Week 7 Angles, Triangles, Trigonometry Lecture Note

Notebook:	Computational Mathematics	
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Cornell Notes	Angles, triangles, trigonometry	Course: BSc Computer Science
		Class: Computational Mathematics[Lecture]
		Date: May 14, 2020

Essential Question:

What are angles and what is trigonometry and how are these related to the study of triangles?
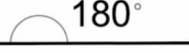
Questions/Cues:

- What is an angle?
- What are some special types of angles, how do we measure radians and how do we convert between radians/degrees?
- What are some properties of triangles?
- What are similar triangles?
- What are properties of right triangles?

**Notes**

What is an angle?

- It is a measure of the separation of two rays emanating from a vertex *v*
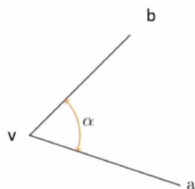
- It is measured in degrees (sexagesimal) or radians

- 1 Degree is 1/180 of a flat angle $180°$

- 1 min is 1/60 of a degree, 1 sec is 1/60 of min

- ex. 35° 23' 12"

  - A flat angle is just the separation between two rays that emanate from the vertex and they depart in opposite direction. A flat angle corresponds to $180°$

- When working with degrees, we are working in base 60

Types of angle

90° right angle

180° flat angle

360° complete angle

Arc length equal to radius

Radius r
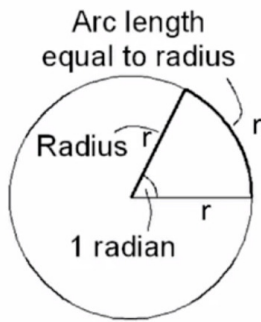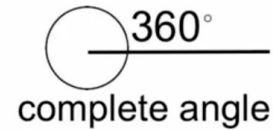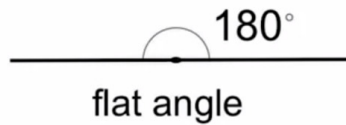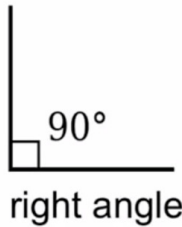
1 radian

$r \rightarrow 1$ radian $\Rightarrow$ (circ.) $2\pi r \rightarrow 2\pi$ radians

$360° = 2\pi$ radians

radians = degrees $\times \pi / 180°$

- The complete angle is the angle that is formed by two rays emanating from a vertex when they coincide
- Angles are periodic, in degree they periodical of $360°$
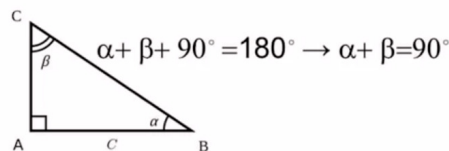
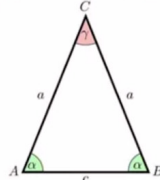# Triangles: properties
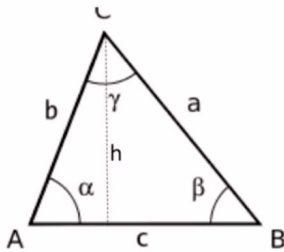
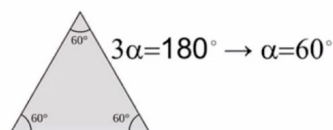$\alpha + \beta + \gamma = 180°$ $S = c \times h/2$ $P = a + b + c$

Triangle: Right trian.

$\alpha + \beta + 90° = 180° \rightarrow \alpha + \beta = 90°$

Isosceles

Equilateral

$3\alpha = 180° \rightarrow \alpha = 60°$

- In a isosceles triangle, two sides are same length and the angles adjacent to the non-equal side are equal
- Similar Triangles = have the same angles but they have sides which are rescaled by the same rescaling factor; they have the same angles and proportional sides. This means the ratio between each side of one triangle and the corresponding side of the similar triangle are the same
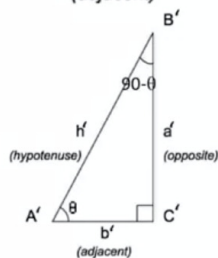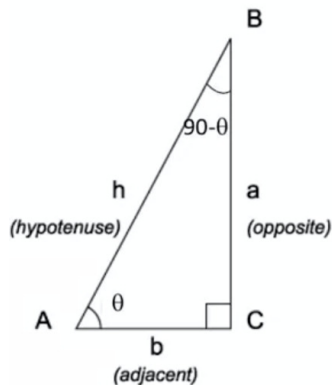
Similar Triangles
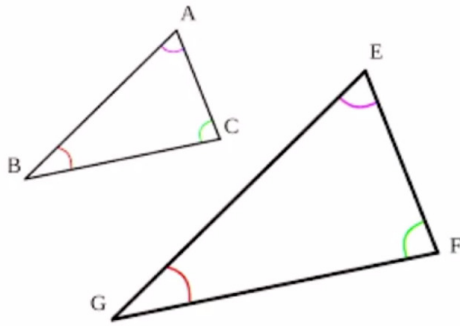
Similar Triangles
rescale one (zoom in or out) and
will coincide with the other

Same angles, proportional sides:

$\rightarrow$ AB/EG=AC/EF=BC/GF

Right Triangles: properties

h/h'=b/b' $\rightarrow$ b/h=b'/h'=Cos(θ)=Sin(90-θ)
h/h'=a/a' $\rightarrow$ a/h=a'/h'=Cos(90-θ)=Sin(θ)

a/a'=b/b' $\rightarrow$ a/b=a'/b'=$\dfrac{Sin(\theta)}{Cos(\theta)}$ =Tan(θ)

1)Sin(θ)=a/h=$\dfrac{opposite}{hypotenuse}$ $\rightarrow$ a=h Sin(θ)
2)Cos(θ)=b/h=$\dfrac{adjacent}{hypotenuse}$ $\rightarrow$ b=h Cos(θ)
3)Tan(θ)=a/b=$\dfrac{opposite}{adjacent}$ $\rightarrow$ a=b Tan(θ)

$a^2+b^2=h^2$ *Pithagora's theorem*

From 1) and 2) it follows

$h^2 Sin^2(\theta)+h^2 Cos^2(\theta)=h^2$

$\rightarrow$ $Sin^2(\theta)+Cos^2(\theta)=1$

→ $a = h\,\mathrm{Sin}(\theta)$
→ $b = h\,\mathrm{Cos}(\theta) = h\,\mathrm{Sin}(90^\circ - \theta)$
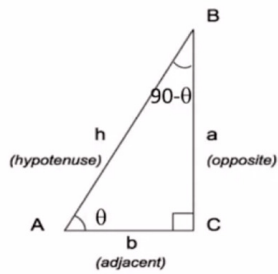
what if $\theta \to 0$? $h \to b$ and $a \to 0$
➡ $\mathrm{Cos}(0^\circ) = \mathrm{Sin}(90^\circ) = 1,\quad \mathrm{Sin}(0^\circ) = \mathrm{Cos}(90^\circ) = 0$

➡ $a/\mathrm{Sin}(\theta) = b/\mathrm{Sin}(90^\circ - \theta) = h = h/\mathrm{Sin}(90^\circ)$
    Sine rule

General triangle

$a/\mathrm{Sin}(\alpha) = b/\mathrm{Sin}(\beta) = c/\mathrm{Sin}(\gamma)$
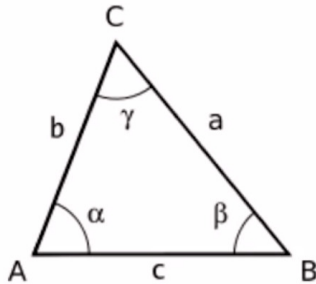
generalized *Pithagora's th.*

→ $a^2 = b^2 + c^2 - 2bc\,\mathrm{Cos}(\alpha)$
   (a.k.a. cosine rule)

Summary

In this week, we learned about what an angle is. Alongside this, we looked at properties of triangles, Pythagoras theorem, the laws of sines and cosines