$$\delta^{(4)} = a^{(4)} - y$$

$$\boxed{h_\theta(x) = g(z) = \frac{1}{1+e^{-z}}}$$

$$\delta^{(4)} = \frac{\partial J(\theta)}{\partial z^{(4)}}$$

$$= -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1 - h_\theta(x))$$

$$= -y \cdot \log(g(z)) - (1-y) \log(1-g(z))$$

$$= -y \frac{1}{g(z)} \cdot g'(z) - (1-y) \cdot \frac{-1}{1-g(z)} \cdot g'(z)$$

$$= -y \cdot (1+e^{-z}) \cdot \frac{e^{-z}}{(1+e^{-z})^2} - (1-y) \cdot \frac{-1}{1-\frac{1}{1+e^{-z}}} \cdot \frac{e^{-z}}{(1+e^{-z})^2}$$

$$= -y \cdot \frac{e^{-z}}{1+e^{-z}} - (1-y) \cdot \frac{-1\ \cancel{1+e^{-z}}}{\cancel{1}+\cancel{e^{-z}}-\cancel{1}} \times \frac{\cancel{e^{-z}}}{(1+e^{-z})^2}$$

$$= -y \cdot \frac{e^{-z}}{1+e^{-z}} - (1-y) \cdot \frac{-1}{1+e^{-z}}$$

$$= \frac{-y \cdot e^{-z}}{1+e^{-z}} - \frac{(y-1)}{1+e^{-z}}$$

$$= \frac{-y \cdot e^{-z} - y + 1}{1+e^{-z}}$$

$$= \frac{1 - y \cdot e^{-z} - y}{1+e^{-z}}$$

$$= \frac{1 - y\left(e^{-z}+1\right)}{1+e^{-z}}$$

$$= \frac{1}{1+e^{-z}} - \frac{y\left(1+e^{-z}\right)}{1+e^{-z}}$$

$$= g\left(z^{(4)}\right) - y$$

$$= a^{(4)} - y$$

$$\delta^{(3)} = \frac{\partial J(\theta)}{\quad} = \frac{\partial J(\theta)}{\quad} \times \frac{\partial z^{(4)}}{\quad} \times \frac{\partial a^{(3)}}{\quad}$$

$$\partial z^{(3)} \qquad \partial z^{(4)} \quad \partial a^{(3)} \quad \partial z^{(3)}$$

$$= (a^{(4)} - y) \times \Theta^{(3)} \times \frac{\partial g(z^{(3)})}{\partial z^{(3)}}$$

$$= (\Theta^{(3)})^T \times \delta^{(4)} \times g'(z^{(3)})$$

$$\delta^{(2)} = \frac{\partial J(\theta)}{\partial z^{(2)}} = \frac{\partial J(\theta)}{\partial z^{(4)}} \times \frac{\partial z^{(4)}}{\partial a^{(3)}} \times \frac{\partial a^{(3)}}{\partial z^{(3)}} \times \frac{\partial z^{(3)}}{\partial a^{(2)}} \times \frac{\partial a^{(2)}}{\partial z^{(2)}}$$

$$= (a^{(4)} - y) \times \Theta^{(3)} \times \frac{\partial g(z^{(3)})}{\partial z^{(3)}} \times \Theta^{(2)} \times g'(z^{(2)})$$

$$= \delta^{(3)}$$

$$= \left(\theta^{(2)}\right)^T \delta^{(3)} g'\left(z^{(2)}\right)$$