return is exchanged.*

- Effectively, the swap restarts at each payment date on a fixed notional amount of assets
- Between payment dates the value is the difference between the value of the floating rate bond and the reference asset.

*This assumes the fair spread on the floating rate does not change.

# Some motivations of TROR Receivers

May gain off-balance sheet exposure to a desired asset class

Create new asset for it to invest in with a specific maturity not available in the market

- E.g., access entire asset classes by receiving total return on an index

Reduce administrative costs (relative to outright purchase of reference asset)

- Especially if underlying is illiquid or regulation prohibits direct ownership

Provides a highly leveraged position, since no cash payments are initially made, and only net return is exchanged

- Leverage is the main reason hedge funds tend to be TROR receivers.
- But generally requires some collateral and/or capital, which reduces leverage

# Example: Creating leverage with TROR swap

| | HEDGE FUND A | HEDGE FUND B | CASH INVESTOR |
|---|---|---|---|
| Asset yield | 8.30% | 8.30% | 8.30% |
| Libor yield | 5.80% | 5.80% | |
| Net asset spread | 2.50% | 2.50% | |
| Spread to LIBOR | -1.00% | -1.00% | |
| Net swap spread | 1.50% | 1.50% | |
| collateral | 5% | 10% | |
| leverage | 20 to 1 | 10 to 1 | 1 to 1 |
| Interest on collateral | 5.80% | 5.80% | |
| Net return | 35.80% | 20.80% | 8.30% |
| | | | |
| | | | |

$$35.80\% = [.083(100) - .068(100) + .058(5)]/5$$

# Some Motivations of TROR Payers

- Hedge price risk and default risk of reference asset
- Avoid sales to speculate on temporary price decline of asset
- Avoid restrictions on shorting
- There may be accounting and regulatory implications
  - Under some regimes it was possible to defer loss recognition while immediately limiting risk exposure
  - Many such loopholes have been closed; one must be careful to understand current rules and regulations
  - TROR payers, often banks, must be sensitive to capital requirement implications.