Paper exercise 1 bonus

a) Give a *formal high-level specification* for a combinational system that calculates the distance between ones in a bit-vector of length *n* that contains exactly two ones and *n-2* zeros. For example, in “10100” the distance would be 2.

b) Determine a switching expression for the following switching function (hint: this is a subset of a well-known number set):
$f(x_3, \dots, x_0) = \text{one} - \text{set}\{k | \forall (1 \leq i, j < k): i \times j \neq k, 2 \leq k \leq 15\}$.

Problem bonus 1a

Give a *formal high-level specification* for a combinational system that calculates the distance between ones in a bit-vector of length *n* that contains exactly two ones and *n-2* zeros. For example, in "10100" the distance would be 2.

Problem bonus 1a

- Formal specification:
- Input: $\underline{x} = (x_{n-1}, x_{n-2}, \cdots, x_0), \exists i: \exists j: 0 \leq i < j < n \wedge x_i = x_j = 1 \ \wedge \forall k: ((0 \leq k < n \wedge k \neq i \wedge k \neq j) \Rightarrow x_k = 0)$
- Output: $y \in \{1,2, \dots, n-1\}$
- Function from input to output: $f(\underline{x}) = y = |i - j|$
- This is *unambiguous* and *concise* but requires understanding logical formulas
- Not the only possible solution

Problem bonus 1b

Determine a switching expression for the following switching function (hint: this is a subset of a well-known number set):

$f(x_3, \dots, x_0) = \text{one} - \text{set}\{k | \forall (1 \leq i, j < k): i \times j \neq k, 2 \leq k \leq 15\}.$

Problem bonus 1b

$$f(x_3,\dots,x_0) = \text{one} - \text{set}\{k \mid \forall(1 \leq i, j < k): i \times j \neq k, 2 \leq k \leq 15\}$$

- This formula defines the set of prime numbers between 2 and 15: 2, 3, 5, 7, 11, and 13
- i.e. they cannot be factored into two smaller numbers
- So the switching function is one-set(2,3,5,7,11,13)
- We need four bits to represent all these values in binary: one-set(”0010”,”0011”, ”0101”, ”0111”,”1011”,”1101”)
- So the switching expression is in POS form:

$$F(x_3,\dots,x_0) = x_3'x_2'x_1x_0' + x_3'x_2'x_1x_0 + x_3'x_2x_1'x_0 \\ + x_3'x_2x_1x_0 + x_3x_2'x_1x_0 + x_3x_2x_1'x_0$$