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1 EECS 442 PS10: Optical Flow and Epipolar Geometry

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2 Overview

In this problem set, You will implemente the optical flow network (PWN-Net) for estimating dense motion between a pair of images. You will also visualize epipolar lines for a pair of stereo images.

3 Starting

Run the following code to import the modules you'll need.

[65]:
```
from glob import glob
import math, os, random
import matplotlib.pyplot as plt
from matplotlib.pyplot import imread # alternative to scipy.misc.imread
import matplotlib.patches as patches
import numpy as np
import os.path
from os.path import *
import torch
import torch.utils.data as data
from torch.utils.data import DataLoader
from tqdm import tqdm_notebook
```

We will use GPUs to accelerate our computation in this notebook. Run the following to make sure GPUs are enabled:

[2]:
```
if torch.cuda.is_available():
  print('Good to go!')
else:
  print('Please set GPU via Edit -> Notebook Settings.')
```

```
Good to go!
```

Download and extract the Dataset.

[3]:
```
!wget http://www.eecs.umich.edu/courses/eecs442-ahowens/fa20/psets/ps10_data/
↪MPI-Sintel-442.zip
!unzip -q MPI-Sintel-442.zip # This might take a few minutes.
```

```
--2021-12-08 16:35:34--
http://www.eecs.umich.edu/courses/eecs442-ahowens/fa20/psets/ps10_data/MPI-
Sintel-442.zip
Resolving www.eecs.umich.edu (www.eecs.umich.edu)... 141.212.113.199
Connecting to www.eecs.umich.edu (www.eecs.umich.edu)|141.212.113.199|:80...
connected.
HTTP request sent, awaiting response... 200 OK
Length: 3949561120 (3.7G) [application/zip]
Saving to: MPI-Sintel-442.zip

MPI-Sintel-442.zip  100%[===================>]   3.68G  5.21MB/s    in 10m 16s

2021-12-08 16:45:51 (6.12 MB/s) - MPI-Sintel-442.zip saved
[3949561120/3949561120]
```

Define some flow utility functions. You don't need to understand codes in this part for the problem set.

[4]:
```
TAG_CHAR = np.array([202021.25], np.float32)

def readFlow(fn):
    """ Read .flo file in Middlebury format"""
    # Code adapted from:
    # http://stackoverflow.com/questions/28013200/
↪reading-middlebury-flow-files-with-python-bytes-array-numpy

    # WARNING: this will work on little-endian architectures (eg Intel x86)␣
↪only!
    # print 'fn = %s'%(fn)
    with open(fn, 'rb') as f:
        magic = np.fromfile(f, np.float32, count=1)
        if 202021.25 != magic:
            print('Magic number incorrect. Invalid .flo file')
            return None
        else:
            w = np.fromfile(f, np.int32, count=1)
            h = np.fromfile(f, np.int32, count=1)
            # print 'Reading %d x %d flo file\n' % (w, h)
            data = np.fromfile(f, np.float32, count=2*int(w)*int(h))
            # Reshape data into 3D array (columns, rows, bands)
            # The reshape here is for visualization, the original code is␣
↪(w,h,2)
            return np.resize(data, (int(h), int(w), 2))
```

```
def writeFlow(filename,uv,v=None):
    """ Write optical flow to file.
    Expect input flow as shape H,W,2(u,v) if v is None.
    If v is None, uv is assumed to contain both u and v channels,
    stacked in depth.
    Original code by Deqing Sun, adapted from Daniel Scharstein.
    """
    nBands = 2

    if v is None:
        assert(uv.ndim == 3)
        assert(uv.shape[2] == 2)
        u = uv[:,:,0]
        v = uv[:,:,1]
    else:
        u = uv

    assert(u.shape == v.shape)
    height,width = u.shape
    f = open(filename,'wb')
    # write the header
    f.write(TAG_CHAR)
    np.array(width).astype(np.int32).tofile(f)
    np.array(height).astype(np.int32).tofile(f)
    # arrange into matrix form
    tmp = np.zeros((height, width*nBands))
    tmp[:,np.arange(width)*2] = u
    tmp[:,np.arange(width)*2 + 1] = v
    tmp.astype(np.float32).tofile(f)
    f.close()


# ref: https://github.com/sampepose/flownet2-tf/
# blob/18f87081db44939414fc4a48834f9e0da3e69f4c/src/flowlib.py#L240
def visulize_flow_file(flow_filename, save_dir=None):
    flow_data = readFlow(flow_filename) #HW2
    #print(flow_data)
    img = flow2img(flow_data)
    plt.imshow(img)
    plt.show()
    if save_dir:
        idx = flow_filename.rfind("/") + 1
        plt.imsave(os.path.join(save_dir, "%s-vis.png" % flow_filename[idx:
 ↪-4]), img)
def visualize_flow_array(flow_data, plot=True, title = None):
    """
```

```python
    flow_data: array of shape H,W,2
    """
    img = flow2img(flow_data)
    if plot:
        plt.imshow(img)
        plt.axis('off')
        if title:
          plt.title(title)
        plt.show()
    return img

def flow2img(flow_data):
    """
    convert optical flow into color image
    :param flow_data:
    :return: color image
    """
    # print(flow_data.shape)
    # print(type(flow_data))
    u = flow_data[:, :, 0]
    v = flow_data[:, :, 1]

    UNKNOW_FLOW_THRESHOLD = 1e7
    pr1 = abs(u) > UNKNOW_FLOW_THRESHOLD
    pr2 = abs(v) > UNKNOW_FLOW_THRESHOLD
    idx_unknown = (pr1 | pr2)
    u[idx_unknown] = v[idx_unknown] = 0

    # get max value in each direction
    maxu = -999.
    maxv = -999.
    minu = 999.
    minv = 999.
    maxu = max(maxu, np.max(u))
    maxv = max(maxv, np.max(v))
    minu = min(minu, np.min(u))
    minv = min(minv, np.min(v))

    rad = np.sqrt(u ** 2 + v ** 2)
    maxrad = max(-1, np.max(rad))
    #print(maxrad)
    u = u / maxrad + np.finfo(float).eps
    v = v / maxrad + np.finfo(float).eps

    img = compute_color(u, v)

    idx = np.repeat(idx_unknown[:, :, np.newaxis], 3, axis=2)
```

```
    img[idx] = 0

    return np.uint8(img)


def compute_color(u, v):
    """
    compute optical flow color map
    :param u: horizontal optical flow
    :param v: vertical optical flow
    :return:
    """

    height, width = u.shape
    img = np.zeros((height, width, 3))

    NAN_idx = np.isnan(u) | np.isnan(v)
    u[NAN_idx] = v[NAN_idx] = 0

    colorwheel = make_color_wheel()
    ncols = np.size(colorwheel, 0)

    rad = np.sqrt(u ** 2 + v ** 2)

    a = np.arctan2(-v, -u) / np.pi

    fk = (a + 1) / 2 * (ncols - 1) + 1

    k0 = np.floor(fk).astype(int)

    k1 = k0 + 1
    k1[k1 == ncols + 1] = 1
    f = fk - k0

    for i in range(0, np.size(colorwheel, 1)):
        tmp = colorwheel[:, i]
        col0 = tmp[k0 - 1] / 255
        col1 = tmp[k1 - 1] / 255
        col = (1 - f) * col0 + f * col1

        idx = rad <= 1
        col[idx] = 1 - rad[idx] * (1 - col[idx])
        notidx = np.logical_not(idx)

        col[notidx] *= 0.75
        img[:, :, i] = np.uint8(np.floor(255 * col * (1 - NAN_idx)))
```

```
    return img


def make_color_wheel():
    """
    Generate color wheel according Middlebury color code
    :return: Color wheel
    """
    RY = 15
    YG = 6
    GC = 4
    CB = 11
    BM = 13
    MR = 6

    ncols = RY + YG + GC + CB + BM + MR

    colorwheel = np.zeros([ncols, 3])

    col = 0

    # RY
    colorwheel[0:RY, 0] = 255
    colorwheel[0:RY, 1] = np.transpose(np.floor(255 * np.arange(0, RY) / RY))
    col += RY

    # YG
    colorwheel[col:col + YG, 0] = 255 - np.transpose(np.floor(255 * np.
↪arange(0, YG) / YG))
    colorwheel[col:col + YG, 1] = 255
    col += YG

    # GC
    colorwheel[col:col + GC, 1] = 255
    colorwheel[col:col + GC, 2] = np.transpose(np.floor(255 * np.arange(0, GC) /
↪ GC))
    col += GC

    # CB
    colorwheel[col:col + CB, 1] = 255 - np.transpose(np.floor(255 * np.
↪arange(0, CB) / CB))
    colorwheel[col:col + CB, 2] = 255
    col += CB

    # BM
    colorwheel[col:col + BM, 2] = 255
```

```
    colorwheel[col:col + BM, 0] = np.transpose(np.floor(255 * np.arange(0, BM) /
↪ BM))
    col += + BM

    # MR
    colorwheel[col:col + MR, 2] = 255 - np.transpose(np.floor(255 * np.
↪arange(0, MR) / MR))
    colorwheel[col:col + MR, 0] = 255

    return colorwheel

# fram utils
def read_gen(file_name):
    ext = splitext(file_name)[-1]
    if ext == '.png' or ext == '.jpeg' or ext == '.ppm' or ext == '.jpg':
        im = imread(file_name)
        if im.shape[2] > 3:
            return im[:,:,:3]
        else:
            return im
    elif ext == '.bin' or ext == '.raw':
        return np.load(file_name)
    elif ext == '.flo':
        return readFlow(file_name).astype(np.float32)
    return []
```

Define the data augmentation functions, dataset and dataloader.

[5]:
```
class StaticRandomCrop(object):
    def __init__(self, image_size, crop_size):
        self.th, self.tw = crop_size
        h, w = image_size
        self.h1 = random.randint(0, h - self.th)
        self.w1 = random.randint(0, w - self.tw)

    def __call__(self, img):
        return img[self.h1:(self.h1+self.th), self.w1:(self.w1+self.tw),:]

class StaticCenterCrop(object):
    def __init__(self, image_size, crop_size):
        self.th, self.tw = crop_size
        self.h, self.w = image_size
    def __call__(self, img):
        return img[(self.h-self.th)//2:(self.h+self.th)//2, (self.w-self.tw)//2:
↪(self.w+self.tw)//2,:]

class MpiSintel(data.Dataset):
```

```
    def __init__(self,crop_size=[384, 512], render_size=[384, 1024], train =␣
↪False, root = '', dstype = 'clean'):

        flow_root = join(root, 'flow')
        image_root = join(root, dstype)

        self.crop_size = crop_size
        self.render_size = render_size
        self.train = train
        file_list = sorted(glob(join(flow_root, '*/*.flo')))
        #Randomly select out 100 samples for test set
        import random
        random.seed(30)
        random.shuffle(file_list)
        if self.train:
            file_list = file_list[:-100]
        else:
            file_list = file_list[-100:]
        self.flow_list = []
        self.image_list = []

        for file in file_list:
            if 'test' in file:
                # print file
                continue

            fbase = file[len(flow_root)+1:]
            fprefix = fbase[:-8]
            fnum = int(fbase[-8:-4])

            img1 = join(image_root, fprefix + "%04d"%(fnum+0) + '.png')
            img2 = join(image_root, fprefix + "%04d"%(fnum+1) + '.png')

            if not isfile(img1) or not isfile(img2) or not isfile(file):
                continue

            self.image_list += [[img1, img2]]
            self.flow_list += [file]

        self.size = len(self.image_list)

        self.frame_size = read_gen(self.image_list[0][0]).shape

        if (self.render_size[0] < 0) or (self.render_size[1] < 0) or (self.
↪frame_size[0]%64) or (self.frame_size[1]%64):
            self.render_size[0] = ( (self.frame_size[0])//64 ) * 64
            self.render_size[1] = ( (self.frame_size[1])//64 ) * 64
```

```python
        assert (len(self.image_list) == len(self.flow_list))

    def __getitem__(self, index):

        index = index % self.size

        img1 = read_gen(self.image_list[index][0])
        img2 = read_gen(self.image_list[index][1])

        flow = read_gen(self.flow_list[index]) # H,W,2

        images = [img1, img2]
        image_size = img1.shape[:2]

        if self.train:
            #cropper = StaticCenterCrop(image_size, self.crop_size)
            #print(image_size,self.render_size)
            cropper = StaticRandomCrop(image_size, self.crop_size)
        else:
            #print(image_size,self.render_size)
            cropper = StaticCenterCrop(image_size, self.render_size)
        images = list(map(cropper, images)) #2,H,W,3
        flow = cropper(flow)
        images = np.array(images).transpose(0,3,1,2)
        flow = flow.transpose(2,0,1) # 2,H,W

        images = torch.from_numpy(images.astype(np.float32))/255.
        flow = torch.from_numpy(flow.astype(np.float32))

        return images, flow # 2,3,H,W and 2,H,W

    def __len__(self):
        return self.size

class MpiSintelClean(MpiSintel):
    def __init__(self, crop_size=[384, 512], render_size=[384, 1024], train =␣
 ↪False, root = ''):
        super(MpiSintelClean, self).__init__(train = train, root = root, dstype␣
 ↪= 'clean')
```

[6]:
```python
train_dataset = MpiSintelClean(crop_size = [384, 512],render_size = [384,␣
 ↪1024], train = True, root='/content/MPI-Sintel-442/training') # Return crops␣
 ↪of size 384, 512
```

```
val_dataset = MpiSintelClean(crop_size = [384, 512],render_size = [384, 1024],␣
↪train = False, root='/content/MPI-Sintel-442/training') # Return images of␣
↪size 384,1024
train_loader = DataLoader(train_dataset, batch_size = 4, shuffle = True)
val_loader = DataLoader(val_dataset, batch_size=4, shuffle = False)

overfit_dataset = torch.utils.data.Subset(val_dataset,[0])
overfit_dataloader = DataLoader(overfit_dataset, batch_size = 1, shuffle =␣
↪False)
```

Define Loss and evaluation metrics.

[7]:
```
import torch
import torch.nn as nn
import math

def EPE(input_flow, target_flow):
    """
    Calculate the end point error between the input_flow and target_flow.
    """
    return torch.norm(target_flow-input_flow,p=2,dim=1).mean()

class L1(nn.Module):
    def __init__(self):
        super(L1, self).__init__()
    def forward(self, output, target):
        lossvalue = torch.abs(output - target).mean()
        return lossvalue

class L2(nn.Module):
    def __init__(self):
        super(L2, self).__init__()
    def forward(self, output, target):
        lossvalue = torch.norm(output-target,p=2,dim=1).mean()
        return lossvalue

class MultiScale(nn.Module):
    """
    PWC-Net outputs optical flow at multiple scales. Network is trained by␣
↪minimizing a multi-scale regression loss between the predicted multi-scale␣
↪flows and multi-scale groundtruth flows.
    """
    def __init__(self, startScale = 4, numScales = 5, l_weight= 0.32, norm=␣
↪'L1'):
        super(MultiScale,self).__init__()

        self.startScale = startScale
        self.numScales = numScales
```

```
        self.loss_weights = torch.FloatTensor([(l_weight / 2 ** scale) for␣
↪scale in range(self.numScales)]).cuda()
        self.l_type = norm
        self.div_flow = 0.05
        assert(len(self.loss_weights) == self.numScales)

        if self.l_type == 'L1':
            self.loss = L1()
        else:
            self.loss = L2()

        self.multiScales = [nn.AvgPool2d(self.startScale * (2**scale), self.
↪startScale * (2**scale)) for scale in range(self.numScales)]
        self.loss_labels = ['MultiScale-'+self.l_type, 'EPE'],

    def forward(self, output, target):
        lossvalue = 0
        epevalue = 0

        if type(output) is tuple:
            target = self.div_flow * target
            for i, output_ in enumerate(output):
                #Total loss is the weighted average of losses at each scale.
                target_ = self.multiScales[i](target)
                epevalue += self.loss_weights[i]*EPE(output_, target_)
                lossvalue += self.loss_weights[i]*self.loss(output_, target_)
            return [lossvalue, epevalue]
        else:
            epevalue += EPE(output, target)
            lossvalue += self.loss(output, target)
            return  [lossvalue, epevalue]
```

3.1 (a) PWC-Net

The warping layer at each pyramid level warps the feature of the second image towards the first image using the coarse flow estimated from the previous level. Fill in the scaling factors for `self.warp(...)` appearing inside the `forward function` of the `PWCNet`.

[8]:
```
"""
implementation of the PWC-DC network for optical flow estimation by Sun et al.,␣
↪2018
Jinwei Gu and Zhile Ren
"""

import torch
import torch.nn as nn
from torch.autograd import Variable
```

```python
import os
import torch.nn.functional as F

def upsample2d_as(inputs, target_as, mode="bilinear"):
    _, _, h, w = target_as.size()
    return F.interpolate(inputs, [h, w], mode=mode, align_corners=True)

def conv(in_planes, out_planes, kernel_size=3, stride=1, padding=1, dilation=1):
    return nn.Sequential(
            nn.Conv2d(in_planes, out_planes, kernel_size=kernel_size,␣
↪stride=stride,
                        padding=padding, dilation=dilation, bias=True),nn.
↪BatchNorm2d(out_planes),
            nn.LeakyReLU(0.1))

def predict_flow(in_planes):
    return nn.Conv2d(in_planes,2,kernel_size=3,stride=1,padding=1,bias=True)

def deconv(in_planes, out_planes, kernel_size=4, stride=2, padding=1):
    return nn.ConvTranspose2d(in_planes, out_planes, kernel_size, stride,␣
↪padding, bias=True)


class PWCNet(nn.Module):
    """
    PWCNet.
    """
    def __init__(self, md=4):
        """
        input: md --- maximum displacement (for correlation. default: 4), after␣
↪warpping
        """
        super(PWCNet,self).__init__()

        self.conv1a  = conv(3,   16, kernel_size=3, stride=2)
        self.conv1aa = conv(16,  16, kernel_size=3, stride=1)
        self.conv1b  = conv(16,  16, kernel_size=3, stride=1)
        self.conv2a  = conv(16,  32, kernel_size=3, stride=2)
        self.conv2aa = conv(32,  32, kernel_size=3, stride=1)
        self.conv2b  = conv(32,  32, kernel_size=3, stride=1)
        self.conv3a  = conv(32,  64, kernel_size=3, stride=2)
        self.conv3aa = conv(64,  64, kernel_size=3, stride=1)
        self.conv3b  = conv(64,  64, kernel_size=3, stride=1)
        self.conv4a  = conv(64,  96, kernel_size=3, stride=2)
        self.conv4aa = conv(96,  96, kernel_size=3, stride=1)
        self.conv4b  = conv(96,  96, kernel_size=3, stride=1)
        self.conv5a  = conv(96, 128, kernel_size=3, stride=2)
```

```
        self.conv5aa = conv(128,128, kernel_size=3, stride=1)
        self.conv5b  = conv(128,128, kernel_size=3, stride=1)
        self.conv6aa = conv(128,196, kernel_size=3, stride=2)
        self.conv6a  = conv(196,196, kernel_size=3, stride=1)
        self.conv6b  = conv(196,196, kernel_size=3, stride=1)

        self.corr  = cost_volume(md)
        #self.corr    = Correlation(pad_size=md, kernel_size=1, max_displacement=md, stride1=1, stride2=1, corr_multiply=1)
        self.leakyRELU = nn.LeakyReLU(0.1)

        nd = (2*md+1)**2
        dd = np.cumsum([128,128,96,64,32])

        od = nd
        self.conv6_0 = conv(od,      128, kernel_size=3, stride=1)
        self.conv6_1 = conv(od+dd[0],128, kernel_size=3, stride=1)
        self.conv6_2 = conv(od+dd[1],96,  kernel_size=3, stride=1)
        self.conv6_3 = conv(od+dd[2],64,  kernel_size=3, stride=1)
        self.conv6_4 = conv(od+dd[3],32,  kernel_size=3, stride=1)
        self.predict_flow6 = predict_flow(od+dd[4])
        self.deconv6 = deconv(2, 2, kernel_size=4, stride=2, padding=1)
        self.upfeat6 = deconv(od+dd[4], 2, kernel_size=4, stride=2, padding=1)

        od = nd+128+4
        self.conv5_0 = conv(od,      128, kernel_size=3, stride=1)
        self.conv5_1 = conv(od+dd[0],128, kernel_size=3, stride=1)
        self.conv5_2 = conv(od+dd[1],96,  kernel_size=3, stride=1)
        self.conv5_3 = conv(od+dd[2],64,  kernel_size=3, stride=1)
        self.conv5_4 = conv(od+dd[3],32,  kernel_size=3, stride=1)
        self.predict_flow5 = predict_flow(od+dd[4])
        self.deconv5 = deconv(2, 2, kernel_size=4, stride=2, padding=1)
        self.upfeat5 = deconv(od+dd[4], 2, kernel_size=4, stride=2, padding=1)

        od = nd+96+4
        self.conv4_0 = conv(od,      128, kernel_size=3, stride=1)
        self.conv4_1 = conv(od+dd[0],128, kernel_size=3, stride=1)
        self.conv4_2 = conv(od+dd[1],96,  kernel_size=3, stride=1)
        self.conv4_3 = conv(od+dd[2],64,  kernel_size=3, stride=1)
        self.conv4_4 = conv(od+dd[3],32,  kernel_size=3, stride=1)
        self.predict_flow4 = predict_flow(od+dd[4])
        self.deconv4 = deconv(2, 2, kernel_size=4, stride=2, padding=1)
        self.upfeat4 = deconv(od+dd[4], 2, kernel_size=4, stride=2, padding=1)

        od = nd+64+4
        self.conv3_0 = conv(od,      128, kernel_size=3, stride=1)
        self.conv3_1 = conv(od+dd[0],128, kernel_size=3, stride=1)
```

```
        self.conv3_2 = conv(od+dd[1],96,  kernel_size=3, stride=1)
        self.conv3_3 = conv(od+dd[2],64,  kernel_size=3, stride=1)
        self.conv3_4 = conv(od+dd[3],32,  kernel_size=3, stride=1)
        self.predict_flow3 = predict_flow(od+dd[4])
        self.deconv3 = deconv(2, 2, kernel_size=4, stride=2, padding=1)
        self.upfeat3 = deconv(od+dd[4], 2, kernel_size=4, stride=2, padding=1)

        od = nd+32+4
        self.conv2_0 = conv(od,      128, kernel_size=3, stride=1)
        self.conv2_1 = conv(od+dd[0],128, kernel_size=3, stride=1)
        self.conv2_2 = conv(od+dd[1],96,  kernel_size=3, stride=1)
        self.conv2_3 = conv(od+dd[2],64,  kernel_size=3, stride=1)
        self.conv2_4 = conv(od+dd[3],32,  kernel_size=3, stride=1)
        self.predict_flow2 = predict_flow(od+dd[4])
        self.deconv2 = deconv(2, 2, kernel_size=4, stride=2, padding=1)

        self.dc_conv1 = conv(od+dd[4], 128, kernel_size=3, stride=1, padding=1,␣
↪ dilation=1)
        self.dc_conv2 = conv(128,      128, kernel_size=3, stride=1, padding=2,␣
↪ dilation=2)
        self.dc_conv3 = conv(128,      128, kernel_size=3, stride=1, padding=4,␣
↪ dilation=4)
        self.dc_conv4 = conv(128,      96,  kernel_size=3, stride=1, padding=8,␣
↪ dilation=8)
        self.dc_conv5 = conv(96,       64,  kernel_size=3, stride=1,␣
↪padding=16, dilation=16)
        self.dc_conv6 = conv(64,       32,  kernel_size=3, stride=1, padding=1,␣
↪ dilation=1)
        self.dc_conv7 = predict_flow(32)

        for m in self.modules():
            if isinstance(m, nn.Conv2d) or isinstance(m, nn.ConvTranspose2d):
                nn.init.kaiming_normal(m.weight.data, mode='fan_in')
                if m.bias is not None:
                    m.bias.data.zero_()


    def warp(self, x, flo):
        """
        warp an image/tensor (im2) back to im1, according to the optical flow
        x: [B, C, H, W] (im2)
        flo: [B, 2, H, W] flow. The first channel should be motion along u and␣
↪second channel should be motion along v.
        """
        flo = flo*20 # targets are divided by 20 during training
        B, C, H, W = x.size()
```

```
        # mesh grid
        xx = torch.arange(0, W).view(1,-1).repeat(H,1)
        yy = torch.arange(0, H).view(-1,1).repeat(1,W)
        xx = xx.view(1,1,H,W).repeat(B,1,1,1)
        yy = yy.view(1,1,H,W).repeat(B,1,1,1)
        grid = torch.cat((xx,yy),1).float()

        if x.is_cuda:
            grid = grid.cuda()
        vgrid = Variable(grid) + flo

        # scale grid to [-1,1]
        vgrid[:,0,:,:] = 2.0*vgrid[:,0,:,:].clone() / max(W-1,1)-1.0
        vgrid[:,1,:,:] = 2.0*vgrid[:,1,:,:].clone() / max(H-1,1)-1.0

        vgrid = vgrid.permute(0,2,3,1)
        output = nn.functional.grid_sample(x, vgrid)
        mask = torch.autograd.Variable(torch.ones(x.size())).cuda()
        mask = nn.functional.grid_sample(mask, vgrid)

        # if W==128:
            # np.save('mask.npy', mask.cpu().data.numpy())
            # np.save('warp.npy', output.cpu().data.numpy())

        mask[mask<0.9999] = 0
        mask[mask>0] = 1

        return output*mask


    def forward(self,x):
        im1 = x[:,:3,:,:]
        im2 = x[:,3:,:,:]

        #Feature extractor, cmn indicates feature extracted from image m at␣
↪level n.
        #Features at pyramid level l = 1 (downsasmping rate = 2)
        c11 = self.conv1b(self.conv1aa(self.conv1a(im1)))
        c21 = self.conv1b(self.conv1aa(self.conv1a(im2)))

        #Features at pyramid level l = 2 (downsasmping rate = 4)
        c12 = self.conv2b(self.conv2aa(self.conv2a(c11)))
        c22 = self.conv2b(self.conv2aa(self.conv2a(c21)))

        #Features at pyramid level l = 3 (downsasmping rate = 8)
        c13 = self.conv3b(self.conv3aa(self.conv3a(c12)))
        c23 = self.conv3b(self.conv3aa(self.conv3a(c22)))
```

```
        #Features at pyramid level l = 4 (downsasmping rate = 16)
        c14 = self.conv4b(self.conv4aa(self.conv4a(c13)))
        c24 = self.conv4b(self.conv4aa(self.conv4a(c23)))

        #Features at pyramid level l = 5 (downsasmping rate = 32)
        c15 = self.conv5b(self.conv5aa(self.conv5a(c14)))
        c25 = self.conv5b(self.conv5aa(self.conv5a(c24)))

        #Features at pyramid level l = 6 (downsasmping rate = 64)
        c16 = self.conv6b(self.conv6a(self.conv6aa(c15)))
        c26 = self.conv6b(self.conv6a(self.conv6aa(c25)))


        corr6 = self.corr(c16, c26)
        corr6 = self.leakyRELU(corr6)


        x = torch.cat((self.conv6_0(corr6), corr6),1)
        x = torch.cat((self.conv6_1(x), x),1)
        x = torch.cat((self.conv6_2(x), x),1)
        x = torch.cat((self.conv6_3(x), x),1)
        x = torch.cat((self.conv6_4(x), x),1)
        flow6 = self.predict_flow6(x)
        up_flow6 = self.deconv6(flow6)
        up_feat6 = self.upfeat6(x)

        ##Fill in scaling factor here##
        warp5 = self.warp(c25, up_flow6/1.6) #Hint: up_flow6 has a downsampling␣
↪rate of 32
        corr5 = self.corr(c15, warp5)
        corr5 = self.leakyRELU(corr5)
        x = torch.cat((corr5, c15, up_flow6, up_feat6), 1)
        x = torch.cat((self.conv5_0(x), x),1)
        x = torch.cat((self.conv5_1(x), x),1)
        x = torch.cat((self.conv5_2(x), x),1)
        x = torch.cat((self.conv5_3(x), x),1)
        x = torch.cat((self.conv5_4(x), x),1)
        flow5 = self.predict_flow5(x)
        up_flow5 = self.deconv5(flow5)
        up_feat5 = self.upfeat5(x)

        ##Fill in scaling factor here##
        warp4 = self.warp(c24, up_flow5/0.8) #Hint: up_flow5 has a downsampling␣
↪rate of 16
        corr4 = self.corr(c14, warp4)
        corr4 = self.leakyRELU(corr4)
```

```
        x = torch.cat((corr4, c14, up_flow5, up_feat5), 1)
        x = torch.cat((self.conv4_0(x), x),1)
        x = torch.cat((self.conv4_1(x), x),1)
        x = torch.cat((self.conv4_2(x), x),1)
        x = torch.cat((self.conv4_3(x), x),1)
        x = torch.cat((self.conv4_4(x), x),1)
        flow4 = self.predict_flow4(x)
        up_flow4 = self.deconv4(flow4)
        up_feat4 = self.upfeat4(x)

        ##Fill in scaling factor here##
        warp3 = self.warp(c23, up_flow4/0.4) #Hint: up_flow4 has a downsampling␣
↪rate of 8
        corr3 = self.corr(c13, warp3)
        corr3 = self.leakyRELU(corr3)


        x = torch.cat((corr3, c13, up_flow4, up_feat4), 1)
        x = torch.cat((self.conv3_0(x), x),1)
        x = torch.cat((self.conv3_1(x), x),1)
        x = torch.cat((self.conv3_2(x), x),1)
        x = torch.cat((self.conv3_3(x), x),1)
        x = torch.cat((self.conv3_4(x), x),1)
        flow3 = self.predict_flow3(x)
        up_flow3 = self.deconv3(flow3)
        up_feat3 = self.upfeat3(x)

        ##Fill in scaling factor here
        warp2 = self.warp(c22, up_flow3/0.2) #Hint: up_flow3 has a downsampling␣
↪rate of 4
        corr2 = self.corr(c12, warp2)
        corr2 = self.leakyRELU(corr2)
        x = torch.cat((corr2, c12, up_flow3, up_feat3), 1)
        x = torch.cat((self.conv2_0(x), x),1)
        x = torch.cat((self.conv2_1(x), x),1)
        x = torch.cat((self.conv2_2(x), x),1)
        x = torch.cat((self.conv2_3(x), x),1)
        x = torch.cat((self.conv2_4(x), x),1)
        flow2 = self.predict_flow2(x)

        x = self.dc_conv4(self.dc_conv3(self.dc_conv2(self.dc_conv1(x))))
        flow2 = flow2 + self.dc_conv7(self.dc_conv6(self.dc_conv5(x)))

        if self.training:
            return flow2,flow3,flow4,flow5,flow6
        else:
            return flow2
```

3.2 TODO: (b) Cost-volume

Implement the cost-volume layer used in the PWC-Net. Please refer to the Fig.2 in the problem text for the illustration of the cost-volume computation.

[9]:
```
import torch.nn as nn
import torch.nn.functional as F

class cost_volume(nn.Module):
    def __init__(self,maxD):
        super(cost_volume,self).__init__()
        self.maxD = maxD
    def forward(self, img_1, img_2):
        B, C, H, W = img_1.shape
        xgrid,ygrid = np.meshgrid(range(-self.maxD,self.maxD+1), range(-self.
↪maxD,self.maxD+1))
        shifts = np.array(list(zip(xgrid.flatten(),ygrid.flatten()))) # Shift␣
↪choices, an array of shape [(2*max_disp+1)^2, 2], where shifts[:,0] are␣
↪x-shifts, shifts[:,1] are y-shifts

        img_1 = F.pad(img_1, (self.maxD,self.maxD,self.maxD,self.maxD),␣
↪mode='constant', value=0)
        volume = img_1.new_zeros([B, shifts.shape[0], H, W])

        # print(img_1.shape)
        # print(shifts.shape)
        for i, shift in enumerate(shifts):
            shift_w, shift_h = shift[0], shift[1]
            # print(shift_w,shift_h)
            ### TODO: Your code here ####
            # Single line of code #
            volume[:, i, :, :] = torch.mean(img_1[:, :, (self.maxD + shift_h):
↪(self.maxD + shift_h)+H, (self.maxD + shift_w):(self.maxD + shift_w)+W] *␣
↪img_2, dim=1)
            ###### End of code ########
        volume = volume.contiguous()
        return volume
```

[10]:
```
# Sanity check
A=torch.ones(1,3,20,25).cuda()
B=torch.ones(1,3,20,25).cuda()
result = cost_volume(4)(A,B)
# Download and load cost_volume_check solution array
!gdown https://drive.google.com/uc?id=1QOG3xdueWMa6gTq6ozhvirMd0XvK0Chj
Solution = torch.load('cost_volume_check.pt')
print('Sanity check Passed!' if torch.all(torch.eq(Solution,result)) else␣
↪'Sanity check failed!')
```

```
Downloading...
From: https://drive.google.com/uc?id=1QOG3xdueWMa6gTq6ozhvirMd0XvK0Chj
To: /content/cost_volume_check.pt
100% 163k/163k [00:00<00:00, 25.9MB/s]
Sanity check Passed!
```

3.3 (C) Overfit on single image pair

Train the implemented PWC-Net using a single pair of image. You don't need to implement anything here.

[11]:
```
def train(model, optimizer, criterion, dataLoader, num_epoch=200):
    model.train()
    for i_epoch in tqdm_notebook(range(num_epoch)):
        for i_batch, sample_batched in enumerate(dataLoader):
            images, gt_flow = sample_batched # B, 2,3,H,W and B,2,H,W
            im_1, im_2 = images[:,0,:,:,:], images[:,1,:,:,:]
            input_imgs = torch.cat((im_1,im_2),1) #B,6,H,W
            output = model(input_imgs.cuda()) #a list of flow at different␣
 ↪scale
            optimizer.zero_grad()
            loss, epe = criterion(output, gt_flow.cuda())
            loss.backward()
            optimizer.step()
        if i_epoch % 10 == 0:
            print("Epoch {}, loss {:.3f}".format(i_epoch, loss.item()))
    print('Final loss {:.3f}:'.format(loss.item()))
def validate(model, dataLoader, visualize = False):
    epe_val = 0
    test_sample_counter = 0
    model.eval()
    with torch.no_grad():
        for i_batch, sample_batched in enumerate(tqdm_notebook(dataLoader)):
            images, gt_flow = sample_batched # B, 2,3,H,W and B,2,H,W
            im_1, im_2 = images[:,0,:,:,:], images[:,1,:,:,:]
            input_imgs = torch.cat((im_1,im_2),1) #B,6,H,W
            output = model(input_imgs.cuda()) #a list of flow at different␣
 ↪scale
            output_full_resolution = upsample2d_as(output, gt_flow)
            epe = EPE(output_full_resolution * 20, gt_flow.cuda())

            N = gt_flow.shape[0]
            test_sample_counter += N
            epe_val += epe.item() * N
            #print(test_sample_counter, epe_val)

            if visualize and i_batch % 5 == 0:
```

```
                 visualize_flow_array(gt_flow[0].cpu().detach().numpy().
↪transpose(1,2,0), title = 'Ground Truth Flow')
                 visualize_flow_array(output[0].cpu().detach().numpy().
↪transpose(1,2,0), title = 'Estimated Flow')
         epe_val /= test_sample_counter
         print("epe_val {:.3f}".format(epe_val))
```

[12]:
```
model = PWCNet().cuda()
optimizer =  torch.optim.Adam(model.parameters(), 1e-4)
criterion = MultiScale(startScale = 4, numScales = 5, l_weight = 0.32, norm =␣
↪'L2').cuda()
train(model,optimizer, criterion, overfit_dataloader, num_epoch = 200)
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:124: UserWarning:
nn.init.kaiming_normal is now deprecated in favor of nn.init.kaiming_normal_.
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:3:
TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`
  This is separate from the ipykernel package so we can avoid doing imports
until

  0%|          | 0/200 [00:00<?, ?it/s]


/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:4004: UserWarning:
Default grid_sample and affine_grid behavior has changed to align_corners=False
since 1.3.0. Please specify align_corners=True if the old behavior is desired.
See the documentation of grid_sample for details.
  "Default grid_sample and affine_grid behavior has changed "

Epoch 0, loss 2.089
Epoch 10, loss 1.008
Epoch 20, loss 0.581
Epoch 30, loss 0.395
Epoch 40, loss 0.290
Epoch 50, loss 0.229
Epoch 60, loss 0.190
Epoch 70, loss 0.163
Epoch 80, loss 0.144
Epoch 90, loss 0.128
Epoch 100, loss 0.117
Epoch 110, loss 0.108
Epoch 120, loss 0.100
Epoch 130, loss 0.091
Epoch 140, loss 0.085
Epoch 150, loss 0.085
Epoch 160, loss 0.077
Epoch 170, loss 0.074
```

```
Epoch 180, loss 0.070
Epoch 190, loss 0.067
Final loss 0.065:
```

3.4 (d) Evaluate and visualize trained model

[13]:
```
!gdown https://drive.google.com/uc?id=1xj_oTXRuK2L9W1v0xc6MbzK7_yTe7nfR #␣
↪Download trained model
model = PWCNet().cuda()
model.load_state_dict(torch.load('model.pth'))
validate(model, val_loader, visualize = True)
```

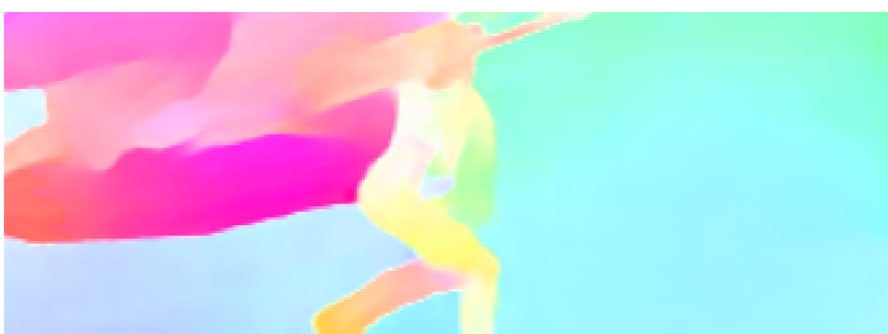
```
Downloading...
From: https://drive.google.com/uc?id=1xj_oTXRuK2L9W1v0xc6MbzK7_yTe7nfR
To: /content/model.pth
100% 37.6M/37.6M [00:00<00:00, 73.1MB/s]

/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:124: UserWarning:
nn.init.kaiming_normal is now deprecated in favor of nn.init.kaiming_normal_.
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:21:
TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`

  0%|          | 0/25 [00:00<?, ?it/s]


/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:4004: UserWarning:
Default grid_sample and affine_grid behavior has changed to align_corners=False
since 1.3.0. Please specify align_corners=True if the old behavior is desired.
See the documentation of grid_sample for details.
  "Default grid_sample and affine_grid behavior has changed "
```

Ground Truth Flow

Estimated Flow

Ground Truth Flow

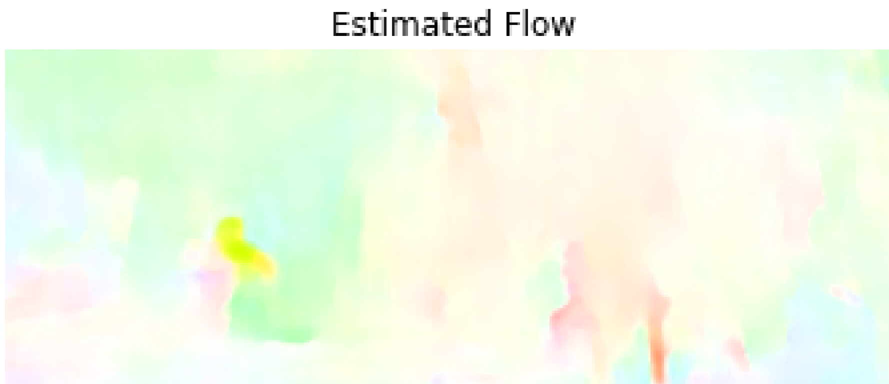
Estimated Flow

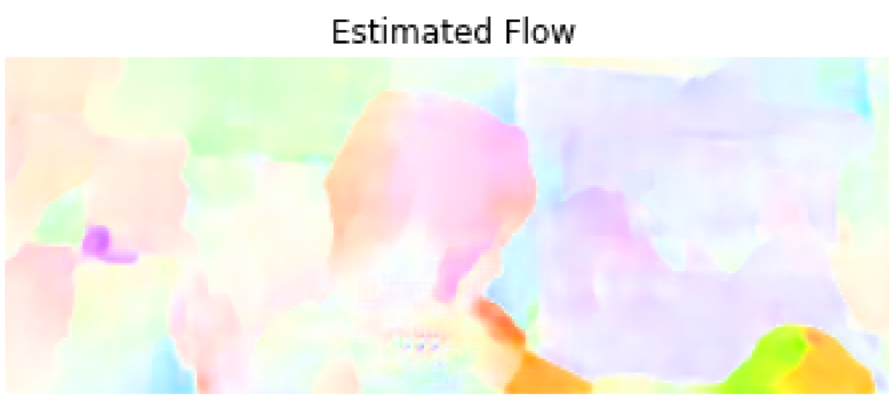


Ground Truth Flow

Estimated Flow

Ground Truth Flow






```
epe_val 12.273
```

3.5 (e) Motion Magnification

Run the provided codes below visualize the magnified movie. You don't need to implement anything here.

[14]:
```
from  scipy import ndimage
def warp_flow(im2, vx, vy):
    """
    im2 warped according to (u,v).
    You'll use this function to warp an image (im2) to make it look like␣
 ↪another image (im1), using
    the given flow field, i.e. we'll make the assumption that
    im1[y, x] = im2[y + vy[y, x], x + vx[y, x]]
    """
    # this code is confusing because we assume vx and vy are the negative
    # of where to send each pixel, as in the results by ce's siftflow code
    y, x = np.mgrid[:im2.shape[0], :im2.shape[1]]
    dy = (y + vy).flatten()[np.newaxis, :]
    dx = (x + vx).flatten()[np.newaxis, :]
    # this says: a recipe for making im1 is to make a new image where im[y, x]␣
 ↪= im2[y + flow[y, x, 1], x + flow[y, x, 0]]
    return np.concatenate([ndimage.map_coordinates(im2[..., i], np.
 ↪concatenate([dy, dx])).reshape(im2.shape[:2] + (1,)) \
                                for i in range(im2.shape[2])], axis = 2)

class MpiSintel_mag(data.Dataset):
    def __init__(self,render_size=[384, 1024], train = False, root = ''):
        image_root = root
        self.render_size = render_size
        self.train = train
        self.image_list = sorted(glob(join(image_root, '*.png')))
        self.size = len(self.image_list)

        self.frame_size = read_gen(self.image_list[0]).shape

        if (self.render_size[0] < 0) or (self.render_size[1] < 0) or (self.
 ↪frame_size[0]%64) or (self.frame_size[1]%64):
            self.render_size[0] = ( (self.frame_size[0])//64 ) * 64
            self.render_size[1] = ( (self.frame_size[1])//64 ) * 64

    def __getitem__(self, index):
        #print(self.image_list[index])
        img1 = read_gen(self.image_list[index])
        #print(img1)
        image_size = img1.shape[:2]
        cropper = StaticCenterCrop(image_size, self.render_size)
        image = list(map(cropper, [img1])) #2,H,W,3
        image = np.array(image).transpose(0,3,1,2)
```

```
        image = torch.from_numpy(image.astype(np.float32))/255.
        return image  # 1,3,H,W

    def __len__(self):
        return self.size
```

[15]:
```
ds = MpiSintel_mag(root='/content/MPI-Sintel-442/training/clean/shaman_2')
numFrames = len(ds)
H,W = ds.render_size
vx = np.zeros([numFrames,H,W])
vy = np.zeros([numFrames,H,W])

#parameters magnification
magnification = 5; #gain for small velocities
th = 30; # not magnify velocities with magnitude above this value
#Estimation motion
I0 = ds[0]
for n in range(1,5):
    In = ds[n]
    input_imgs = torch.cat((In,I0),1) #B,6,H,W
    output = model(input_imgs.cuda()) #a list of flow at different scale
    output_full_resolution = upsample2d_as(output, torch.zeros([1,2,384,1024])).
 ↪cpu().detach().numpy() * 20
    #visualize_flow_array(output_full_resolution[0].transpose(1,2,0))
    vx[n-1,:,:], vy[n-1,:,:] = output_full_resolution[0,0],␣
 ↪output_full_resolution[0,1]

import imageio
images = []
I0 = ds[0][0].numpy().transpose(1,2,0)
#Magnification and creation of new sequence.
#Top half of the gif is the original movie. Bottom half of the gif is the␣
 ↪magnified version.
for n in range(1,5):
    In = ds[n][0].numpy().transpose(1,2,0) # 1,3, H,W

    V = np.sqrt(vx[n-1,:,:]**2+vy[n-1,:,:]**2)
    vx_mag = vx[n-1,:,:]*(V>th) + magnification*vx[n-1,:,:]*(V<=th)
    vy_mag = vy[n-1,:,:]*(V>th) + magnification*vy[n-1,:,:]*(V<=th)
    In_hat = warp_flow(I0, vx_mag, vy_mag)

    In_hat = In*(V>th)[:,:,np.newaxis] + In_hat*(V<=th)[:,:,np.newaxis]

    frame = np.concatenate([In*255**2, In_hat*255**2])
    images.append(np.clip(frame,0,255).astype(np.uint8))

imageio.mimsave('movie.gif', images,format='GIF', fps=3)
```

```
#Load and visualize the original (top half) and manified movie (bottom half).
from IPython.display import Image
Image(open('movie.gif','rb').read())
```

Output hidden; open in https://colab.research.google.com to view.

4 Epipolar Geometry

Download the temple stereo images from the Middlebury stereo dataset.

[64]:
```
!wget https://vision.middlebury.edu/mview/data/data/templeSparseRing.zip
!unzip templeSparseRing.zip
```

```
--2021-12-08 17:45:22--
https://vision.middlebury.edu/mview/data/data/templeSparseRing.zip
Resolving vision.middlebury.edu (vision.middlebury.edu)... 140.233.20.14
Connecting to vision.middlebury.edu
(vision.middlebury.edu)|140.233.20.14|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 4004383 (3.8M) [application/zip]
Saving to: templeSparseRing.zip.1

templeSparseRing.zi 100%[===================>]   3.82M  5.22MB/s    in 0.7s

2021-12-08 17:45:23 (5.22 MB/s) - templeSparseRing.zip.1 saved
[4004383/4004383]

Archive:  templeSparseRing.zip
```

[66]:
```
def process_parameters():
    """
    Reads the parameters for the Middlebury dataset
    :return: an intrinsics matrix containing the camera parameters and
    a list of extrinsics matrices representing mapping from the world to camera␣
 ↪coordinates
    """
    intrinsics = []
    extrinsics = []
    with open(os.path.join("templeSparseRing", "templeSR_par.txt"), 'r') as f:
        _ = f.readline()
        for line in f:
            raw_data = line.split()
            # Read camera parameters K (intrinsics matrix)
            camera_params = np.array(raw_data[1:10]).reshape((3, 3)).
 ↪astype(float)
```

```
            intrinsics.append(camera_params)

            # Read homogeneous transformation (extrinsics matrix)
            rotation = np.array(raw_data[10:19]).reshape((3, 3)).astype(float)
            translation = np.array(raw_data[19:]).reshape((3, 1)).astype(float)
            extrinsics.append(np.hstack([rotation, translation]))

    return intrinsics[0], extrinsics
```

We will select the first image as the reference frame and transform all the extrinsics matrices.

[67]:
```
def set_reference(extrinsics):
    """
    Set the first image as reference frame such that its transformation
    becomes the identity, apply the inverse of the extrinsics matrix of
    the reference frame to all other extrinsics matrices
    :param extrinsics: list of original extrinsics matrices
    :return: list of transformed extrinsics matrices
    """
    shifted_extrinsics = []
    stacked = np.vstack([extrinsics[0], [0, 0, 0, 1]])
    inv_ref = np.linalg.inv(stacked)
    for ex in extrinsics:
        stacked = np.vstack([ex, [0, 0, 0, 1]])
        transformed = np.matmul(stacked, inv_ref)
        transformed /= transformed[-1, -1]
        shifted_extrinsics.append(transformed[:3, :])
    return shifted_extrinsics
```

4.1 TODO: (a) Back-projection

You can use the fundamental matrix to find the epipolar line in another image frame. Alternatively, you can also back-project the pixel in the reference image at different depths. The back-projected pixels will all fall along a ray that resembles the epipolar line.

[142]:
```
def coordinate_transform(intrinsics, extrinsics, pixel, d, i):
    """
    Transform image coordiantes from the reference frame to the second image␣
 ↪given a depth d
    :param intrinsics: the matrix K storing the camera parameters
    :param extrinsics: list of 3 x 4 extrinsics matricies [R | t]
    :param pixel: tuple of two ints representing x and y coordinates on the␣
 ↪reference image
    :param d: a float representing a distance
    :param i: int at the end of the image name (4 represents templeSR0004.png)
    :return: a tuple of ints representing the x, y coordinates on the second␣
 ↪image
    """
```

```
    extrinsics_img2 = extrinsics[i - 1]
    ############## TODO ##############
    # Back-project pixel x in reference frame to world coordinates X
    # X = K^-1 * x * d
    v = np.linalg.inv(intrinsics) @ np.array([pixel[0], pixel[1], 1]).T
    X = d * v
    # Forward project point X to the second image's pixel coordinates x
    # pixel_coord = K * extrinsics_img2 * X
    pixel_coord = intrinsics @ extrinsics_img2 @ np.array([X[0], X[1], X[2],␣
↪1]).T
    pixel_coord = np.array([pixel_coord[0]/pixel_coord[-1], pixel_coord[1]/
↪pixel_coord[-1]]).T
    ############## END ##############
    return pixel_coord.astype(int)
```

4.2 (b) Compute fundamental matrix

[143]:
```
def compute_fundamental_matrix(intrinsics, extrinsics, i):
    """
    Compute the fundamental matrix between the i-th image frame and the
    reference image frame
    :param intrinsics: the intrinsics camera matrix K
    :param extrinsics: list of original extrinsics matrices
    :param i: int at the end of the image name (2 represents templeSR0002.png)
    :return: list of transformed extrinsics matrices
    """
    rot = extrinsics[i - 1][:3, :3]
    trans = extrinsics[i - 1][:3, 3]
    # Compute the epipole and fundamental matrix
    # e = K R^T t
    epipole = intrinsics @ rot.T @ trans
    epipole_cross = np.array([[0, -epipole[2], epipole[1]], [epipole[2], 0,␣
↪-epipole[0]], [-epipole[1], epipole[0], 0]])
    # F = K'^(-T)RK^T[e]_x
    fundamental = np.linalg.inv(intrinsics).T @ rot @ intrinsics.T @␣
↪epipole_cross
    fundamental /= fundamental[-1, -1]
    return fundamental
```

4.3 (c) Visualize epipolar line

You will then visualize the epipolar line with both the fundamental matrix and the back-projected ray.

[144]:
```
def visualize_epipolar_line(pixel, intrinsics, extrinsics, fundamental, i):
    """
    Visualizes the pixel in the reference frame, and its corresponding
```

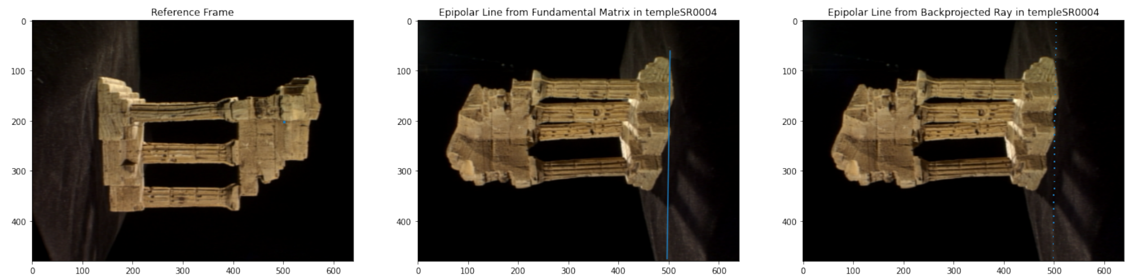
```
    epipolar line in the i-th image frame
    :param pixel: a tuple of (x, y) coordinates in the reference image
    :param fundamental: fundamental matrix
    :param i: int at the end of the image name (4 represents templeSR0004.png)
    """
    img1 = imread(os.path.join("templeSparseRing", "templeSR0001.png"))
    img2 = imread(glob(os.path.join("templeSparseRing", "*.png"))[i - 1])

    # Plot reference image with a chosen pixel
    _, ax = plt.subplots(1, 3, figsize=(img1.shape[1] * 3 / 80, img1.shape[0] /␣
↪80))
    ax[0].imshow(img1)
    ax[0].add_patch(patches.Rectangle(pixel, 5, 5))
    ax[0].title.set_text('Reference Frame')

    # Compute epipolar_line from fundamental matrix and the pixel coordinates
    # Hartley Zisserman page 246: "I' = Fx is the epipolar line corresponding␣
↪to x"
    # Epipolar line l' in image 2's coordinates
    epipolar_line = fundamental @ np.array([pixel[0], pixel[1], 1]).T

    # Plot epipolar line from fundamental matrix in second image
    x = np.arange(img2.shape[1])
    y = np.array((-epipolar_line[0] * x - epipolar_line[2]) / epipolar_line[1])
    indices = np.where(np.logical_and(y >= 0, y <= img2.shape[0]))
    ax[1].imshow(img2)
    ax[1].plot(x[indices], y[indices])
    ax[1].title.set_text('Epipolar Line from Fundamental Matrix in templeSR000'␣
↪+ str(i))

    # Epipolar line from backprojected ray of different depths
    ax[2].imshow(img2)
    ax[2].title.set_text('Epipolar Line from Backprojected Ray in templeSR000'␣
↪+ str(i))
    for d in np.arange(0.4, 0.8, 0.005):
      pixel_coord = coordinate_transform(intrinsics, extrinsics, pixel, d, i)
      if pixel_coord[0] >= 0 and pixel_coord[1] >= 0 and pixel_coord[0] + 3 < \
              img1.shape[1] and pixel_coord[1] + 3:
          ax[2].add_patch(patches.Rectangle((pixel_coord[0], pixel_coord[1]),␣
↪3, 3))
    plt.show()
```

[145]:

```
# TODO: Feel free to try different images and pixel coordinates
# image_frame is the image number (i.e. 4 is templeSR0004.png)
image_frame = 4

# pixel location (x, y) in the reference frame
```

```
pixel = (500, 200)
```

[146]:
```
intrinsics, extrinsics = process_parameters()
shifted_extrinsics = set_reference(extrinsics)
fundamental = compute_fundamental_matrix(intrinsics, shifted_extrinsics,␣
 ↪i=image_frame)
visualize_epipolar_line(pixel, intrinsics, shifted_extrinsics, fundamental,␣
 ↪i=image_frame)
```



5 Convert to PDF

If the below cell doesn't work, try this alternative.

[ ]:
```
# generate pdf
# %%capture
!git clone https://gist.github.com/bc5f1add34fef7c7f9fb83d3783311e2.git
!cp bc5f1add34fef7c7f9fb83d3783311e2/colab_pdf.py colab_pdf.py
from colab_pdf import colab_pdf
# change the name to your ipynb file name shown on the top left of Colab window
# Important: make sure that your file name does not contain spaces!
colab_pdf('cktran_09859713.ipynb')
```

```
Reading package lists... Done
Building dependency tree
Reading state information... Done
texlive-fonts-recommended is already the newest version (2017.20180305-1).
texlive-generic-recommended is already the newest version (2017.20180305-1).
texlive-xetex is already the newest version (2017.20180305-1).
0 upgraded, 0 newly installed, 0 to remove and 57 not upgraded.
[NbConvertApp] Converting notebook /content/drive/My Drive/Colab
Notebooks/cktran_09859713.ipynb to pdf
```