Week 13 Exponential and Logarithmic functions lecture note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** **Exponential and Logarithmic functions**	Course: BSc Computer Science Class: Computational Mathematics[Lecture] Date: July 21, 2020

Essential Question:

What are the exponential and logarithmic functions?

Questions/Cues:

- What are laws of indices/exponents used to simplify exponential expressions?
    - From this, what is the form of the exponential function?
- What are various graphs of the exponential function like?
- What are the properties of the exponential function?

Notes

Definitions

Integer $a^n = a \times a \times \ldots \times a$ ($\rightarrow \sqrt[n]{\ }$ inverse)

$\rightarrow$ $a^{n+m} = a^n \times a^m$ $a \geq 0$

prove also: 2) $(a^n)^m = a^{nm}$

3) $(a\,b)^n = a^n b^n$

Rational power $a^{n/m} = (\sqrt[m]{a})^n$ or $\sqrt[m]{a^n}$

First is defined only for m odd if a<0,

second for any m and a if n even

$\rightarrow$ Always defined if a>0

$(4)^{3/2} = (\sqrt{4})^3 = \pm 8$ by convention 8

Definitions

$a^{n/m+q/k}$?

$a^{n/m+q/k} = a^{(nk+mq)/(mk)} = \sqrt[mk]{a^{nk} a^{mq}} = \sqrt[mk]{a^{nk}}\ \sqrt[mk]{a^{mq}} = a^{n/m}\, a^{q/k}$

Extension to all real numbers:

for any real number x you can truncate it

Ex. $\pi = 3.1415\ldots \approx 31415/10000$

$x \rightarrow y(x) = a^x$ with a>0

Properties:

$a^{x+y} = a^x \times a^y$

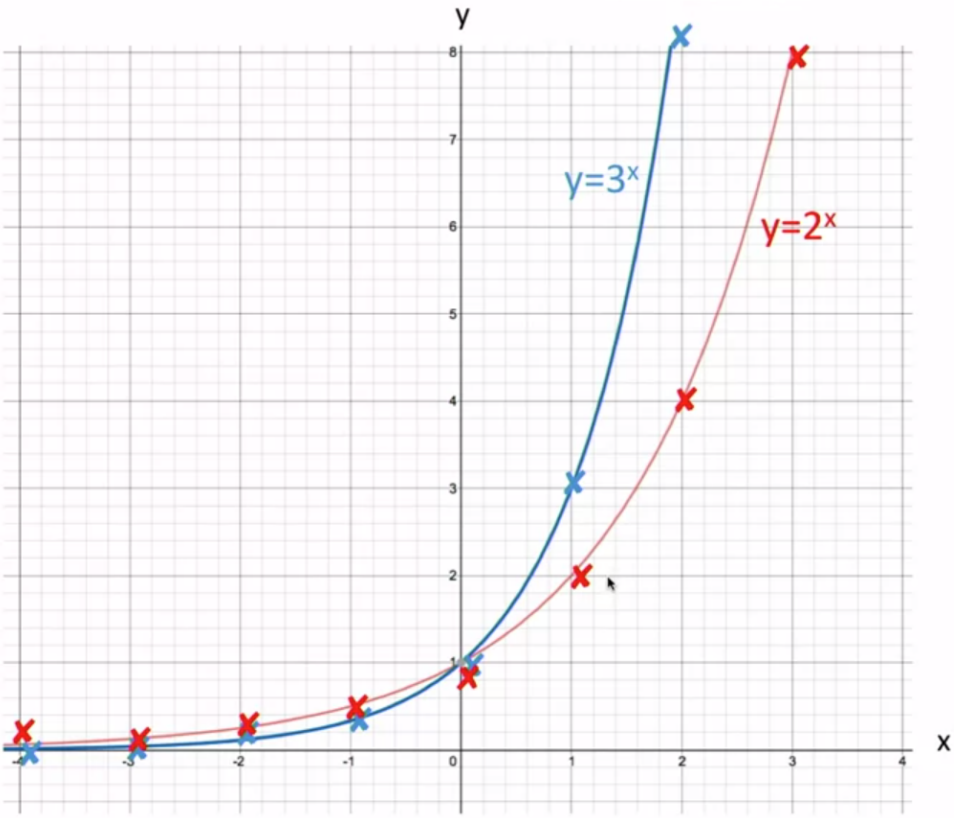
$(a^x)^y = a^{xy}$ $(ab)^x = a^x b^x$

Graphs

Using a table of values plot graphs of $f(x) = a^x$ for $a = 2, 3$ for $-4 \leq x \leq 4$

x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = 2^x$	$\frac{1}{16} \cong 0.06$	$\frac{1}{8} \cong 0.13$	$\frac{1}{4} = 0.25$	$\frac{1}{2} = 0.5$	1	2	4	8	16

x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = 3^x$	$\frac{1}{81} \cong 0.01$	$\frac{1}{27} \cong 0.04$	$\frac{1}{9} \cong 0.1$	$\frac{1}{3} \cong 0.33$	1	3	9	27	81



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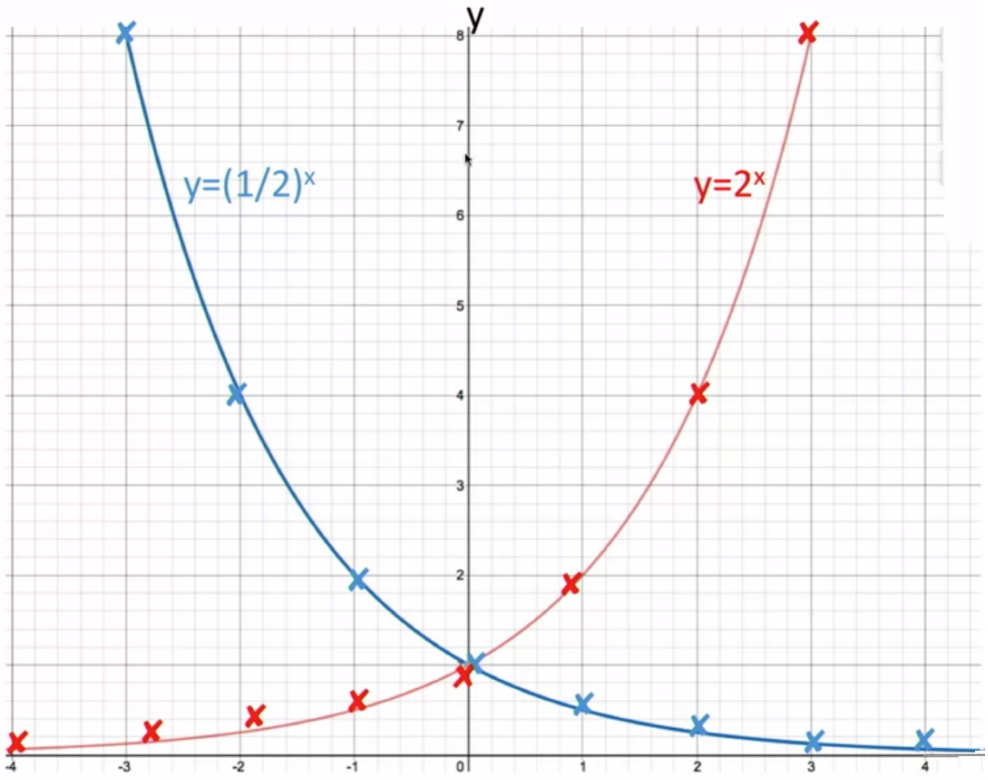
x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = \left(\frac{1}{2}\right)^x$	16	8	4	2	1	$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{8} \cong 0.13$	$\frac{1}{16} \cong 0.06$



Summary properties

a) $f(x) = a^x$ only defined $a > 0$ (so $f(x) = a^x$ is not defined for $a < 0$ (what happens for $a = 0$?))
b) For all a, $f(x) = a^x$ has a $y-$ intercept of 1, that is the graph passes through (0,1)
c) For all a, the graph of $f(x) = a^x$ pass through (1, a)
d) For a > 1 $f(x) = a^x$ is increasing
e) For a < 1 $f(x) = a^x$ is decreasing (and for a = 1?)
f) $f(x) = a^x > 0$ for all x
g) the x$-axis$ is an asymptote
h) For $a > 1$ the bigger a is the more rapidly $f(x) = a^x$ increases
i) For $a < 1$ the smaller a is the more rapidly $f(x) = a^x$ decreases

Summary

In this week, we learned about the laws of exponents, what the exponential function is, the different graphs of the exponential function and the properties of the exponential function.