15.415x Foundations of Modern Finance

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Lecture 19: Investment and Financing

Key concepts

- Interaction between investment and financing
- Leverage without tax shield (review)
- Leverage with tax shield: APV and WACC with tax shield
- Implementing APV
- Implementing WACC
- APV versus WACC

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Interaction btw investment and financing

So far in analyzing financing, we considered:

- How to choose capital structure, taking investment decisions as given.

Now we consider:

- How financing may affect investment decisions.
  - Debt capacity of a project's assets may bring tax shield, which adds value.

Main question:

- How to value a project, taking into account how it can be financed.

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Leverage and taxes

In order to find out the proper valuation/discount rates for future cash flows, we need to properly specify their evolution over time.

Let:

- X_t – CF from the firm's assets at time t (independent of leverage),
- $V_{U,t}$ – value of firm without leverage at t,
- $V_{L,t}$ – value of the firm with leverage at t,
- D_t – value of its debt,
- E_t – value of its equity,
- r_A – required rate of return on the firm′s assets/unlevered firm,
- r_L – required rate of return on the levered firm,
- r_D – required rate of return (interest) on debt,
- r_E – required rate of return on equity,
- τ – corporate tax rate.

Leverage without tax shield

In the absence of taxes impact from leverage, MM implies:

- A firm's asset can be viewed as a portfolio of its debt and equity:

$$V = D + E$$

- Asset return equals weighted average of debt and equity returns:

$$r_A = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E = WACC$$

- The value of the firm is given by:

$$V = D + E = \sum_{s=1}^{\infty} \frac{(1-\tau)\, X_s}{(1+r_A)^s} = V_U = \sum_{s=1}^{\infty} \frac{(1-\tau)\, X_s}{(1+WACC)^s} = V_L$$

Leverage without tax shield

- MM II: Cost of equity with leverage (D/E) is:

$$r_E = r_A + \frac{D}{E}(r_A - r_D)$$

  - r_A is independent of D/E (leverage),
  - r_E increases with D/E (assuming riskless debt),
  - r_D may also increase with D/E as debt becomes risky.

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Leverage with tax shield - APV

As in the case with tax shield, MM implies:

- The value of a firm/project with leverage consists of the following:

$$V_L = E + D = V_U + PVTS - PVDC = APV$$

  For simplicity, we will ignore the cost of default (PVDC) for the remainder of our discussion here, assuming debt is mostly riskless.

- We can then view the firm/project with leverage in two equivalent ways:

$$V_L = E + D = V_U + PVTS = APV$$

  1) A portfolio of debt and equity, or
  2) A portfolio of two assets:
     a) an asset with payoff identical to the project (after tax),
     b) an asset with payoff identical to the tax shield.

Leverage with tax shield - WACC

In order to find the proper valuation/discount rate(s) for future cash flows, we need to properly specify their evolution over time.

Assume: Leverage ratio remains constant over time. That is:

$$\frac{D_t}{V_{L,t}} = w_D, \qquad \frac{E_t}{V_{L,t}} = w_E$$

First, we have:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = w_D r_D + w_E r_E$$

Next, we have:

$$\begin{aligned}(1 + r_L)\, V_{L,t} &= (1 - \tau)\, X_{t+1} \;+\; \tau\, r_D\, D_t \;+\; V_{L,t+1} \\ &= (1 - \tau)\, X_{t+1} \;+\; \tau\, r_D \frac{D_t}{V_{L,t}} V_{L,t} \;+\; V_{L,t+1}\end{aligned}$$

We can rewrite the above as:

$$(1 + r_L - w_D\, \tau\, r_D)\; V_{L,t} \;=\; (1 - \tau)\; X_{t+1} \;+\; V_{L,t+1}$$

Leverage with tax shield - WACC

From

$$(1 + r_L - w_D \tau r_D)\, V_{L,t} \;=\; (1-\tau)\, X_{t+1} \;+\; V_{L,t+1}$$

define:

$$\begin{aligned} WACC \;&= r_L - w_D\, \tau\, r_D \;= w_D r_D + w_E r_E - w_D\, \tau\, r_D \\ &= w_D (1-\tau)\, r_D \;+\; w_E\, r_E \end{aligned}$$

We then have:

$$(1 + WACC)\; V_{L,t} \;=\; (1-\tau)\, X_{t+1} + V_{L,t+1}$$

or

$$V_{L,t} \;=\; \frac{(1-\tau) X_t + V_{L,t+1}}{1 + WACC} \;=\; \sum_{s=1}^{\infty} \frac{(1-\tau) X_{t+s}}{(1 + WACC)^s}$$

WACC with taxes - WACC

Thus, with leverage and taxes, we can value a firm/project in two equivalent ways (assuming constant leverage ratio):

1. NPV adjusted for the impact of leverage, Adjusted Present Value (APV):

$$V_L = E + D = V_U + PVTS = \sum_{s=1}^{\infty} \frac{(1-\tau)\, X_s}{(1+r_A)^s} + PVTS$$

   (Still ignore the cost of default, assuming debt mostly riskless.)

2. NPV discounted by WACC (which adjusts for tax shield):

$$V_L = \sum_{s=1}^{\infty} \frac{(1-\tau)\, X_s}{(1+WACC)^s}$$

   where the weighted average cost of capital is given by:

$$WACC = w_D\,(1-\tau)\,r_D + w_E\,r_E = \frac{D}{D+E}\,(1-\tau)\,r_D + \frac{E}{D+E} r_E$$

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Implementing APV

1. Find a traded firm with the same business risk:
   - Debt to equity ratio D/E,
   - Equity return r_E (by CAPM or APT),
   - Debt return r_D,
   - Tax rate τ.
2. Uncover r_A (the discount rate without leverage).
3. Apply r_A to the after-tax cash flow of the project to get V_U.
4. Compute PV of debt tax shield.
5. Compute APV.

Implementing APV: Uncovering r_A

A key step in this approach is uncovering r_A from data on a levered firm.

In order to do so, we need to make several simplifying assumptions:

- Assume that pre-tax payout of the firm grows geometrically:

$$X_t = X_{t-1}\, g_t, \qquad g_t \text{ are IID growth rates with mean } g.$$

- Time periods are short.
- The discount rate on total payout r_A is constant.
- The firm rebalances its debt each period to maintain a constant debt/equity ratio.

The value of the firm's assets is then given by:

$$V_{U,t} = \sum_{s=1}^{\infty} \frac{(1-\tau)\, X_t\, g^s}{(1+r_A)^s} = \frac{(1-\tau)\, X_t}{r_A - g}$$

which evolves proportionally with X_t.

Implementing APV: Uncovering r_A

Now consider the value of firm's assets and liabilities:

$$D_t + E_t = V_{U,t} + PVTS_t$$

where $PVTS_t$ is the present value of the firm's tax shield.

- From this, we relate average returns on assets and liabilities:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = \frac{V_{U,t}}{V_{U,t} + PVTS_t} r_A + \frac{PVTS_t}{V_{U,t} + PVTS_t} r_{TS}$$

- The critical observation at this point is that, because of the constant debt/equity ratio, the risk of PVTS is virtually the same as the risk of the firm's asset value $V_{U,t}$ (if the periods are short). That is,

$$r_{TS} = r_A$$

- The same logic implies that their market betas satisfy:

$$\beta_{TS} = \beta_A$$

Implementing APV: Uncovering r_A

- We then conclude that:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = w_D r_D + w_E r_E = r_A$$

- The required rate of return on equity, as stated by MM II with taxes, is:

$$r_E = r_A + \frac{D}{E}(r_A - r_D)$$

- Under CAPM, we have:

$$\beta_L = w_D \beta_D + w_E \beta_E = \beta_A$$

- Note that this is the same formula we obtained previously under the assumption of no taxes.

- Here, there are corporate taxes. Yet, the formulas still holds because the risk of the tax shield generated by debt, under stated assumptions, has the same risk as firm's assets.

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Implementing APV: Example

Example. Want to buy SuperSoft, a software company:

- Currently privately held and 100% equity financed.
- Projected annual pre-tax cash flow $1.5M perpetually.
- Effective corporate tax rate is 30%.
- SuperSoft's assets can sustain a debt level up to 20% of its asset value without significant distress cost.
- The (risk-free) interest rate SuperSoft pays on its debt is $r_F = 5\%$.
- There are two publicly traded software companies, 1 and 2:

	r_E (%)	r_D (%)	D/E	τ
Firm 1	24.31	6.95	0.5	0.34
Firm 2	29.07	9.88	1.2	0.30

Implementing APV: Example

Example (cont'd)**.**

- Using MM II with taxes, we obtain the required rate of return on assets for firm 1 and 2:

$$r_A^1 = 18.51\%, \qquad r_A^2 = 18.60\%$$

- Take the average: we have $r_A = 18.56\%$.
- The value for SuperSoft, if unlevered, is (in millions):

$$V_U = \frac{(1-0.3)(1.5)}{0.1856} = \$5.66$$

- The value if levered (in millions):

$$V_L = V_U + PVTS = 5.66 + (0.3)(0.2)(0.05)(5.66)/(0.1856) = \$5.75$$

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Implementing WACC

Example (cont'd)**.**

- $V_U = 5.66$ is the value of SuperSoft's assets. The safe debt level is then:

$$D = (0.2)\, V_U$$

- Since the debt is safe, the (instantaneous) interest rate is the risk-free rate $r_F = 5\%$. The present value of tax shield is:

$$PVTS = \tau\,(r_F D)/r_A = (0.3)\left(\frac{0.05}{0.1856}\right)(0.2)\ V_U = (0.016)\, V_U$$

- The value of equity with leverage is:

$$E = V_U + PVTS - D = [1 + 0.016 - 0.2]\, V_U = (0.816)\, V_U$$

- We then have:

$$w_D = \frac{0.2}{0.2 + 0.816} = \frac{0.2}{1.016}, \qquad w_E = \frac{0.816}{1.016}$$

$$r_E = r_A + \frac{D}{E}(r_A - r_D) = 18.56\% + (0.2/0.816)(13.56\%) = 21.88\%$$

Implementing WACC

Example (cont'd)**.**

- We can compute the WACC:

$$WACC = \frac{(0.2)(1-0.3)}{1.016}(5\%) + \frac{0.816}{1.016}(21.88\%) = 18.26\%$$

- Using WACC, the value of the levered firm is:

$$V_L = \frac{(1-0.3)(1.5)}{WACC} = \frac{(1-0.3)(1.5)}{0.1826} = \$5.75$$

  We obtain the same answer as APV.

- With WACC, we can apply to similar projects with the same financing (assuming it is optimal).

WACC with taxes

Selected Industry Capital Structures, Betas, and WACCs

Industry	Debt ratio (%)	Equity beta	Asset beta	WACC (%)
Electric and Gas	43.2	0.58	0.33	8.1%
Food production	22.90	0.85	0.66	11.0%
Paper and plastic	30.40	1.03	0.72	11.4%
Equipment	19.10	1.02	0.83	12.4%
Retailers	21.70	1.19	0.93	13.2%
Chemicals	17.30	1.34	1.11	14.7%
Computer software	3.50	1.33	1.28	16.2%
Average of all industries	21.50	1.04	0.82	12.3%
Assumptions: Risk-free rate 6%; market risk premium 8%; cost of debt 7.5%; tax rate 35%				

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WACC vs. APV

Pros of WACC:

- Widely used.
- Fewer computations needed (important before computers).
- More straightforward, easier to understand and explain (?).

Cons of WACC:

- Mixes up effects of assets and liabilities. Errors/approximations in the effect of liabilities contaminate valuation.
- Not very flexible:
  - Non-constant debt ratios?
  - Cost of hybrid securities (e.g., convertibles)?
  - Other effects of financing (e.g., costs of distress)?
  - Personal taxes?

WACC vs. APV

Advantages of APV:

- No contamination.
- Clearer: Easier to track down where value comes from.
- More flexible: Just add other effects as separate terms.

Cons of APV:

- Almost nobody uses it. But…

Overall:

- For complex, changing or highly leveraged capital structure (e.g., LBO), APV is much better.
- Under the right circumstances, it doesn't matter much which method to use (if applied properly).

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