Recitation 5

Question 1

Now is Year 0. Suppose that firm ABC is expected to pay \$2.50 dividend per share next year (i.e., in Year 1). In Year 2, you expect this dividend to remain at \$2.50. You also expect that ABC's stock price will be \$35 in Year 2. Assume that the appropriate discount rate for ABC's stock is 10%.

(a) What price would you be willing to pay for a share of ABC stock today, if you planned to hold the stock for two years?

(b) Suppose instead you plan to hold the stock for one year. What price would you expect to be able to sell a share of ABC stock for in one year?

(c) Given your answer in part (b), what price would you be willing to pay for a share of ABC stock today, if you planned to hold the stock for one year? How does this compare to your answer in part (a)?

Solutions:

(a) What price would you be willing to pay for a share of ABC stock today, if you planned to hold the stock for two years?

Price of the stock is the value of its future cash flows discounted with appropriate discount rate:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2 + P_2}{(1+r)^2}$$

Plugging the numbers into the equation we get

$$P_0 = \frac{\$2.50}{1+10\%} + \frac{\$2.50 + \$35}{(1+10\%)^2}$$

$$P_0 = \$33.26.$$

(b) Suppose instead you plan to hold the stock for one year. What price would you expect to be able to sell a share of ABC stock for in one year?

The price of this stock in Year 1 is:

$$P_1 = \frac{D_2 + P_2}{1 + 10\%}$$

Plugging the numbers into the equation

$$P_1 = \frac{\$2.50 + \$35}{1 + 10\%} = \$34.09.$$

(c) Given your answer in part (b), what price would you be willing to pay for a share of ABC stock today, if you planned to hold the stock for one year? How does this compare to your answer in part (a)?

$$P_0 = \frac{D_1 + P_1}{1 + 10\%}$$

$$P_0 = \frac{\$2.50 + \$34.09}{1 + 10\%} = \$33.26$$

Note that we got exactly the same answer in part (a) and part (c). That demonstrates that the current price P_0, that the one-year investor and the two-year investor willing to pay for this stock is the same. This is because the one-year investor also cares about the dividend and price in Year 2. Because these determine the price at which she can sell the stock at the end of Year 1.

Question 2

Now is Year 0. CommonApp is a startup company that currently pays no dividends. You expect CommonApp to start paying dividends in Year 5. The first dividend, in Year 5, will be \$1 per share. After that, the dividend is expected to grow at 3% forever. Assume that the appropriate discount rate for CommonApp's stock is 12%.

What is the current share price of CommonApp?

Solutions:

Let us use the Gordon growth model to compute the price of the stock. Since the stock starts paying at Year 5, in Year 4, we can consider the company's dividends as **growing perpetuity**. Therefore, we can find the price of the stock in Year 4 as

$$P_4 = \frac{D_5}{r - g},$$

where g is the growth rate of the dividends, r is the appropriate discount, and D_5 is the dividend of the firm in Year 1.

$$P_4 = \frac{\$1}{12\% - 3\%} = \$11.11.$$

To find the stock price right now, P_0, we need to get a value of P_4 in Year 4 now. In other words, we need to discount it to the present.

$$P_0 = \frac{P_4}{(1+r)^4} = \frac{\$11.11}{(1+12\%)^4} = \$7.06.$$

Just for practice, let us calculate the price in an alternative way. Let us find the cost of the stock in Year 5 just after the moment it paid dividends. This value only counts dividends starting from Year six onwards:

$$P_5 = \frac{D_6}{r-g} = \frac{D_5 \times (1+g)}{r-g} = \frac{\$1 \times (1+3\%)}{12\% - 3\%} = \$11.44.$$

The term $(1+3\%)$ appears in the numerator since the dividends will grow from Year 5 to Year 6. Next, the stock price at Year 0 equals the discounted value of dividends in Year 5 plus the discounted value of stock price in Year 5.

$$P_0 = \frac{P_5 + D_5}{(1+r)^5}.$$

That is what an investor gets if she holds the stock for five years and solves it after.

$$P_0 = \frac{\$1 + \$11.44}{(1+12\%)^5} = \$7.06.$$

Not surprisingly, the answer is the same. The exercise point is to illustrate that though there are different ways to solve the price, as long as you are doing it consistently and counting all dividends correctly, you will get the same answer.

Question 3

Now is Year 0. iDoc, a health service company, just paid a dividend of \$1.50. You expect iDoc to increase its dividends by 10% per year for the next two years (through Year 2), and thereafter by 3% per year. The appropriate annual discount rate (equity cost of capital) for iDoc is 8%.

(a) Use the Gordon Growth model to estimate the share price of iDoc right now, i.e., in Year 0.

(b) What is the expected share price of iDoc in Year 1?

Solutions:

(a) Use the Gordon Growth model to estimate the share price of iDoc right now, i.e., in Year 0.

Let us split the timeline into two Growth Stages of IDoc. Growth Stage 1 is between year zero and year two when dividends will grow by $g_1 = 10\%$. Hence

$$D_2 = D_0 \times (1+10\%)^2.$$

Growth Stage 2 is from year three onwards when dividends grow at $g_2 = 3\%$. In particular, we can calculate dividend in Year 3 $D_3 = D_2 \times (1 + 3\%)$. Note that during the growth Stage 2 we can consider the company's dividends as growing perpetuity. Therefore, we can find the price of the stock in Year 2 as

$$P_2 = \frac{D_3}{r - g_2} = \frac{D_2 \times (1 + 3\%)}{8\% - 3\%} = \frac{\$1.5 \times (1 + 10\%)^2 \times (1 + 3\%)}{8\% - 3\%} = \$37.39.$$

The final step is to find the current share price. If an investor keeps the stock for 2 years, then the cash flows from the stock are $D_1 = D_0 \times (1 + 10\%)$ in Year 1 and $D_2 + P_2$ in Year 2. Hence, the value accrued to shareholders during the first growth stage is

$$P_0 = \frac{D_1}{1 + r} + \frac{D_2 + P_2}{(1 + r)^2}.$$

Plugging the numbers we have

$$P_0 = \frac{\$1.5 \times (1 + 10\%)}{(1 + 8\%)} + \frac{\$1.5 \times (1 + 10\%)^2 + \$37.39}{(1 + 8\%)^2} = \$35.14.$$

(b) What is the expected share price of iDoc in Year 1?

To find the share price in Year 1, all we need to do is to discount the dividends and price of the stock in Year 2 to Year 1:

$$P_1 = \frac{D_2 + P_2}{(1 + r)} = \frac{\$1.5 \times (1 + 10\%)^2 + \$37.39}{(1 + 8\%)} = \$36.30.$$

Let us discuss an alternative way of solving this question. The fundamental equation that ties the share price today with the share price in Year 1 is

$$P_0 = \frac{D_1 + P_1}{(1 + r)}.$$

From the equation, we can express P_1 as a function of the current price, expected dividends, and the discount rate:

$$P_1 = P_0(1 + r) - D_1.$$

$$P_1 = \$35.14(1 + 8\%) - \$1.5 \times (1 + 10\%) = \$36.30.$$

The answer is the same.

Question 4

Consider a firm, ABC. The profitability of ABC and its investment policy are summarized in the following table:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
BVPS	\$100					
EPS		\$20				
Plow-back ratio		80%	50%	50%	0%	0%
Return on investment		20%	15%	10%	10%	10%

Starting in Year 4 the Plow-back ratio and Return on investment are the same going to an infinite horizon. The appropriate discount rate is 12%.

(a) What is the current share price of this stock?

(b) What is the expected return on this stock between Year 0 and Year 1?

(c) What is the present value of ABC's growth opportunities, or PVGO?

Solutions:

(a) What is the current share price of this stock?

Let us start by laying out the main assumptions.

Assumption 1. Assets do not depreciate. Without new investment, EPS stays at the current level forever.

This means that even without new investment the current earnings, or more specifically, earnings per share, will stay constant at the current level.

Assumption 2. Investment at time t starts generating earnings at $t + 1$.

Assumption 3. Investment generates a perpetual earnings stream.

Assumption 4. Investment opportunities are characterized by ROI — return on investment.

Before solving, let us discuss a specific example of an investment opportunity.

Suppose a firm invests \$100 million into a factory today, i.e., in Year 0. Return on this investment, ROI, is 15%.

This means that starting from Year 1, and continuing forever, the firm earns \$15 million:

$$\$100 \times 15\% = \$15.$$

Let us look at the relationship between the key variables. Denote the plow-back ratio by b_t.

Investment per share is the fraction of earnings per share that we retain:

$$I_t = EPS_t \times b_t$$

The next year's earnings per share are the current earnings per share plus return on investment, i.e., the profitability of our investments multiplied by the amount that we reinvested:

$$EPS_{t+1} = EPS_t + ROI_t \times I_t$$

The book value per share will be the prior year's book value per share, plus these new investments:

$$BVPS_{t+1} = BVPS_t + I_{t+1}.$$

Dividends would be the part of EPS that would not be retained, so we pay it out to shareholders:

$$D_t = EPS_t\,(1 - b_t)$$

Collecting all key equations together, we get:

$$\begin{gathered} I_t = EPS_t \times b_t \\ EPS_{t+1} = EPS_t + ROI_t \times I_t \\ BVPS_{t+1} = BVPS_t + I_{t+1} \\ D_t = EPS_t\,(1 - b_t) \end{gathered}$$

The solution we give you in the video is employing these equations. The solution runs the equations iteratively, starting from year 0 and moving forward. Since everything all parameters of the model are constant starting Year 4 and the plow-back ratio is 0, the firm generates constant cash flows starting the year. The video shows that $D_4 = \$26.19$. From a standpoint in Year 3, the stock is just perpetuity, and its price is

$$P_3 = \frac{D_4}{r} = \$218.23.$$

where r is the discount rate. Next, from current time, we can calculate the share price as sum of discounted cash flows from the share held for 3 years, i.e.,

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3 + P_3}{(1+r)^3}.$$

$$P_0 = \frac{\$4}{1+12\%} + \frac{\$11.6}{(1+12\%)^2} + \frac{\$12.47 + \$218.23}{(1+12\%)^3} = \$177.02$$

(b) What is the expected return on this stock between Year 0 and Year 1?

Let us start with this fundamental relationship between the price today and the price and dividend in the next year

$$P_t = \frac{D_{t+1} + P_{t+1}}{1+r}$$

Expected return can be expressed as

$$r = \frac{D_{t+1} + P_{t+1} - P_t}{P_t}$$

or,

$$r = \underbrace{\frac{D_{t+1}}{P_t}}_{\text{dividend yield}} + \underbrace{\frac{P_{t+1} - P_t}{P_t}}_{\text{capital gain}}.$$

The first part of the return is called **dividend yield**. The second part of the return, which comes from the price change, is called **capital gain**. Doing the calculations for every year, we can see that though the dividend yield and capital gain can constitute a different share of the expected return, the total return is the same for all years. It equals to 12%.

Note that the return equals the discounting rate we used for our calculations.

(c) What is the present value of ABC's growth opportunities or PVGO?

The present value of growth opportunities is the additional value that our investment opportunity adds to the share on top of what would have happened if the company did not invest, i.e., it just continued the status quo from year 1 onwards. Mathematically, it is

$$PVGO = P_0 - \frac{EPS_1}{r}$$

The last term is the perpetuity paying EPS_1 in every period since earnings do not change without investment.

$$PVGO = \$177.02 - \frac{\$20}{12\%} = \$10.36$$

The current value (per share) of ABC's growth opportunities is \$10.36.

Question 5

Consider firm ABC. Expected EPS in Year 1 is \$20. ABC's ROI is 20% and is expected to remain constant at that level forever. ABC's current payout ratio is 55%. ABC is expected to maintain this payout ratio at 55% forever. The appropriate discount rate is 12%.

(a) What is the current share price of ABC?

(b) What is ABC's PVGO?

Solutions:

(a) What is the current share price of ABC?

The difference between this question and question 4 is that ROI was not constant. For the solution, we follow the approach we have established in Question 4 using Excel. What we can see is that dividends actually do not stop growing. Nevertheless, if we look at the growth ratio of dividends at every period t, we will find numerically that the growth rate is constant for all years:

$$\frac{D_{t+1} - D_t}{D_t} = 9\%.$$

That is not a coincidence and can be shown analytically. Let b be the plow-back ratio. Then Dividends are the part of EPS that would not be retained, and

$$D_t = (1-b) \times EPS_t$$

Next period's dividend:

$$D_{t+1} = (1-b) \times EPS_{t+1}.$$

Now, recall the formulas for EPS and investment:

$$\begin{aligned} EPS_{t+1} &= EPS_t + ROI \times I_t \\ I_t &= EPS_t \times b. \end{aligned}$$

Combining all four equations together, we get,

$$EPS_{t+1} = EPS_t(1 + ROI \times b)$$

$$D_{t+1} = (1-b) \times EPS_t(1 + ROI \times b)$$

Finally, let us see what is the dividend growth:

$$\frac{D_{t+1} - D_t}{D_t} = \frac{D_{t+1}}{D_t} - 1 = \frac{(1-b) \times EPS_t(1 + ROI \times b)}{(1-b) \times EPS_t} - 1 = ROI \times b$$

Now, let us check that this formula correctly applies to our problem:

$$g = \frac{D_{t+1} - D_t}{D_t} = ROI \times b = 20\% \times (1 - 55\%) = 9\%.$$

That confirms our numerical finding!

Now, we have already learned how to solve the share price since it is just simple growing perpetuity:

$$P_0 = \frac{D_1}{r - g} = \frac{\$11}{12\% - 9\%} = \$366.67$$

(b) What is ABC's PVGO?

The present value of growth opportunities is

$$PVGO = P_0 - \frac{EPS_1}{r}$$

The last term is the perpetuity paying EPS_1 in every period since earnings do not change without investment.

$$PVGO = \$366.67 - \frac{\$20}{12\%} = \$200$$

Note that it is a positive number. That makes sense since the return of investment is higher for the company than its discount rate.

Question 6

Firm XYZ has two lines of business, organized as two divisions, A and B; both divisions generate risky cash flows. Division A expects to generate a cash flow of \$2 million next year; and it will grow at a rate of 3% each year thereafter forever. Division B expects to generate a cash flow of \$2 million next year; and it will grow at a rate of 5% each year thereafter forever. Currently, the total market value of XYZ is \$87 million. The cost of capital for the first line of business is 10%.

(a) What is the cost of capital for the second line of business?

Assume the company is planning on making an investment which will improve profitability of both divisions. This project requires an initial investment of \$7 million right now, and will increase cash flow for both divisions by \$0.5 million starting in year 3. This additional cash flow will grow at the same rate as other cash flow in each division (i.e., at 3% rate in division A and at 5% rate in division B).

(b) Assume that the cost of capital for both divisions is not affected by this project. If management decides to invest in this new technology, by how much will the current market value of XYZ increase?

Solutions:

(a) What is the cost of capital for the second line of business?

The question is interesting because it makes us realize that a firm's divisions can be different and may have different business risks. Specifically, in this problem, we are given division A with a cost of capital (COC) equal to 10%, but as we can see in a moment, division B has different COC, i.e., different levels of risk.

Firm value is the sum of the value of two divisions:

$$\boldsymbol{V} = \boldsymbol{V_A} + \boldsymbol{V_B}$$

Each division produces a cash flow that is a growing perpetuity, hence:

$$87 = \frac{\$2}{10\% - 3\%} + \frac{\$2}{r_B - 5\%}$$

Solving this equation for r_B gives 8.42%.

Assume the company is planning on making an investment that will improve the profitability of both divisions. This project requires an initial investment of \$7 million right now and will increase cash flow for both divisions by \$0.5 million starting in year 3. This additional cash flow will grow at the same rate as other cash flow in each division (i.e., at 3% rate in division A and at 5% rate in division B).

(b) Assume that the cost of capital for both divisions is not affected by this project. If management decides to invest in this new technology, by how much will the current market value of XYZ increase?

The NPV of this project equals the initial investment that we have to make right now of \$7 million plus the new cash flow that this project will generate.

$$NPV = -\$7 + \frac{1}{(1+10\%)^2}\left[\frac{0.5}{10\% - 3\%}\right] + \frac{1}{(1+8.42\%)^2}\left[\frac{0.5}{8.42\% - 5\%}\right]$$

The second and third terms of the equation are the present value of the increase in the cash flows that accrue to division A and division B, respectively. We discount each of the terms by the correct COC for the division. Since the cash flows start in year 3, we can consider them as growth perpetuities for each division.

The NPV of the project is \$11.33 million. If management decides to invest in this new technology, The new market value of XYZ will increase by the amount and achieve \$98.33 million:

$$\$87\text{million} + \$11.33\text{million} = \$98.33\text{million}.$$

This is an increase of

$$\frac{\$11.33}{\$87} = 13.02\%,$$

i.e., by engaging in this investment project, the management of the company increases its value by 13.02%.