Week 1 Number Bases Lecture Note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** Number Bases	Course: BSc Computer Science
		Class: Computational Mathematics[Lecture]
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Essential Question:

What is a number base and what are the different number systems available for use in computing?

Questions/Cues:

- What does digit mean?
- What is Base 10?
- What is Base 2?
- What is Base 16 (hexadecimal)?
- How do we convert a number in a generic base b to decimal?
- How do we convert a decimal number to another base?

Notes

- Digit = finger in Latin
- Base 10 = uses digits 0-9 and when you count to 9, you go back to 0 and a digit to the left starting with 1 to denote the continuation of the counting
    - Largest number with single digit in Base 10 is 9, with two digits is 99 and lastly three digits 999
- Base 2 = uses digits 0 and 1, where when you reach 1 and wish to continue, you "roll" the rightmost digit(s) to 0 and place a "1" digit in the leftmost space to continue counting, eg. 1, 2 = 10, 3 = 11, 4 = 100
    - Powers of 10 are rep'ed by multiples of 2 when viewing binary to decimal. Ex. 2 = 10, 4 = 100, 8 = 1000, 16 = 1000

decimal	binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

Notice that $10 = 2, 100 = 4, 1000 = 8, 10000 = 16$

*** **For base n, use digits 0, 1,... n-1**

Number bases: Examples

Base 10 digits 0,1,…9

$127_{10} = 1\times10^2 + 2\times10^1 + 7\times10^0 = 100+20+7$

Base 2 digits 0,1

$1001_2 = 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0 =$
$= 1\times8 + 0\times4 + 0\times2 + 1\times1 = 9_{10}$

Number bases: Examples (cont'd)

Base 2 to base 10: convert 100101011_2

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1
1	0	0	1	0	1	0	1	1

$256 + 0 + 0 + 32 + 16 + 8 + 4 + 2 + 1$
$= 299_{10}$

- Base 16 (hexadecimal) = uses 16 digits from 0-9 and A-F (denoting numbers 10 to 15). When counting in base 16, just like in other number systems, when you run of digits, you place a 0 in the rightmost digit(s) and place "1" digit to continue counting.

Base 16 hexadecimal

16 digits
0,1,…..9,A,B,C,D,E,F

Counting 0,1,…9,A,…F, 10,..,19,1A,…
..1E,1F, 20,21…1FE,1FF,200..

Examples 1F= $1x16^1+15x16^0=31_{10}$

Generic base *b*, conversion to decimal

$a_n\ a_{n-1}\ a_{n-2}\ldots.a_0$

In decimal units corresponds to

$a_n \times b^n + a_{n-1} \times b^{n-1}+\ldots+a_0 \times b^0$

- To convert a decimal number to another base, we use the method of repeated division, where we divide the decimal number by the desired base until the decimal number is reduced to zero and then when listing our answer in the new base form we read the remainders bottom to top; placing each digit of the remainders left to right

Method: repeated division

General example: convert 58_{10} in base 2

58/2 =29 r 0
29/2 =14 r 1
14/2 = 7 r 0
7/2 = 3 r 1
3/2 = 1 r 1
1/2 = 0 r 1

read the remainders bottom up
→ 58_{10} =111010_2

Summary

In this week, we learned about the different number systems and how to convert between them.