Problem P1: Chapter 7 R-lab

I had to use the textbook and solution manual to finish this homework, as I am total new to R programming.

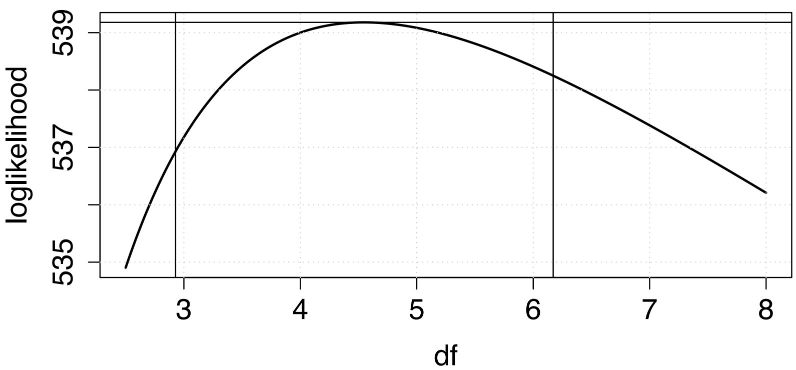
Problem 1

```
w = matrix( c(0.5,0.3,0.2,0) )
S = as.matrix(cov(Berndt))
t(w) %*% (S %*% w) # computes the variance of the linear combination
```

variance = 0.004408865

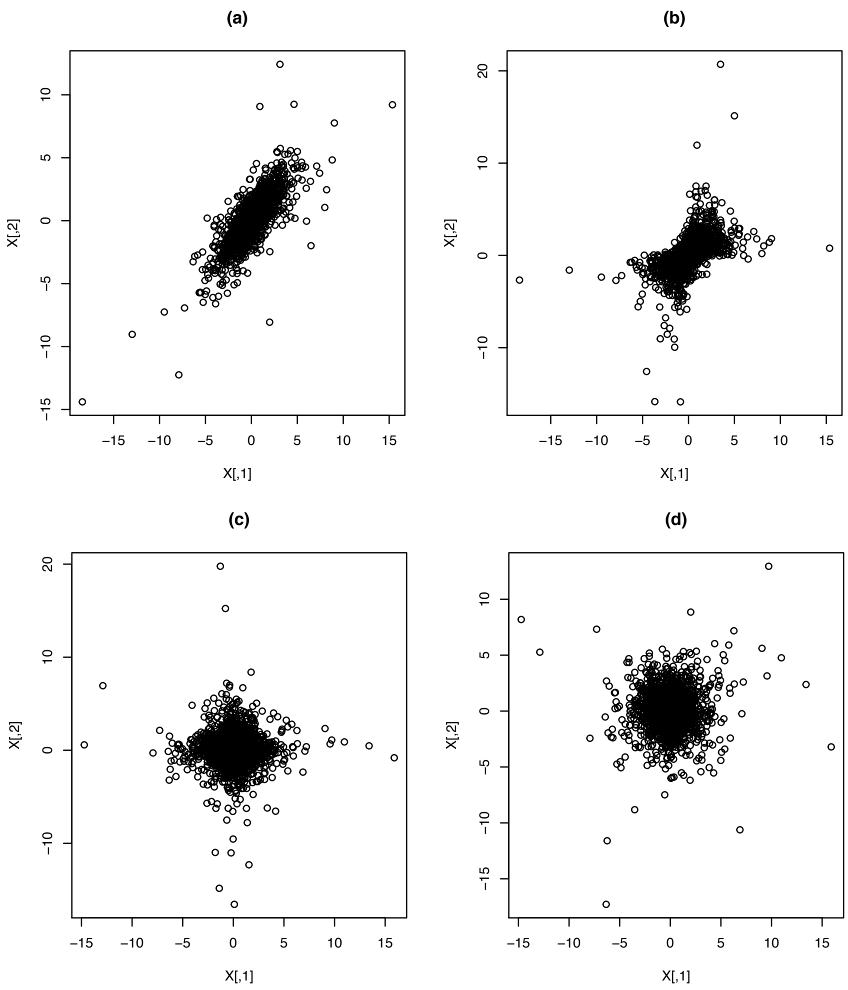
Problem 2

```
source('ml_fit_multivariate_t.R')
result = ml_fit_multivariate_t(Berndt)
df = result$df_range          # extract results ...
max_index = result$max_index
loglik = result$logliks

if( FALSE ){
  # Compute the derivative of the loglikelihood from the discrete samples evaluated above
  #
  h = df[2]-df[1]
  d2_LL_nu2 = ( loglik[max_index+1] - 2*loglik[max_index] + loglik[max_index-1] ) / h^2
}else{
  # Use the function fdHess in the nlme package to numerically evaluate the derivive of the loglikelihood:
  #
  library(nlme)
  res = fdHess( df[max_index], function (x) loglik_fn(Berndt,x) )
  d2_LL_nu2 = res$Hessian
}

s_nu = sqrt( 1/(-d2_LL_nu2) ) # the standard error in nu

alpha = 0.10
z_crit = qnorm( 1-0.5*alpha )

plot( df, loglik, type="l", cex.axis=1.5, cex.lab=1.5, ylab="loglikelihood", lwd=2 )
abline(h = max(loglik))
abline(v = df[max_index] - z_crit * s_nu)
abline(v = df[max_index] + z_crit * s_nu)
grid()
```



Problem 3
Which sample has independent variates? Explain your answer.

Sample (c) has independent variates, since there is no dependency between the two variates with the w.



Problem 4
Which sample has variates that are correlated but do not have tail dependence? Explain your answer.

Sample (b) has variates that are correlated but do not have tail dependence, since the diagonal elements are non-zeros and the variates are independent with w.

Problem 5
Which sample has variates that are uncorrelated but with tail dependence? Explain your answer.

Sample (d) has variates that are uncorrelated but with tail dependence, since the diagonal elements are zeros but the variates are dependent with w.

Problem 6
(a) What is the distribution of R?

R = (X+Y)/2 has a t-distribution with 5 degrees of freedom. Its mean is [1/2 1/2][0.01 0.02]^T=0.0015, and variance (3/5)[1/2 1/2]{0.1, 0.03, 0.03, 0.15}[1/2 1/2]^T=0.0465

(b) Write an R program to generate a random sample of size 10,000 from the distribution of R. Your program should also compute the 0.01 upper quantile of this sample and the sample average of all returns that exceed this quantile.

The 0.01 upper quantile: 0.6579784;
Using N = 10000 has a mean of upper 0.990000 quantile of = 0.745559.

Problem 7
(a) What does the code A = chol(cov(Y)) do?

chol does Cholesky decomposition on the covariance matrix Y.

A is an upper triangular matrix, and, as can be seen below, the sample covariance matrix of Y is equal to A^TA.

```
> A
        ibm     crsp
ibm   0.0175  0.003773
crsp  0.0000  0.006779

> cov(Y)
           ibm        crsp
ibm   3.061e-04  6.602e-05
crsp  6.602e-05  6.019e-05
```

```
> t(A)%*%A
           ibm       crsp
ibm  3.061e-04 6.602e-05
crsp 6.602e-05 6.019e-05
```

(b) Find θ_{ML}, the MLE of θ.
The MLE of θ is given in the R output below.

```
> fit_mvt$par
[1] 0.0003789 0.0008317 0.0126907 0.0026859 0.0051011 4.2618395
```

We see that the estimated mean vector is (0.0003789, 0.0008317), the estimated Cholesky factor of the covariance matrix is

(0.01269   0.00268;   0   0.00510),

and the estimated degrees of freedom parameter is 4.26.

(c) Find the Fisher information matrix for θ. (Hint: The Hessian is part of the object fit_mvt. Also, the R function solve will invert a matrix.)

The Fisher information matrix is printed below.

```
> fisher = fit_mvt$hessian
> fisher
           [,1]       [,2]      [,3]       [,4]      [,5]      [,6]
[1,]  1.533e+07 -1.572e+07   -337423  1.804e+05     87121   -474.73
[2,] -1.572e+07  7.444e+07    145737  1.014e+06    838199    938.50
[3,] -3.374e+05  1.457e+05  23313101 -1.365e+07  -8785834  -7420.14
[4,]  1.804e+05  1.014e+06 -13648160  7.138e+07  -5213629   -608.94
[5,]  8.712e+04  8.382e+05  -8785834 -5.214e+06 147902020 -19875.27
[6,] -4.747e+02  9.385e+02     -7420 -6.089e+02    -19875     21.77
```

(d) Find the standard errors of the components of θ_{ML} using the Fisher information matrix.

The standard error are below:

```
> se = sqrt(diag(solve(fisher)))
> se
 [1] 2.887e-04 1.310e-04 2.506e-04 1.292e-04 9.373e-05 2.570e-01
```

(e) Find the MLE of the covariance matrix of the returns.

The MLE of the covariance matrix is printed as COV_Y in the output below. For comparison, the sample covariance is printed in the last line.

```
> ML = fit_mvt$par
> Ahat = matrix(c(ML[3:4],0,ML[5]),nrow=2,byrow=TRUE)
> Ahat
        [,1]     [,2]
[1,] 0.01269 0.002686
[2,] 0.00000 0.005101
> COV_Y = t(Ahat)%*%Ahat * ML[6]/(ML[6]-2)
> COV_Y
          [,1]      [,2]
[1,] 3.035e-04 6.423e-05
[2,] 6.423e-05 6.262e-05

> cov(Y)
           ibm      crsp
ibm  3.061e-04 6.602e-05
crsp 6.602e-05 6.019e-05
```

(f) Find the MLE of ρ, the correlation between the two returns (Y1 and Y2).

The MLE of ρ is 0.4659. For comparison, the sample correlation is 0.4864.

```
> rho = COV_Y[1,2]/sqrt(COV_Y[1,1]*COV_Y[2,2])
> rho
[1] 0.4659
> cor(Y)
        ibm   crsp
ibm  1.0000 0.4864
crsp 0.4864 1.0000
```