


Week 3 Sequences and Series Lecture Note

Notebook: Computational Mathematics

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Cornell Notes	Topic: Sequences and Series	Course: BSc Computer Science
		Class: Computational Mathematics[Lecture]
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Essential Question:		
What are sequences?		
Questions/Cues:		
<ul style="list-style-type: none">• What is a geometric sequence?• What is an arithmetic sequence?• What is the formal definition of a sequence?• What is the Fibonacci Sequence?		
Notes		
<ul style="list-style-type: none">• Geometric Sequence = has the property that the ratio between a value in the sequence and its previous value is constant, it can any number q with a being the initial value of the sequence. The general form of a geometric sequence is as followed:<div><div>Geometric sequence</div><div></div><div>$a, a \times q, a \times q^2, \dots a \times q^n ..$</div></div>• Arithmetic sequence = has the property where the nth term in the sequence is the sum of the previous term before it and a constant number q. In other words, the difference between each element of the sequence and its previous is constant.		

What is a sequence?

- Formal definition: given a set X a sequence is a function $a: \mathbb{N} \rightarrow X$
i.e. a set $a(0), a(1) \dots a(n) \dots$ denoted with $\{a_n\}_{n \in \mathbb{N}}$
- Can be defined explicitly $a_n = f(n)$
example: $a_n = 2n + 1 \rightarrow 1, 3, 5, 7, 9 \dots$
- or by recursion i.e. $a_n = f(a_{n-1}, \dots, a_{n-k})$
example 1: *Arithmetic sequence* $a_n = q + a_{n-1}$
 $\rightarrow a_0, a_0 + q, a_0 + 2q, \dots \rightarrow a_n = n \times q + a_0$
example 2: *Geometric sequence* $a_n = q \times a_{n-1}$
 $\rightarrow a_0, a_0 q, a_0 q^2, a_0 q^3 \dots a_0 q^n \dots \rightarrow a_n = a_0 \times q^n$
- A sequence is said to be convergent, when upon increasing n it approaches a finite constant value

$$\lim_{n \rightarrow \infty} a_n = L < \infty$$

example: geometric seq. $q < 1 \rightarrow q = 1/2$
 $1/2, 1/4, 1/8, \dots, 1/64, \dots$ converges to 0

- A sequence is said to be divergent when increasing n it never reaches a constant finite value (either goes to ∞ or oscillates)
example: geometric seq. $q > 1 \rightarrow q = 2$
 $2, 4, 8, 16, 32, \dots, 256, \dots$

Examples: Fibonacci Sequence

- Definition by recursion: $a_0 = 0, a_1 = 1, a_n = a_{n-1} + a_{n-2}$
- $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots$
- $\frac{a_n}{a_{n-1}} \rightarrow \varphi = \frac{1 + \sqrt{5}}{2} = 1.618 \dots$ Golden Ratio



Summary

In this week, we learned about what a sequence is and the two different kinds of sequences, Geometric and Arithmetic. Finally, in the end we look at a famous example of a sequence

called the Fibonacci Sequence.