



# 15.415x Foundations of Modern Finance

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## **Lecture 16: Capital Budgeting II**

# Key concepts

- Review of capital budgeting (real investment) decisions
- Discount rates
- Risk and horizon
- Real options
- Taxonomy of real options
- Valuing real options
- Insights from real options

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# Corporate financial decisions

Questions answered so far: How the financial market values assets.

- Time value of money – interest rates,
- Risk premium (price of risk) – APT, CAPM, option pricing models.

Questions to answer next: How firms make financial decisions using market valuations and instruments.

- Real investments (capital budgeting): What projects to invest in?
  - Cash flows.
  - Discount rates.
  - Strategic/real options.
- Financing: How to finance a project?
  - Selling financial assets (bank loans, public debt, stocks, convertibles, ...)
- Payout: What to pay back to shareholders?
  - Paying dividends, buyback shares, ...
- Risk management: What risk to avoid and how?

# Investment decisions

Investment/capital budgeting criteria:

- For a single project, take it if and only if its NPV is positive.
- For many independent projects, take all those with positive NPV.
- For mutually exclusive projects, take the one with highest (positive) NPV.

In order to compute the NPV of a project, we need to analyze:

1. Cash flows,
2. Discount rates,
3. Strategic options.

We have examined 1 in Part I of this course. Now examine 2 and 3 with more asset pricing tools.

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# Discount rates

## So far, we have learned that:

- A project's discount rate (required rate of return or COC) is the expected rate of return demanded by investors/market for the project.
- Discount rate in general depends on the timing and risk of the project's cash flow.
- Discount rate is usually different for different projects (e.g., different risks).
- It is in general incorrect to use a company-wide “cost of capital” to discount cash flows of different projects.

## What is the required rate of return on a project?

- Simple case: A single discount rate can be used for all cash flows of a project (the term structure of discount rates is flat).
- General case: Different discount rates for different cash flows:
  - Term structure of interest rates is not flat,
  - Different pieces of a cash flow may have different risks.

# Discount rates

## Use CAPM to estimate cost of capital (discount rate)

- Project's required rate of return is determined by the **project beta**:

$$\bar{r}_{project} = r_F + \beta_{project} \times (\bar{r}_M - r_F)$$

- What matters is the project beta, not the company beta!
- What if project beta is unknown?
  - Find comparable “pure-play” company and use its beta.
  - Find comparable projects and use its cash flows to estimate beta.
  - Use fundamental analysis and judgment to “guesstimate” beta.



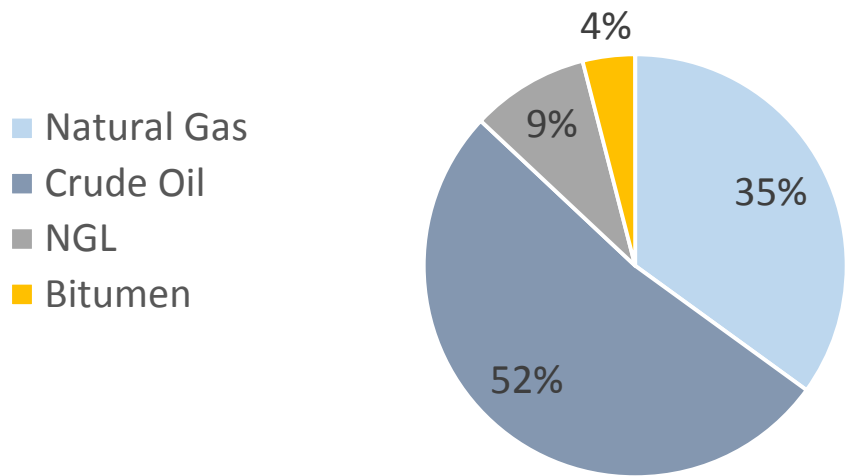
## Discount rates

**Example.** Berkshire Hathaway is considering entering the natural gas and oil business, and must evaluate the NPV of the estimated cash flow from this business. The beta for Berkshire Hathaway's assets is 0.48. What cost of capital should it use for the NPV calculation of its new business?

- Berkshire Hathaway should not use its own beta to discount cash flows from energy business.
- Berkshire Hathaway should use the beta of an energy company, e.g., ConocoPhillips.
- What about using ExxonMobil's beta?

# Discount rates

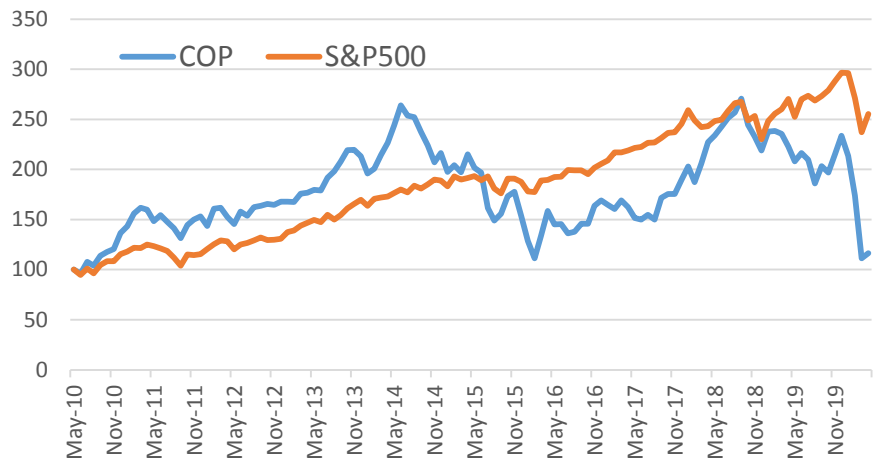
ConocoPhillips 2019 Production Mix



Stock Price, NYSE: COP, (04/2010-04/2020)



Cumulative Total Return (04/2010-04/2020)



# Discount rates

## Example (cont'd).

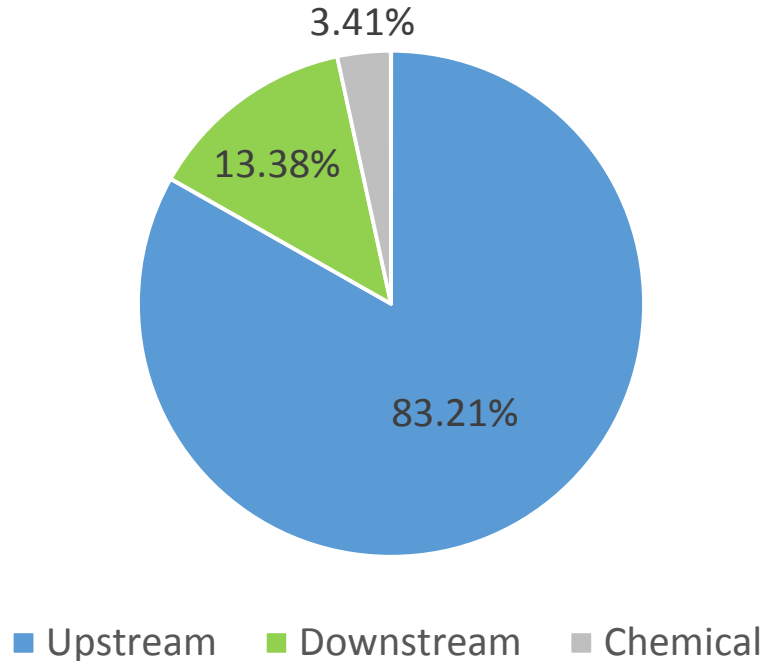
- Beta for COP's assets (from <http://finance.yahoo.com>): 0.77 (April 2020).
  - COP's equity beta is 1.49, debt/equity ratio is 0.94.
  - Assuming COP's debt has zero beta, its asset beta is 0.77.
- Risk-free rate: 5%.
- Market risk premium: 6%.

$$\bar{r}_{project} = r_F + 0.77 \times (\bar{r}_M - r_F) = 0.05 + 0.77 \times 0.06 = 9.62\%$$

- Use judgment in interpreting and adjusting these estimates.
- Estimates are merely approximations!
- How good is the approximation?

# Discount rates

2019 ExxoMobil Earnings



- Upstream: Unconventional, Deepwater, LNG, Heavy oil, Conventional
- Downstream: Fuels, Lubes
- Chemical: Basic Chemicals, Commodity and Performance Products

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# Discount rates and time horizon

Discount rates over different horizons are in general different.

The term structure of discount rates arises from two sources:

1. Term structure of interest rates.
  - Time value of money (without risk) is different for different dates.
2. Term structure of risk premia.
  - The risk of cash flows at different dates is different.
  - The price of risk can be different for different dates.

## Discount rates and time horizon

**Example.** Suppose that the market premium is 8% and the risk-free rate is 4%. Consider a financial contract that pays two years from now ( $t = 2$ ) the ratio:

$$CF_2 = \frac{S\&P_2}{S\&P_1} = 1 + r_{M2}$$

What should its current price be?

- At  $t = 1$ , the price of this contract is:

$$PV_1(CF_2) = \frac{1 + E_1[r_{M2}]}{1 + E_1[r_{M2}]} = \frac{1 + \bar{r}_M}{1 + \bar{r}_M} = 1$$

- Since the value of this contract at  $t = 1$  (the end of the first year) is certain, the value of this contract today is:

$$PV_0(CF_2) = \frac{1}{1 + 0.04} = 0.9615$$

or

$$PV_0(CF_2) = \frac{E[CF_2]}{(1 + r_F)(1 + \bar{r}_M)} = 0.9615$$

- The risks over the two periods are different, so are the discount rates.

## Discount rates and time horizon

**Example (cont'd).** Now consider another financial contract that pays two years from now ( $t = 2$ ) the following ratio:

$$CF_2 = \frac{S\&P_2}{S\&P_0} = (1 + r_{M1})(1 + r_{M2})$$

- At  $t = 1$ , the price of this contract is:

$$PV_1(CF_2) = \frac{(1 + r_{M1})(1 + E_1[r_{M2}])}{1 + E_1[r_{M2}]} = \frac{(1 + r_{M1})(1 + \bar{r}_M)}{1 + \bar{r}_M} = 1 + r_{M1}$$

- The value of this contract at  $t = 1$  has the same risk as the market, its value today is:

$$PV_0(CF_2) = \frac{1 + E_0[r_{M1}]}{1 + E_0[r_{M1}]} = 1$$

or

$$PV_0(CF_2) = \frac{E_0[CF_2]}{(1 + E_0[r_{M1}])(1 + E_1[r_{M2}])} = \frac{E_0[CF_2]}{(1 + \bar{r}_M)(1 + \bar{r}_M)} = 1$$

- In this case, the risks over the two periods are the same and **compounding** on each other, and we use the same risk-adjusted discount rates for both periods (with a flat term structure for risk-free rates).



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# Introduction to real options

In real investment decisions:

- Managers often have options to adapt and revise decisions in response to unexpected developments/new information (real options).
- Flexibility is valuable and should be included in valuing a project.
  - E.g., ability to adjust production in response to demand increases project value compared to fixed production before learning about demand.
- “Strategic grounds” are sometimes added to simple NPV analysis.

Two issues in analyzing real options:

## 1. Identification

- Are there real options imbedded in a project?
- What type of options?

## 2. Valuation

- How to value these options?

# Identifying real options

How to identify real options?

- There are options imbedded in all but the most trivial projects.
- All the “art” consists of:
  - Identifying those that are “significant”,
  - It takes practice, and sometimes “vision”.

Two conditions for real/strategic options:

1. New information possibly arrives in the future,
2. The news, when arrives, may affect decisions.

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# Identifying real options

**Example.** UW Inc. is deciding whether to buy a copper mine:

- The mine can produce 10 million kilograms of copper.
- It takes one year to extract the copper, at a cost of \$5.4/kg, paid up front.
- The copper price is now \$6/kg.
- Next year, the copper price can either increase by 30% or decrease by 20%, with equal probability.
- All uncertainty about copper price is resolved next year.
- The copper price risk is totally diversifiable.
- The risk-free rate is 5%.

What is the current value of this project?

Consider two strategies:

- 1) Extract now,
- 2) Extract next year.

# Identifying real options

## Example (cont'd).

1) Extract now

■ Expected cash flow next year:

$$\frac{1}{2} (7.8 + 4.8) = 6.3$$

■ NPV (per kg) using simple DCF:

$$NPV_0 = -5.4 + \frac{6.3}{1 + 0.05} = 0.6$$

What if the risk is not diversifiable?

# Identifying real options

## 2) Extract next year

- In the up state, the NPV of the project in year 1:

$$NPV_1(\text{up}) = -5.4 + \frac{7.8}{1.05} = 2.03$$

- In down state, the NPV of the project in year 1:

$$NPV_1(\text{down}) = -5.4 + \frac{4.8}{1.05} = -0.83$$

- The NPV in year 0:

$$\begin{aligned} NPV_0 &= \frac{[NPV_1(\text{up}) + NPV_1(\text{down})]/2}{1.05} \\ &= \frac{1}{1.05} \left[ -5.4 + \frac{(7.8 + 4.8)/2}{1.05} \right] = \frac{1}{1.05} (0.6) = 0.57 \end{aligned}$$

# Identifying real options

Using the naive DCF analysis, we have:

- Buying the mine has positive NPV,
- Should start extraction now,
- NPV of the mine is  $(10 \text{ million}) (0.6) = \$6 \text{ million}$ .

Is this the best we can do?



# Identifying real options

**Example 1 (cont).** Take into account the flexibility in responding to new information about copper price:

- In the up state, the NPV of the project in year 1:

$$NPV_1(\text{up}) = -5.4 + \frac{7.8}{1.05} = 2.03$$

- In down state, the NPV of the project in year 1 if we extract:

$$NPV_1(\text{down}) = -5.4 + \frac{4.8}{1.05} = -0.83$$

Since it does not payoff, we will not extract in this state (abandon).

- The NPV in year 0:

$$NPV_0 = \frac{[NPV_1(\text{up}) + 0]/2}{1.05} = 0.97$$

Thus, the NPV for the copper mine is  $(10)(0.97) = \$9.7$  million.

- Wait until year 1 to decide,
- Extract if the copper price goes up and not otherwise.

Two ways to value: (1) “Dynamic NPV”, (2) Option pricing.

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# Taxonomy of real options

- Consider project characteristics such as “Phases”, “Strategic investments”, “Scenarios”, ...
- Examine the pattern of cash flows and expenditures over time.
  - For instance, large expenditures are likely to be discretionary.
- **Taxonomy of frequently encountered options:**
  - Growth option,
  - Abandonment option,
  - Option to expand or contract scale,
  - Timing option,
  - Option to switch (inputs, outputs, processes, etc.)...

# Growth options

- An investment includes a **growth option** if it allows follow-on investments, and the decision whether to undertake the follow-on investments will be made later on the basis of new information.
  - Need to take value of the growth option into account.
- Such projects are often presented as having “strategic value.”
  - Examples:
    - R&D → Developing applications if R&D is successful.
    - Movie Production → Sequel...
    - Investment in emerging markets, exploratory oil drills ...

# Growth options

- Growth options are akin to **call options**: You have the option, not the obligation, to get an asset by incurring a cost.
- Growth options can be “nested”, i.e., series of related choices:
  - Rocky 1 → Rocky 2 → Rocky 3 → ...
- Growth options can be very valuable and account for over half of the market value of some industries.
  - Industries with heavy R&D,
  - Industries with multiple product generations (e.g. computers, software, pharmaceuticals),
  - Platforms.

# Abandonment options

- An investment includes an abandonment option if, under certain circumstances, it can be shut down if so chosen.
- Abandonment options may be hidden in aggregated forecasts: While it may be preferable to continue operations on average, shutting down may be better under some scenarios.
- Abandonment options are akin to **put options**: You have the option (but no obligation) to get rid of something and receive a payment (the liquidation value).

## Scale options

- If conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization.
- If conditions are less favorable than expected, the firm can contract the scale of operations. In extreme cases, production can temporarily halt and start again.
- Similar to growth and abandonment options.
- Examples:
  - Ability to slow the rate of mineral extraction from a mine,
  - Ability to add a temporary third shift at a factory.

# Timing options

- Flexibility about the timing of an investment (possibly including “never”) can be very valuable.
  - Example: The value for a building permit should account for the timing option, i.e., when buying the permit, you are buying the right to build whenever you want (during the permit’s lifetime).
- Akin to an **American call option**: You have the option (but no obligation) to get something at any time by paying a cost (exercise price).
- Note: Only those investment timing choices for which relevant information is likely to arrive in the future have “option value.”

Important in:

- All R&D intensive industries, e.g., pharmaceuticals;
- Long-development capital-intensive projects;
  - E.g., large-scale construction or energy-generating plants...
- Start-up ventures...



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# Valuing growth potential

**Example.** StartUp Inc. is a new company whose only asset is a patent on a new vaccine.

- If produced, the drug will generate **sure** profits of \$1 million per year for 17 years (after then, competition will drive profits to zero).
- It will cost \$10 million of initial investment to produce the drug.
- The interest rates have a flat term structure, which is now at 8% per year.
- The interest rates can either increase to 10% or decrease to 5% next year, and will stay constant afterwards.
  - Note: The binomial model for interest rates above is only an approximation for convenience in computation.

What is the value of the patent?

# Valuing growth potential

## Example (cont'd).

The NPV of investing in the drug today (time 0) is:

$$NPV = \frac{\$1 \text{ million}}{0.08} \left[ 1 - \frac{1}{(1.08)^{17}} \right] - \$10 \text{ million} = -\$878,362$$

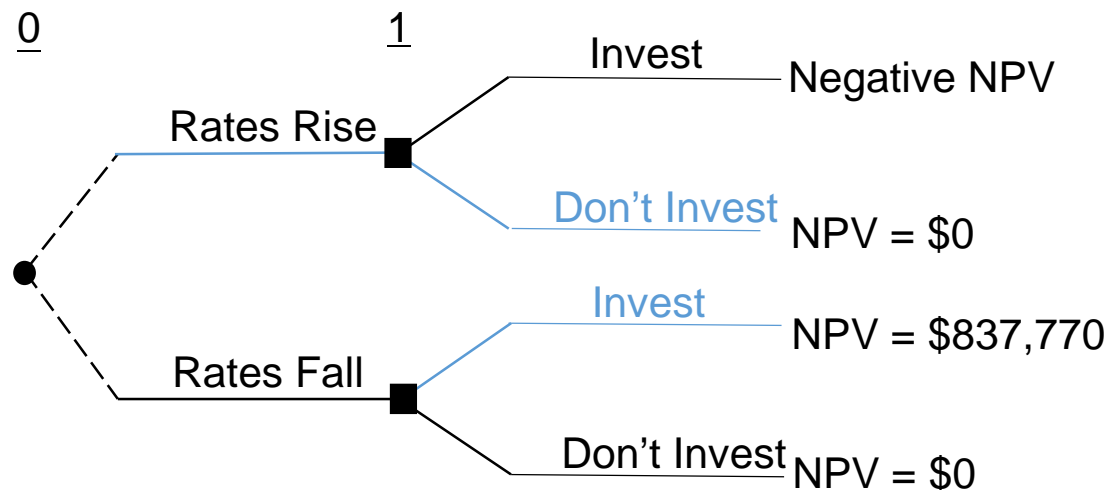
- Given today's interest rates, it does not make sense to produce the drug today.
- What if interest rates fall (rise) to 5% (10%) next year?

## Valuing growth potential

- If interest rates rise to 10%, the NPV is still negative, and it does not make sense to invest in the drug.
- If interest rates fall to 5%, the NPV of investing in the drug (at time 1) is:

$$NPV = \frac{\$1 \text{ million}}{0.05} \left[ 1 - \frac{1}{(1.05)^{16}} \right] - \$10 \text{ million} = \$837,770$$

Thus, if interest rates fall to 5%, the NPV becomes positive, and it may make sense to invest in the drug today.



## Valuing growth potential

To value the two scenarios, we need to find risk-neutral probabilities, the probabilities that set the value of a financial asset today equal to the present value of its future payoffs.

- In this case, a 17-year risk-free annuity that pays \$1,000 per year is used.

- The value of the annuity today is:

$$S = \frac{\$1,000}{0.08} \left[ 1 - \frac{1}{(1.08)^{17}} \right] = \$9,122$$

- If interest rates rise to 10% in one year, the value of the annuity will be:

$$S_d = \$1,000 + \frac{\$1,000}{0.10} \left[ 1 - \frac{1}{(1.10)^{16}} \right] = \$8,824 = d S, \quad \text{where } d = 0.967$$

- If interest rates fall to 5% in one year, the value of the annuity will be:

$$S_u = \$1,000 + \frac{\$1,000}{0.05} \left[ 1 - \frac{1}{(1.05)^{16}} \right] = \$11,838 = u S, \quad \text{where } u = 1.298$$

## Valuing growth potential

- Recall that the risk-neutral probability of interest rates decreasing to 5%,  $q$ , is the probability such that the expected return of the annuity is equal to the one-year risk-free rate (assumed to be 8%).

$$q = \frac{(1 + r_F) - d}{u - d} = \frac{1.080 - 0.967}{1.298 - 0.967} = 0.341$$

- The value today of the investment opportunity is the present value of the expected cash flows (using risk-neutral probabilities) discounted at the risk-free rate:

$$PV = \frac{(837,770)(0.341) + (0)(1 - 0.341)}{1.08} = \$264,518$$

- In this example, even though the cash flows of the project are known with certainty, the uncertainty regarding future interest rates creates substantial option value for the firm.
  - The firm's ability to use the patent if interest rates fall is worth \$221,693 today.

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# Valuation of real options

**Example. Real options in follow-up projects.** In 1990, MC Inc. considers entering PC business:

- R&D has come up with model-A --- a new PC model
- CFs of model-A, if introduced, are as follows (after tax):

Cash flow (\$ million)	1990	1991	1992	1993	1994	1995
Investment	- 450	- 50	- 100	- 100	125	125
Operating CF (after tax)		140	159	259	185	
Net CF	- 450	90	59	159	310	125

NPV at cost of capital of 20% is -\$42 million:

$$-450 + \left[ \frac{90}{1 + 0.2} + \frac{59}{(1 + 0.2)^2} + \frac{159}{(1 + 0.2)^3} + \frac{310}{(1 + 0.2)^4} + \frac{125}{(1 + 0.2)^5} \right]$$



# Valuation of real options

However,

- Launch model-A would allow MC Inc. to introduce model-B in 1993;
- Expected CFs from model-B are twice that of model-A;
- In expectation, model-B is a loser too;
- But there are scenarios in which model-B really pays off.

## Valuation of real options

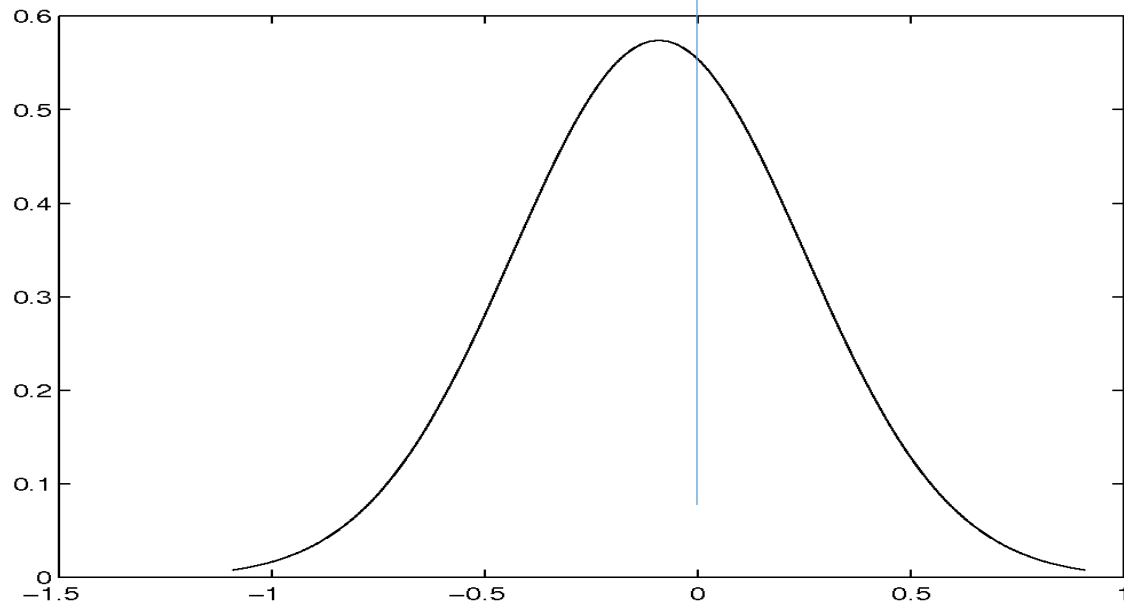
Different scenarios for model B	PV (model B) in 1993 (\$ M)
Benchmark scenario	- 84
Initial investment reduced by 30%	178
Sales increase by 40%	368
Profit margin increases by 50%	302

Should MC Inc. start model-A?

The expected value of model-B is -\$84 million. Could this prospect justify the \$42 million sacrifice to enter the market with model-A?

# Valuation of real options

Probability Distribution of NPV in 1993 for Model-B (in billion dollars)



- Starting model-B in 1993 is an option.
- So long as MC can abandon the business in 1993, only the right-hand-side of the distribution is relevant.
- NPV of the right-hand-side is huge even if the chance of ending up there is less than 50%.

# Valuation of real options

Assume:

- 1) Model-B decision has to be made in 1993.
- 2) Entry in 1993 with Model-A is prohibitively expensive.
- 3) MC has the option to stop in 1993 (possible loss limited).
- 4) Investment needed for model-B is \$900M (twice that of A, fixed).
- 5) PV of operating profits from model-B is \$472 million in 1990:
- 6) 
$$\frac{1}{(1+0.2)^3} \left[ \frac{180}{1+0.2} + \frac{118}{(1+0.2)^2} + \frac{318}{(1+0.2)^3} + \frac{620}{(1+0.2)^4} + \frac{250}{(1+0.2)^5} \right] = 472$$
- 7) PV evolves with annual standard deviation of 35% (from prices of traded companies in the same business).
- 8) Interest rate is 9%.

## Valuation of real options

The opportunity to invest in model-B is a 3-year call option:

- The strike price is \$900 million,
- The underlying asset is the profit stream from model-B,
- The present value (in 1990) of the underlying asset is \$472 million,
- With volatility 35% and interest rate 9%, Black-Scholes formula gives:

Value of call = \$54 million.

Total NPV of model-A (\$M):

	A	A+B
DCF	-42	-91
Option	54	
Total	12	

# Valuation of real options

**Endnote:** Black-Scholes calculation (in \$ million).

- The underlying asset is the future profit from model-B, now worth  $A = 472$ ,
- The strike price is  $K = 900$ ,
- The maturity is 3 years,
- The volatility of the underlying asset value is  $\sigma = 35\%$ ,
- The interest rate 9%.

$$x = \frac{\ln \left[ \frac{A}{PV(K)} \right]}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T} = \frac{\ln \left( \frac{472}{900 \times 1.09^{-3}} \right)}{(0.35) \sqrt{3}} + \frac{1}{2} (0.35) \sqrt{3} = -0.3351$$

$$\begin{aligned} C &= A N(x) - PV(K) N(x - \sigma \sqrt{T}) \\ &= (472) N(-1.6115) - (900)(1.09^{-3}) N(-0.3351 - 0.35\sqrt{3}) \\ &= 54 \end{aligned}$$

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# Insights from real options

- Naive DCF analysis tends to under-estimate the value of strategic options:
  - Follow-on projects as options (American call),
  - Abandonment options (American put),
  - Scale options (conversion options),
  - Timing options (American call).
- It is difficult to apply DCF for option valuation.
- Options can be valued (sometimes, e.g., when the underlying asset is traded).

# Insights from real options

Think of investment/strategic planning as a process of:

1. Acquiring and disposing of options, and
2. Exercising options optimally.

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