Statistics for Financial Engineering - Refresher Seminar, Homework 1

Radoš Radoičić

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Homework has two types of problems: theoretical (T) and practical (P).

You need to show all your work. It is due on 07/31/17.

Problem T1: "Student" Find the expectation and the variance of the t-distribution with n degrees of freedom. (Note: You should get some "strange" answers for $n = 1$ and $n = 2$.)

Problem T2: "Kurtosis" Find the moment generating function, skewness and kurtosis of $X \sim \Gamma(\alpha, \beta)$.

Problem T3: "Mixing it up" Problem 5.5 in the textbook.

Problem T4: "Fisher" (A) Find MLE of the uknown parameter λ for Poisson distribution $\Pi(\lambda)$. Compute the Fisher information and state asymptotic normality property of MLE.

(B) Find MLE of the unknown parameter μ for the normal distribution $N(\mu, \sigma^2)$, where σ^2 is known. Compute the Fisher information $I(\mu)$ and state asymptotic normality property of MLE.

(C) Find MLE of the unknown parameter σ^2 for the normal distribution $N(0, \sigma^2)$. Compute the Fisher information $I(\sigma^2)$ and state asymptotic normality property of MLE.

Problem T5: "Invariance" Suppose that $X_1, \ldots, X_n$ form a random sample from $U(a, b)$, where both endpoints a and b are unknown. Find the MLE of the mean of the distribution. (hint: Invariance property of MLE states that if $\hat{\theta}$ is the MLE of θ, then $g(\hat{\theta})$ is the MLE of $g(\theta)$ for any one-to-one function g.)

Problem P1: Chapter 4 R-lab.

Problem P2: Chapter 5 R-lab.