Assignment

Question 3. Prove that for a system with Hamiltonian H, the value of the Hamiltonian (the internal energy of the system) remains constant over time, along solutions of the equations of motion generated by H.

1 Conservation

Represent L (Lagrangian) and H (Hamiltonian) as:

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \equiv \frac{\partial L}{\partial q_i} = 0 \to p_i = const.,$$

which leads to constant motion. Along a path of motion

$$\frac{dH}{dt} = \frac{\partial H}{\partial q_i}\dot{q}_i + \frac{\partial H}{\partial p_i}\dot{p}_i + \frac{\partial H}{\partial t}$$

$$= \frac{\partial H}{\partial q_i}\Big(\frac{\partial H}{\partial p_i}\Big) + \frac{\partial H}{\partial p_i}\Big(-\frac{\partial H}{\partial q_i}\Big) + \frac{\partial H}{\partial t}$$

$$= \frac{\partial H}{\partial t}$$

So if H does not explicitly depend on t, $\frac{\partial H}{\partial t} = 0$ and so $H = const$.

2 Phase Space

A particular motion from Hamilton's equation can give a path or curve in space. A classical state of a system can be defined at time t by a point in 2f dimension $(\vec{q}, \vec{p})$. The phase velocity, $\vec{v}$, is represented as

$$\vec{v} \equiv \begin{bmatrix} \dot{\vec{q}} \\ \dot{\vec{p}} \end{bmatrix} = \begin{bmatrix} \vec{\nabla}_p H \\ -\vec{\nabla}_q H \end{bmatrix},$$

If H is independent of t, then path lines are on constant energy surfaces.

(Not sure if this is enough,"sighs")

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