Unit-3, Magneto Statics

Magneto statics or Static Magnetic fields are having some similarities with Electro static fields. As $\vec{E}$ and $\vec{B}$ are related according to $\vec{D} = \varepsilon \vec{E}$ in Electro statics. The $\vec{H}$ and $\vec{B}$ are related according to $\vec{B} = \mu \vec{H}$ in Magneto statics. Where $\vec{H}$ is the Magnetic field intensity and $\vec{B}$ is Magnetic flux Density and μ is the permeability of the medium given as $\mu = \mu_0 \mu_v$. μ_0 is having a value of $\mu_0 = 4\pi \times 10^{-7}$ H/m. The source for Magnetic fields is charges moving with Constant Velocity or current.

There are two major laws governing Magnetostatic fields. They are ① Biot-Savart's Law and ② Ampere's Circuit Law. Like Coulomb's law, Biot-Savart's law is the general law of Magneto statics. Just as Gauss's law is a Special case of Coulomb's Law, Ampere's law is a special case of Biot-Savart's Law and is easily applied in problems involving symmetrical Current distribution.

Biot-Savart's Law :— It states that the magnetic field intensity $d\vec{H}$ produced at a point P, as shown in below figure, by the differential Current element $I \, d\vec{l}$ is propotional to the

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product $\vec{I}d\vec{l}$ and sine of the angle, α between the current

element and the line joining P

to the element and is inversely

proportional to the square of the

distance, R between the point P

and the element. That is $d\vec{H} \propto \dfrac{\vec{I}d\vec{l}\,\sin\alpha}{R^\nu}$, or

$d\vec{H} = \dfrac{K\vec{I}d\vec{l}\,\sin\alpha}{R^\nu}$ where K is the Constant of proportionality

and is given by $K = \dfrac{1}{4\pi}$. So $d\vec{H} = \dfrac{\vec{I}d\vec{l}\,\sin\alpha}{4\pi R^\nu}$. From the

definition of Cross product, we can write $\boxed{d\vec{H} = \dfrac{\vec{I}d\vec{l} \times \vec{a}_R}{4\pi R^\nu} = \dfrac{\vec{I}d\vec{l} \times \vec{R}}{4\pi R^3}}$
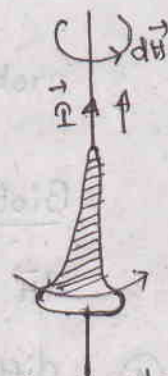
where $R = |\vec{R}|$ and $\vec{a}_R = \dfrac{\vec{R}}{R}$.

The direction of $d\vec{H}$ can be determined by the

right hand thumb rule with the right-hand thumb pointing

in the direction of current, the right hand fingers encircling

the wire in the direction of $d\vec{H}$ as shown in figure. Alternatively

we can use the right handed screw

rule to determine the direction

of $d\vec{H}$. With the screw placed along

the wire and pointed in the direction

of Current flow, the direction of

Thumb rule   Screw rule

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advance of the screw is the direction of $d\vec{H}$ as shown in figure. It is customary to represent the direction of the magnetic field intensity $\vec{H}$ (or current $\vec{I}$) by a small circle with a dot or cross sign depending on whether $\vec{H}$ (or $\vec{I}$) is out of or into the page as $\odot$ and $\otimes$ respectively.

Similar to the Continous charge distributions in Electrostatics there are 3-types of current sources in Magneto statics. They are ① line Current, $\vec{I}\,d\vec{l}$ ② Surface Current, $\vec{K}\,d\vec{s}$ and ③ Volume Current, $\vec{J}\,dv$. In terms of these currents, the Biot-Savart law becomes as

$$\vec{H} = \int_L \frac{\vec{I}\,d\vec{l} \times \vec{a_R}}{4\pi R^\nu} \dots \text{line current}$$

$$\vec{H} = \int_S \frac{\vec{K}\,d\vec{s} \times \vec{a_R}}{4\pi R^\nu} \dots \text{Surface Current}$$

$$\vec{H} = \int_V \frac{\vec{J}\,dv \times \vec{a_R}}{4\pi R^\nu} \dots \text{Volume Current.}$$

Field due to Infinite Line Current :- Let us Consider a conductor is placed along the Z-axis of length AB in which a uniform line Current of $\vec{I}\,d\vec{l}$ is flowing as shown in the figure. The upper end B Subtending an angle α_2 and lower end A is subtending an angle α_1 to the point, P.

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Considering the Contribution of $d\vec{H}$ at point P due to an elementary current filament $d\vec{l}$ at $(0,0,3)$, from Biot-Savart Law, $d\vec{H} = \dfrac{I\, d\vec{l} \times \vec{R}}{4\pi R^3}$.

Here, $d\vec{l} = d\vec{3}$, $\vec{R} = \rho\,\vec{a_\rho} - 3\,\vec{a_3}$. So

$$d\vec{l} \times \vec{R} = \rho\, d3\,\vec{a_\phi}$$

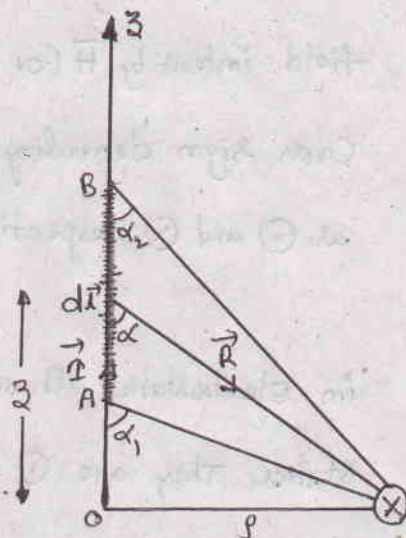Hence, $\vec{H} = \displaystyle\int_A^B \dfrac{I\rho\, d3}{4\pi \left[\rho^2 + 3^2\right]^{3/2}}\,\vec{a_\phi}$

Let $3 = \rho \cot\alpha$ then $d3 = -\rho \csc^2\alpha\, d\alpha$.

$$\therefore\ \vec{H} = \int_{\alpha_1}^{\alpha_2} \dfrac{-I\rho^2 \csc^2\alpha\, d\alpha}{4\pi\left[\rho^2 + \rho^2\cot^2\alpha\right]^{3/2}}\,\vec{a_\phi}$$

$$= \dfrac{-I}{4\pi}\int_{\alpha_1}^{\alpha_2} \dfrac{\rho^2 \csc^2\alpha\, d\alpha}{\rho^3 \csc^3\alpha}\,\vec{a_\phi}$$

$$= \dfrac{-I}{4\pi\rho}\,\vec{a_\phi}\int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

$$\boxed{\vec{H} = \dfrac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)\vec{a_\phi}}$$ This expression is applicable to any filamentary Conductor of finite length. As a special case, when the Conductor is semiinfinite so that point A is at $(0,0,0)$ and point B is at $(0,0,\infty)$ with $\alpha_1 = 90°$ and $\alpha_2 = 0°$. Then

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$$\vec{H} = \frac{\vec{I}}{4\pi\varsigma} \vec{a_\phi} (\cos\dot{0} - \cos 9\dot{0}) = \frac{\vec{I}}{4\pi\varsigma} \vec{a_\phi}.$$ Another special case is

when the conductor is infinite in length. For this case point A

is at $(0,0,-\infty)$ while B is at $(0,0,\infty)$ with $\alpha_1 = 180°$, and $\alpha_2 = 0°$.

$$\therefore \vec{H} = \frac{\vec{I}}{4\pi\varsigma} \vec{a_\phi} (\cos\dot{0} - \cos 180\dot{0})$$

$$\vec{H} = \frac{\vec{I}}{4\pi\varsigma} \vec{a_\phi} (1 - (-1))$$

$$\boxed{\vec{H} = \frac{\vec{I}}{2\pi\varsigma} \vec{a_\phi}}$$ where $\vec{a_\phi} = \vec{a_L} \times \vec{a_\varsigma}$. $\vec{a_L}$ is unit vector

along the line current and $\vec{a_\varsigma}$ is a unit vector along the

perpendicular line from line current to the field point.

Ampere's Circuit Law :- It states that the closed line

integral of $\vec{H}$-field around a closed path is same as the total

current, $\vec{I}_{encl}$ enclosed by the path. i.e $\boxed{\oint_L \vec{H} \cdot d\vec{l} = \vec{I}_{encl}}$.

Ampere's Law is similar to Gauss's Law and it is easily

applied to determine $\vec{H}$ when the current distribution is

symmetrical. By applying Stoke's theorem to the Left-hand

side of above equation, we get $\vec{I}_{encl} = \oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{s}$

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But $\vec{I}_{encl} = \int\limits_{S} \vec{J} \cdot d\vec{s}$

Comparing the two surface integrals, we can write

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$ − Maxwell's 3rd equation.

This is called the Maxwell's 3rd equation showing that magneto static field is not Conservative.

Applications of Ampere's Law :− We can apply this Law to determine $\vec{H}$ for an infinite line current, infinite surface or sheet of current. and infinitely long co-axial transmission line.

(A). Infinite Line Current:− Consider an infinitely long filamentary current $\vec{I}$ along the z-axis as shown in figure. To determine $\vec{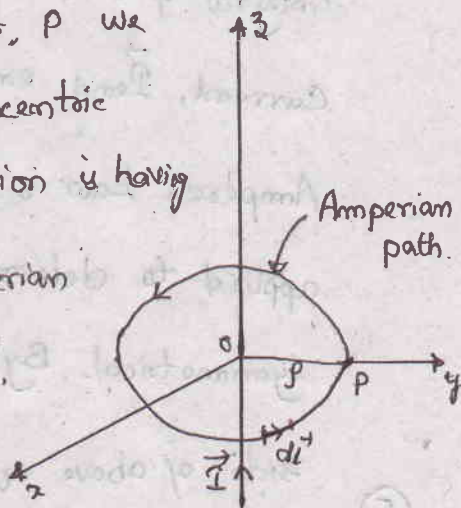H}$ at any general point, P we select the Amperian path as a Concentric Circle because the current distribution is having Circular Symmetry, Since the Amperian path encloses the whole current, $\vec{I}$.

according to Ampere's Law

$$\vec{I} = \int_{\phi=0}^{2\pi} H_\phi \, \vec{a_\phi} \cdot \mathcal{S} \, d\phi \, \vec{a_\phi} = H_\phi \int_0^{2\pi} \mathcal{S} \, d\phi = 2\pi \mathcal{S} \, H_\phi$$

$$H_\phi = \frac{I}{2\pi \mathcal{S}} \quad \text{or in general} \quad \boxed{\vec{H} = \frac{\vec{I}}{2\pi \mathcal{S}} \, \vec{a_\phi}} \quad \text{as expected.}$$

(B). <u>Infinite Sheet of Current</u> :– Consider an infinite current

sheet in the $z=0$ plane. In this sheet assume a uniform current density $\vec{K} = K_y \vec{a_y}$ A/m as shown in figure. Applying Ampere's Law to Rectangular Amperian path gives $\oint_L \vec{H} \cdot d\vec{l} = I_{encl.} = K_y b$

For the current flowing in y direction, the $\vec{H}$ will be in the x direction only. i.e. $\vec{H} = \begin{cases} H_x \vec{a_x} & \text{for } z>0 \\ -H_x \vec{a_x} & \text{for } z<0. \end{cases}$

Evaluating the line integral of $\vec{H}$ along the closed path gives

$$\oint_L \vec{H} \cdot d\vec{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \vec{H} \cdot d\vec{l}$$
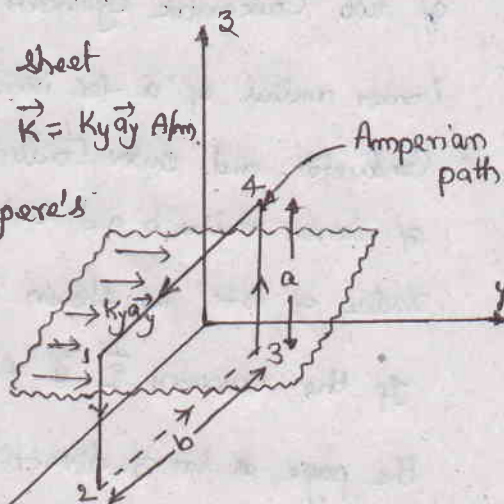
$$= 0(-a) + (-H_x)(-b) + 0(a) + H_x b$$

$$= 2 H_x b.$$

or $H_x = \frac{1}{2} K_y$. or in general $\vec{H} = \begin{cases} \frac{1}{2} K_y \vec{a_x} & \text{for } z>0 \\ -\frac{1}{2} K_y \vec{a_x} & \text{for } z<0. \end{cases}$
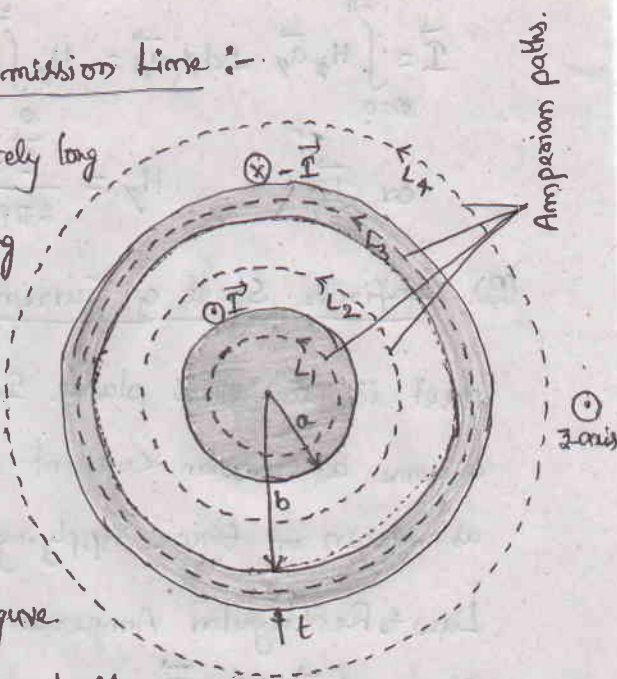
or for the current in any direction, we can write $$\boxed{\vec{H} = \frac{1}{2} \vec{K} \times \vec{a_n}}$$ where $\vec{a_n}$ is the unit normal vector directed from current sheet to the point, P.

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(C) <u>Infinitely long Co-axial Transmission Line</u> :-

Consider an infinitely long co-axial transmission line consisting of two concentric cylinders having inner radius of 'a' for inner conductor. and outer conductor of inner radius 'b' and outer radius of 'b+t' as shown in figure.

If the current $\vec{I}$ is flowing out of the page as in z-direction, the the current of $-\vec{I}$ will flows into the page in -z-direction. Since the current distribution is symmetrical, we apply Ampere's Law along the four possible Amperian paths: $0 < \rho < a$, $a < \rho < b$, $b < \rho < b+t$ and $\rho > b+t$.

For the first Amperian path (L_1): $0 < \rho < a$, Apply Ampere's Law we got $\oint_{L_1} \vec{H} \cdot d\vec{l} = \vec{I}_{encl}$

$$\vec{I}_{encl} = \int_S \vec{J} \cdot d\vec{S} \quad \text{and} \quad \vec{J} = \frac{\vec{I}}{\pi a^2} \vec{a_z}, \quad d\vec{S} = \rho \, d\rho \, d\phi \, \vec{a_z}$$

$$\vec{I}_{encl} = \int_S \vec{J} \cdot d\vec{S} = \frac{\vec{I}}{\pi a^2} \int_{\rho=0}^{\rho} \int_{\phi=0}^{2\pi} \rho \, d\rho \, d\phi = \frac{\vec{I}}{\pi a^2} \pi \rho^2 = \frac{\vec{I} \rho^2}{a^2}$$

$$\therefore H_\phi \int_{L_1} dl = H_\phi \, 2\pi\rho = \frac{\vec{I} \rho^2}{a^2}, \quad \boxed{H_\phi = \frac{\vec{I} \rho}{2\pi a^2}}$$

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For the second Amperian path (L_2): $a < \rho < b$, Applying Ampere's

Law, we get $\oint_{L_2} \vec{H} \cdot \vec{dl} = \vec{I}_{encl}$

$\vec{I}_{encl} = \vec{I}$

$\therefore \oint_{L_2} \vec{H} \cdot \vec{dl} = \vec{I}$

$\int_{\phi=0}^{2\pi} H_\phi \, \rho \, d\phi = \vec{I}$, $H_\phi \, \rho \, [\phi]_0^{2\pi} = \vec{I}$, $2\pi \rho \, H_\phi = \vec{I}$

$$\boxed{H_\phi = \frac{\vec{I}}{2\pi\rho}}$$

For the third Amperian path (L_3): $b < \rho < b+b$, Applying Ampere's

Law, we get $\oint_{L_3} \vec{H} \cdot \vec{dl} = \vec{I}_{encl}$

$\vec{I}_{encl.} = \vec{I} + \int_S \vec{J} \cdot \vec{ds}$ Here $\vec{J}$ is the current density

of the outer Conductor along $-\vec{a_3}$ direction that is

$$\vec{J} = - \frac{\vec{I}}{\pi\left[(b+t)^v - b^v\right]} \vec{a_3}$$

Thus $\vec{I}_{encl} = \vec{I} - \frac{\vec{I}}{\pi\left[(b+t)^v - b^v\right]} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\rho} \rho \, d\rho \, d\phi$

$$\vec{I}_{encl} = \vec{I}\left[1 - \frac{\rho^v - b^v}{2bt + t^v}\right]$$

$\oint_{L_3} \vec{H} \cdot \vec{dl} = \int_{\phi=0}^{2\pi} H_\phi \, \rho \, d\phi = 2\pi\rho \, H_\phi = \vec{I}\left[1 - \frac{\rho^v - b^v}{2bt + t^v}\right]$

$$\boxed{H_\phi = \frac{\vec{I}}{2\pi\rho}\left[1 - \frac{\rho^v - b^v}{2bt + t^v}\right]}$$

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For the fourth Amperian path, (L_4): $\rho > b+t$, Applying Ampere's Law

we get $\oint_{L_4} \vec{H} \cdot d\vec{l} = \vec{I}_{encl}$

$$\vec{I}_{encl} = \vec{I} - \vec{I} = 0$$

$$\oint_{L_4} \vec{H} \cdot d\vec{l} = 0$$

$$\int_{\phi=0}^{2\pi} H_\phi \, \rho \, d\phi = 0$$

$$\boxed{H_\phi = 0}$$

Finally the magnetic field intensity, $\vec{H}$ for an infinite co-axial line

$$\vec{H} = \begin{cases} \dfrac{\vec{I}\rho}{2\pi a^2} \, \vec{a}_\phi & \text{for } 0 < \rho < a \\[2mm] \dfrac{\vec{I}}{2\pi\rho} \, \vec{a}_\phi & \text{for } a < \rho < b \\[2mm] \dfrac{\vec{I}}{2\pi\rho} \left[1 - \dfrac{\rho^2 - b^2}{2bt + t^2} \right] & \text{for } b < \rho < b+t \\[2mm] 0 & \text{for } \rho > b+t. \end{cases}$$

In this case it is observed that the magnetic field at the outside of the cable is zero.

Magnetic flux density :- The magnetic flux density $\vec{B}$ is similar to the electric flux density $\vec{D}$. As $\vec{D} = \varepsilon_0 \vec{E}$ in free space, the magnetic flux density, $\vec{B} = \mu_0 \vec{H}$. Where $\mu_0 = 4\pi \times 10^{-7}$ H/m is
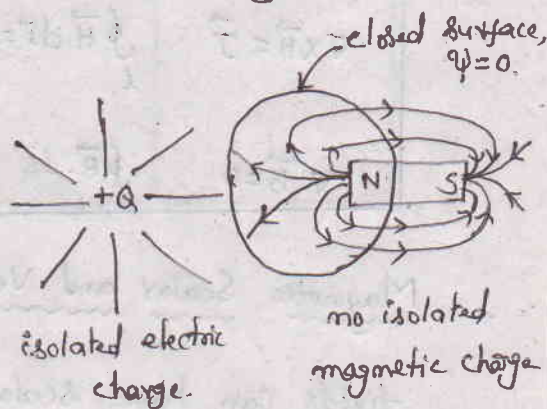
the permeability of free space. The magnetic flux, ψ through a surface, S is given by $\psi = \int_S \vec{B} \cdot d\vec{s}$. where ψ is in webers (wb) and the magnetic flux density is in webers/square meter (wb/m²) or tesla. If we consider the surface as a closed one then the magnetic flux will be zero because all the magnetic flux lines are closed lines and there is no isolated magnetic charge & pole as we are having isolated electric charge in electrostatics. as shown in figure.

$$\therefore \psi = \oint_S \vec{B} \cdot d\vec{s} = 0.$$

Applying Divergence theorem

we get $\oint_S \vec{B} \cdot d\vec{s} = \int_V \nabla \cdot \vec{B} \, dv. = 0$

or $\boxed{\nabla \cdot \vec{B} = 0}$. Maxwell's fourth equation.

closed surface, $\psi = 0$.

isolated electric charge.

no isolated magnetic charge

The above equation is called the Maxwell's fourth equation for static fields which says that the divergence of magnetic flux density is zero.

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Now we can write all the 4 Maxwell's equations in different forms as in below table.

Differential form (or Point form)	Integral form	Concept
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{S} = \int_V (\nabla \cdot \vec{D}) \, dv = \int_V \rho_v \, dv$	Gauss's Law
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Conservativeness of electro static field (Electric potential).
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \, d\vec{S} = \int_S \vec{J} \cdot d\vec{S}$	Ampere's Law.
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{S} = 0$	Non existence of isolated magnetic charge or pole.

Magnetic Scalar and Vector potentials :- The magnetic fields can have scalar potential V_m and Vector potential, $\vec{A}$.

To define V_m and $\vec{A}$ we will make use of the scalar and vector identities given by $\nabla \times \nabla V = 0$ and $\nabla \cdot (\nabla \times \vec{A}) = 0$. These two identities must be satisfied to any scalar or vector field.

Just as $\vec{E} = -\nabla V$, we define magnetic scalar potential V_m as

$$\vec{H} = -\nabla V_m$$

we know that $\nabla \times \vec{H} = \vec{J}$

$$\nabla \times \vec{H} \quad \nabla \times (-\nabla V_m) = 0 \quad \text{if } \vec{J} = 0.$$

We Know that for a magnetic field $\nabla \cdot \vec{B} = 0$. We can define the vector magnetic potential $\vec{A}$ (wb/m) such that

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

Just as we defined $V = \int \frac{dQ}{4\pi \varepsilon_0 r}$ in Electrostatics. we can define

$$\vec{A} = \int_L \frac{\mu_0 \vec{I} \, dl}{4\pi R} \quad \text{for line current}$$

$$\vec{A} = \int_S \frac{\mu_0 \vec{K} \, d\vec{s}}{4\pi R} \quad \text{for Surface Current}$$

$$\vec{A} = \int_V \frac{\mu_0 \vec{J} \, dv}{4\pi R} \quad \text{for Volume current.}$$

Forces due to Magnetic fields : There are Three ways in which force due to magnetic fields can be happened. They are

ⓐ due to a moving charged particle in $\vec{B}$ field

ⓑ on a current element in an external $\vec{B}$ field.

ⓒ between two current elements.

ⓐ force on a charged particle : If the charges are not moving at the initial stage, According to Coulomb's Law, we have $\vec{F_e} = Q\vec{E}$. If these charges start moving with constant velocity then the magnetic force can be expressed as

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$\vec{F}_m = Q \vec{u} \times \vec{B}$. where $\vec{u}$ is the velocity of charges. Force due to a moving charge, Q in the presence of both electric and magnetic fields will be $\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = Q[\vec{E} + \vec{u} \times \vec{B}]}$. This is called Lorentz force equation or Ampere's force equation.

ⓑ. force on a Current element :— If we are having a differential line Current $\vec{I} \, d\vec{l}$ in a $\vec{B}$ field, then the force on this current element can be expressed as $d\vec{F} = \vec{I} \, d\vec{l} \times \vec{B}$.

If the current, I is through a closed path L, the total force is given by $\boxed{\vec{F} = \oint_L \vec{I} \, d\vec{l} \times \vec{B}}$. Similarly for a surface current element, $\boxed{\vec{F} = \oint_S \vec{K} \, d\vec{s} \times \vec{B}}$, and for a volume current element $\boxed{\vec{F} = \int_V \vec{J} \cdot dv \times \vec{B}}$.

ⓒ. force between two Current elements :— Let us now consider the force between two current elements $\vec{I}_1 \, d\vec{l}_1$ and $\vec{I}_2 \, d\vec{l}_2$. According to Biot-Savart's Law both currents produce magnetic fields. we can find the differential

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force $d(d\vec{F_1})$ on differential current element $d\vec{l_1}$ due to the differential magnetic field $d\vec{B_2}$ produced by $\vec{I_2}d\vec{l_2}$ as shown in figure.

$$d(d\vec{F_1}) = \vec{I_1}d\vec{l_1} \times d\vec{B_2}$$

From Biot-Savart's Law,

$$d\vec{B_2} = \frac{\mu_0 \, \vec{I_2}d\vec{l_2} \times \vec{a}_{R_{21}}}{4\pi R_{21}^{\,v}}$$
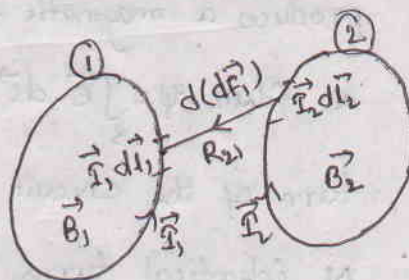
$$\therefore \quad d(d\vec{F_1}) = \frac{\mu_0 \, \vec{I_2}d\vec{l_2} \times \vec{I_1}d\vec{l_1} \times \vec{a}_{R_{21}}}{4\pi R_{21}^{\,v}} \qquad \text{or}$$

$$d(d\vec{F_1}) = \frac{\mu_0 \, \vec{I_1}d\vec{l_1} \times (\vec{I_2}d\vec{l_2} \times \vec{a}_{R_{21}})}{4\pi R_{21}^{\,v}} \qquad \text{or}$$

$$\boxed{\vec{F_1} = \frac{\mu_0 \, \vec{I_1}\,\vec{I_2}}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\vec{l_1} \times (d\vec{l_2} \times \vec{a}_{R_{21}})}{R_{21}^{\,v}}}$$
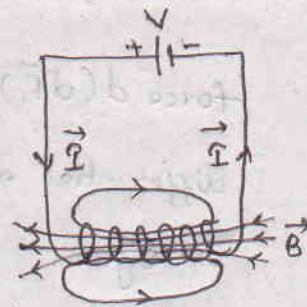
Similarly the force $\vec{F_2}$ on loop ② due to magnetic field $\vec{B_1}$ from loop ① is obtained by interchanging the subscripts 1 and 2.

$$\text{i.e.} \quad \boxed{\vec{F_2} = \frac{\mu_0 \, \vec{I_2}\,\vec{I_1}}{4\pi} \oint_{L_2} \oint_{L_1} \frac{d\vec{l_2} \times (d\vec{l_1} \times \vec{a}_{R_{12}})}{R_{12}^{\,v}}} \quad \text{i.e.} \quad \vec{F_2} = -\vec{F_1}.$$

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Inductors and Inductances :— A circuit or a closed conducting path carrying current $\vec{I}$ produces a magnetic field $\vec{B}$ which causes a flux $\psi = \int_S \vec{B} \cdot d\vec{s}$. to pass through each turn of the circuit as shown in figure. If the circuit has N identical turns, we define the flux linkage, λ as $\lambda = N\psi$. This flux linkage λ is proportional to the current, $\vec{I}$ producing it. That is $\lambda \propto \vec{I}$ or $\lambda = L\vec{I}$. where L is a constant of proportionality called <u>inductance</u> of the ckt. A ckt or part of a ckt. that has inductance is called an <u>inductor</u>. From the above equation we can define $\boxed{L = \dfrac{\lambda}{\vec{I}} = \dfrac{N\psi}{\vec{I}}}$. the unit of inductance is the Henry (H) which is same as webers/ampere. This type of inductance can also be called as <u>Self inductance</u> Since the flux linkages are produced by the inductor itself.
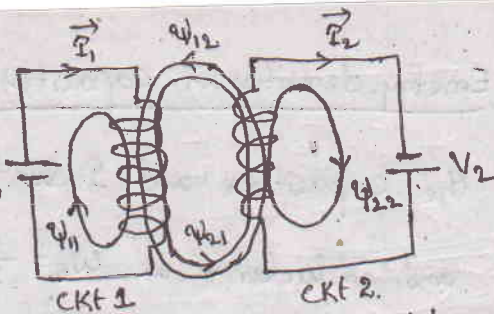
Mutual Inductance :— Instead of having a single circuit, If we have two ckt's carrying current $\vec{I}_1$ and $\vec{I}_2$ as shown in the figure, a magnetic interaction exists b/w the ckts.

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Four component fluxes $\psi_{11}, \psi_{12},$ ψ_{21} and ψ_{22} are produced. The flux ψ_{12} is the flux passing through CKT 1 due to current $\vec{I_2}$ in CKT 2. This flux is mutually shared by the two CKts. Similarly ψ_{21} is also mutually induced in the two Circuits. We define the mutual inductance M_{12} as the ratio of flux linkage

$\lambda_{12} = N_1 \psi_{12}$ on CKT 1 due to current $\vec{I_2}$. That is

$$M_{12} = \frac{\lambda_{12}}{\vec{I_2}} = \frac{N_1 \psi_{12}}{\vec{I_2}}$$

Similarly the mutual inductance M_{21} is defined as the flux linkages of CKT 2 due to current $\vec{I_1}$ in CKT 1. That is

$$M_{21} = \frac{\lambda_{21}}{\vec{I_1}} = \frac{N_2 \psi_{21}}{\vec{I_1}}$$

We can define the self inductances of CKT 1 and 2 as

$$L_1 = \frac{\lambda_{11}}{\vec{I_1}} = \frac{N_1 \psi_1}{\vec{I_1}} \quad \text{and} \quad L_2 = \frac{\lambda_{22}}{\vec{I_2}} = \frac{N_2 \psi_2}{\vec{I_2}}$$

where $\psi_1 = \psi_{11} + \psi_{12}$ and

$\psi_2 = \psi_{22} + \psi_{21}$.

(17)

Energy densities in capacitors and Inductors :- The Energies in the capacitors and Inductors can found from their capacitance and inductance as $W_E = \frac{1}{2} CV^2$ in Electrostatics. and

$$W_{EM} = \frac{1}{2} L I^2 \text{ in Magneto statics.}$$

The same Energies can also be expressed in terms of the fields they are producing as

$$W_E = \frac{1}{2} \int_V \vec{D}.\vec{E} \, dv = \frac{1}{2} \int_V \varepsilon \vec{E}^2 \, dv \quad \text{in electro statics.}$$

and

$$W_{Em} = \frac{1}{2} \int_V \vec{B}.\vec{H} \, dv = \frac{1}{2} \int_V \mu \vec{H}^2 \, dv \quad \text{in Magneto statics.}$$