Week 14 Exponential and Logarithmic functions continued lecture note

Notebook:	Computational Mathematics
Created:	2020-04-21 2:48 PM
Updated:	2020-07-21 4:24 PM
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URL:	https://www.coursera.org/learn/uol-cm1015-computational-mathematics/home/week/13

Cornell Notes	**Topic:** Exponential and Logarithmic functions continued	Course: BSc Computer Science Class: Computational Mathematics[Lecture] Date: July 21, 2020

Essential Question:

What are the exponential and logarithmic functions?

Questions/Cues:

- What are some definitions/properties pertaining to logarithms?
- What are the different graphs of the logarithmic function?
- What are the properties of the logarithmic function?

Notes

Definitions

if $x = f(y) = a^y \xrightarrow{\text{inverse}} y = f^{-1}(x) = \log_a x$

$\rightarrow f(f^{-1}(x)) = f^{-1}(f(x)) = x \quad \rightarrow a^{\log_a x} = x \quad \rightarrow \log_a(a^x) = x$

Always defined if a>0

Properties

$\log_a(x \times y) = \log_a(x) + \log_a(y)$

proof: replace identities $a^{\log_a x} = x \quad a^{\log_a y} = y$

$\log_a(x \times y) = \log_a(a^{\log_a x} \times a^{\log_a y}) = \log_a(a^{(\log_a x + \log_a y)}) = 1$

Properties

$\log_a(x^b) = b \times \log_a(x)$

proof: replace $a^{\log_a x} = x$

$\rightarrow \log_a((a^{\log_a x})^b) = \log_a(a^{b \log_a x}) = b \log_a(x)$

$$log_a x = \frac{log_c(x)}{log_c(a)}$$

proof: replace identity $a^{\log_a x} = x$

$\rightarrow \log_c(a^{\log_a x}) = \log_a(x) \log_c(a)$

$$\rightarrow \frac{log_c(x)}{log_c(a)} = \frac{\log_a(x) \log_c(a)}{\log_c(a)} = \log_a(x)$$

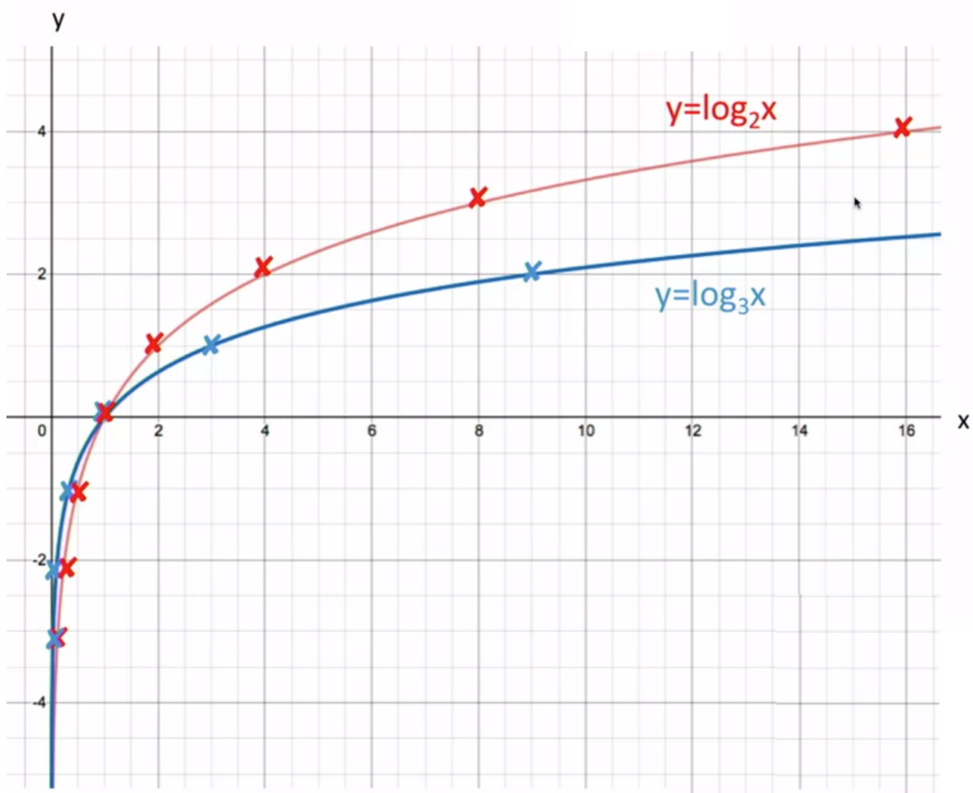
Graphs

Using a table of values plot graphs of $f(x) = \log_a x$ for a = 2, 3,

x	-2	0	1	2	4	8	16	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$f(x) = \log_2 x$	undefined	undefined	0	1	2	3	4	-1	-2	-3

x	-3	0	1	3	9	27	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$
$f(x) = \log_3 x$	undefined	undefined	0	1	2	3	-1	-2	-3

$\log_a x$ only defined for x>0



Graphs

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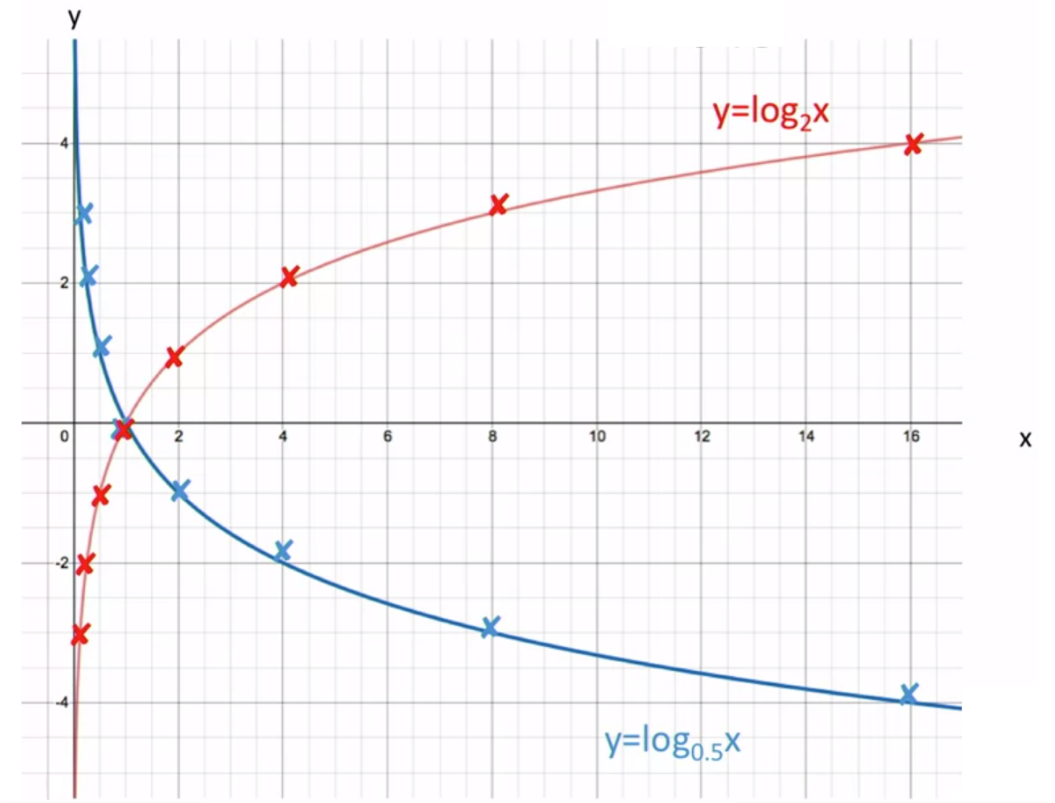
x	-2	0	1	2	4	8	16	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$f(x) = \log_{\frac{1}{2}} x$	undefined	undefined	0	-1	-2	-3	-4	1	2	3



Summary properties

a) $f(x)=\log_a x$ is only defined for $a>0$
b) $f(x)=\log_a x$ is only defined for $x>0$
c) For all a, $f(x)=\log_a x$ has an $x-$intercept of 1, that is the graph passes through (1,0)
d) For all a, the graph of $f(x)=\log_a x$ passes through $(a,1)$
e) For a>1 $f(x)=\log_a x$ is increasing
f) For a<1 $f(x)=\log_a x$ is decreasing (and for a=1?)
g) the $y-axis$ is an asymptote
h) For $a>1$ the bigger a is the more slowly $f(x)=\log_a x$ increases
i) For $a<1$ the smaller a is the more slowly $f(x)=\log_a x$ decreases
j) lg(x) or Log(x) indicates $\log_{10}$(x)
and ln(x) or log(x) denotes $\log_e$ (x) e=2.71828…

$a^x \quad a>0$

1) $a^{x+y} = a^x \cdot a^y$
2) $(a^x)^y = a^{x\cdot y}$
3) $a^0 = 1$
4) $a^{-n} = \frac{1}{a^n}$

• $\frac{e^{3x}}{e^x} = e^{3x}\underbrace{\left(\frac{1}{e^x}\right)}_{=e^{-x}} = e^{3x}\cdot e^{-x} = e^{3x-x} = e^{2x}$

• $\sqrt{e^{6t}} = (e^{6t})^{\frac{1}{2}} = e^{6\cdot t\cdot\frac{1}{2}} = e^{3t}$

• $\left(\frac{1}{4}\right)^{-x} = \left(\frac{1}{4}\right)^{-1\cdot x} = \left(\left(\frac{1}{4}\right)^{-1}\right)^x = 4^x$

$3^{x+1} = 9 \rightarrow x$?

$3^{x+1} = 3^{2} \Rightarrow x+1 = 2 \Rightarrow x = 1$

$3^{x+1} = 10$,

$\log_3 x \rightleftarrows 3^x \Rightarrow \log_3(3^{\alpha}) = \alpha$

$\log_3(3^{x+1}) = \log_3 10$

$x + 1 = \log_3 10 \Rightarrow x = \log_3 10 - 1$

$\log_a x \quad \begin{cases} x > 0 \\ a > 0 \end{cases}$

1) $\log_a(b \cdot c) = \log_a b + \log_a c$

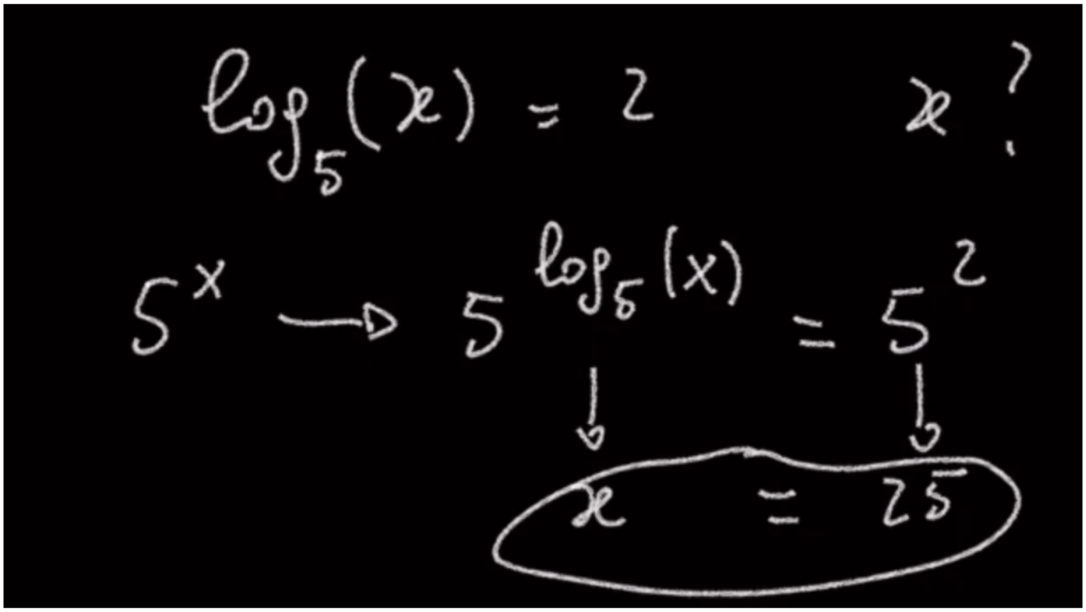
2) $\log_a(b^c) = c \cdot \log_a b$

3) $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

4) $\log_a b = \dfrac{\log_c b}{\log_c a}$

•) $\log_{10}(10 \cdot x) = \log_{10} 10 + \log_{10} x$
$= 1 + \log_{10} x$

•) $\log_3 10 = \dfrac{\log_{10} 10}{\log_{10} 3} = \dfrac{1}{\log_{10} 3}$



Summary

In this week, we learned about some definitions/properties pertaining to logarithms, the form of a logarithmic function, the different graphs of the logarithmic function and the properties of the logarithmic function.