Week 9 – Credit risk

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Outline

- Statistical approach to default risk
  - Decomposing the credit spread
- Structural approach to default risk
  - Simple binomial example
  - Merton model and extensions
- Credit derivatives

Corporate debt

- Divided into two broad credit quality categories by rating agencies
  1. Investment grade (IG)
  2. Non-investment grade, high yield (HY), or “junk”
     - Much higher historical default rates
     - Original issue vs. “fallen angels”
     - Strong historical return performance
- Default risk manifests itself as:
  - Downgrade risk
  - Event risk
    - E.g., legal changes, surprise actions by managers
  - Liquidity risk
    - Riskier bonds tend to be less liquidity

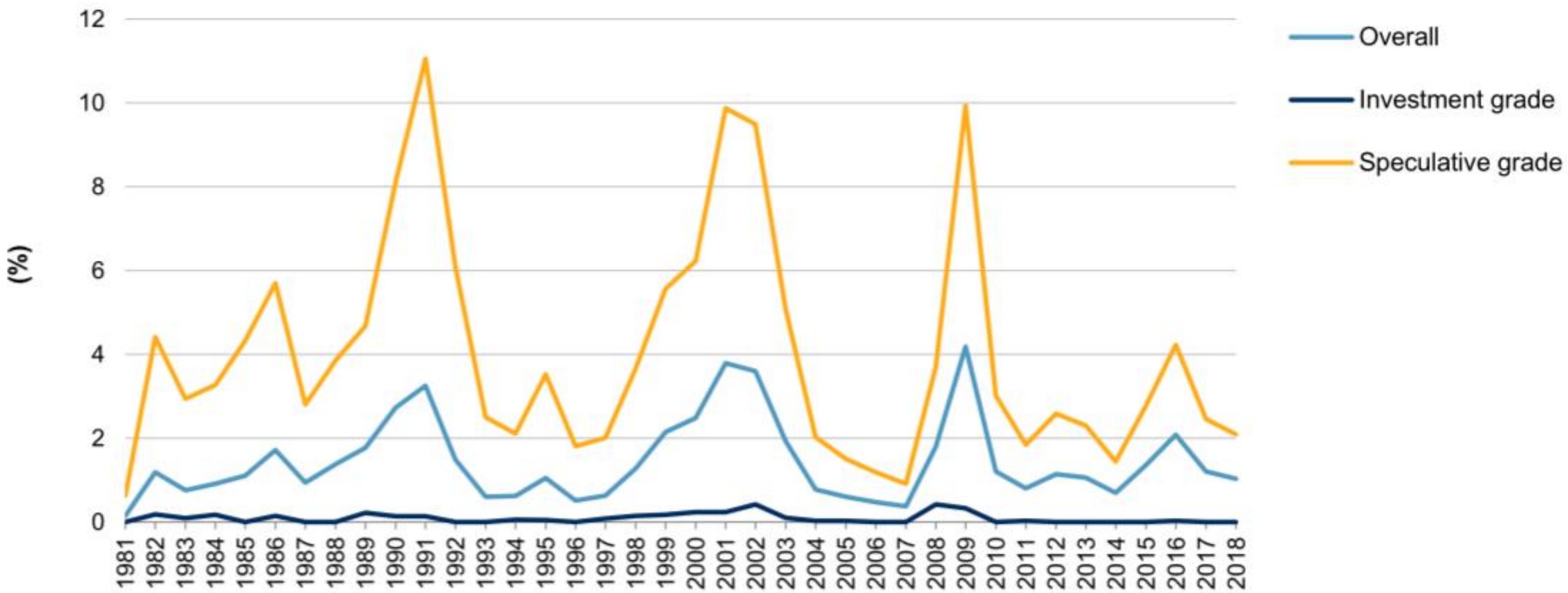
Default and recovery rates:
key statistics for assessing credit risk

- Default rate
- Probability of a default event
  - Usually stated as an annual rate
  - What is a default event depends on who is defining it; rating agencies provide definitions
- Recovery rate
  - Amount expected to be recovered as a fraction of what is owed
  - Sometimes expressed in present value terms
    - Can be based on price of a defaulted bond relative to principal
  - recovery rate = (1 – loss rate)
  - Loss rate is also called “loss given default”
- Term structure of default and recovery
  - Expected rates may vary over the life of a security

Default rates vary enormously:
over time, by credit rating, and with the business cycle

Chart 1

Global Default Rates: Investment Grade Versus Speculative Grade

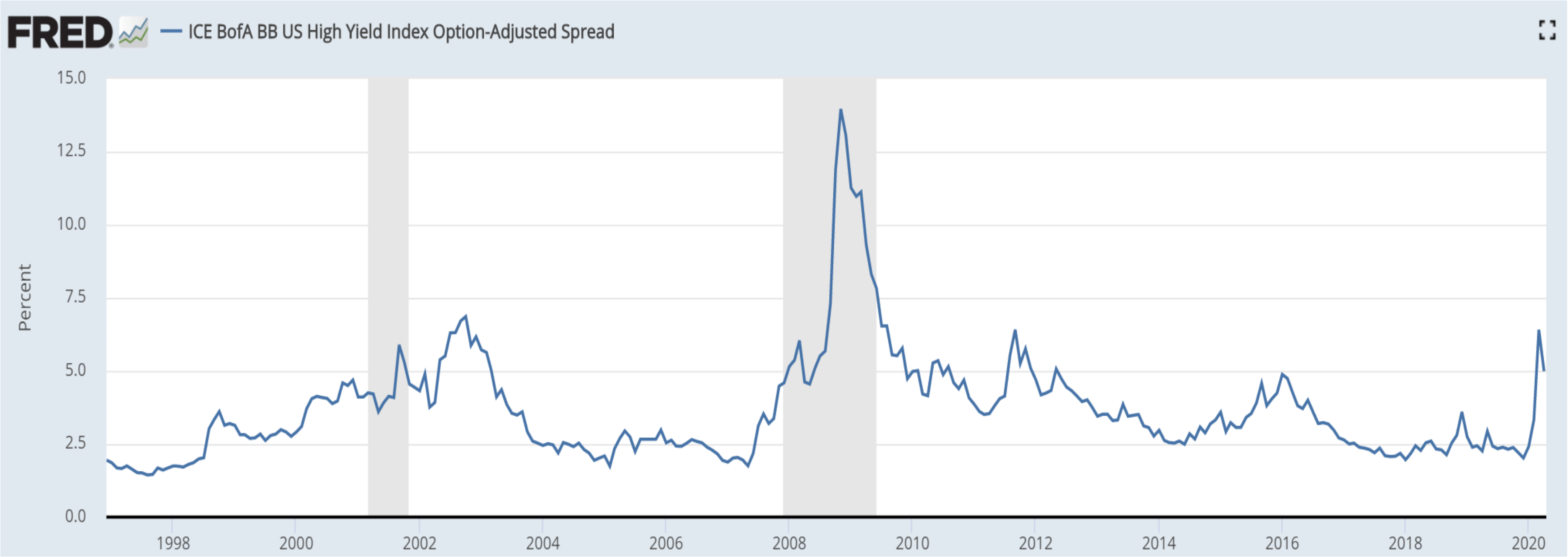


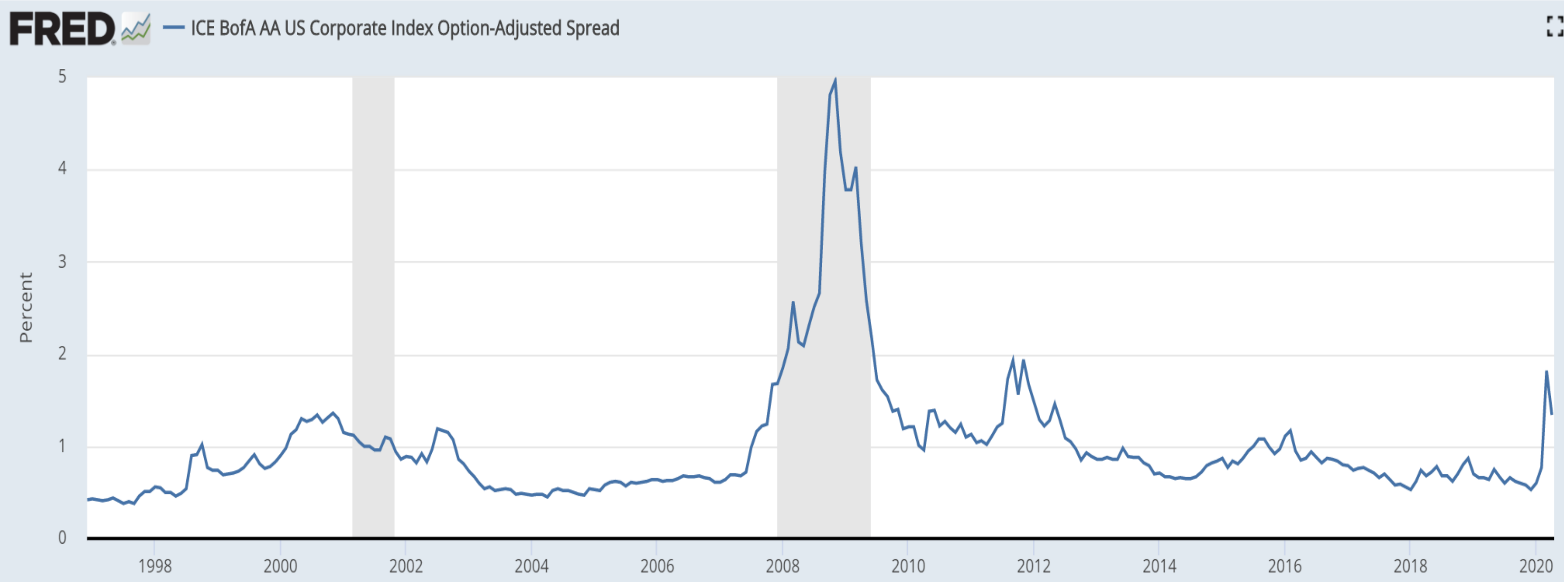
Sources: S&P Global Fixed Income Research and S&P Global Market Intelligence's CreditPro®.

Credit spreads on risky debt

- The “credit spread” (or yield spread) is the difference in the yield to maturity on a risky bond and on a Treasury bond of similar maturity
  - If it is an “options-adjusted spread” it assumes no other embedded options (e.g., no prepayment or call option)

- For a given credit rating, observed yield spreads vary with maturity
  - The yield spread usually increases with maturity over moderate horizons
  - Sometimes credit spreads are decreasing at very long maturities
  - These patterns partially reflect compositional differences in firms borrowing at different maturities





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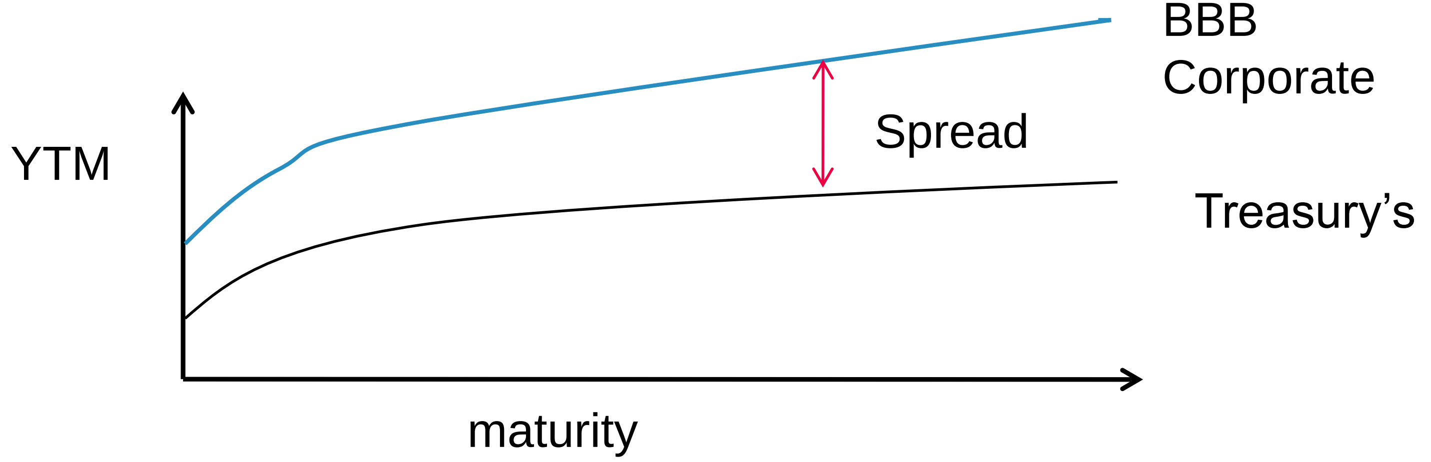
Decomposition of Credit Spreads

- What determines the YTM spread between Treasury's and defaultable securities of the same maturity?

- Conceptually the spread has several components:
  - Expected losses
  - Premium for market risk (β)
  - "Liquidity premium"
  - Non credit features (e.g. tax treatment, embedded options)

$\left.\begin{array}{l}\text{Expected losses}\\ \text{Premium for market risk } (\beta)\end{array}\right\}$ Prob*(Default) x E*(Loss given default)

("*" denotes a risk-neutral variable)



A simple valuation model for risky bonds

- This simple model illustrates that **expected cash flows are not the promised cash flows**, and the **yield to maturity is not the expected return**.

- In the model, the difference between the expected return, r, and the YTM, y, is influenced by the default rate, the recovery rate, and the bond maturity.

- The model is implemented in the spreadsheet default.xls (on webpage)

A simple valuation model for risky bonds

- Consider a risky **T**-period coupon bond, with coupon rate, **c**, and face value **F=1**.
- Assumptions
- A constant default rate each period, **d**
  - The probability of no default from time 0 to time t is $(1-d)^{t-1}$
- A constant recovery rate, **g**
- An expected return **r** equal to the risk-free rate plus a premium for market risk,

e.g., $r = r_f + \beta(E(r_m) - r_f)$

- Investors discount **expected** cash flows at **r** to determine the price of the bond

Basis for numerator in bond Sharpe ratio

A simple valuation model for risky bonds

- Default rate d
- Recovery rate g
- Expected return r
- Coupon rate c
- Maturity T
- face value 1; *P* = price per $1 face value

$$P = \sum_{i=1}^{T}\left((1-d)^{i-1}\frac{(dg(1+c)+(1-d)c)}{(1+r)^{i}}\right)+\frac{(1-d)^{T}}{(1+r)^{T}}$$

$(1-d)^{i-1}$ is the probability that the bond is still outstanding at time i

$\frac{(dg(1+c)+(1-d)c)}{(1+r)^{i}}$ is the pv of the expected coupon plus recovery at time i

$\frac{(1-d)^{T}}{(1+r)^{T}}$ is the pv of the expected $1 principal payment at time T

The yield to maturity takes promised payments as certain:

$$P = \sum_{i=1}^{T}\left(\frac{c}{(1+y)^{i}}\right)+\frac{1}{(1+y)^{T}}$$

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Structural models of credit risk

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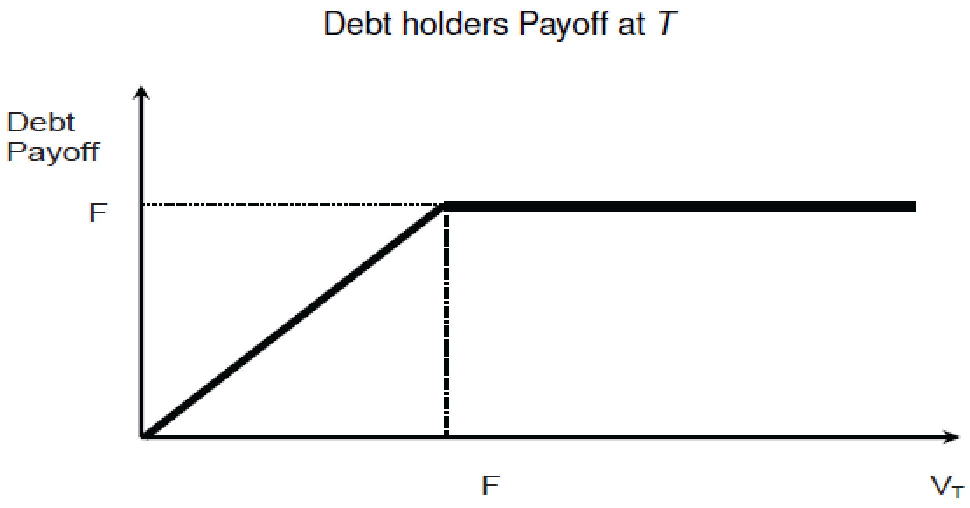
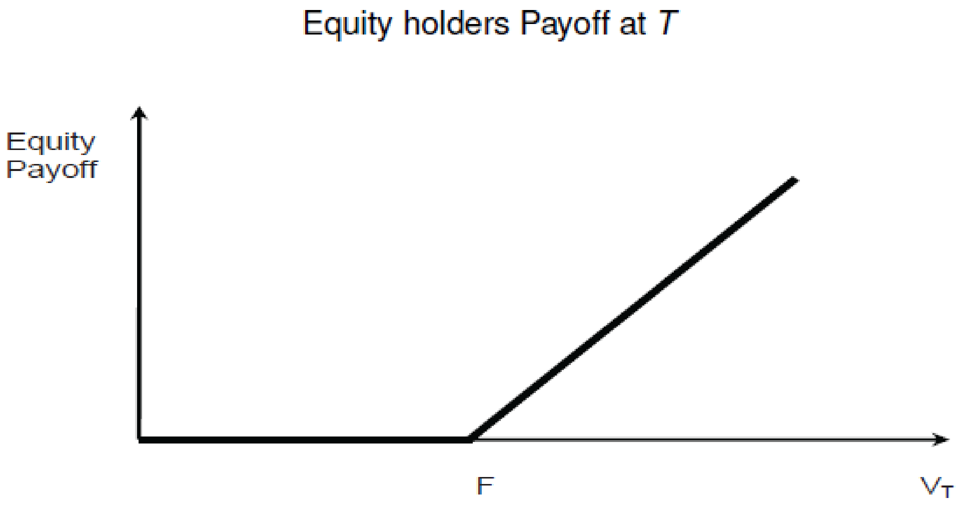
Risky corporate debt and equity payoff diagrams



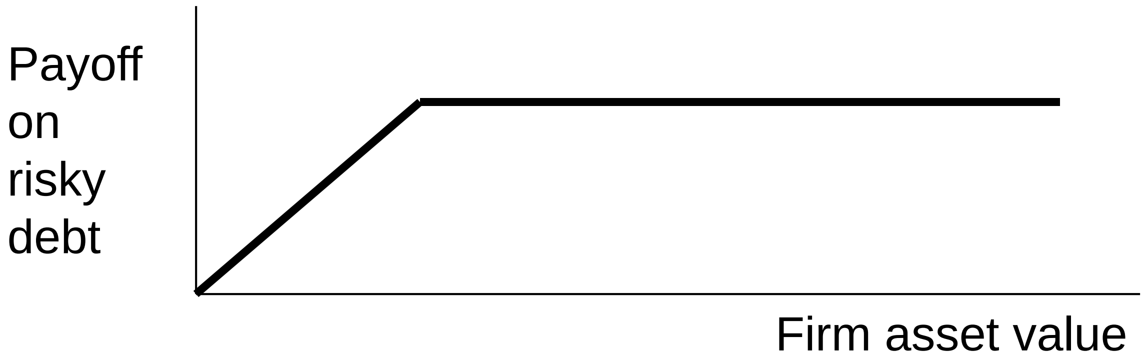


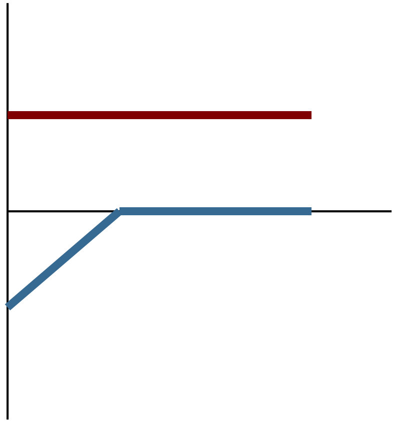
Notes:

1. Sum of debt and equity is asset value
2. Underlying in these diagrams is firm asset value (not equity)

Decomposing payoff on risky debt

- A risky zero coupon corporate bond is equivalent to a portfolio that includes:
  - a long position in a risk-free zero coupon bond
  - a short position in a put option on the assets of the firm, with a strike price equal to the face value of the bond





- A loan guarantee is equivalent to a put option **on firm assets.** The guarantor writes the put option in exchange for a fee (premium).

Valuing loan guarantees as put options

<u>Example</u>

Binomial guarantee pricing:

Today XYZ Co. has a market value of $100 million, and next year the company's assets will take on one of two values: $140 million or $70 million:

$$\begin{array}{ll} & \$140 \\ \$100 & \\ & \$70 \end{array}$$

XYZ also has guaranteed debt with a face value of $90 million (covering principal and interest), coming due next year.

What is the value of the guarantee (from perspective of guarantor)**?**

The payments of the guarantor will be $0 if the assets are worth $140 or -$20 if the assets are worth $70.

Then the payoffs for the guarantor look like:

$$\begin{array}{ll} & \$0 \\ P & \\ & -\$20 \end{array}$$

This is a written put option <u>on the company's assets</u> with a strike price of $90.

Valuing loan guarantees as put options

Assume the price of a 1-year risk-free bond is \$95 per \$100 face. Its value is represented by:

\$100

\$95

\$100

The value of the option can be replicated using the information on asset value and the risk-free bond:

We require that the payments match in the good and bad state of the world:

X100 + Y140 = 0

X100 + Y70 = -20

These two linear equations in two unknowns can be solved for X and Y to yield:

X = -.4
Y = .2857

The price of this portfolio, based on the \$95 price of the bond and the \$100 current asset value, is -.4(\$95) + .2857(\$100) **= -\$9.43**

(The guarantor has a highly levered position in the assets of the firm!)

The Merton Model

- Today is t = 0 and consider a firm with current assets $V_0 = E_0 + D_0$
- Assume the firm's (market value) return on assets is log-normally distributed:

$$V_T = V_0 \times e^{\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\epsilon}$$

- The assets for financed with equity and debt. The debt is a zero coupon bond with face value F and maturity T

- There are two possible outcomes for debt holders at maturity T:
  1. If $V_T > F \implies$ the firm can sell some of its assets and pay the debt holders
     - Debt holders get F; Equity holders get the residual $V_T - F$
  2. If $V_T < F \implies$ the firm will be unable to pay its debt, and therefore defaults
     - The debt holders take possession of the assets of the firm $\implies$ their payoff is V_T; Equity holders get nothing

The Merton Model: Valuing equity as a call option

- The payoff to equity holders is then the one of a call option

$$\max\left(V_T - F, 0\right)$$

- If we denote E_0 the value of equity today,

$$E_0 = \text{Call}\left(V_0, F, r, T, \sigma\right)$$

where

$$\text{Call}\left(V_0, F, r, T, \sigma\right) = V_0\, N(d_1) - F\, e^{-rT}\, N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r + \sigma^2/2\right) T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

Notice the ***volatility in this formula is for assets****, not equity*

The Merton model: Volatility of levered equity

- What is the volatility of levered equity?

- We know the relation between the volatility of a call option and its underlying volatility is given by its vega. Applying the same formula implies:

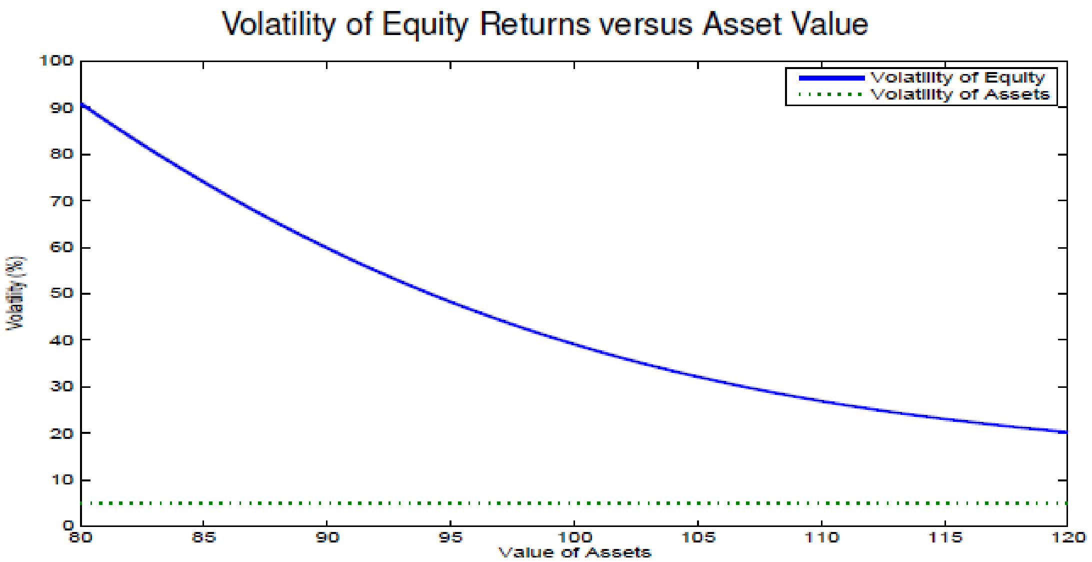
$$\text{Volatility of Equity Returns} = \sigma_E = \left(\frac{VN(d_1)}{VN(d_1) - Ke^{-rT}N(d_2)} \right) \times \sigma$$

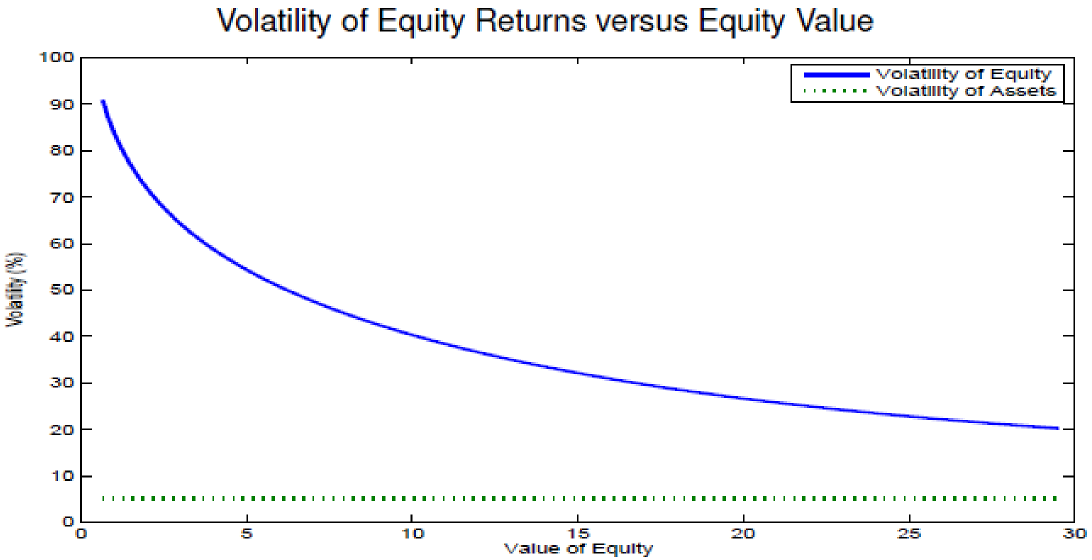
  - The term in parenthesis can be much larger than 1, implying that equity return volatility can be many times higher than the volatility of the underlying assets

- This can also be written as $\sigma_e = \dfrac{V \times N(d_1)}{E} \sigma$

- As the value of equity relative to assets falls, its volatility increases
- This is known as the "leverage effect" on equity volatility, first noted by Fischer Black
- *Important implication*: As a firm becomes distressed, analyzing its expected returns using the CAPM becomes highly problematic because its beta not well-approximated by a constant

The Merton model: Volatility of levered equity





The Merton Model: Inferring asset value and volatility

- We have two non-linear equations in two unobservable variables, V_0 and σ

$$E = \text{Call}\,(V_0, F, r, T, \sigma) = V_0\, N(d_1) - F\, e^{-rT}\, N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + (r + \sigma^2/2)\, T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

And

$$\sigma_e = \frac{V_0 \times N(d_1)}{E}\sigma$$

- Solve simultaneously given other parameters to find V_0 and σ

The Merton model: Value of debt

What is the value of defaultable debt in the model?

- The payoff to debt holders is

$$\min(V_T, F) = V_T - \max(V_T - F, 0)$$

- The value today of this payoff is then

$$D_0 = V_0 - E_0 = V_0 - Call(V_0, F, r, T, \sigma)$$

- Note that this expression also comes immediately from the balance sheet identity

$$\text{Assets of a Firm} = \text{Debt} + \text{Equity}$$

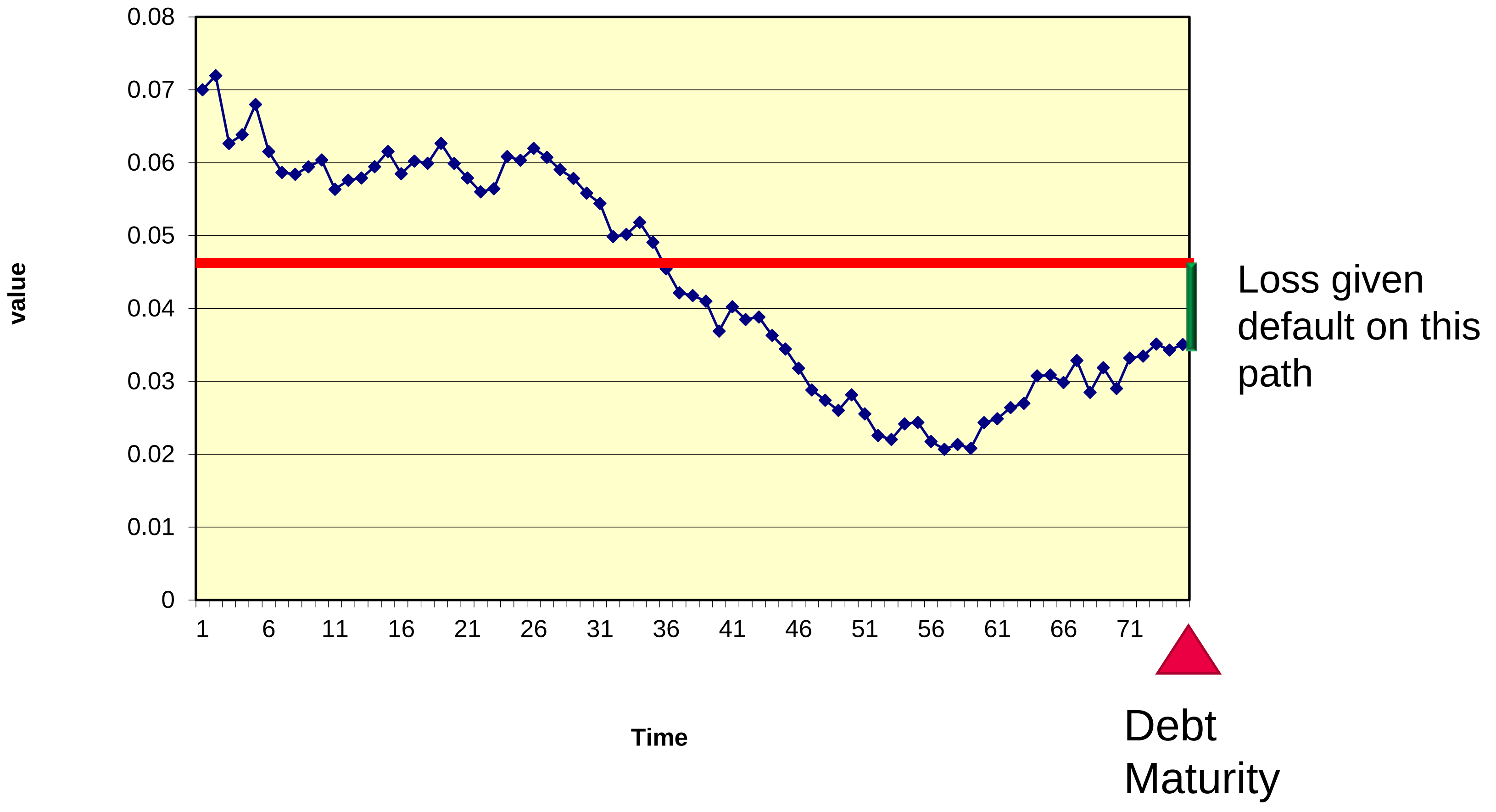
- Exploiting put call parity, we can express the value of debt alternatively, and more intuitively, as

$$D_0 = Fe^{-r \times T} - Put(V_0, F, r, T, \sigma)$$

- The value of debt is equal to the risk free debt minus a put option. The put option represents the (risk adjusted) expected losses due to default (when assets in the firm are insufficient to pay the debt at T)

<u>Note</u>: The put option also represents the value of a guarantee on that debt.

Sample Time Path of Firm Assets



The Merton model: Credit spreads

- We can then use the Merton's model to compute a corporate bond credit spread
- From the definition of yield to maturity y for a corporate bond, we have:

$$D_0 = e^{-y\times T} \times F \quad \Longrightarrow \quad Fe^{-r\times T} - Put(V_0, F, r, T, \sigma) = e^{-y\times T}F$$

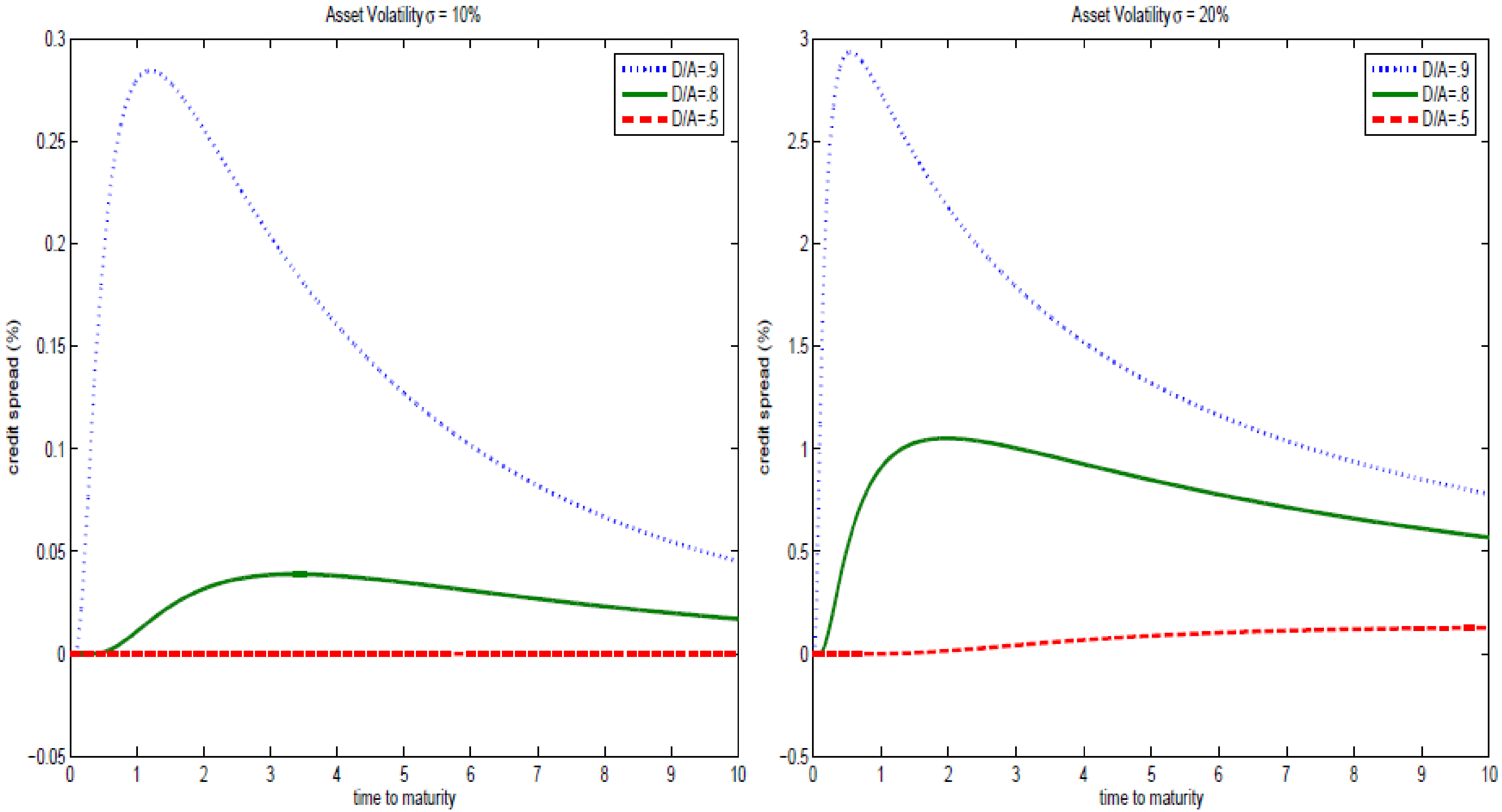
which implies

$$e^{-r\times T} - Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-y\times T}$$

$$1 - e^{r\times T} \times Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-(y-r)\times T}$$

$$\text{Credit Spread} = y - r = -\frac{1}{T} log\left[1 - e^{r\times T} Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right)\right]$$

The Merton model: Credit spread over time



- Issues: (A) They are small; (B) They converge to zero at $T \to 0$

The Merton model: Relative pricing of junior and senior debt

- The Merton model can be used to find the price of bonds with different priorities
- For example, suppose that a firm issues two bonds: one senior and one junior (also called subordinated), with face value F_S and F_J
- At maturity we have the following "waterfall" of payoffs

Payoffs

	$0 < V_T < F_S$	$F_S < V_T < F_S + F_J$	$F_S + F_J < V_T$
Senior	V_T	F_S	F_S
Junior	0	$V_T - F_S$	F_J
Equity	0	0	$V_T - (F_S + F_J)$

Merton model: Relative pricing of junior and senior debt

- That is, senior and junior debt and equity must satisfy

$$\begin{aligned}
\text{Payoff of Senior Debt} &= V - \max\left(V - F_S, 0\right) \\
\text{Payoff of Junior Debt} &= \max\left(V - F_S, 0\right) - \max\left(V - \left(F_S + F_J\right), 0\right) \\
\text{Payoff of Equity} &= \max\left(V - \left(F_S + F_J\right), 0\right)
\end{aligned}$$

- We have then have

$$\begin{aligned}
D_{S,0} &= V - BSC\left(V, F_S, r, T, \sigma\right) \\
D_{J,0} &= BSC\left(V, F_S, r, T, \sigma\right) - BSC\left(V, F_S + F_J, r, T, \sigma\right) \\
E_0 &= BSC\left(V, F_S + F_J, r, T, \sigma\right)
\end{aligned}$$

The payoffs on more complicated structures can also be valued using Monte Carlo simulation of the risk-neutral asset process

The Merton model: Extensions

- Many extensions of this basic model exist, including:
- Early bankruptcy
  - American put option: there is a lower bound V_b to assets so that if $V(t) < V_b$ the firm is bankrupt
- Stationary leverage
  - Merton model indicates often counterfactual decline in leverage over time
- Unobservable firm value
  - Investors can only rely on noisy accounting information to estimate $V(t)$: the default barrier could be closer than you think
  - Can be modeled by incorporating jumps in asset process
- Stochastic interest rates
  - interest rates follow processes like the ones we saw last week

Example: KMV model and distance to default

- A generalization of the Merton model is the KMV model, used commercially by Moody’s
- Inputs are stock prices, **book value of liabilities**, stock volatility, interest rates.
- To get from book liabilities to **market liabilities** and therefore initial market assets, solve two simultaneous non-linear equations:

Equity value = F[**asset value, asset vol**, cap str, r]

Equity volatility = F[**asset value, asset vol**, cap str, r]

  - Unknowns are asset value and asset vol

- “**Default point**” defined by a rule for assets relative to liabilities
  - An abstraction, proxy for the complicated question of when a firm really is likely to default
  - E.g., assets fall to < 70% of short-term + ½ long-term book liabilities
- These quantities are used as inputs to find **“distance to default**.”

Example: KMV model and distance to default

- The likelihood and severity of default depends on asset volatility and leverage
- The "**Distance to Default**" measure is defined by:

$$\text{DTD} = \frac{[(\text{mkt value assets}) - (\text{default point})]}{[(\text{mkt value assets})(\text{asset volatility})]}$$

- Interpretation: the number of standard deviations the firm is away from default.
- KMV has a proprietary algorithm to map DTD into the "**Expected Frequency of Default**" (EDF)
- Simple implementations of the model tend to under-estimate default probabilities over short horizons
- Can address by modeling shocks as having a **jump component**
  - Jumps often modeled as Poisson process
  - Also can address with a stochastic default barrier

Valuing loan guarantees on a binomial tree

- When the default barrier is more complicated and default can occur at any time, a binomial pricing approach is a natural choice.
- For example, consider the case of America West Airlines (AWA), that received a loan guarantee after 9/11 from the U.S. government.
  - In 2002, AWA was the eighth largest passenger airline in the U.S. Softening economic conditions had already severely reduced airline revenues. Following 9/11, Moody's downgraded AWA from B1 in April 2001 to Ca on November 21, 2001.
  - After the terrorist attacks, Congress enacted the Air Transportation Safety and System Stabilization Act, allowing airlines to apply for credit guarantees.
  - AWA received final approval from the Air Transportation Stabilization Board (ATSB) for a **government loan guarantee of $380 million in January 2002**.
  - It paid fees and gave the government warrants in compensation.
- What was that assistance worth on net?

Valuing loan guarantees as put options

- This example is based on analysis in “Estimating the Value of Subsidies for Federal Loans and Loan Guarantees,“ A Congressional Budget Office Study available on its website
- Key inputs:
  - Expected return on equity (based on estimated equity beta)
  - Volatility equity return (estimated from historical data)
  - risk-free rate
  - default/prepayment rules
  - loan maturity
  - Loan coupon rate
  - Rest of firm capital structure

Valuing Loan Guarantees as Put Options

- Stage 1 estimation: use equity data to find asset stats
  - volatility of firm asset value
  - current value of firm assets

Estimated first stage using Merton model

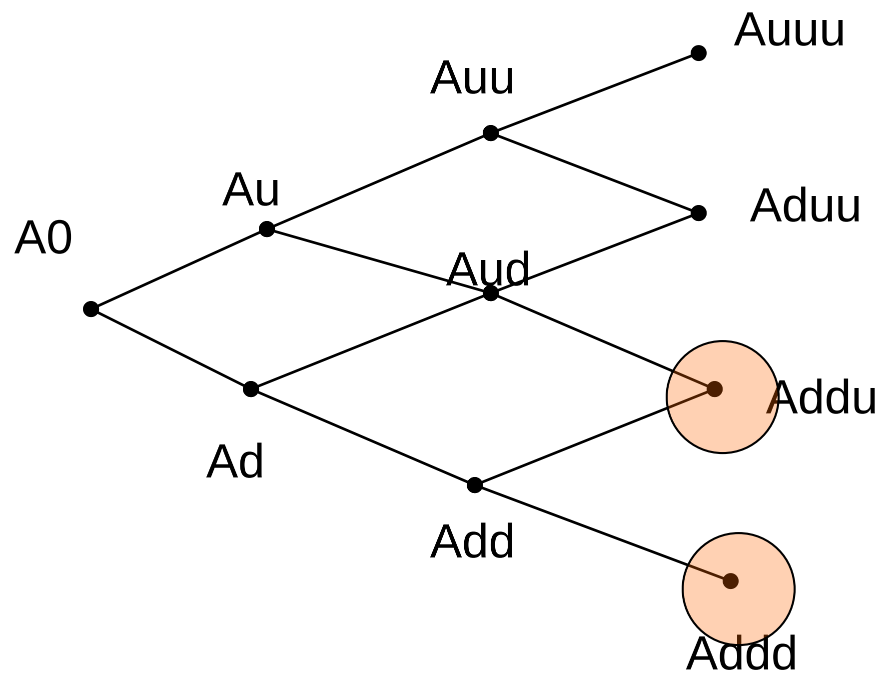
  - average return (physical) on firm assets
    - Derived from implied asset beta; used for risk assessment, e.g., VaR

$$dA_t / A_t = (r_A - \delta)dt + \sigma_A dZ_t$$

  - risk-neutral return on firm assets
    - Used for pricing guarantees

$$dA_t / A_t = (r_f - \delta)dt + \sigma_A dZ_t$$

Pricing



$$A_u = A_0(1+(r_f-\delta)\Delta t+\sigma_A\sqrt{\Delta t})$$

$$A_d = A_0(1+(r_f-\delta)\Delta t-\sigma_A\sqrt{\Delta t})$$

Denotes nodes where a loss occurs according to trigger rule.

Loss(t) = F – A(t)

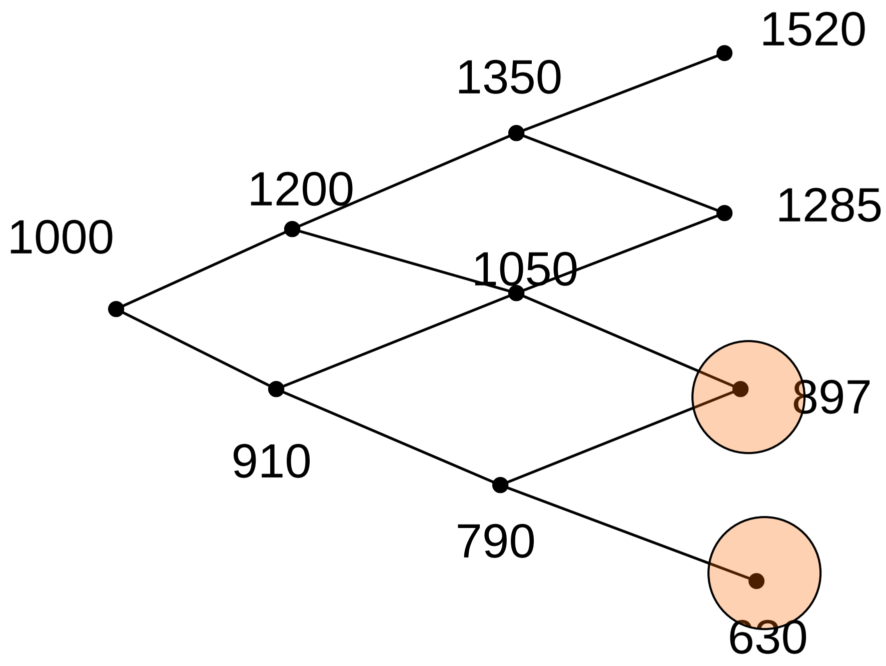
where F is payoff value of guaranteed debt

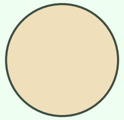
Losses are weighted by probability of occurrence and discounted to the present.

Probabilities are "risk-neutral", so discounting is at risk-free rate.

Pricing

e.g., 0 coupon bond F = 900, T=3; and r_f=.05, p*(up) =.5





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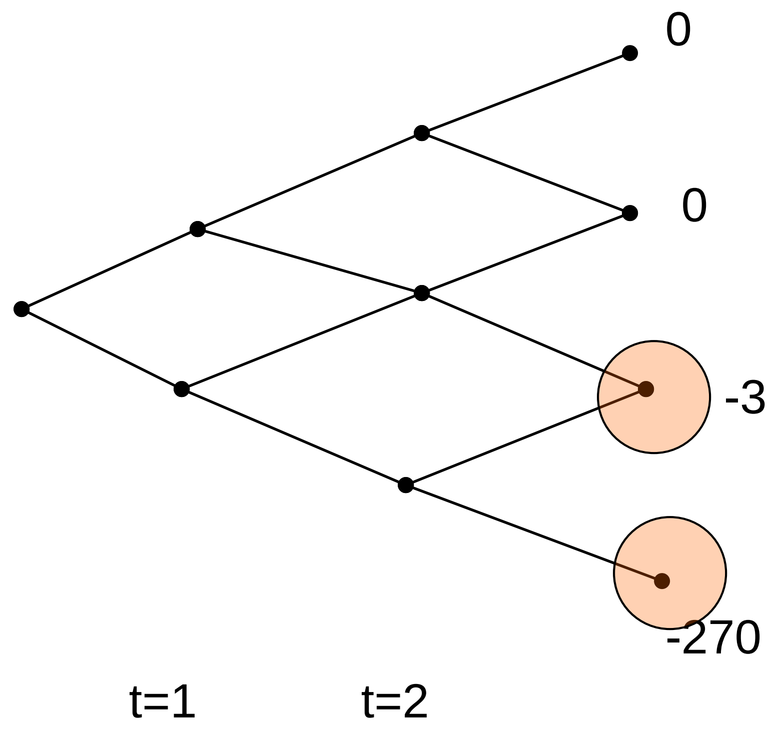

30.12634	0.680272	0
	62.58503	1.428571
		130

Losses are weighted by probability of occurrence and discounted to the present.

Probabilities are "risk-neutral", so discounting is at risk-free rate.

PV(losses) = pr(3d)(-270)/$(1.05)^3$ + pr(2d1u)(-3)/$(1.05)^3$ =

$-30.126

Risky bond value = 900/$(1.05)^3$ – 30.13

= $777.46 - $30.13 = $747.33

Valuing loan guarantees as put options

Terms

	AWA
Warrants	10 years; $3/share; 18.8 million shares
Guarantee Fees	8% per year
Loan Guarantee	7 years; LIBOR+0.4%
Loan size	$380 million

No prepayment penalties.

Valuing loan guarantees as put options

Note: The same model can be used to value guarantee fees and warrants, whose payoffs are also contingent on the firm's asset value.

Results

Table 1: Net Market Value Subsidies (millions of dollars)

	AWA
Warrants	50.2
Guarantee Fees Paid	56.6
Loan Guarantee	(150.5)
Net Profit (Loss) to the Government	(43.7)

Credit derivatives

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Credit derivatives

- Includes the many types of contracts whose payoffs depend on credit events:
    - Loan and bond guarantees
    - Credit default swaps (CDS)
    - Structured credit products (e.g., CLOs, MBS)
    - Total rate of return swaps
    - Credit-linked notes
- Trading is mostly over-the-counter
- Used primarily by financial institutions to manage credit risk exposures
- Exponential growth prior to 2008 financial crisis. After that sharp falloff, and now slower growth
    - Excessive CDS exposure brought down insurance giant AIG, which was bailed out

Credit derivatives

- Credit derivatives generally involve two counterparties
  - Protection buyer
  - Protection seller

- There is also an underlying "reference entity" whose behavior determines the cash flows on the credit derivatives
- E.g., a skipped payment on a debt obligation by Boeing would trigger a payment on Boeing CDS

Question: Under what conditions is the protection buyer hedging? Under what conditions is the protection buyer speculating?

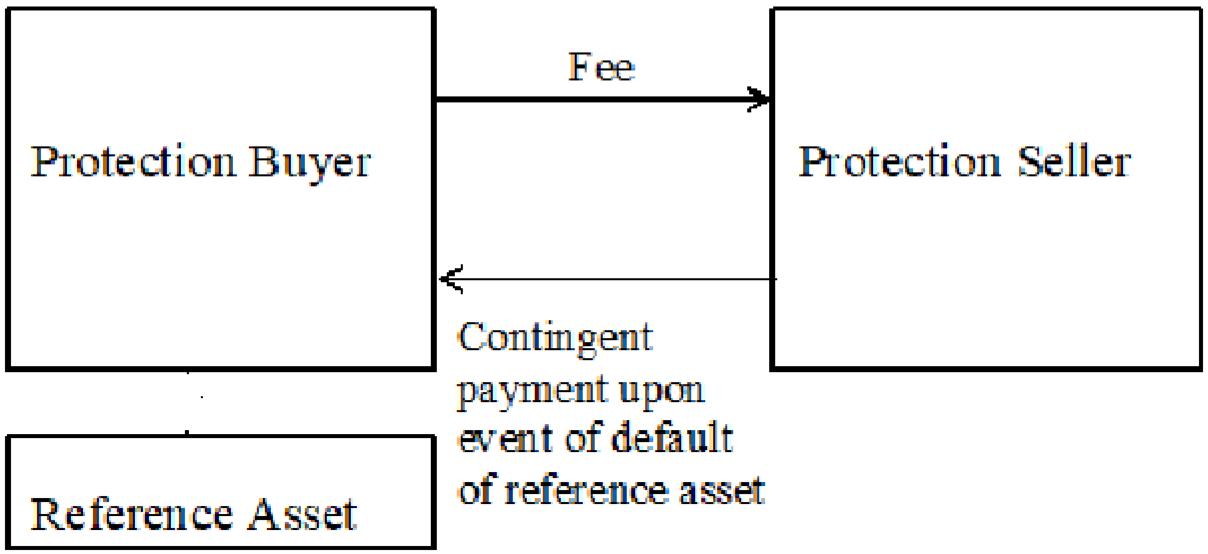
Credit Default Swaps (CDS)

If insurance fee is paid over time, it's a "swap"

If insurance fee is paid up front, it's an "option"

This type of contract can be written on anything

- e.g., loan, bond, sovereign risk, credit exposure on derivative contract



CDS structures

- **Fixed maturity**
- **Fee paid until maturity or default**
- **Various triggering events:**
  - Bankruptcy
  - Credit event upon merger
  - Downgrade
  - Failure to pay
  - Repudiation
  - Restructuring
  - Payment acceleration
  - Must be verifiable public announcement of the event
- **Various alternative settlement rules; contracts can differ:**
- Cash settlement = face value – market value at trigger event
  - Market value determined by average of dealer quotes
- Physical delivery: deliver defaulted bond for face value (may be multiple deliverables, and hence cheapest to deliver option)
- Digital settlement: fixed payment in event of trigger event
- **Contracts usually governed by ISDA rules**

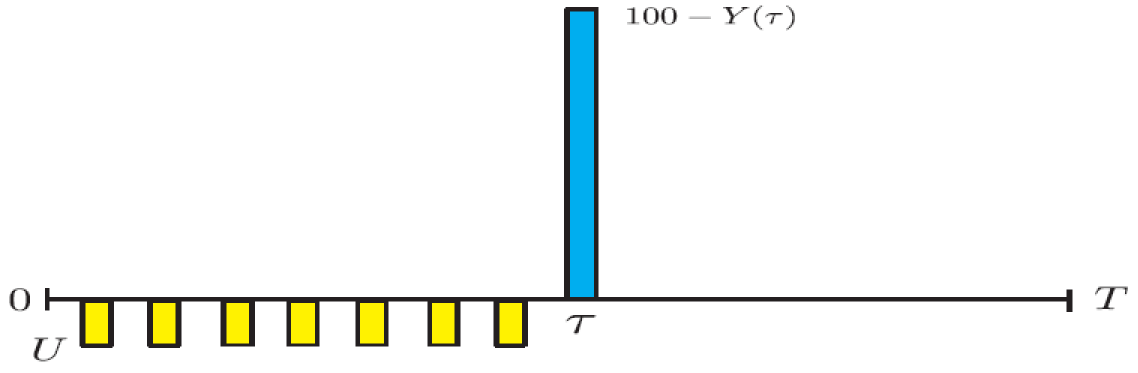
CDS structures

- Single name and indices
  - “Single name” means debt is from one company
  - Indices
    - Payoff based on defaults on a pool of bonds
    - CDX (e.g., iTraxx)
  - Structured products based on index give rise to
    - synthetic CDOs
    - Nth to default bonds
    - Other repackaging of risk

- To read about credit indices, you can download “Credit Indices: A Primer” from IHS Markit’s website

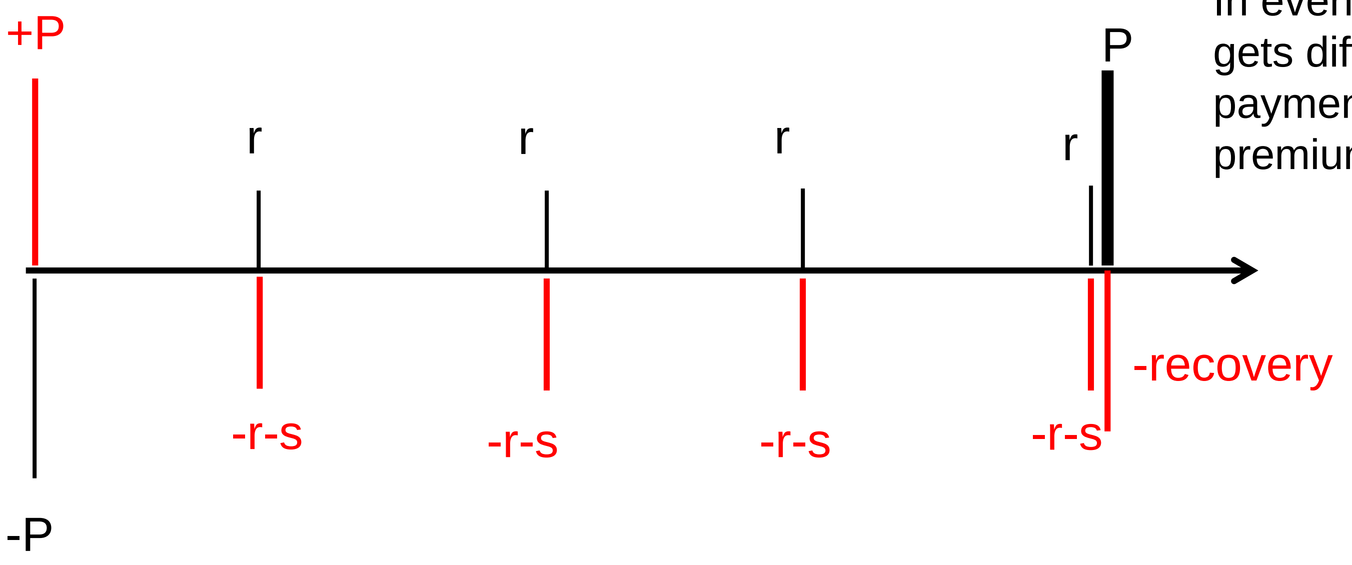
Single-name credit default swap

- This is essentially an insurance contract.
- Protection buyer pays periodic premium *U* on a notional amount of protection for a period of length *T*, say on $100 face value of the underlying entity's debt
- In the event the underlying entity defaults on the debt at time τ and the recovery amount is *Y(τ)* per $100 face value, the protection buyer is made whole
  - If the contract calls for cash settlement the protection buyer gets 100 - *Y(τ)*
  - If the contract specifies delivery the protection buyer exchanges $100 face value of the bond which is worth *Y(τ)* for $100.



CDS pricing

- Bottom line: The credit spread is approximately the fair CDS premium.
  - As for loan guarantee, writing protection is like being long the risky bond and short a risk-free bond
  - Conversely, buying a CDS is like shorting the risky bond and buying a risk-free bond (see picture)
  - Simple characterization is only precisely true in special cases; more complicated model is needed to account for different liquidity, counterparty risk, fixed vs. floating



In event of default, protection buyer gets difference between promised payment P and recovery net of premium

CDS Pricing

- Approach 1: A CDS is like a credit guarantee. The present value of the insurance can be estimated using options pricing methods (e.g., structural models like the extensions of the Merton model discussed earlier).

- Approach 2: Price delivery-settled swap by "no arbitrage" with reference to underlying securities:* (corresponds to graph on previous slide)
  - Assume CDS written on floating rate corporate bond "C" with spread S over risk-free floating rate bond, originally priced at par.
  - Ignoring transactions costs, the same protection is obtained by the CDS buyer by shorting the risky bond C, and investing in a par value default-free floating rate note.
  - Hold portfolio through maturity or credit event.
  - Net spread paid is S until termination.
  - In event of credit event, liquidate portfolio and get face value of risk-free bond – value of defaulted bond. Same is CDS payoff!
    - This assumes credit event occurs on a coupon reset date, so risk-free bond is priced at par
  - **It follows that S, the credit spread, is the fair premium rate on the swap.**

Multi-name CDS

- Suppose an investor holding a portfolio of defaultable bonds is worried about default
- The investor can

  (1) Purchase a CDS for each bond in the portfolio; or

  (2) Purchase insurance on the portfolio itself

Question: Which is more expensive?

Multi-name CDS

- **Example**: Nth-to-default basket default swaps
  - Not all firms in a portfolio will default at the same time, particularly if the credits are diversified across country, industry, etc.
  - It is popular to purchase protection that just pays off after some number of defaults have occurred
  - This is much cheaper than buying protection for each individual credit
- The basket spread (premium) depends on:
  - Number of credits: more credits => credit event more likely => more costly to insure
  - Credit quality and recovery rates
  - Default correlation across underlying reference entities (important!)

Summary

MANAGEMENT SLOAN SCHOOL

- This week we looked at how statistical and structural models can be used to price credit risk
- Statistical models derive default and recovery rates from data on borrower characteristics, including leverage ratios, credit ratings, tangible collateral, etc.
- Structural models infer default and recovery rates based on the stochastic structure of a borrower's assets and a default barrier
- Both approaches predict the effect of credit risk on the value of credit-sensitive securities
  - For defaultable bonds, both should produce similar answers if properly calibrated
  - The structural approach is more flexible and better suited for pricing more complex credit derivatives
- In practice, hybrids of the two approaches are often used and give better results.