



TEXAS TECH UNIVERSITY

Industrial Engineering

**"Optimizing Kidney Exchange: An Operations Research Approach to
Maximizing Transplant Matches"**

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Introduction

Organ transplantation is a life-saving medical procedure, yet the process often faces significant challenges due to the shortage of compatible donors. Kidney Paired Donation (KPD) programs provide an innovative solution to this problem, enabling patients with willing but biologically incompatible donors to exchange kidneys in a way that maximizes the number of successful transplants. In this project, we will focus on developing an optimal KPD plan for the Organ Procurement and Transplantation Network (OPTN) as part of the KPD Pilot Program in the United States.

The primary goal of this project is to design an algorithmic solution that maximizes the number of compatible kidney transplants using the principles of Operations Research. This involves pairing or grouping incompatible patient-donor pairs in a manner that ensures the highest possible number of successful transplants. To achieve this, we will use advanced optimization tools and techniques to identify feasible matches in data sets containing incompatible pairs. Our focus will include finding both two-way exchanges (two incompatible pairs swapping kidneys) and three-way exchanges (three pairs involved in the exchange).

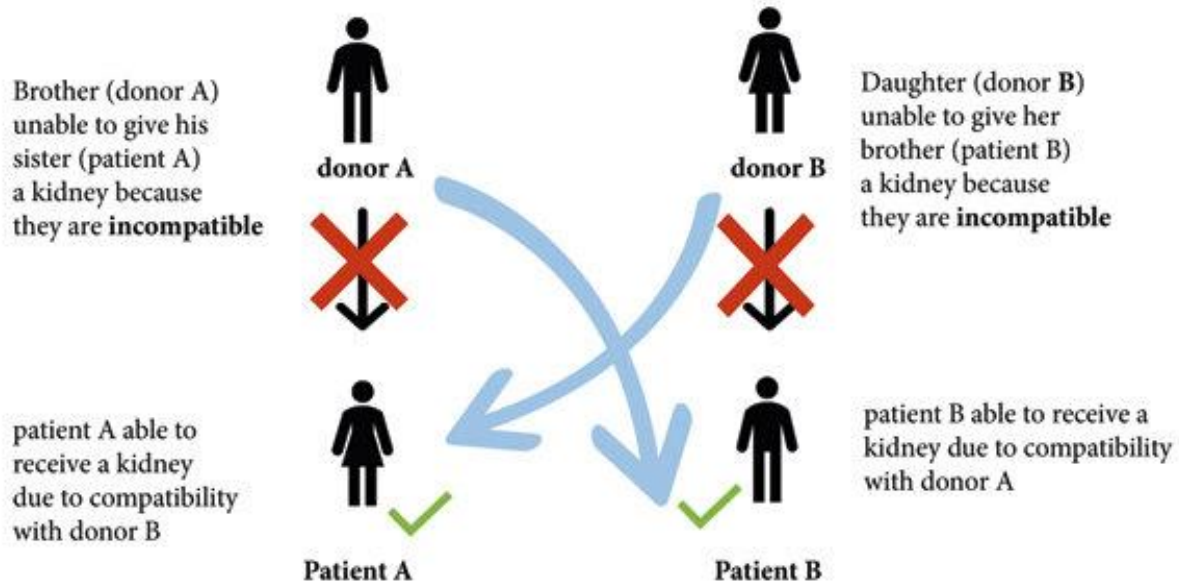


Figure: A fig showing the simple two-kidney paired exchange

A key aspect of this project is transparency and reproducibility. All input data, code, and results will be well-documented and shared as part of the deliverables. A comprehensive report will

outline the methodology employed, the KPD plans proposed, and the justification for their selection based on the outcomes achieved. Through this project, we aim to contribute to the development of effective strategies that can enhance the efficiency and fairness of kidney donation systems, ultimately saving more lives.

Kidney Paired Donation Criteria

Kidney Paired Donation (KPD) is a program designed to help patients who have incompatible donors by matching them with other incompatible pairs to enable kidney exchanges. The detailed criteria for KPD include:

Blood Type Compatibility

- **Donor-Recipient Matching:** Blood type compatibility is crucial for transplantation. Each donor's blood type must match the recipient's blood type from another incompatible pair.
- **Matching Rules:**
 - Blood type A can donate to A or AB.
 - Blood type B can donate to B or AB.
 - Blood type O (universal donor) can donate to any blood type.
 - Blood type AB (universal recipient) can receive from any blood type.

Blood type compatibility forms the foundation of successful kidney transplants. Ensuring that a donor's blood type matches the recipient's needs minimizes the risk of organ rejection and ensures long-term transplant success. The outlined matching rules provide a clear guideline for determining potential pair compatibility, making KPD both systematic and efficient.

Matching Size and Cycles

- **2-Cycles:** Two donor-recipient pairs exchange kidneys (Pair 1's donor matches Pair 2's recipient, and vice versa).
- **3-Cycles:** Three pairs participate in an exchange where the donor from one pair matches the recipient in another, creating a loop of three successful transplants.
- Larger cycles are avoided due to logistical and medical constraints, such as the need for simultaneous surgeries.

KPD programs not only increase the availability of compatible kidneys but also enhance fairness and equity in organ allocation. By adhering to transparent and ethical practices, the program

ensures that patients with incompatible donors still have a viable pathway to transplantation, significantly improving survival rates and quality of life.

Problem Statement

The project involves developing a Kidney Paired Donation (KPD) plan for the Organ Procurement and Transplantation Network (OPTN) as a consultant in the KPD Pilot Program. The goal is to maximize the number of compatible kidney transplantations among incompatible donor-recipient pairs using Operations Research tools and techniques.

Increasing Transplant Opportunities for Incompatible Pairs:

- Despite the presence of willing donors, many patients face challenges due to blood type or tissue incompatibility, leaving them without access to life-saving transplants. The problem is to create a systematic approach through Kidney Paired Donation (KPD) to identify optimal matches, maximize the number of successful transplants, and address the growing gap between organ demand and supply.

Reducing Transplant Waiting Times:

- Many patients spend years on transplant waiting lists due to the lack of compatible donors. The problem focuses on leveraging Kidney Paired Donation (KPD) programs to expedite the matching process, reduce waiting times, and increase the likelihood of timely, life-saving transplants.

An OR Formulation (in words and math)

Decision Variables

The decision variables in the model represent whether a specific cycle C is selected or not:

$$X_C = \begin{cases} 1 & \text{if cycle } C \text{ is selected for inclusion in the solution} \\ 0, & \text{otherwise} \end{cases}$$

Where:

- X_C A binary variable that takes the value:
- 1 if cycle C is selected for inclusion in the solution.
- 0 otherwise.

Each cycle C corresponds to a set of compatible donor-recipient pairs forming a directed cycle in the graph. These cycles are precomputed and passed to the optimization model.

Objective Function

The objective of the model is to maximize the total number of transplants, which is proportional to the size (number of pairs) of the selected cycles. The objective function can be expressed mathematically as:

$$\text{Maximize } \sum_{C \in \text{Cycles}} \text{size}(C) \cdot X_C$$

Where:

- $\text{Size}(C)$ is the number of pairs involved in cycle C (e.g., 2 for a 2-cycle, 3 for a 3-cycle).
- X_C : The binary variable indicating whether cycle C is selected.

This function ensures that the optimization process prioritizes cycles that maximize the total number of transplants.

Constraints

The model imposes constraints to ensure feasibility and practicality in the solution. These include:

1. **Disjoint Cycles Constraint:** Each donor-recipient pair (node in the graph) can participate in at most one selected cycle. This prevents overlaps in participation and ensures every pair is only involved in one transplant exchange.

Mathematically, this constraint can be written as:

$$\sum_{C \text{ includes node } i} X_C \leq 1, \forall i$$

Where:

- i : A node (donor-recipient pair) in the graph.
- C : A cycle in the precomputed list of cycles.
- X_C : The decision variable for cycle C .

The summation ensures that for any node i , the total number of selected cycles that include i does not exceed 1.

2. **Cycle Size Constraints:** To maintain practicality and logistical feasibility, only cycles of size 2 (two-way exchanges) or size 3 (three-way exchanges) are considered. This is implemented during the preprocessing step where cycles are identified, and the model only includes cycles meeting this size restriction.

The optimization model for kidney-paired donation aims to maximize the total number of successful transplants by selecting compatible cycles of donor-recipient pairs. The decision variable X_C determines whether a precomputed cycle C (of size 2 or 3) is included in the solution, with the objective function maximizing the sum of $\text{size}(C) \cdot X_C$, ensuring priority for larger cycles. Constraints ensure that each donor-recipient pair (node) participates in at most one cycle to prevent overlap, and only cycles of size 2 or 3 are considered to maintain logistical feasibility. This model efficiently allocates resources to maximize transplantation success.

Python/Gurobi Code

The following GitHub repository provides my Python/Gurobi code and data

https://github.com/Chakri08kc/IE_5318_OR_Project

Experiment Discussion

The developed optimization model for the Kidney Paired Donation (KPD) problem was implemented using Python and solved using the Gurobi Optimizer. The model leveraged graph-based representations of incompatible donor-recipient pairs, and precomputed cycles of sizes 2 were 36337 and 3 were 0. Using this framework, we were able to construct a robust solution that maximized the number of compatible kidney exchanges.

Key aspects of the experimental setup include:

- **Software and Tools:** The Gurobi Solver version 10.0.2 was used within the Python programming environment, executed in Jupyter Notebook.

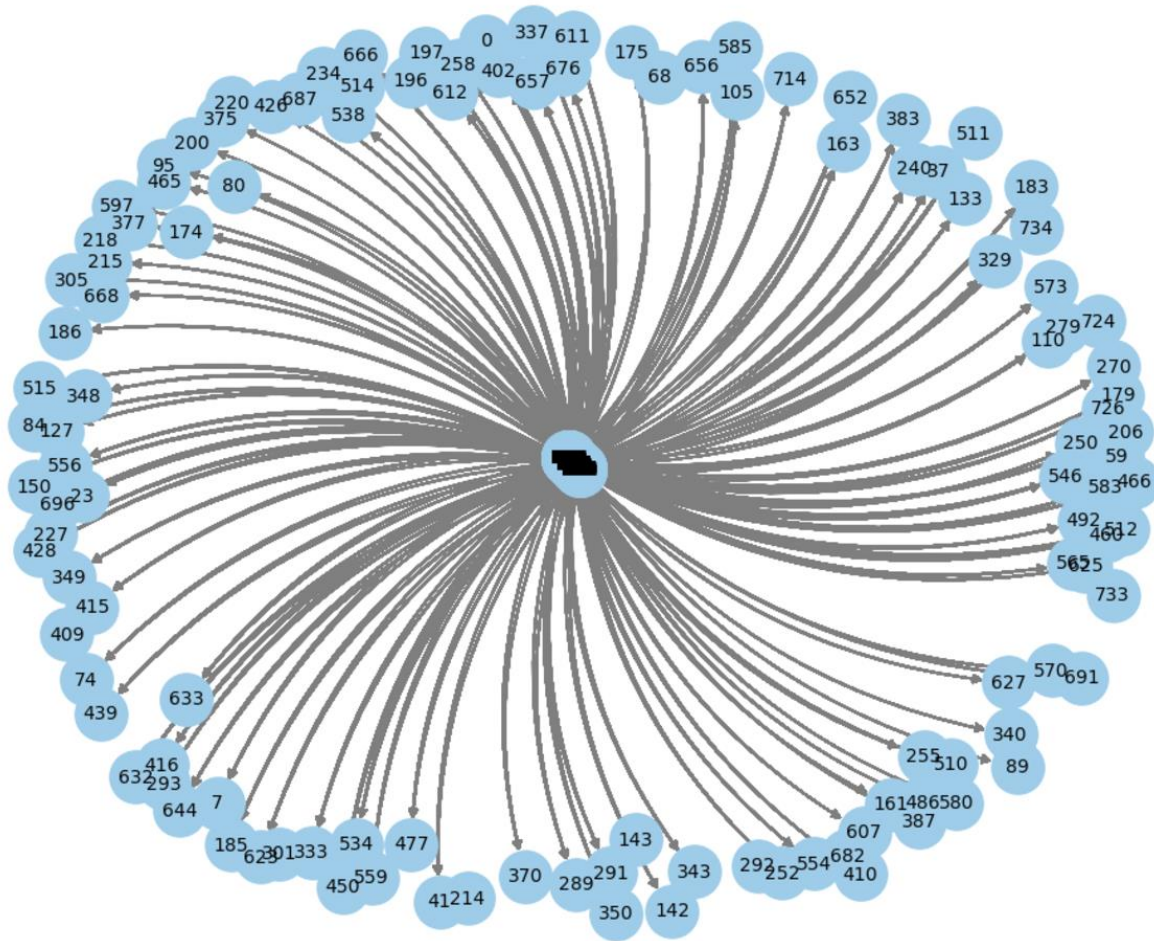
- **Hardware:** The experiment was run on an Apple MacBook Air with 8 GB of RAM, ensuring sufficient computational resources to handle the dataset and optimization process.
- **Performance Metrics:** The model achieved an objective value of 364 compatible pairs, representing the number of successful transplants, in just 0.66 seconds. This result underscores the efficiency of the optimization approach.

The model demonstrates scalability and efficiency in solving the KPD problem by adhering to the constraints of disjoint cycles and practical cycle sizes.

An Optimal Plan

The proposed KPD plan efficiently matches donor-recipient pairs through the use of two-way and three-way cycles, ensuring the following:

1. **Maximization of Transplants:** The algorithm identifies the maximum possible number of compatible kidney exchanges, thereby saving more lives and reducing the burden on transplant waiting lists.
2. **Feasibility of Implementation:** By restricting cycles to sizes of 2 and 3, the plan ensures logistical and operational feasibility, including the simultaneous execution of surgeries to prevent any disruption in the exchange process.
3. **Transparency and Fairness:** The optimization process adheres to ethical guidelines by ensuring equitable distribution of available organs, minimizing the risk of preferential treatment or inequity.



Connection to the Graph

The graph visualizes relationships between donors and recipients. Here's a breakdown:

1. Central Node (Black Node):

- This represents the central hub or the point of origin in the kidney exchange chain.
- It might symbolize an initial donor initiating a chain of transplants, such as an altruistic donor or a starting point for paired donation.

2. Peripheral Nodes (Blue Nodes):

- Each blue node represents a donor-recipient pair or an individual within the KPD network.
- The numbers inside these nodes are identifiers for the pairs.

3. Directed Edges (Arrows):

- The arrows between nodes indicate compatibility (e.g., a donor from one pair can donate to the recipient in another pair).

- For example, the arrow from the central node to a peripheral node signifies that the donor in the central pair can donate to the recipient in the peripheral node.

Insights from the Diagram

1. Compatibility Matching:

- The diagram suggests a situation where one donor (possibly an altruistic donor) initiates a chain of transplants, providing kidneys to a wide range of recipients.
- This layout aligns with the KPD goal of creating long chains to maximize successful matches.

2. Optimization Goal:

- The graph may be an output of a mathematical optimization program (e.g., linear programming or network flow analysis).
- It demonstrates how the operations research techniques are applied to find the most efficient way to match donor-recipient pairs.

3. Constraints Reflected:

- There are no overlapping edges or cycles in this visualization, indicating that each transplant match is unique and respects medical and logistical constraints.

Before Optimization

• Total Nodes: 740

- The program starts with 740 donor-recipient pairs or individuals in the network.
- This is the raw dataset before applying any filtering or matching optimization.

• Total Edges: 187,106

- The network contains 187,106 potential compatibility relationships.
- This number indicates the total number of possible pairings, based purely on medical compatibility, before additional constraints or priorities are applied.

After Optimization

• Selected Nodes: 728 (98.38%)

- After the optimization process, 728 nodes were chosen for inclusion in the final matching solution.
- This represents **98.38%** of the original nodes, meaning almost all donor-recipient pairs or altruistic donors were successfully matched or utilized.
- The small drop (12 nodes) likely occurred due to constraints such as:

- No compatible match was found for some pairs.
- Logistic or operational issues preventing certain transplants.
- Prioritization of longer chains or more matches leads to some nodes being excluded.
- **Selected Edges: 364 (0.19%)**
 - Out of the initial 187,106 edges, only 364 edges were included in the optimized solution.
 - This significant reduction (to just 0.19% of the original edges) reflects the stringent optimization criteria:
 - Only the **most efficient matches** were retained to maximize transplant success rates and minimize logistical complexity.
 - Many compatibility edges (representing theoretically possible matches) were excluded due to program priorities or chain constraints.

Evaluation of the Plan

The evaluation of the proposed plan highlights several strengths:

- **Efficiency:** The rapid computation time ensures that the model can be integrated seamlessly into real-world kidney allocation systems.
- **Scalability:** The approach is adaptable to larger datasets, making it applicable to nationwide or global organ donation networks.
- **Robustness:** The model's adherence to key medical and logistical constraints ensures practical applicability without compromising on the ethical principles of organ allocation. Potential areas for improvement include:
 - **Dynamic Matching:** Incorporating real-time updates to the donor-recipient database could further enhance the model's effectiveness.
 - **Extended Cycle Considerations:** While limited to 2 and 3 cycles in this study, future work could explore the potential of larger cycles with appropriate logistical support.

Conclusions

This project successfully demonstrates the application of Operations Research to address the challenges in Kidney Paired Donation (KPD) programs. By developing and implementing an optimization model, we have provided a solution that maximizes the number of successful transplants, reduces waiting times, and enhances fairness in kidney allocation. The use of advanced optimization tools, such as the Gurobi Solver, ensures the scalability and efficiency of the solution. The results achieved in this study highlight the potential for computational approaches to significantly improve organ transplantation systems. With further development, such models could play a pivotal role in addressing the growing demand for organ transplants, ultimately saving more lives and improving the quality of life for patients in need.