S V COLLEGE OF ENGINEERING

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II B.Tech-II Sem (EEE, ECE)

MATHEMATICS – IV (OBJECTIVE)

UNIT - I

1. If n is a positive integer then write the value of $\Gamma(n+1)$ [b]

a) $(n-1)\Gamma(n-1)$ b) $n\Gamma(n)$ c) $(n+1)\Gamma(n)$ d) none

2. Write the value of $\int_0^\infty e^{-x^2}dx$ [a]

a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\sqrt{\pi}}{4}$ d) none

3. Express $\int_0^{\frac{\pi}{2}} sin^p x\, cos^q x\, dx$ in terms of Gamma function where $p>1,\ q>-1$. [b]

a) $\frac{\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}+\frac{q+1}{2}\right)}$ b) $\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+q+2}{2}\right)}$ c) $\frac{\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+1+q+1}{2}\right)}$ d) None

4. Write the value of $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx,\ \ m>0,\ \ n>0$ in terms of Beta function. [d]

a) $\beta(m)$ b) $\Gamma(n)$ c) $\Gamma(m,n)$ d) $\beta(m,n)$

5. Express $\int_0^2 \left(8-x^3\right)^{\frac{-1}{3}}dx$ in terms of Beta function [a]

a) $\frac{1}{3}\beta\left(\frac{1}{3},\frac{2}{3}\right)$ b) $\beta\left(\frac{1}{3},\frac{2}{3}\right)$ c) $\Gamma\left(\frac{2}{3}\right)$ d) $\Gamma\left(\frac{1}{3},\ \frac{2}{3}\right)$

6. $\frac{\beta(m+1,\ n)}{\beta(m,\ n)},\ \ m>0,\ \ n>0=$ ___________________ [c]

a) $\frac{n}{m+n}$ b) $\frac{1}{m+n}$ c) $\frac{m}{m+n}$ d) None

7. $\beta(\frac{1}{2},\frac{1}{2})$ = [a]

(a)3.1416 (b) 2.146 (c) 4.1416 (d) 1.1416

8. $\Gamma(3.5)$ =_______________ [c]

(a)$\sqrt{\Pi}$ (b) $\frac{\sqrt{\Pi}}{8}$ (c)$\frac{15}{8}\sqrt{\Pi}$ (d) $\frac{10}{8}\sqrt{\Pi}$

9. The value of $\beta(2,1)+\beta(1,2)$= __________ [c]

(a) $\beta(3,3)$ (b)0 (c)1 (d) $\beta(1,2)$

10.$\Gamma(n)$ = ____________ [a]

(a)$(n-1)\Gamma(n-1)$ (b) $\Gamma(n+1)$ (c)n! (d) $\Gamma(n-1)$

11. $\Gamma(-\frac{1}{2})$ = ________ [c]

(a)$\sqrt{\Pi}$ (b) $\frac{\sqrt{\Pi}}{8}$ (c) $-2\sqrt{\Pi}$ (d) $\frac{10}{8}\sqrt{\Pi}$

12. $\Gamma(0)$ = __________ [d]

(a) 1 (b) $\sqrt{\Pi}$ (c) $\frac{\Pi}{2}$ (d) none

13. $\Gamma(-5)$ = _________________ [d]

(a) 5 (b) -5 (c) 0 (d) not defined

14. ___________________ also known as Euler's integral of first kind. [c]

(a)Bessel function (b)Gamma function (c)Beta function (d) onto function

15. ___________________ also known as Euler's integral of second kind. [b]

(a)Bessel function (b)Gamma function (c)Beta function (d) onto function

16.$\Gamma(n+1) =$ ___________ [c]

(a)$(n-1)\Gamma(n-1)$ (b) $\Gamma(n+1)$ (c)n! (d) $\Gamma(n-1)$

17. $\Gamma(-1089) =$ ________________ [d]

(a) 1089 (b) -1089 (c) 0 (d) not defined

18. $\Gamma\left(\frac{1}{2}\right) =$ ________ [a]

(a)$\sqrt{\Pi}$ (b) $\frac{\sqrt{\Pi}}{8}$ (c) $-2\sqrt{\Pi}$ (d) $\frac{10}{8}\sqrt{\Pi}$

19. $\Gamma\left(\frac{1}{4}\right).\Gamma\left(\frac{3}{4}\right) =$ ________________ [d]

(a)$\sqrt{\Pi}$ (b) $\frac{\sqrt{\Pi}}{8}$ (c) $-2\sqrt{\Pi}$ (d)$\Pi\sqrt{2}$

20. $\Gamma(4) =$ _____________ [a]

(a) 6 (b) 5 (c) 2 (d)1

UNIT – II

1. $\frac{d}{dx}\left[x^n J_n(x)\right] =$ ________ [a]

a) $x^n J_{n-1}(x)$ b) $-x^{-n} J_{n+1}(x)$ c) $x^n J_{n+1}(x)$ d) None

2. $\left[J_{\frac{1}{2}}(x)\right]^2 + \left[J_{\frac{-1}{2}}(x)\right]^2 =$ ________ [c]

a) $\frac{\pi x}{2}$ b) $\frac{1}{\pi x}$ c) $\frac{2}{\pi x}$ d) None

3. $P_n(-1) =$ _______ [b]

a) 1 b) $(-1)^n$ c) $(2)^n$ d) None

4.$(2n+1)P_n(x) + P_{n-1}^{1}(x) =$ ________________ [a]

a) $P'_{n+1}(x)$ b) $P'_n(x)$ c) $n\ P_{n-1}(x)$ d) None

5.Value of $\int_0^{\infty} \frac{e^{-ax}}{\sqrt{x}}dx$ is [b]

a) $\frac{\sqrt{\pi}}{a}$ b) $\sqrt{\frac{\pi}{a}}$ c) $2\sqrt{\frac{\pi}{a}}$ d) $\frac{3\sqrt{\pi}}{a}$

6. $\int_0^1 \frac{(1-x)^{n-1}}{x^n}dx$ $(0 < n < 1)$ is [c]

a) $\frac{n}{\sin n\pi}$ b) $\frac{n}{\sin\frac{n\pi}{2}}$ c) $\frac{\pi}{\sin n\pi}$ d) $\frac{\pi}{\sin\frac{n\pi}{2}}$

7. $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx =$ [a]

a) r(n) b) r(n – 1) c) B(n, n) d) B(n – 1, n – 1)

8. $\int_0^{\pi/2} \tan^{1/2}\theta\ d\theta =$ [a]

a) $\pi/\sqrt{2}$ b) $\sqrt{2}\,\pi$ c) $\sqrt{3\pi}$ d) $3\sqrt{\pi}$

9.Write the value of $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$ where $m > 0, n > 0$ interms of Beta function:

a) $\beta(m, n)$ b) $\frac{1}{2}\beta(m, n)$ c) $2\beta(m, n)$ d) none [a]

10. $\frac{d}{dx}\left[x^{-n} J_n\right] =$ [c]

a) $x^{n-1} J_n^{(x)}$ b) $x^n J_{n-1}^{(x)}$ c) $-x^n J_{n+1}^{(x)}$ d) none

11.Generating function of Bessel functions is [c]

a) $e^{\frac{x}{2}\left(t+\frac{1}{t}\right)}$ b) $e^{-\frac{x}{2}\left(t+\frac{1}{t}\right)}$ c) $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ d) $e^{-\frac{x}{2}\left(t-\frac{1}{t}\right)}$

12.$x[(J_{-1/2}^{(x)})^2 + (J_{1/2}^{(x)})^2] =$ [a]

a) $\frac{2}{\pi}$ b) $\frac{\pi}{2}$ c) $\frac{1}{\pi}$ d) π

13. $P_n(1) =$ ______________________ [a]

a) 1 b) –1 c) 0 d) $(-1)^n$

14. $\int_{-1}^{1} [P_n(x)]^2\,dx =$ [b]

a) 0 b) $\frac{2}{2n+1}$ c) $\frac{2n}{2n+1}$ d) none

15. $J_{-\frac{1}{2}}(x) =$ ______________________ [a]

a) $\sqrt{\frac{2}{\Pi x}}$ cosx b)) $\sqrt{\frac{2}{\Pi x}}$ sinx c) $\frac{2}{\Pi x}$ cosx d) $\frac{2}{\Pi x}$ sinx

16. $\frac{d}{dx}J_0 =$ ______________________ [b]

a) J_1 b) $-J_1$ c) J_0 d) J_2

17. $\int J_3(x)dx =$__________________ [b]

a) $J_2(x) + \frac{2}{x}J_2(x)$ b) $-J_2(x) - \frac{2}{x}J_2(x)$ c) $J_0(x) + J_1(x)$ d) none

18. The polynomial $2x^2 - 4x + 2$ interms of legendre polynomials is [b]

a) $P_2(x) + P_1(x) + P_o(x)$ b) $\frac{4}{3}P_2(x) - 4P_1(x) + \frac{8}{3}P_0(x)$ c) $P_0 + P_1$ d) $P_0 - P_1$

19. $P_n(-x)$=________________________ [c]

a) $P_n(x)$ b) $P_n(-1)$ c) $(-1)^n P_n(x)$ d) $P_n(x+1)$

20. In terms of Legendre polynomials $x^2 =$ ________________________ [a]

a) $\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)$ b) $\frac{1}{3}P_0(x) - \frac{2}{3}P_0(x)$

c) $P_2(x) + P_1(x) + P_o(x)$ d) $\frac{8}{3}P_0(\frac{4}{3}P_2(x) - 4P_1(x) + x)$

UNIT – III

1. The derivative of $w = f(z) = z^3 - 2z$ at the point $z = -1$ is __________ [a]

a) 1 b) -1 c) i d) -i

2. If $f(z)$ is analytic at Z_0 then it is [c]

a) Continuous every where b) Domain is continuous every where
c) Continuous at $Z = Z_0$ d) None

3. $W = f(z) = \frac{1+z}{1-z}$ is [c]

a) analytic every where
b) nowhere analytic
c) analytic for all finite values of Z except at Z =1
d) None

4. Cauchy – Riemann equations are [a]

a) $u_x = v_y$ & $v_x = -u_y$
b) $u_x = vy,\ v_x = uy$
c) $u_x = v_y$ & $v_x = v_y$
d) None

5. An analytic function with constant modulus is a [c]

a) function of x
b) function of y
c) constant function
d) function of x & y

6. If the derivative exists at all points of Z of region R, Then $f(z)$ is said to be _________in R

a) non analytic b) analytic c) not differentiable d) None [b]

7.Functions which satisfy Laplace's equation in a region R are called __in R [a]

a) Harmonic b) not Harmonic c) analytic d) None

8.To construct on a analytic function according to Milne-Thomson method, replace… [a]

a) x by z and y by 0 b) x by y and y by 0 c) x by z and y by 1 d) none

9.The C – R equations in polar form……….. [a]

a) $u_r = \frac{1}{r} v_\theta,\ v_r = -\frac{1}{r} u_\theta$
b) $u_r = r v_\theta,\ v_r = -\frac{1}{r} u_\theta$
c) $u_r = \frac{1}{r} v_\theta,\ v_r = r u_\theta$
d) none

10.If $u = e^{-x}(x \sin y - y \cos y)$ is harmonic then……… [b]

a) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ c) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ d) none

11. The points which are mapped onto themselves under a conformal mapping are called [a]

a) fixed points b) identity points c) conformal points d) None

12. The fixed points of $w = \frac{z}{2-z}$ are [a]

a) 0, 1 b) 1, -1 c) -1, 0 d) 0

13. The fixed points of $w = \frac{z-1}{z+1}$ are [b]

a) ± 1 b) $\pm i$ c) 0 d) -1

14. The Transformation $w = z + \frac{1}{z}$ transforms circles with radius constant into [a]

a) family of confocal ellipses
b) hyperbolas
c) Circles
d) None

15. The transformation $w = \cosh z$ transforms line Parallel to x-axis into [c]

a) ellipses b) circles c) hyperbolas d) none

16. The Mapping $w = 3z^2 - 4z + 1$ is [b]

a) Conformal every where
b) no where conformal at $z = \frac{2}{3}$
c) not conformal at $z = 2$
d) none of these

17. The transformation which maps the interior of a polygon of the w-plane into the upper half of the z-plane is given by [d]

a) Inversion
b) Bilinear transformation

c) Joukowski's transformation d) Schwarz-christoffel transformation

18. The fixed points of the transformation $w=\frac{1}{z-2i}$ are [a]

a) $z=i$ b) $z=-i$ c) $z=\pm i$ d) $z=\pm 1$

19. The cross-ratio of four points z_1, z_2, z_3, z_4 is -------------------------- [b]

a) $\frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_3)}$ b) $\frac{(z_1+z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_3)}$ c) $\frac{(z_1-z_2)(z_3+z_4)}{(z_1-z_4)(z_3-z_3)}$ d) $\frac{(z_1-z_2)(z_3-z_4)}{(z_1+z_4)(z_3+z_3)}$

20. Bilinear transformation always transforms circles into ------------- [d]

a)straight lines b) parabolas c)hyperbolas d) circles

UNIT – IV

1.The value of $\int_c \frac{z^3}{(z-2)^2}dz$ where c is $|z|=1$ is [a]

a) 0 b) $2\pi i$ c) $-2\pi i$ d) $6\pi i$

2. If $c:|z|=4$ then $\int_c \frac{dz}{z^2+9}=$ [c]

a) $2\pi i$ b) 1 c) 0 d) 4π

3.Singular points of $f(z)=\frac{z^2+1}{z^2-3z+2}$ [a]

a) 1, 2 b) -1, 2 c) 1, -2 d) -1, -2

4.If $f(z)$ is analytic within and on a closed curve c and if 'a' is any point

within c, then $\int_c \frac{f(z)}{z-a}dz=$ [b]

a) $f(a)$ b) $2\pi i\ f(a)$ c) $\frac{f(a)}{2\pi i}$ d) $2\pi f(a)$

5.The valve of $\int \frac{\sin^6 z}{\left(z-\frac{\pi}{2}\right)^3}dz$ around the circle $c:|z|=1$ [d]

a) $2\pi i$ b) $16\pi i$ c) $4\pi i\cos z$ d) Zero

6. The value of $\int_0^{1+i}(x-y+ix^2)dz$ along the straight line from z = 0 to z = 1 + i is [b]

a) $\frac{1+i}{3}$ b) $\frac{-1+i}{3}$ c) $\frac{i}{3}$ d) $-\frac{i}{3}$

7.The value of $\int_c \frac{e^z}{z-3}dz$ where c is the circle |z| = 4 is [a]

a) $2\pi ie^3$ b) $\frac{2\pi i}{e^3}$ c) $2\pi i$ d) e^3

8.If f(z) is an analytic function of z and if $f^1(z)$ is continuous at each point within and on a closed contour c then……… [a]

a) $\int_c f(z)dz=0$ b) $\int_c f(z)dz\neq 0$ c) $\int_c f(z)dz=2\pi i$ d) none

9.The value of $\int_c \frac{e^z}{z^3} dz$ where c : |z| = 2 ……….. [c]

a) $2\pi i$ b) $2\pi ie$ c) πi d) $2\pi ie^2$

10.Generalization of Cauchy's integral formula is [c]

a) $f^n(a) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{z-a} dz$ b) $f^n(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{(z-a)^{n+1}} dz$

c) $f^n(a) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-a)^{n+1}} dz$ d) $f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} dz$

11. Find Taylor's series of $\log(1+z)$, about $z=0$ [d]

a) $1-z+z^2-z^3+---, |z|<1$ b) $1+z+z^2+z^3+---, |z|<1$

c) $1-2z+3z^2-4z^3+---, |z|<1$ d) None

12. Find the Laurent's series of $\frac{1}{z-3}$, when $|z|<3$_____ [a]

a) $-\frac{1}{3}-\frac{1}{9}z-\frac{1}{27}z^2----$ b) $\frac{1}{3}+\frac{1}{z}+\frac{1}{z^2}+----$

c) $z+z^2+z^3+z^4+----$ d) None

13. By Taylor's series, $f(z)=---$ [a]

a) $\sum_{n=0}^{\infty} \frac{f^n(a)}{\lfloor n}(z-a)^n$ b) $\sum_{n=0}^{\infty} \frac{f^n(a)}{\lfloor n}$

c) $\sum_{n=0}^{\infty} (z-a)^n$ d) None

14. $Cos\ z =$_____ [a]

a) $1-\frac{z^2}{\lfloor 2}+\frac{z^4}{\lfloor 4}-----$ b) $1+\frac{z^2}{\lfloor 2}+\frac{z^4}{\lfloor 4}-----$

c) $z-\frac{z^3}{\lfloor 3}+-----$ d) None

15. Expand e^z as Taylor's series about Z= 1,------ [a]

a) $e\left[1+(z-1)+\frac{(z-1)^2}{\lfloor 2}+----\right]$ b) $1+\frac{z}{\lfloor 1}+\frac{z^2}{\lfloor 2}+----$

c) $1-\frac{z}{\lfloor 1}+\frac{z^2}{\lfloor 2}+----$ d) None

16. In Laurent's series, If $f(z)=\sum_{h=0}^{\infty} a_n(z-a)^n+\sum_{h=1}^{\infty} b_n(z-a)^{-n}$ then $a_n =$___ [b]

a) $\frac{1}{2\pi i}\int_C \frac{f(w)}{(w-a)^{-n+1}} dw$ b) $\frac{1}{2\pi i}\int_C \frac{f(w)}{(w-a)^{n+1}} dw$

c) 0 d) None

17. Radius of convergence of Taylor's series of f(z) = $\frac{z}{z-1}$ about z = 2 is [c]

a) 2   b) 0   c) 1   d) None

18. Radius of convergence of Taylor's expansion of e^z about $z = 1$ is [c]

a) 0   b) 1   c) ∞ d) none

19. Laurent expansion of f(z) about z = a in an annular region $0 < |z - a| < r$ contains[c]

a) positive integer powers of (z – a) only

b) positive integer powers of (z + a) only

c) positive and negative integer powers of (z – a) only

d) positive and negative integer powers of (z + a) only

20. If $f(z) = \dfrac{1}{z^2 - 1}$ then expansion of f(z) in $|z| > 1$ is [c]

a) $(1 + z^2 + z^4 + z^6 + \ldots\ldots\ldots\ldots)$ b) $(1 - z^2 + z^4 - z^6 + \ldots\ldots\ldots\ldots)$

c) $\left(\dfrac{1}{z^2} + \dfrac{1}{z^4} + \dfrac{1}{z^6} + \ldots\ldots\ldots\ldots\right)$ d) $\left(\dfrac{1}{z^2} - \dfrac{1}{z^4} + \dfrac{1}{z^6} - + \ldots\ldots\ldots\ldots\right)$

UNIT – V

1. Write the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2 (z+2)}$ [b]

a) 1, 1   b)1, -2   c) 1, 2   d) 0,2, -2

2. Write the Poles of the function $f(z) = \dfrac{z^2}{z^2 + a^2}$ [b]

a) a, a   b) $\pm ai$ c) $a, -a$ d) $-a, -a$

3. If $z = a$ is a pole of order 1 of $f(z)$, then $\operatorname{Res}_{z=a} f(z) =$ ___ [c]

a) 1   b) $\underset{Z \to a}{Lt} f(z)$ c) $\underset{Z \to a}{Lt} (z-a) f(z)$ d) None

4. If $f(z)$ has a pole of order n at $z = a$, then $\operatorname{Res}_{z=a} f(z) =$ ___ [c]

a) 1   b) $\underset{Z \to a}{Lt} f(z-a) f(z)$ c) $\dfrac{1}{(n-1)!} \underset{z \to a}{Lt} \dfrac{d^{n-1}}{dz^{n-1}} \left[(z-a)^n . f(z) \right]$ d) None

5. $\displaystyle\int_C \frac{e^{iz}}{z + 3i} dz =$ ___, where $C: |z + 3i| = 1$ [d]

a) 0   b) $2\pi i$ c) $2\pi i . e^{-3}$ d) $2\pi i . e^3$

6. $\displaystyle\int_C \frac{1-2z}{z(z-1)(z-2)} dz =$ ___, where $C: |Z| = 1.5$ [d]

a) 0   b) 1   c) $2\pi i$ d) $3\pi i$

7. Find the Residue at $z = -1$ for $f(z) = \dfrac{z}{(z+1)(z-3)^2}$ _____ [a]

a) $-\dfrac{1}{16}$ b) 1   c) -1   d) $\dfrac{1}{14}$

8. Find the Residue at $z = 2i$ of $f(z) = \dfrac{z^2}{(z^2 + 9)(z^2 + 4)}$ ______ [a]

a) $-\frac{1}{5i}$ b) $\frac{1}{4i}$ c) 1 d) -1

9. $\int_C \frac{z^2-z+1}{z-1}dz =$_______, where $c:|z|=1/2$ [c]

a) 1 b) ½ c) 0 d) None

10. $\int_c \frac{e^{2z}}{(z+1)^4}dz =$_____, where $c:|z|=3$ [c]

a) 1 b) 2 c) $8\pi i\frac{e^{-2}}{3}$ d) None

11. The zeros of the function $\sin\frac{1}{z}$ is [b]

a) z = nπ (n is integer) b) $z = \pm\frac{1}{n\pi}$, n = 1, 2, 3,......

c) $z = \pm\frac{2}{n\pi}$, n = 1, 3, 5, 7,...... d) $z = (2n + 1)\frac{\pi}{2}$, n = 0, 1, 2,......

12. Write the poles of the f(z)= $\frac{z^2}{(z^2+9)(z^2+4)}$ [a]

a) 3i,-3i,2i,-2i b) 9i,-9i,4i,-4i

c) i,-I,4i,-4i d)None

13. Residue of $\frac{e^{iz}}{z^2+1}$ at z=-i [a]

a) $-\frac{e}{2i}$ b) $\frac{e}{2i}$ c) $-\frac{e}{3i}$ d) None

14. The poles of the function f(z) = $\frac{z}{\cos z+\sin z}$ are [d]

a) $z = \frac{\pi}{2}$ b) $z = \frac{n\pi}{2}$ c) $z = \frac{(2n+1)\pi}{2}$ d) $z = \frac{(4n+3)\pi}{4}$

15. Residue of f(z) = $\frac{z^2}{(z-1)^2(z+2)}$ at z = – 2 is [c]

a) 4 b) 9 c) 4/9 d) – 4/9

16. Residue of f(z) = $\frac{z+1}{z^2(z-2)}$ at z = 0 is [b]

a) 3/4 b) – 3/4 c) 4/3 d) None

17. The residue of $\frac{e^z-1}{z^4}$ at z = 0 is [d]

a) 0 b) 1/2 c) 1/4 d) 1/6

18. The value of $\oint_c \frac{z-3}{z^2+2z+5}dz$, where c is the circle |z| = 1 is [b]

a) 2πi b) 0 c) – 2πi d) None

19. The value of $\oint_c \frac{dz}{z^3(z+4)}$, where c is the circle |z| = 2 is [b]

a) 0 b) $\frac{\pi i}{32}$ c) $-\frac{\pi i}{32}$ d) 32 πi

20. Find the residue of $\frac{z e^z}{(z-1)^3}$ at z=1 [a]

a) $\frac{3e}{2}$ b) $\frac{5e}{2}$ c) $\frac{7e}{2}$ d) $\frac{9e}{2}$