The Electrical Analogous elements for the elements of mechanical system are given below

$f(t) \rightarrow e(t)$ (or) $v(t)$

$B \rightarrow R$

$M \rightarrow L$

$K \rightarrow 1/C$

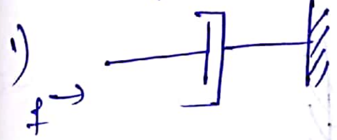
Each node in the mechanical system corresponds to a closed loop in electrical system. A mass is considered as node. The number of meshes in electrical analogous is same as that of the number of nodes (masses) in mechanical system.

Force Current Analogy :-

Mechanical System

input = force ; output = velocity

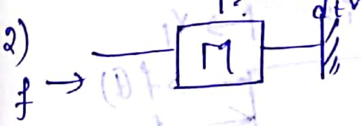
1) $\mapsto x$ $\quad \mapsto v = \frac{dx}{dt}$



$f = B\frac{dx}{dt}$

$$\boxed{f = Bv}$$

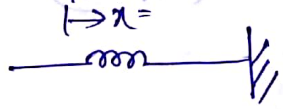
2) $\mapsto x$ $\quad \mapsto a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$



$f = M\frac{d^2x}{dt^2}$

$$\boxed{f = M\frac{dv}{dt}}$$

3) $\mapsto x = \int v\,dt$



$f = Kx$

$$\boxed{f = K\int v\,dt}$$

Electrical System

input = current

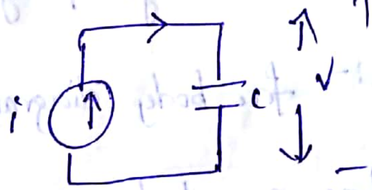
output = voltage

1)

$$\boxed{i = V/R}$$

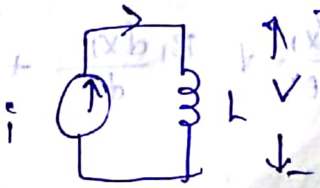
$\therefore i = 1/R\ V$

2)



$$\boxed{i = C\frac{dv}{dt}}$$

3)



$$\boxed{i = \frac{1}{L}\int v\,dt}$$

The electrical analogous elements in Force - current analogy for the elements of mechanical system is given below

$f(t) \rightarrow i(t)$ $\quad B \rightarrow 1/R$

$M - C$

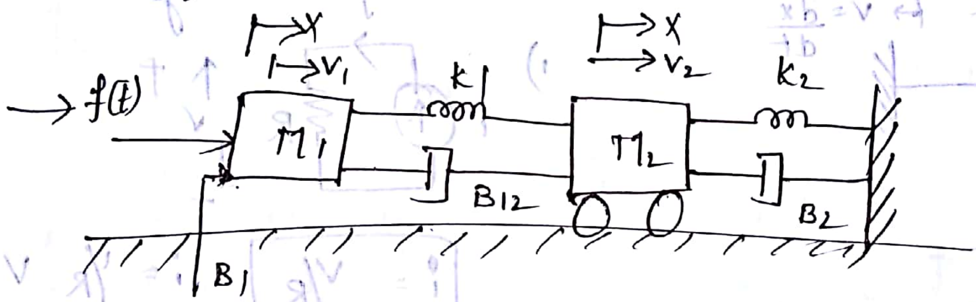
$K \rightarrow 1/L$

Each node in the mechanical system corresponds to a node in an electrical system. A mass is considered as node.

The no. of nodes in electrical analogous is same as that of no. of nodes in mechanical system.

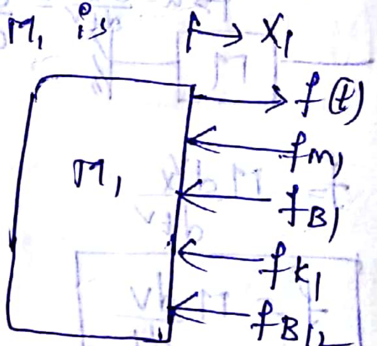
Problem :-

① Write the differential equations governing the mechanical system as shown in fig. Draw the force - voltage and force-current in electrical analogous ckt and verify by writing mesh and node equations.



sol Mechanical system given has two nodes

Mass ① :- free body diagram for mass M_1 is

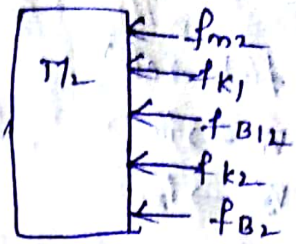


by Newtons second law

$$f(t) = f_{m_1} + f_{b_1} + f_{k_1} + f_{b_{12}}$$

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) \quad ①$$

Mass 2 :- free body diagram of M_2



by Newton's second law

$$0 = f_{m_2} + f_{b_{12}} + f_{b_2} + f_{k_1} + f_{k_2}$$

$$= M_2 \frac{d^2 x_2}{dt^2} + B_{12} \frac{d}{dt}(x_2 - x_1) + B_2 \frac{dx_2}{dt} + k_1 (x_2 - x_1) + k_2 x_2 \quad -②$$

on replacing the displacements by velocity in the differential equations ① & ② of an mechanical system

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} \quad ; \quad \frac{dx}{dt} = v \text{ and } x = \int v \, dt$$

① ⇒

$$f(t) = M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + k_1 \int (v_1 - v_2) dt = 0 \quad -③$$

② ⇒

$$0 = M_2 \frac{dV_2}{dt} + B_2 V_2 + B_{12}(V_2 - V_1) + k_1 \int (V_2 - V_1) dt + k_2 \int V_2 dt \quad -④$$

FORCE VOLTAGE ANALOGOUS ckt :-

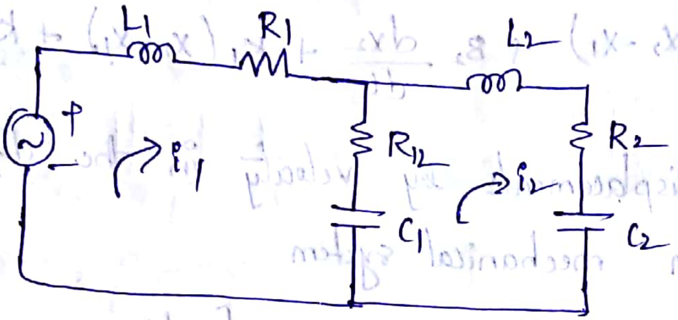
two nodes in mechanical system so we have two meshs in electrical ckt.

The force applied to mass M_1 is represented by voltage source in first mesh. The elements M_1, B_1, k_1 and B_{12} are connected to first node. Hence they are represented as analogous element in mesh 1 forming closed path.

The elements k_1 B_{12} M_2 k_2 & B_2 are connected to second node. They are represented by analogous elements in mesh 2 forming a closed path.

The elements k_1 and B_{12} are common b/w node 1 & 2, so they are represented as common elements b/w two meshes.

$f(t) \rightarrow e(t)$ | $M_1 \rightarrow L_1$ | $B_1 \rightarrow R_1$ | $K_1 \rightarrow 1/C_1$
$V_1 \rightarrow i_1$ | $M_2 \rightarrow L_2$ | $B_2 \rightarrow R_2$ | $K_2 \rightarrow 1/C_2$
$V_2 \rightarrow i_2$ | | $B_{12} \rightarrow R_{12}$ |



① → $M_1 \; B_1 \; K_1 \; B_{12}$
$L_1 \; R_1 \; 1/C_1 \; R_{12}$

② → $K_1 \; B_{12} \; M_2 \; K_2 \; B_2$
$1/C_1 \; R_{12} \; L_2 \; 1/C_2 \; R_2$

③ → $$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\, dt = e(t) \quad \text{— ⑤}$$

④ → $$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\, dt + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\, dt = 0 \quad ⑥$$

The equations ③ & ④ are similiar to that ⑤ & ⑥ eq^n's

<u>Force - Current Analagous ckt</u>

Two nodes in mechanical system so two nodes in force-current analagous ckt.

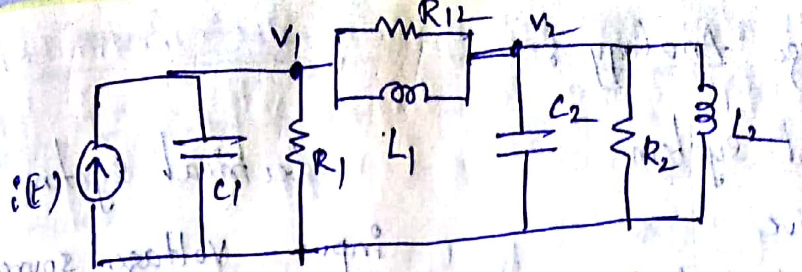
Force applied to M_1 is represented as current source connected to a node in analogous electrical circuit.

$M_1 \; B_1 \; K_1$ & B_{12} —— connected to Node ①

$K_1 \; B_{12} \; M_2 \; K_2$ & B_2 ⟶ connected to Node ②

where K_1 & B_{12} are common b/w node ① & ②

$f(t) \rightarrow i(t)$ | $M_1 - C_1$ | $B_1 - 1/R_1$ | $K_1 - 1/L_1$
$V_1 \rightarrow V_1$ | $M_2 - C_2$ | $B_2 - 1/R_2$ | $K_2 - 1/L_2$
$V_2 \rightarrow V_2$ | | $B_{12} - 1/R_{12}$ |



by nodal equations

at node ①

$$C_1\frac{dV_1}{dt} + \frac{1}{R_1}V_1 + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{L_1}\int(V_1 - V_2)\,dt = i(t) \quad (7)$$

at node ②

$$C_2\frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{L_1}\int(V_2 - V_1)\,dt = 0 \quad (8)$$

eq ⑦ & ⑧ are similar to that of ③ & ④ eq'ns

Electrical Analogous to Mechanical Rotational System

The three basic elements 1) Moment of inertia (J) 2) Rotational dashpot (B) and 3) Torsional spring (K) that are used in modelling mechanical rotational systems are analogous to resistance, inductance and capacitance of electrical system.

The input in mechanical system (Rotational) i.e, torque is analogous to either voltage or current source in electrical system. The output i.e, angular velocity in mechanical rotation system is analogous to either current or voltage in electrical system.

Since electrical system has two types of inputs i.e, voltage source and current source there are two types of analogies mainly

→ Torque - voltage analogy

→ Torque - current analogy

Torque - voltage Analogy :-

Mechanical system

Input - torque

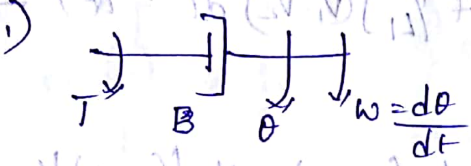
output - angular velocity

electrical system

input - Voltage source

output - current

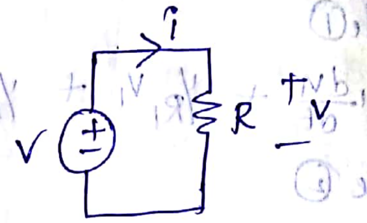
1)



$$T = T_b = B\frac{d\theta}{dt}$$

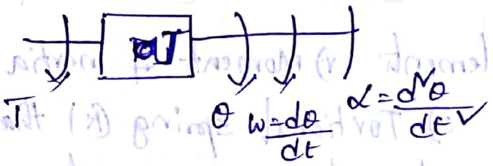
$$\boxed{T = B\omega}$$

1)



$$\boxed{V = iR}$$

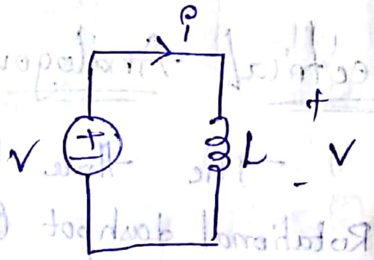
2)



$$T = T_J = J\frac{d^2\theta}{dt^2}$$

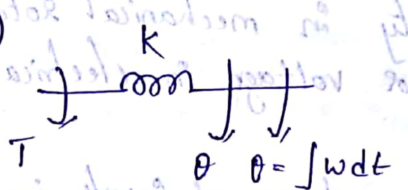
$$\boxed{T = J\frac{d\omega}{dt}}$$

2)



$$\boxed{V = L\frac{di}{dt}}$$

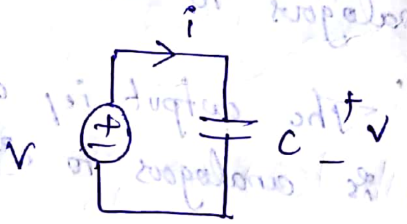
3)



$$T = T_K = K\theta$$

$$\boxed{T = K\int \omega \, dt}$$

3)



$$\boxed{V = \frac{1}{C}\int i \, dt}$$

The electrical analogous elements for the elements of mechanical rotational system are given below

$T \rightarrow e(t) \,|\, v(t)$

$\omega \rightarrow i$ $\quad$ $J \rightarrow L$

$B \rightarrow R$ $\quad$ $K \rightarrow 1/C$