

Applied Machine Learning

Lecture: 7-1 Model Evaluation

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Slides adapted from Andrew NG, Eric Eaton, Raquel Urtasun, and Patrick Winston

Outline

- 7.1 Machine learning diagnostic
- 7.2 Model selection & evaluation
- 7.3 Diagnosing bias vs. variance
- 7.4 Regularization and bias/variance
- 7.5 Learning curves
- 7.6 Error metrics
- 7.7 Imbalanced data

Machine learning diagnostic

Suppose you have implemented regularized linear regression to predict housing prices.

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^m \theta_j^2 \right]$$

However, when you test your hypothesis on a new set of houses, you find that it makes unacceptably large errors in its predictions. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features (x_1^2, x_2^2, x_1x_2 , etc.)
- Try decreasing λ
- Try increasing λ

Credit: Andrew NG

Machine learning diagnostic:

Diagnostic:

A test that you can run to gain insight what is/isn't working with a learning algorithm, and gain guidance as to how best to improve its performance.

Diagnostics can take time to implement, but doing so can be a very good use of your time.

Credit: Andrew NG

7.2 Model selection & evaluation

Model Selection

Model selection: estimating the performance of different models in order to choose the best one.

1. $h_{\theta}(x) = \theta_0 + \theta_1 x$
2. $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$
3. $h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_3 x^3$
- \vdots
10. $h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{10} x^{10}$

Which one to choose? how a model generalizes to unseen test data.

Credit: Andrew NG

Model Evaluation

- A part of the model development process to find the best model
- Two methods of evaluating models are
 1. Hold-out
 2. Cross validation

1. Hold-out method

Size	Price	
2104	400	$(x^{(1)}, y^{(1)})$
1600	330	$(x^{(2)}, y^{(2)})$
2400	369	\vdots
1416	232	\vdots
3000	540	$(x^{(m)}, y^{(m)})$
1985	300	
<hr/>		
1534	315	
1427	199	$(x_{test}^{(1)}, y_{test}^{(1)})$
1380	212	$(x_{test}^{(2)}, y_{test}^{(2)})$
1494	243	\vdots
		$(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

```

>>> import numpy as np
>>> from sklearn.model_selection import train_test_split
>>> X, y = np.arange(10).reshape((5, 2)), range(5)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7],
       [8, 9]])
>>> list(y)
[0, 1, 2, 3, 4]
>>> X_train, X_test, y_train, y_test = train_test_split(
...     X, y, test_size=0.30, random_state=42)
>>> X_train
array([[4, 5],
       [0, 1],
       [6, 7]])
>>> y_train
[2, 0, 3]
>>> X_test
array([[2, 3],
       [8, 9]])
>>> y_test
[1, 4]

```


1. Hold-out method

- Learn parameter θ from training data (70%)
- Compute test set (30%)

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

Credit: Andrew NG

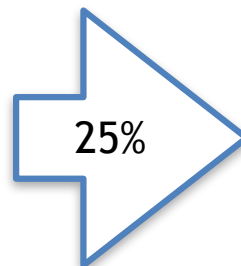
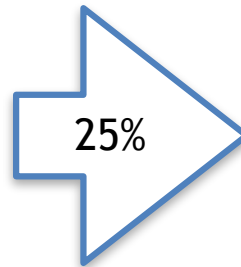
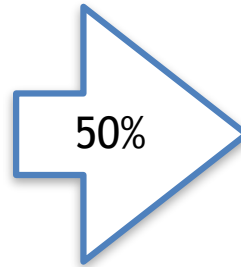
1. Hold-out method

Method	Advantage	Disadvantage
1. Holdout	<ul style="list-style-type: none">• Simple• Takes no longer to compute	<ul style="list-style-type: none">• Waste data (30% in this slide)• Its evaluation can have a high error

2. Cross validation

2. Cross-Validation

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243


 $(x^{(1)}, y^{(1)})$
 $(x^{(2)}, y^{(2)})$
 \vdots
 $(x^{(m)}, y^{(m)})$

 $(x_{cv}^{(1)}, y_{cv}^{(1)})$
 $(x_{cv}^{(2)}, y_{cv}^{(2)})$
 \vdots
 $(x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$

 $(x_{test}^{(1)}, y_{test}^{(1)})$
 $(x_{test}^{(2)}, y_{test}^{(2)})$
 \vdots
 $(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

Train/validation/test error

Training error:

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

Test error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

2. Cross Validation (K-fold)

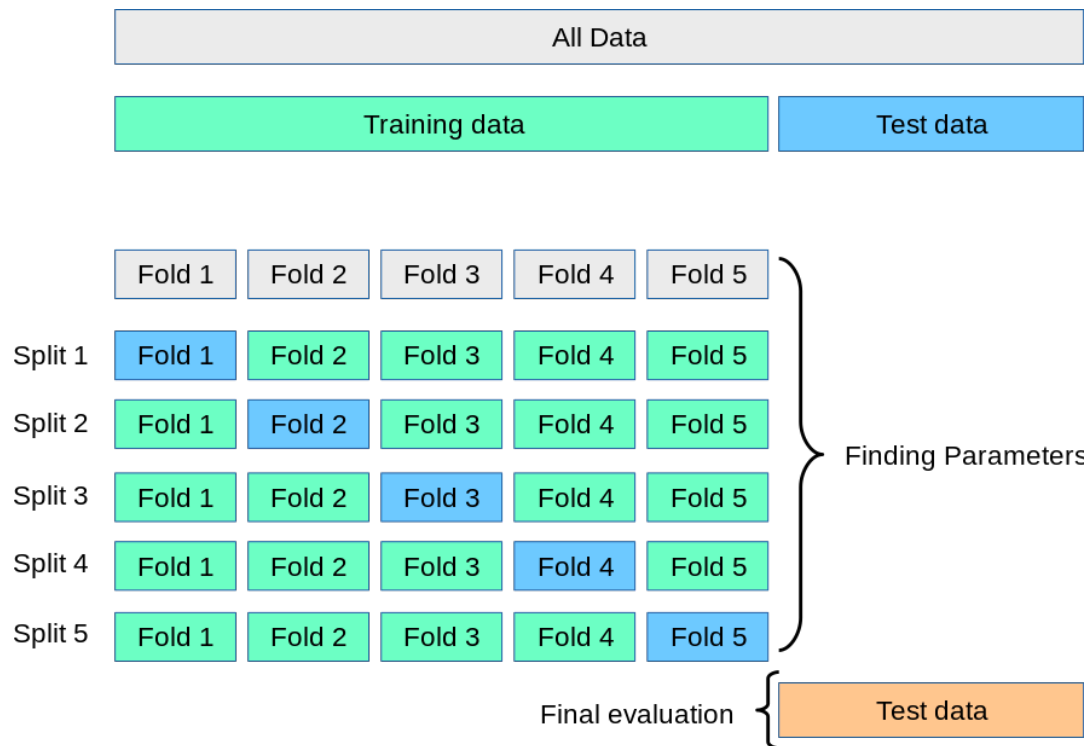


Figure from Hands-on
Machine Learning

The training set is divided into k subsets, and the **holdout method** is repeated k times.

Each time, one of the k subsets is used as the test set and the other $k-1$ subsets are put together to form a training set.

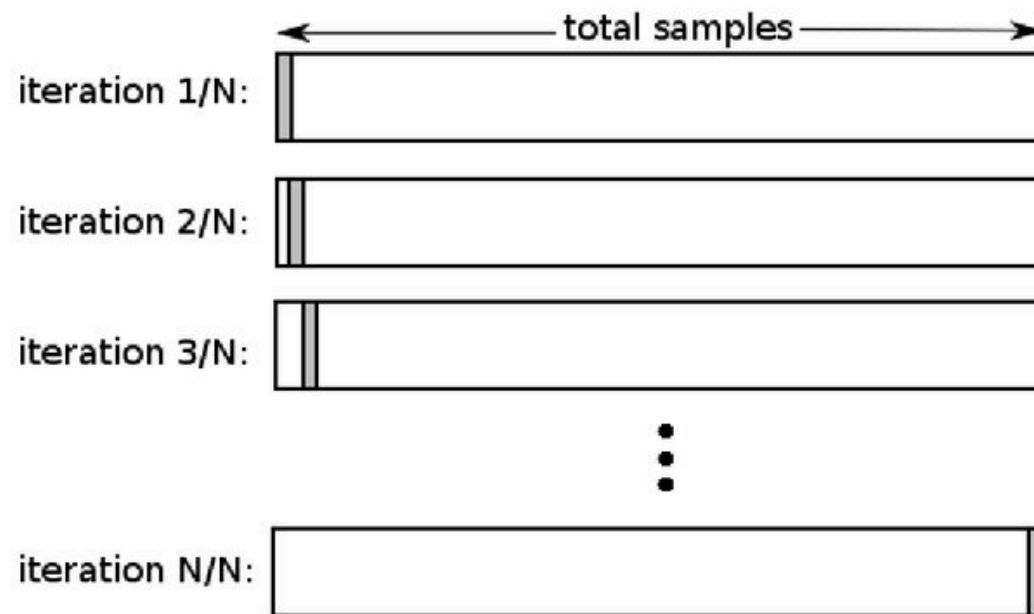
Then the average error across all k trials is computed.

```
>>> import numpy as np
>>> from sklearn.model_selection import KFold
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4])
>>> kf = KFold(n_splits=2)
>>> kf.get_n_splits(X)
2
>>> print(kf)
KFold(n_splits=2, random_state=None, shuffle=False)
>>> for train_index, test_index in kf.split(X):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3] TEST: [0 1]
TRAIN: [0 1] TEST: [2 3]
```

Evaluation method

Method	Advantage	Disadvantage
1. Holdout	<ul style="list-style-type: none">• Simple• Low computation	<ul style="list-style-type: none">• Waste data (30% in this slide)• Its evaluation can have a high variance
2. Cross validation	<ul style="list-style-type: none">• Every data point gets to be in a test set exactly once, and gets to be in a training set $k-1$ times.• The variance of the resulting estimate is reduced as k is increased	<ul style="list-style-type: none">• High computation

LOOCV (Leave-one-out Cross Validation)



K-fold cross validation taken to its logical extreme, with K equal to N.

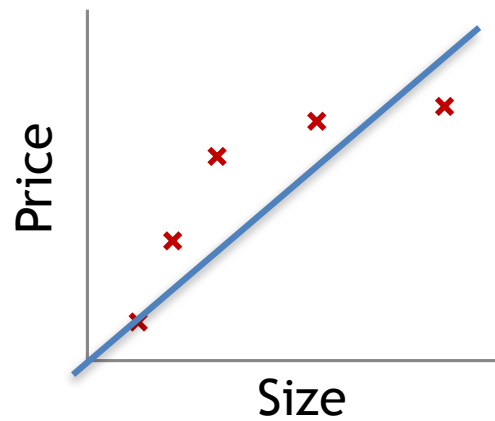
```
>>> from sklearn.model_selection import LeaveOneOut  
  
>>> X = [1, 2, 3, 4]  
>>> loo = LeaveOneOut()  
>>> for train, test in loo.split(X):  
...     print("%5 %5" % (train, test))  
[1 2 3] [0]  
[0 2 3] [1]  
[0 1 3] [2]  
[0 1 2] [3]
```


Evaluation method

Method	Advantage	Disadvantage
1. Holdout	<ul style="list-style-type: none">• Simple• Low computation	<ul style="list-style-type: none">• Waste data (30% in this slide)• Its evaluation can have a high variance
2. Cross validation	<ul style="list-style-type: none">• Every data point gets to be in a test set exactly once, and gets to be in a training set $k-1$ times.• The variance of the resulting estimate is reduced as k is increased	<ul style="list-style-type: none">• High computation
3. Leave-One-Out	<ul style="list-style-type: none">• Does not waste data	<ul style="list-style-type: none">• High computation

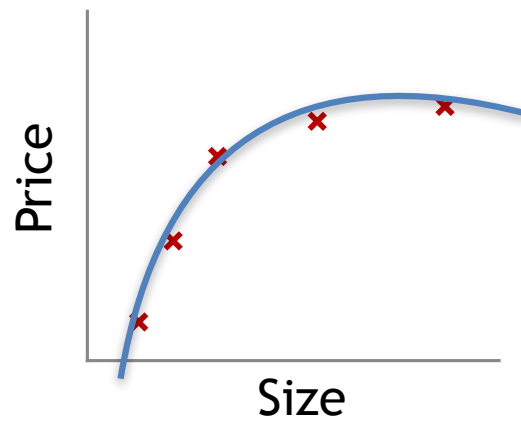
7.4 Diagnosing bias vs. variance

Bias/variance



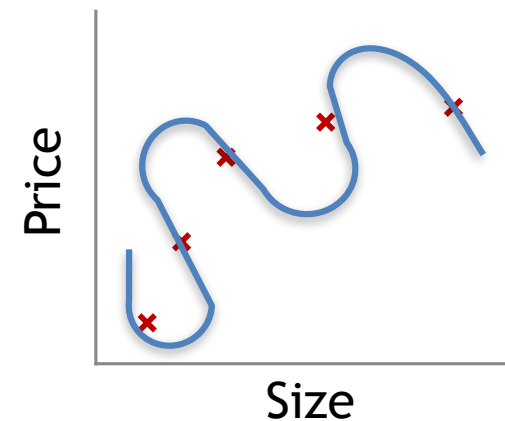
$$\theta_0 + \theta_1 x$$

High bias
(underfit)



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

“Just right”



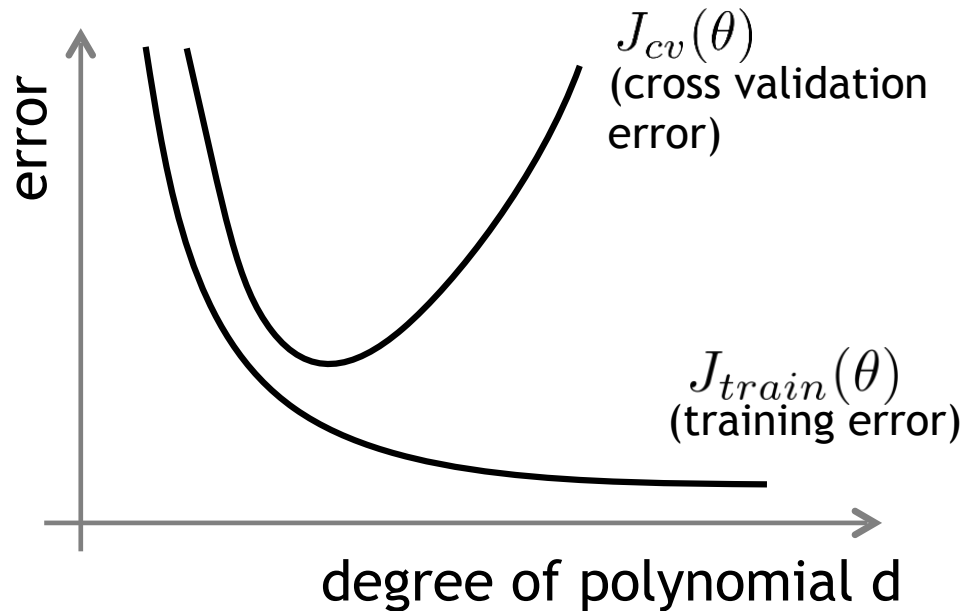
$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

High variance
(overfit)

Credit: Andrew NG

Diagnosing bias vs. variance

Suppose your learning algorithm is performing less well than you were hoping ($J_{cv}(\theta)$ or $J_{test}(\theta)$ is high.). Is it a bias problem or a variance problem?



Bias (underfit):

Variance (overfit):

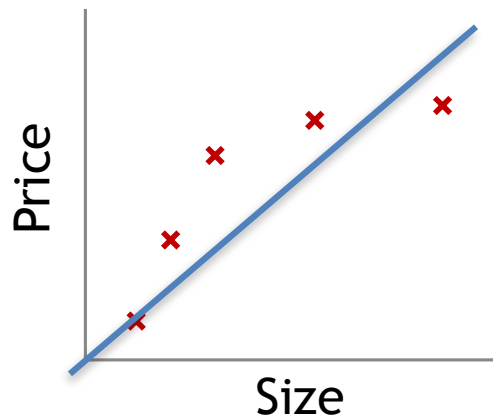
Credit: Andrew NG

7.5 Regularization and bias/ variance

Linear regression with regularization

Model: $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$

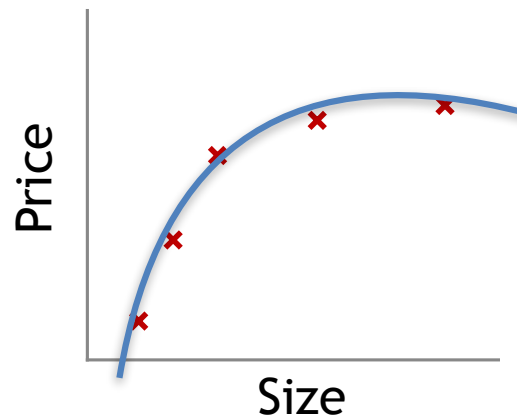


Large λ

High bias (underfit)

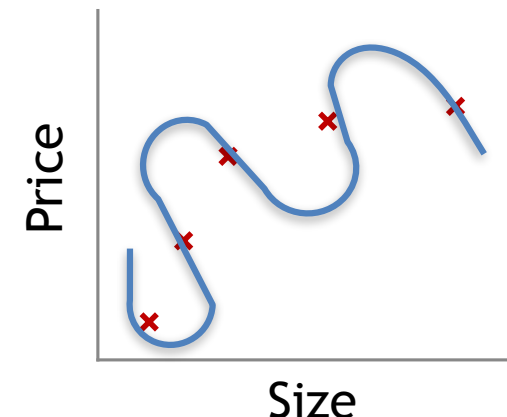
$\lambda = 10000$. $\theta_1 \approx 0, \theta_2 \approx 0, \dots$

$h_{\theta}(x) \approx \theta_0$



Intermediate λ

“Just right”



Small λ

High variance (overfit)

Credit: Andrew NG

Choosing the regularization parameter λ

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

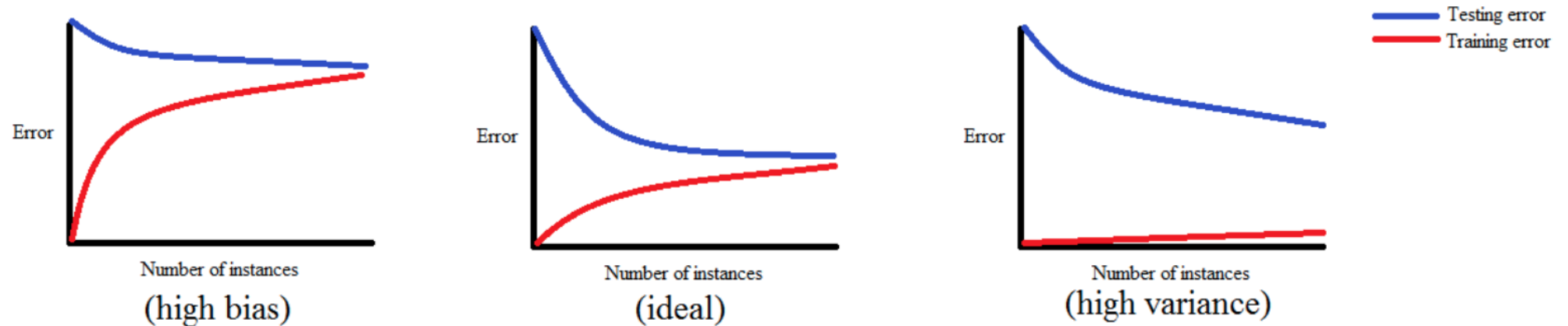
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

Credit: Andrew NG

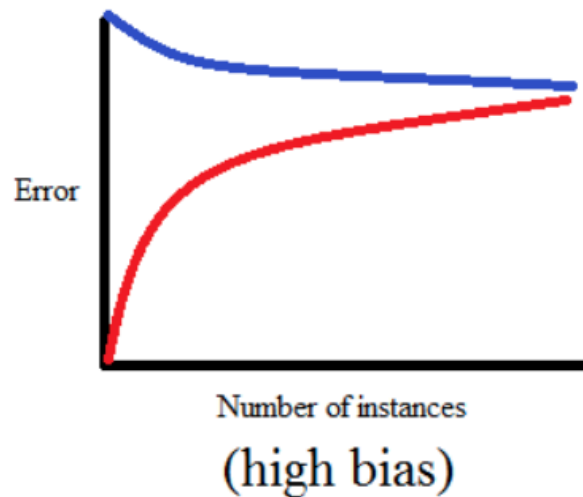
7.6 Learning curves

Error VS #Training data

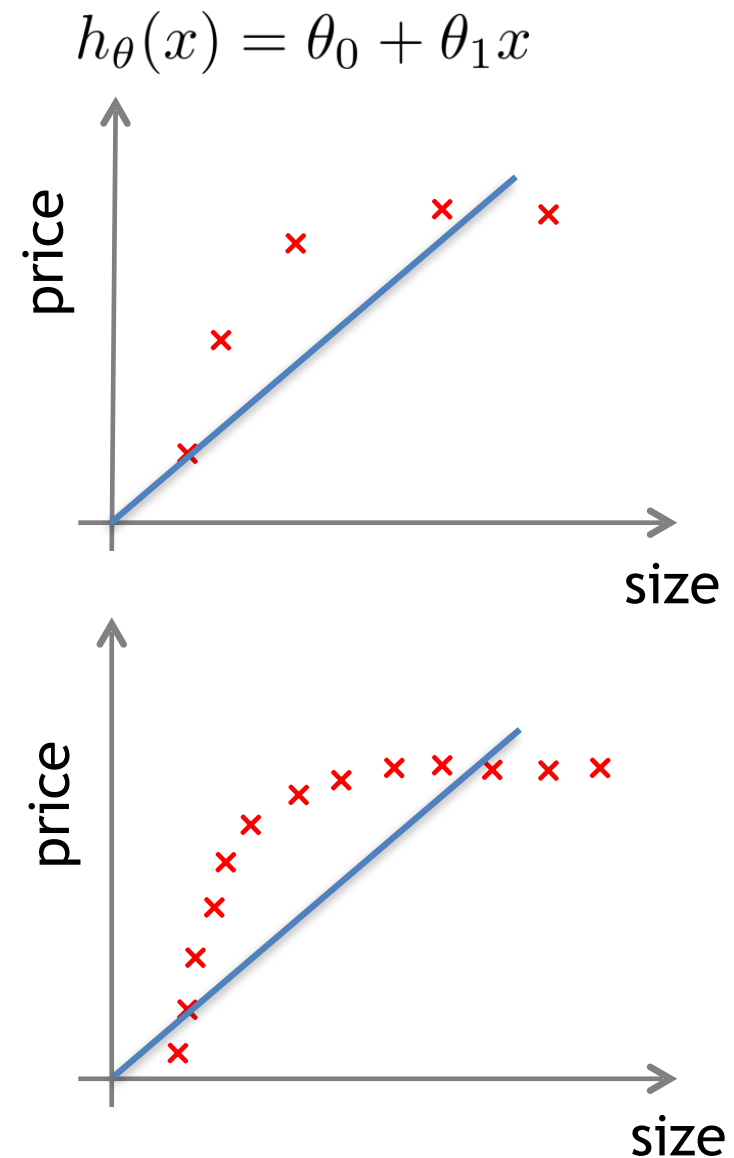


<https://rmartinshort.jimdofree.com/2019/02/17/overfitting-bias-variance-and-leaning-curves/>

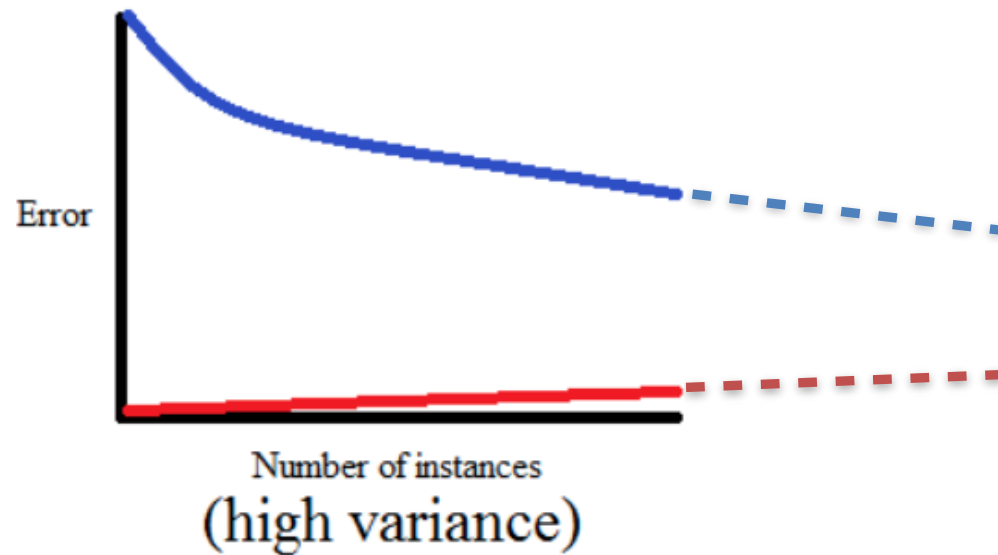
High bias



If a learning algorithm is suffering from high bias, getting more training data will not (by itself) help much.

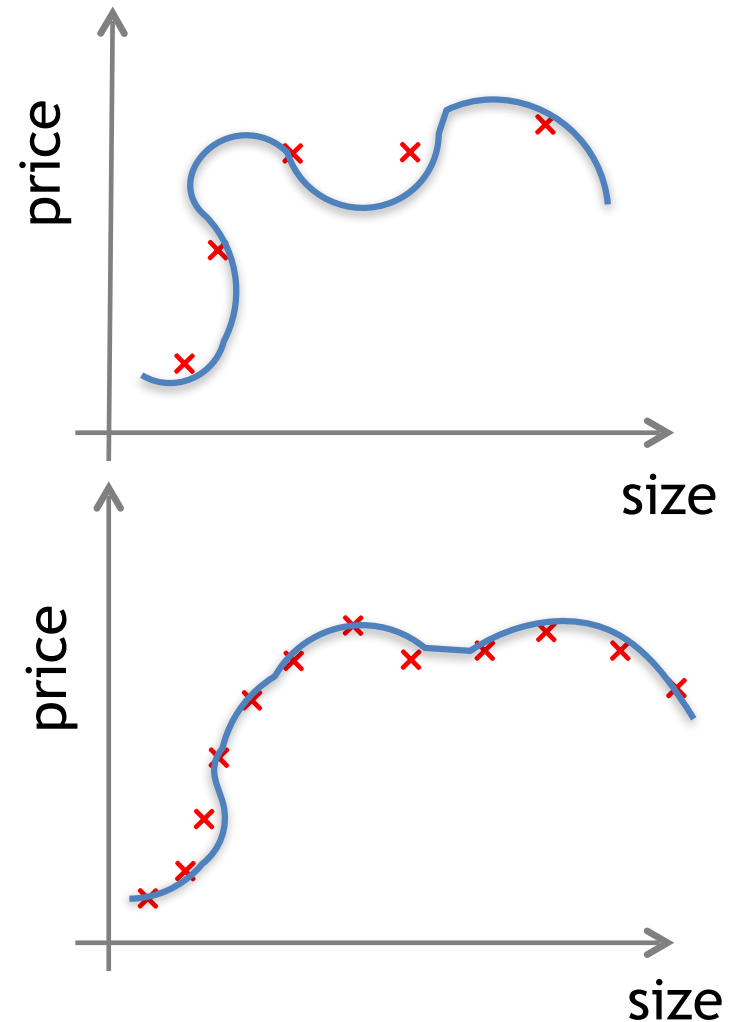


High variance



If a learning algorithm is suffering from high variance, getting more training data is likely to help.

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{100} x^{100} \quad (\text{and small } \lambda)$$



Debugging a learning algorithm:

Suppose you have implemented regularized linear regression to predict housing prices. However, when you test your hypothesis in a new set of houses, you find that it makes unacceptably large errors in its prediction. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features (x_1^2, x_2^2, x_1x_2 , etc)
- Try decreasing λ
- Try increasing λ

7.6 Error metrics

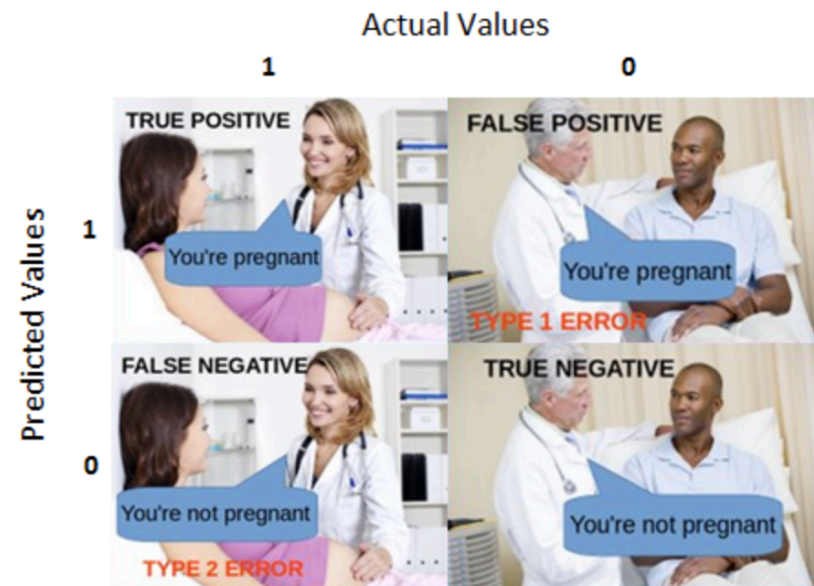
Error metrics

- Confusion Matrix
- Precision , Recall , and Accuracy
- F1-score
- ROC AUC Curve and score

Confusion Matrix

- A performance measurement for ML classification

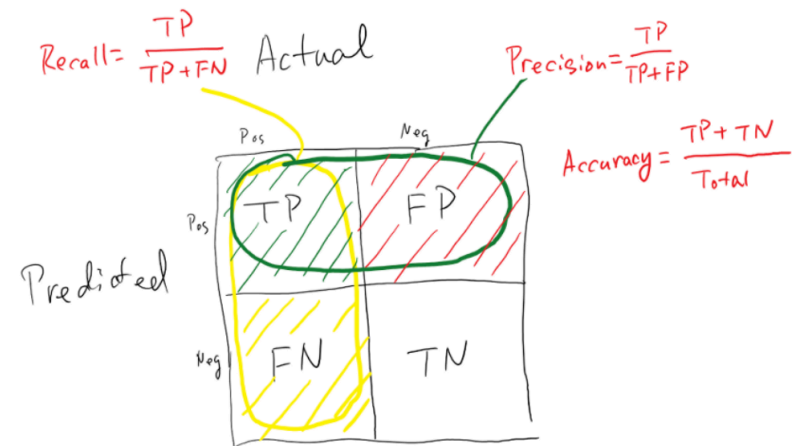
		Actual Values	
		Positive (1)	Negative (0)
Predicted Values	Positive (1)	TP	FP
	Negative (0)	FN	TN



<https://towardsdatascience.com/understanding-confusion-matrix-a9ad42dcfd62>

Confusion Matrix

y	y pred	output for threshold 0.6	Recall	Precision	Accuracy
0	0.5	0	1/2	2/3	4/7
1	0.9	1			
0	0.7	1			
1	0.7	1			
1	0.3	0			
0	0.4	0			
1	0.5	0			



<https://towardsdatascience.com/understanding-confusion-matrix-a9ad42dcfd62>

Precision/Recall

$y = 1$ in presence of rare class that we want to detect

Precision

(Of all patients where we predicted $y = 1$, what fraction actually has cancer?)

Recall

(Of all patients that actually have cancer, what fraction did we correctly detect as having cancer?)

Trading off precision and recall

Logistic regression: $0 \leq h_{\theta}(x) \leq 1$

Predict 1 if $h_{\theta}(x) \geq 0.5$

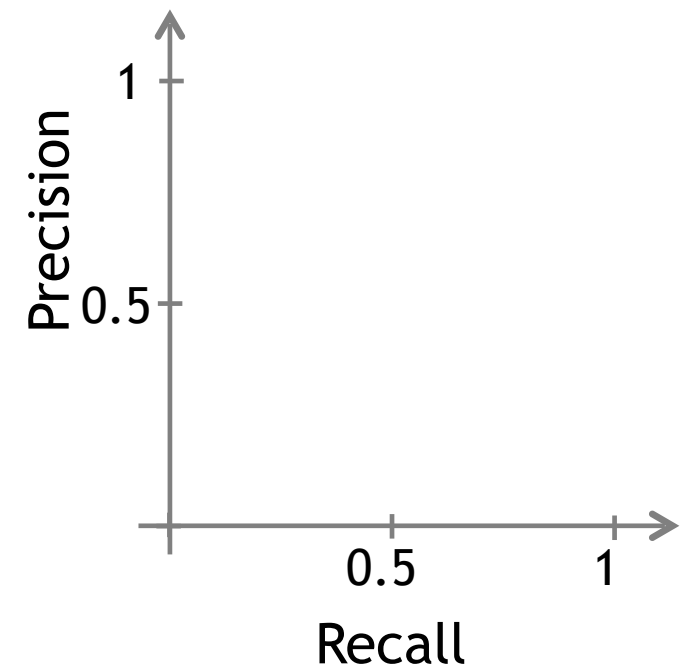
Predict 0 if $h_{\theta}(x) < 0.5$

Suppose we want to predict $y = 1$ (cancer) only if very confident.

Suppose we want to avoid missing too many cases of cancer (avoid false negatives).

More generally: Predict 1 if $h_{\theta}(x) \geq \text{threshold}$.

$$\text{precision} = \frac{\text{true positives}}{\text{no. of predicted positive}}$$
$$\text{recall} = \frac{\text{true positives}}{\text{no. of actual positive}}$$



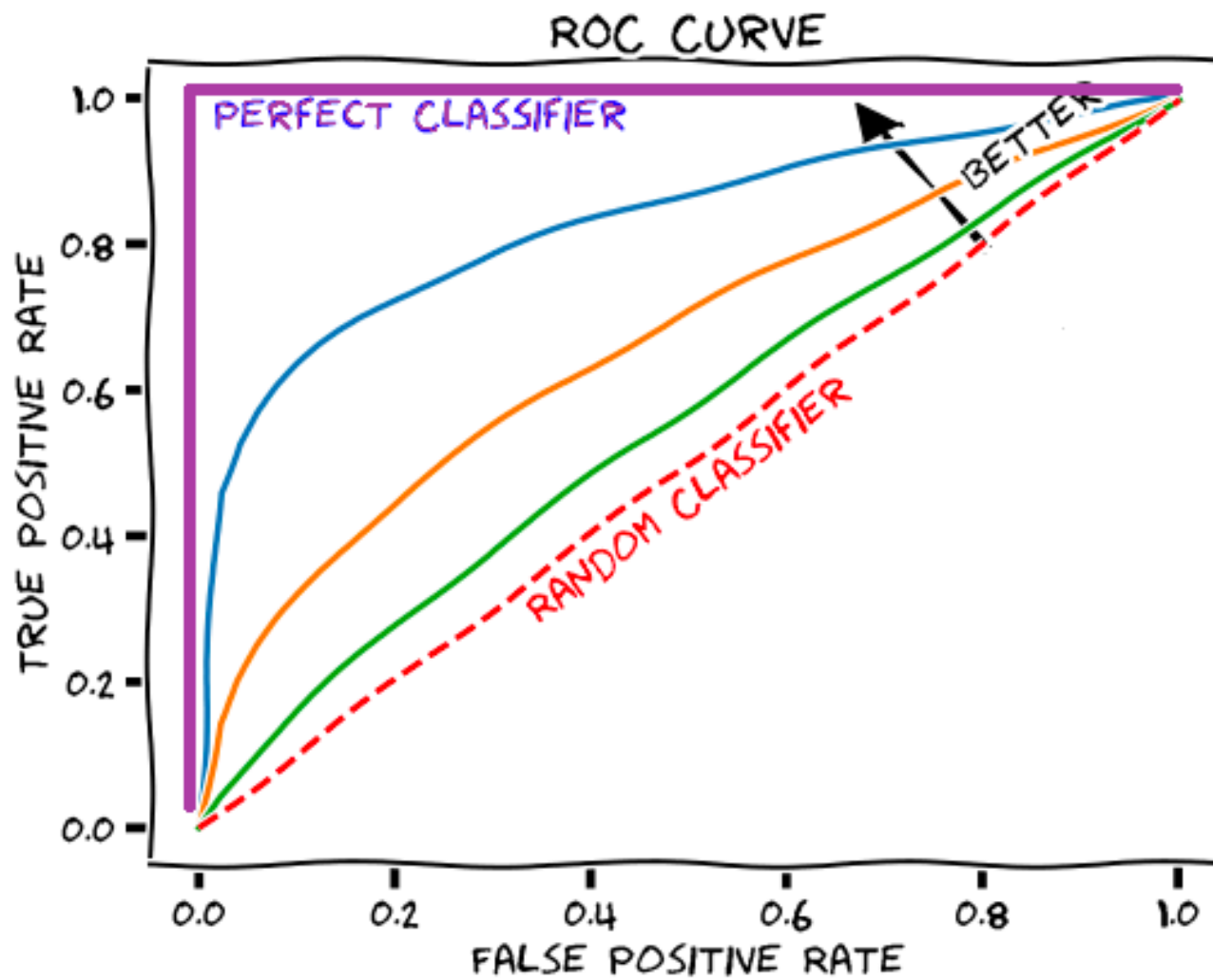
F_1 Score (F score)

How to compare precision/recall numbers?

	Precision(P)	Recall (R)
Algorithm 1	0.5	0.4
Algorithm 2	0.7	0.1
Algorithm 3	0.02	1.0

Average: $\frac{P+R}{2}$

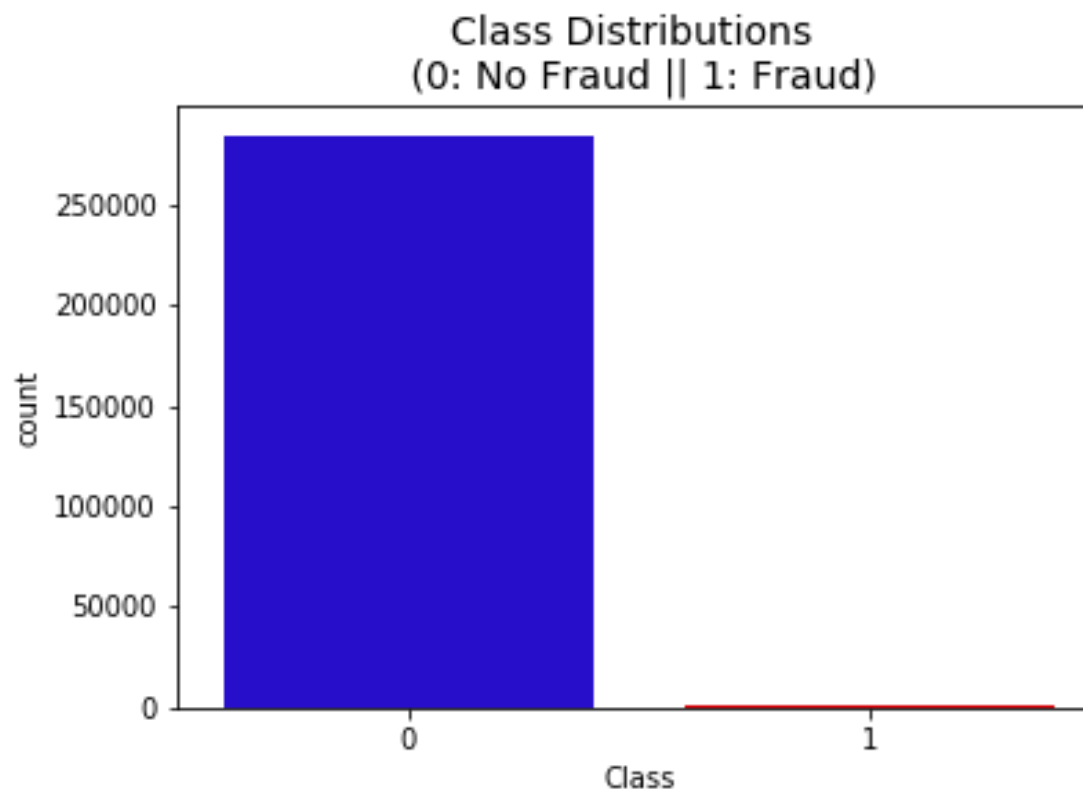
F_1 Score: $2 \frac{PR}{P+R}$



<https://glassboxmedicine.com/2019/02/23/measuring-performance-auc-auoc/>

7.7 Imbalanced data

What is imbalanced data



<https://www.kaggle.com/janiobachmann/credit-fraud-dealing-with-imbalanced-datasets>