Brief Article

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1 Finite element method

1.1 Backward Euler scheme

$$\frac{\boldsymbol{v}^n - \boldsymbol{v}^{n-1}}{\Delta t} + \boldsymbol{v}^{n-1} \cdot \nabla \boldsymbol{v}^n + \nabla p^n = \nu \Delta \boldsymbol{v}^n, \qquad \nabla \cdot \boldsymbol{v}^n = 0$$

$$\frac{1}{\Delta t} \left(\boldsymbol{v}^n - \boldsymbol{v}^{n-1}, \boldsymbol{\phi} \right) + \left(\boldsymbol{v}^{n-1} \cdot \nabla \boldsymbol{v}^n, \boldsymbol{\phi} \right) - (p, \nabla \cdot \boldsymbol{\phi}) + \nu \left(\nabla \boldsymbol{v}^n, \nabla \boldsymbol{\phi} \right) = 0$$

$$-\left(\nabla \cdot \boldsymbol{v}^n, \psi \right) = 0$$

$$\begin{aligned} \frac{1}{\Delta t} \sum_j \left(\boldsymbol{\phi}_i, \boldsymbol{\phi}_j \right) v_j^n + \sum_j \left(\boldsymbol{v}^{n-1} \cdot \nabla \boldsymbol{\phi}_j, \boldsymbol{\phi}_i \right) v_j^n \\ - \sum_j \left(\psi_j, \nabla \cdot \boldsymbol{\phi}_i \right) p_j^n + \nu \sum_j \left(\nabla \boldsymbol{\phi}_j, \nabla \boldsymbol{\phi}_i \right) v_j^n = \frac{1}{\Delta t} \sum_j \left(\boldsymbol{\phi}_i, \boldsymbol{\phi}_j \right) v_j^{n-1} \end{aligned}$$

$$-\sum_j \left(\nabla \cdot \boldsymbol{\phi}_j, \psi_i \right) v_j^n = 0$$

$$\begin{bmatrix} \frac{1}{\Delta t} M + F^{n-1} + A & B \\ B^\top & 0 \end{bmatrix} \begin{bmatrix} v^n \\ p^n \end{bmatrix} = \begin{bmatrix} \frac{1}{\Delta t} M v^{n-1} \\ 0 \end{bmatrix}$$