Recitation 20

Spring 2021

Question 1

Consider a firm whose board of directors just announced a cash dividend of \$100 million to be paid tomorrow. For valuation purposes, ignore this one day period. After this payout, the firm will have assets worth \$1000 million and 10 million shares. The firm's cost of capital is 10%. Ignore corporate and personal taxes. The firm has no debt.

(a) What is the firm's share price before the dividend is paid?

(b) What is the firm's share price after the dividend payment (i.e. the ex-dividend price)?

(c) Suppose that instead of paying out all cash as a single dividend at once, the board announces that it will pay quarterly dividends, with a first dividend of \$25 million to be paid the day after the announcement. What is the firm's share price after the first dividend payment in this case? Assume that unpaid earnings are reinvested back into the firm's operations.

(d) Suppose that the day after the quarterly dividend payment, the firm announces it will buy back \$21.5 million worth of shares to be implemented the following day.

   i. How many shares will the firm buy back?

   ii. What will be the share price after the repurchase?

Solutions:

(a) The share price before the dividend is paid is the dividend plus the value of assets after the payout, divided by the number of shares:

$$\begin{aligned} P &= \frac{\$100M + \$1000M}{10M} \\ &= \$110 \end{aligned}$$

(b) The ex-dividend share price is:

$$P^{EX} = P - D = \$110 - \frac{\$100M}{10M} = \$100$$

(c) With this dividend policy, the quarterly dividend is \$2.50 per share. The ex-dividend share price with a quarterly dividend of \$2.5 per share is:

$$P^{EX} = P - D = \$110 - \$2.50 = \$107.50$$

(d) i. To find the number of shares bought back, divide the value of shares bought back by the share price:

$$N = \frac{\$21.5M}{\$107.5} = 200,000$$

ii. After the repurchase, the firm's equity value is:

$$V = \$1,100 - \$25M - \$21.5M = \$1,053.5M$$

The number of shares outstanding is:

$$N = 10M - 0.2M = 9.8M$$

So the share price is:

$$P = \frac{V}{N} = \frac{\$1,053.5M}{9.8M} = \$107.50$$

Note that there is no change in the share price, since the value of the cash outflow from the share buyback is equal to the cash inflow in the form of surrendered shares.

Question 2

Cartier uses $76,000$ of troy ounces of silver per year to produce their jewelry and would like to hedge the price risk for next year. The current price of silver is $22.35 per troy ounce. Cartier is considering two hedging strategies. First, it can enter into a one-year forward contract with BNP Paribas for 76,000 troy ounces at a price of $25.70 per ounce. Second, it can trade silver futures on CME. The current price of a one-year futures contract is $25.25. The contract unit is 5,000 troy ounces.

(a) Assuming that Cartier does not hedge, what would be its cost of silver consumption if the silver price increases to $23.00? What is it increases to $27.00?

(b) Suppose that you enter into a 1-year forward contract with BNP Paribas. What is the total cost of your silver consumption in Year 1? Is this a perfect hedge?

(c) Suppose you use futures for your hedging needs. Assuming that you can't buy fractions of a standard contract, how many contracts do you need to buy or sell? Is this a perfect hedge?

Solutions:

(a) If the price of silver is $23.00, Cartier's cost of silver consumption is:

$$\$23.00 \times 76,000 = \$1.75M$$

If the price of silver is $27.00, Cartier's cost of silver consumption is:

$$\$27.00 \times 76,000 = \$2.05M$$

(b) Since the offered forward price is \$25.70, Cartier's cost of silver consumption would be:

$$\$25.70 \times 76,000 = \$1.95M$$

This is a perfect hedge. By entering into this contract, Cartier fully hedges its exposure to silver price risk.

(c) If we hedged the position with futures, we would need $\frac{76,000}{5,000} = 15.2$ contracts. Since we can't buy fractions of a contract, we will buy 15 contracts. This guarantees the price of 75,000 ounces of silver. Cartier will need to buy the remaining 1,000 ounces at the market price.

If the silver price is \$23.00, Cartier's cost would be:

$$\$25.75 \times 75,000 + \$23.00 \times 1,000 = \$1.917M$$

If the silver price is \$27.00, Cartier's cost would be:

$$\$25.75 \times 75,000 + \$27.00 \times 1,000 = \$1.921M$$

As shown, we do not achieve a perfect hedge in this case. We are still exposed to the risk, although most of the position (75,000 out of 76,000 ounces) is hedged.

Question 3

An endowment fund owns a fixed income portfolio consisting of US Treasury STRIPS: 1 million units of STRIP A and 1.5 million units of STRIP B. The fund would like to construct a hedge for the interest rate risk using Treasury STRIP C.
Data on the portfolio and bond C is given below:

Bond	Price	Duration (Years)
A	P_A	4.0
B	P_B	8.0
C	P_C	7.0

All STRIPS have a principal of \$100. Assume a flat term structure for interest rates. The current interest rate is 2%.

(a) What is the modified duration of the portfolio?

(b) Using bond C, construct a hedge to protect the value of the portfolio against parallel shifts in interest rates. How many units of bond C should we go long/short?

(c) Is this a perfect hedge?

Solutions:

(a) The duration of the portfolio is a weighted average of durations of assets in the portfolio. We need to:

1. Find the total value invested in bonds A and B

2. Take the weighted average of durations of bonds A and B

To find the value invested in A and B, we need to find the prices of these bonds:

$$P_A = \frac{\$100}{(1+2\%)^4} = \$92.38$$
$$P_B = \frac{\$100}{(1+2\%)^8} = \$85.35$$

The value of the portfolio is:

$$\begin{aligned} V_P &= n_A P_A + n_B P_B \\ &= 1M \times \$92.38 + 1.5M \times \$85.35 \\ &= \$220.41M \end{aligned}$$

The duration of the portfolio is:

$$\begin{aligned} D_P &= \frac{n_A P_A}{V_P} D_A + \frac{n_B P_B}{V_P} D_B \\ &= \frac{1M \times \$92.38}{\$220.41M} \times 4 + \frac{1.5M \times \$85.35}{\$220.41M} \times 8 \\ &= 6.32 \end{aligned}$$

Note that the modified duration of the portfolio is:

$$MD_P = \frac{D_P}{1+y} = \frac{6.32}{1+2\%} = 6.20$$

(b) Let's denote our position in bond C by n_C. If this number is positive, we will interpret this as going long; if it is negative, it will mean we are going short.

To hedge against parallel (and small) shifts in interest rates, we need to establish a position on bond C so that the duration of our portfolio is zero. Let's denote this portfolio by H.

The duration of portfolio H is:

$$\begin{aligned} D_H = \frac{n_A P_A}{V_H} D_A + \frac{n_B P_B}{V_H} D_B + \frac{n_C P_C}{V_H} D_C &= 0 \\ n_A P_A D_A + n_B P_B D_B + n_C P_C D_C &= 0 \\ D_P \times V_P + n_C P_C D_C &= 0 \end{aligned}$$

Note that $P_C = \frac{100}{(1+2\%)^7} = \$87.06$.

So, solving for n_C:

$$\begin{aligned} n_C &= \frac{-D_P V_P}{P_C D_C} \\ &= \frac{-6.32 \times \$220.41M}{\$87.06 \times 7} \\ &= -2.29M \end{aligned}$$

(c) Duration matching allows us to allows us to hedge only small and parallel shifts in the yield curve:

1. Due to convexity of bond prices, large changes in interest rates will not be perfectly hedged with just duration matching. We would also need to match convexity.
2. In practice, interest rates at different maturities may be affected differently by economic events. This would affect prices of bonds at different maturities differently.

Question 4

A life insurance company has existing liabilities that obligate it to pay out \$5 million per year for the next 10 years. Assume that the term structure of interest rates is flat and the current interest rate is 2%.
The insurance company would like to invest in safe government bonds in order to make it immune to fluctuations in interest rates. It considers 1-year and 10-year Treasury STRIPS. Construct an investment portfolio for the insurance company that hedges the interest rate risk.

Solutions:

We will first find the value of liabilities and the duration of liabilities. The liabilities are a 10-year annuity:

$$\begin{aligned} V_L &= \frac{\$5M}{2\%}(1 - \frac{1}{(1+2\%)^10}) \\ &= \$44.91M \\ D_L &= \frac{1}{V_L}\sum_{t=1}^{10}\frac{\$5M \times t}{(1+2\%)^t} \\ &= 5.34 \end{aligned}$$

To hedge liabilities, we want to allocate funds across 1-year and 10-year STRIPS such that the duration of the portfolio matches the duration of liabilities. We will denote V_1 as the value invested in 1-year STRIPS, and V_{10} as the value invested in 1-year STRIPS.
The hedging portfolio should have the same value and duration of the liabilities. This gives us two equations:

$$\begin{aligned} V_1 + V_{10} &= V_L \\ \frac{V_1}{V_1+V_{10}}D_1 + \frac{V_{10}}{V_1+V_{10}}D_{10} &= D_L \end{aligned}$$

So, we have:

$$\begin{aligned} V_1 + V_{10} &= \$44.91M \\ \frac{V_1}{V_1+V_{10}} \times 1 + \frac{V_{10}}{V_1+V_{10}} \times 10 &= 5.34 \end{aligned}$$

Solving yields:

$$V_1 = \$23.27M$$
$$V_{10} = \$21.64M$$

To immunize against changes in interest rates, the insurance company should purchase \$23.27 million worth of 1-year STRIPS and \$21.64 million worth of 10-year STRIPS.

Question 5

Consider a market with two possible states a year from now: Good or Bad. In the Good state, the return on the market portfolio will be 10%, and in the Bad state it will be −20%. The one-year risk-free rate is 5%.
A biotechnology startup in Cambridge, MA works on a new vaccine delivery technology. Next year, this technology will be sold to Moderna. If the market is in the Good state, the startup will generate \$80 million in cash flow. If the market is in the Bad state, the cash flow will only be \$20 million.
The startup is considering borrowing \$15 million to fund the development. If the startup fails to pay the loan back, it faces bankruptcy costs of \$5 million. In the event of default, these costs will reduce the value of the remaining assets, and therefore the payout to the debtholders. Suppose that the loan is issued at par and that the interest is due at maturity (in Year 1). Assume that the tax rate is 25%.

(a) You were hired as a financial advisor to the startup. Your goal is to advise on the best financing arrangement.

(b) The CFO of the startup considers a possibility of hedging default risk in order to avoid bankruptcy costs. She approached the investment bank Lazard, which offered to sell a put option that pays \$750,000 in the Bad state of the economy (and nothing in the Good). Assuming that the option is fairly priced, what would you advise the biotech startup to do?

Solutions:

(a) The after-tax cash flow in Year 1 in each of the states is given below:

$$\text{Good: } \$80M \times (1 - 25\%) = \$60M$$
$$\text{Bad: } \$20M \times (1 - 25\%) = \$15M$$

We need to find the risk-neutral probabilities of each state (u and d). Here, $u = 1.10$ and $d = 0.8$, as given in the question (the market return in each of the states). So,

$$q_u = \frac{(1 + r_f) - d}{u - d} = \frac{1.05 - 0.80}{1.10 - 0.80} = 0.833$$
$$q_d = 1 - q_u = 0.167$$

If the firm is 100% equity financed, its value is:

$$V_U = \frac{q_u \times \$60M + q_d \times \$15M}{1 + r_f}$$
$$= \$50.00M$$

We now need to find the value of the levered firm. We start by finding the interest rate on the loan, which we will later use to compute the present value of tax shields. We know the borrowed amount should equal the expected value of the payout under risk-neutral measures, discounted back one year at the risk-free rate. Note that the value in the good state is $\$15M \times (1+y)$, where y is the interest rate. The value of the debt in the bad state is the value of assets in the bad state, minus the \$5M of bankruptcy costs. So we have:

$$\begin{aligned}\$15M &= \frac{0.833 \times (1+y) \times \$15M + 0.167 \times (\$15M - \$8M)}{1+r_f} \\ y &= 12.67\%\end{aligned}$$

Solving for y yields $y = 12.67\%$. So, if the loan is issued at par, the annual interest rate on this loan will be 12.67%, and FV = \$15M.

We can now determine the present value of interest tax shields generated by this loan. There is only a tax shield in year 1, since that is the only year where interest is due. Furthermore, the tax shield only exists in the up state, as the firm effectively turns over its remaining assets to debtholders in the down state when it defaults. We have:

$$\begin{aligned}PVTS &= \frac{q_u \times \tau \times r \times \$15M}{1+r_f} \\ &= \frac{0.833 \times 25\% \times 12.67\% \times \$15M}{1+5\%} \\ &= \$0.38M\end{aligned}$$

We also need to determine the present value of the costs of financial distress in order to get to the value of the levered firm. The firm pays \$5M in bankruptcy costs if it is distressed, which only happens in the down state and in Year 1. So the present value of financial distress is:

$$\begin{aligned}PVDC &= \frac{q_d \times \$5M}{1+r_f} \\ &= \frac{0.167 \times \$5M}{1+5\%} \\ &= \$0.79M\end{aligned}$$

We can put this all together to arrive at the value of the levered firm if it borrows \$15M:

$$\begin{aligned}V_L &= V_U + PVTS - PVDC \\ &= \$50.00M + \$0.38M - \$0.79M \\ &= \$49.59M\end{aligned}$$

Since the costs of financial distress outweigh the tax shield benefits, we have that $V_L < V_U$, so the firm should not take out this loan.

(b) Having an additional \$750,000 in the Bad state allows the startup to borrow \$15 million risk free. In this case, the startup has enough funds to pay both principal (\$15M) and interest ($\$15M \times 5\% = \$0.75M$) in both Good and Bad states.

What are the dollar costs and benefits of this hedging strategy?

Cost of Hedging Strategy

The cost of the strategy is the price of the option. Since the option is fairly priced, we can find the option price using risk-neutral probabilities:

$$P = \frac{q_d \times \$0.75M}{1 + r_f} = \$0.12M$$

Benefit of Hedging Strategy

The benefit of this strategy is the interest tax shield from debt and the fact that we are avoiding bankruptcy costs.

The PVTS is:

$$PVTS = \frac{25\% \times r_f \times \$15M}{1 + r_f} = \$0.18M$$

There would be no costs of financial distress, since the firm won't default. So the value of the levered firm is:

$$V_L = \$50.00M + \$0.18M - \$0.12M = \$50.06M$$