

Project 1

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Abstract

This paper will analyze the behaviour of the equations that define how

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Part 1

Models under Consideration

1.1 Discrete Model for Account Balance

$$A_d(t) = A_0\left(1 + \frac{r}{n}\right)^{nt} \quad (1)$$

1.2 Continuous Model for Account Balance

$$A_c(t) = A_0e^{rt} \quad (2)$$

1.3 Rate of Change in Account Balance in Continuous Model

$$\frac{dA_c}{dt} = rA_c \quad (3)$$

1.4 Installment Debt Rate Model

$$\frac{dA_c}{dt} = rA_c - P \quad (4)$$

1.4.1 Classification

Part 2

Comparison of Discrete and Continuous Account Balance Models

2.1 Comparison of Discrete model as compounding rate increases

2.2 Compounding rate calculated annually

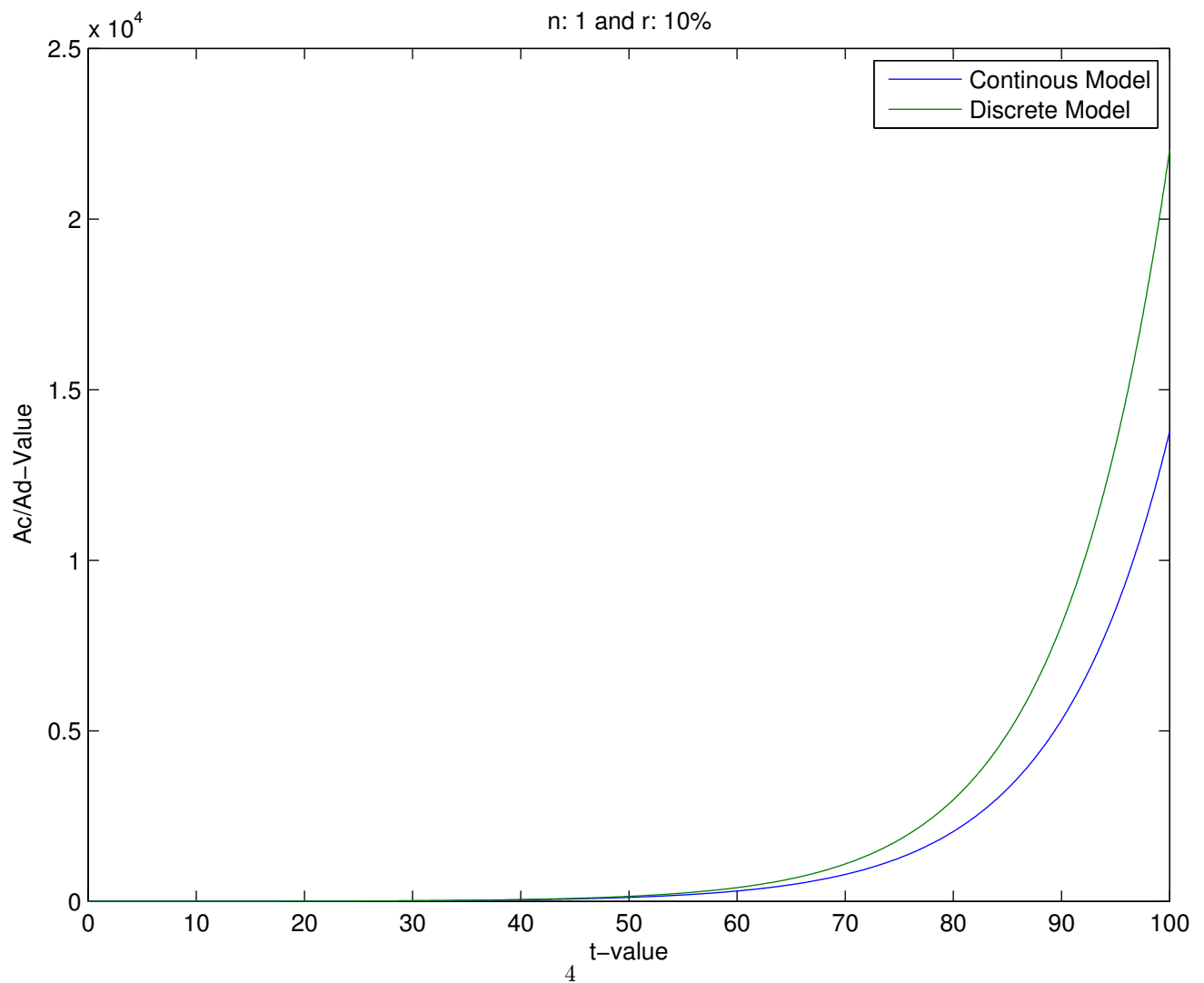


Figure 1: Annual Compounding Rate of 10%

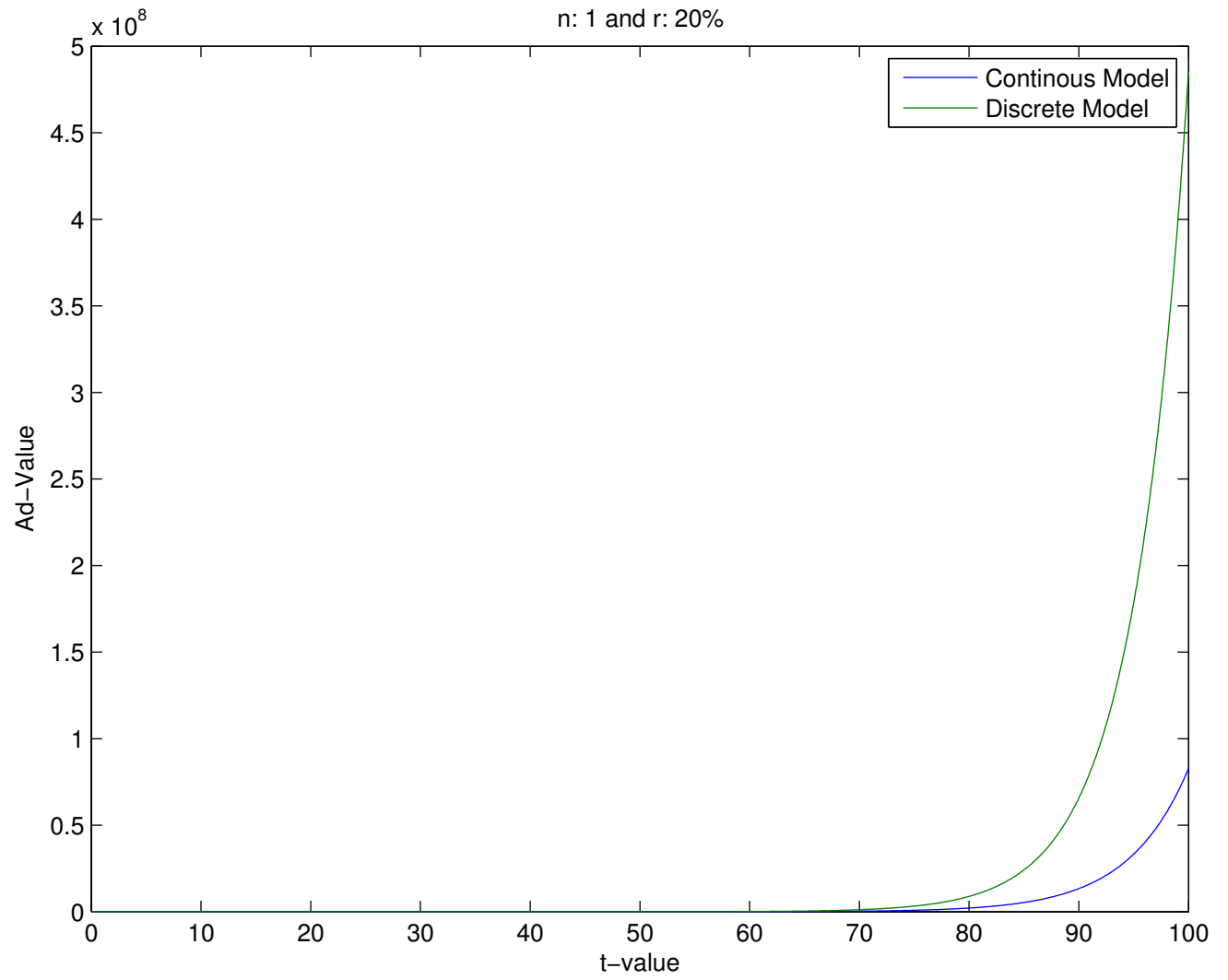


Figure 2: Annual Compounding Rate of 20%

2.3 Compounding rate calculated biannually

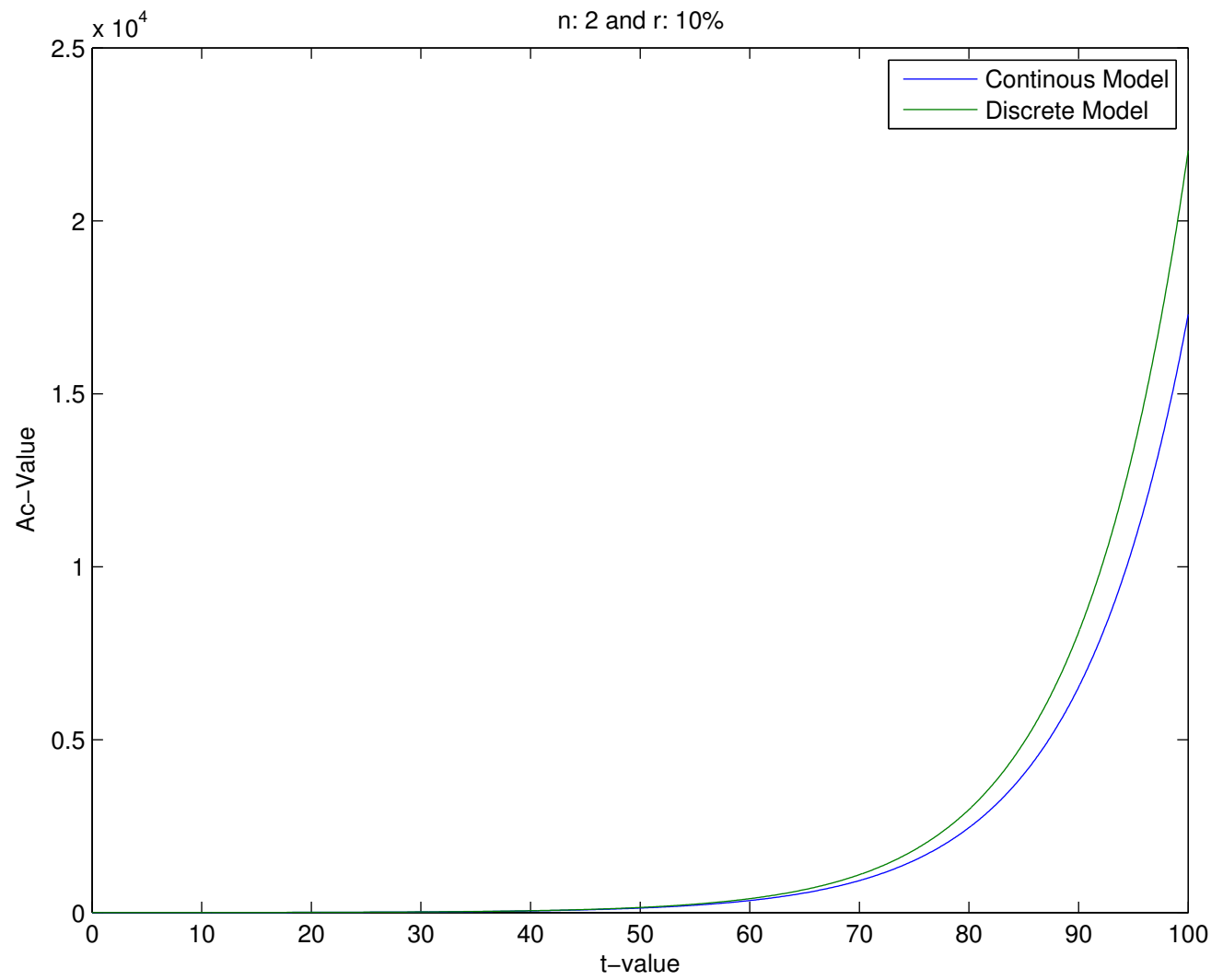


Figure 3: Biannual Compounding Rate of 10%

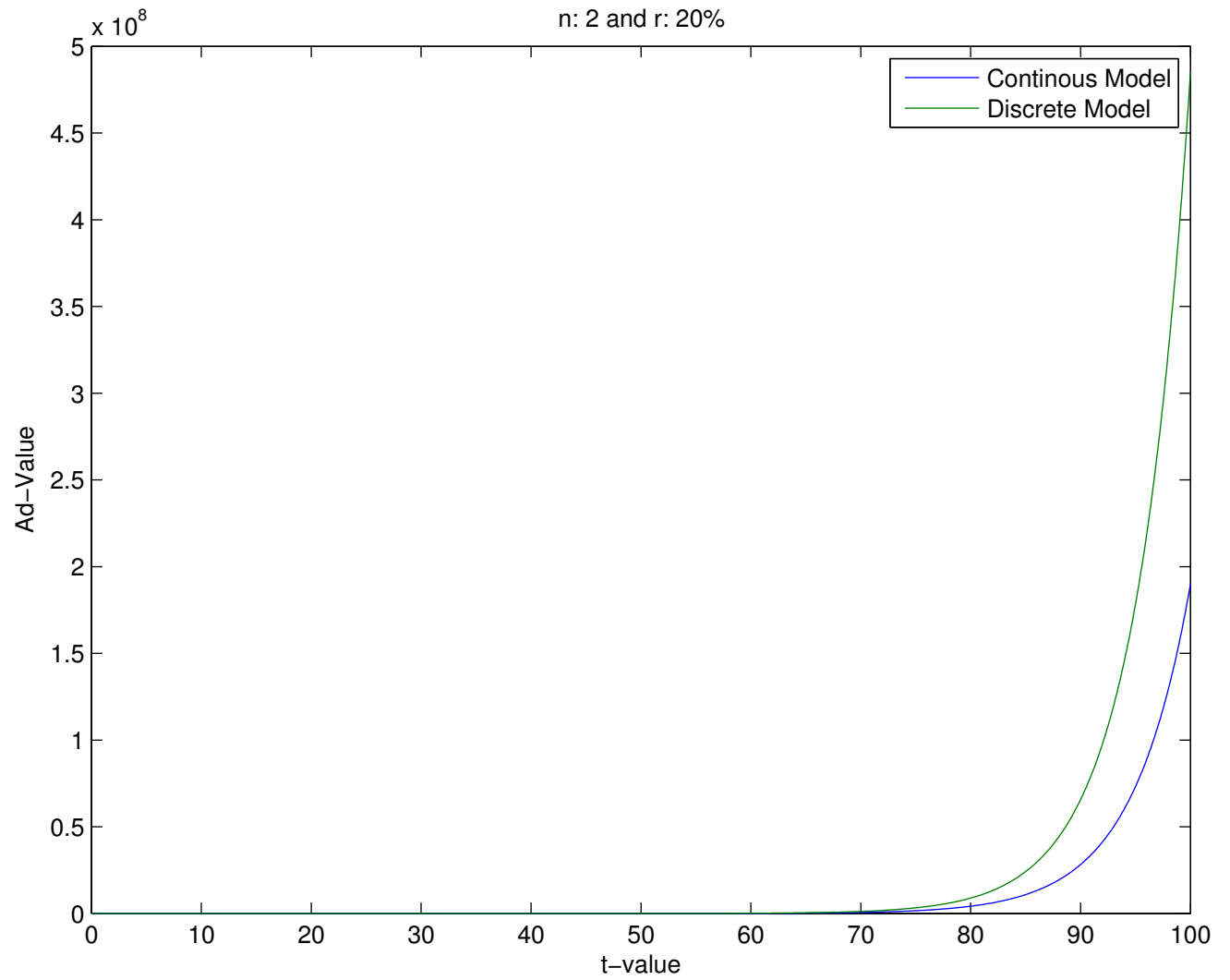


Figure 4: Biannual Compounding Rate of 20%

2.4 Compounding rate calculated quarterly

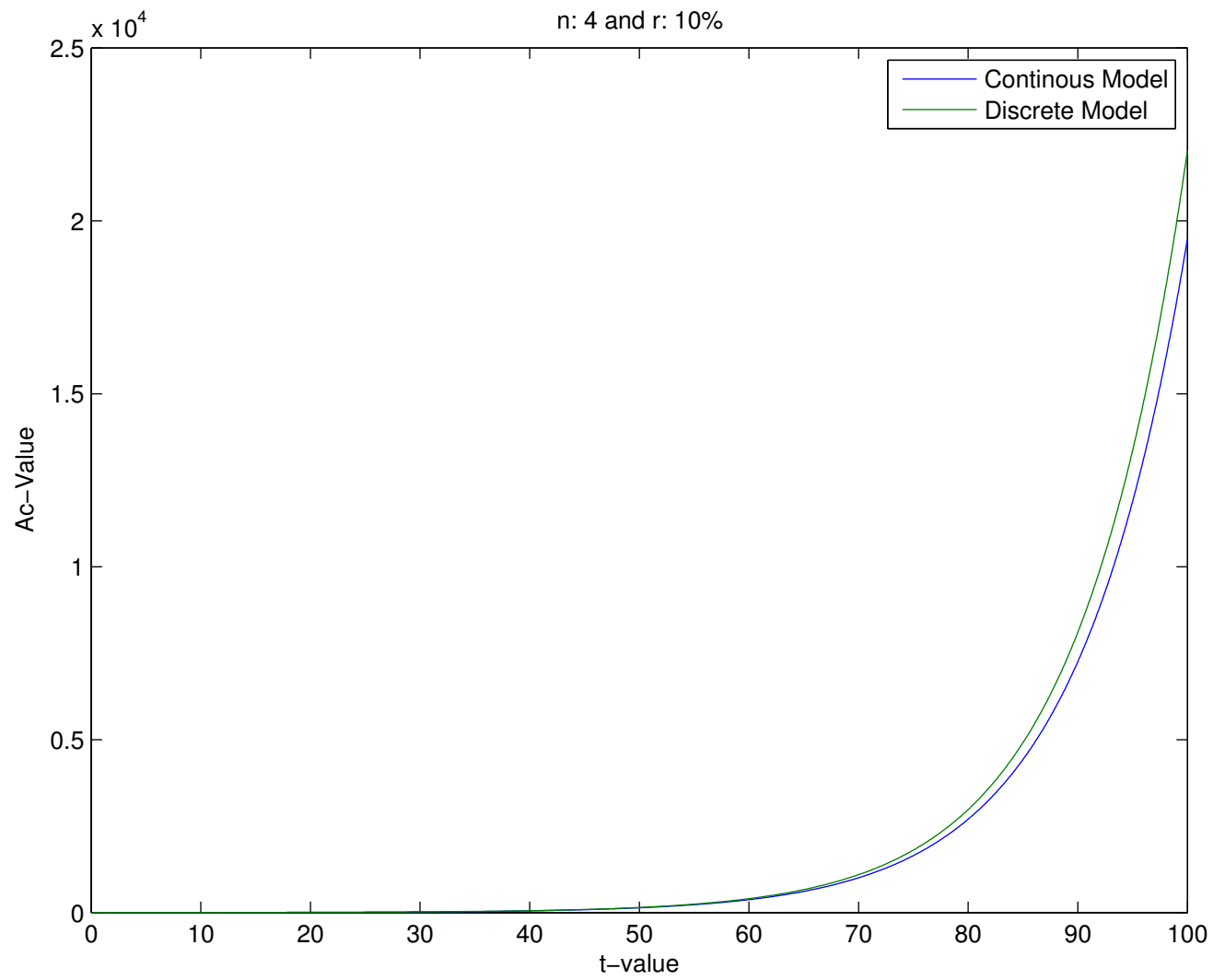


Figure 5: Quarterly Compounding Rate of 10%

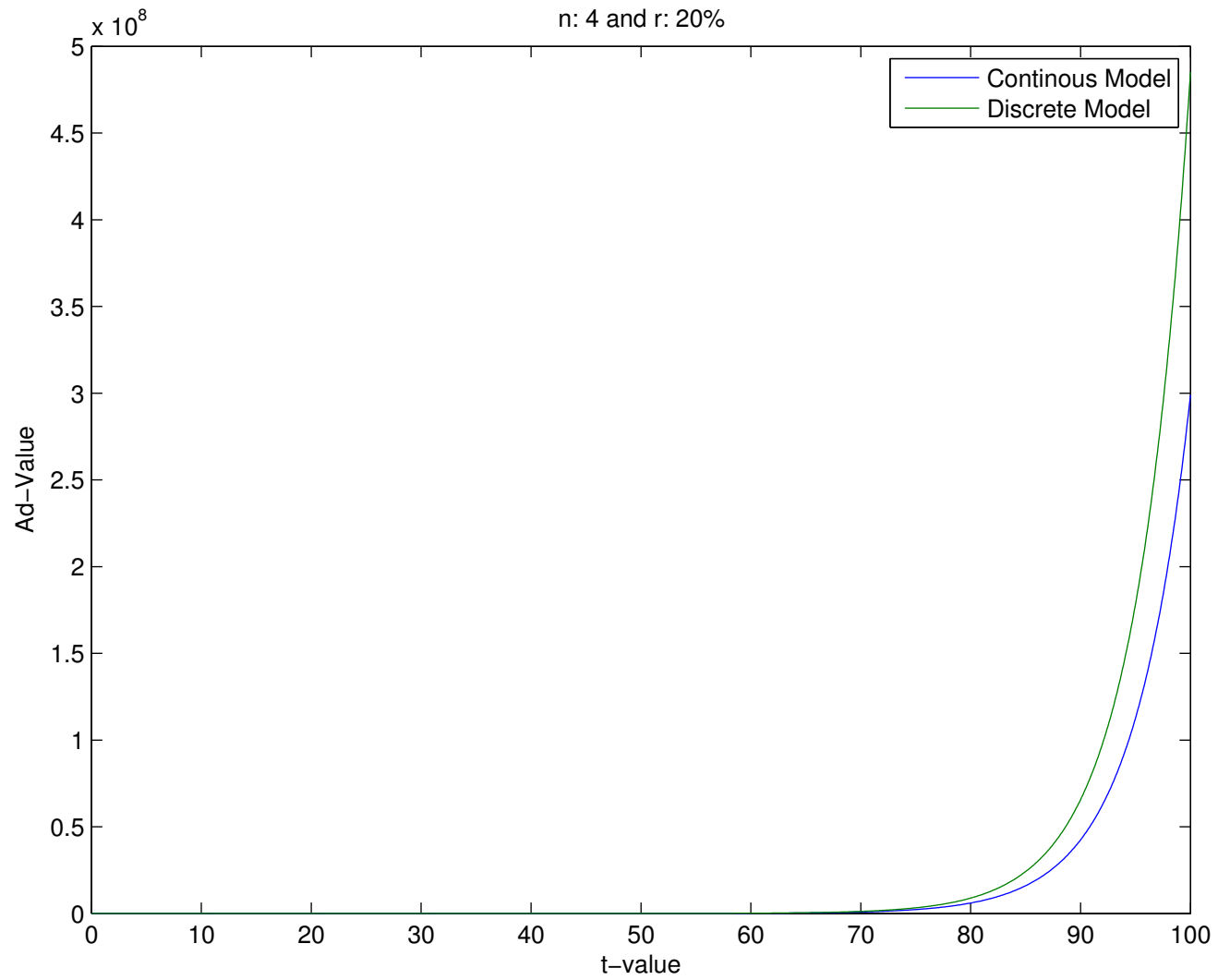


Figure 6: Quarterly Compounding Rate of 20%

2.5 Compounding rate calculated monthly

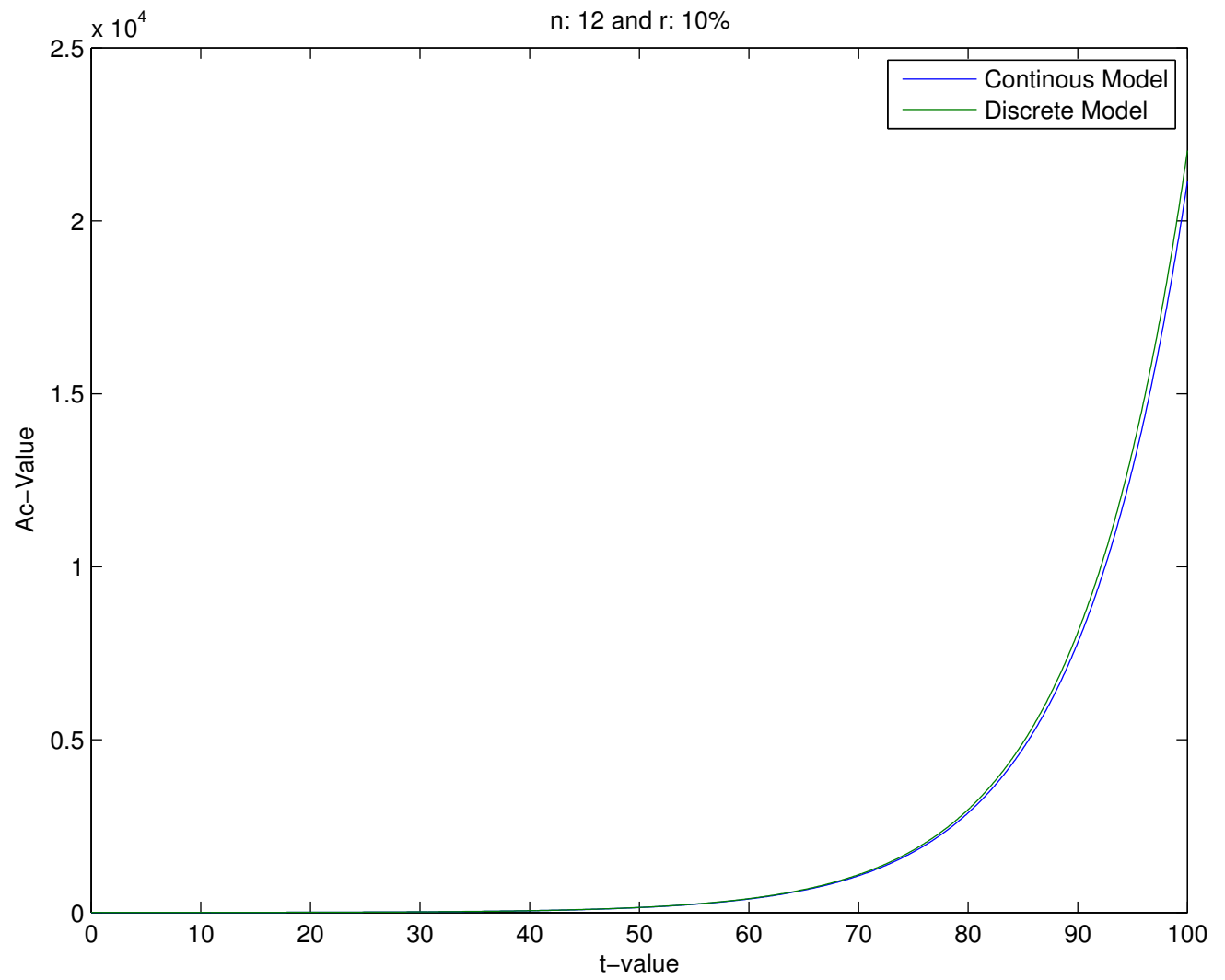


Figure 7: Monthly Compounding Rate of 10%

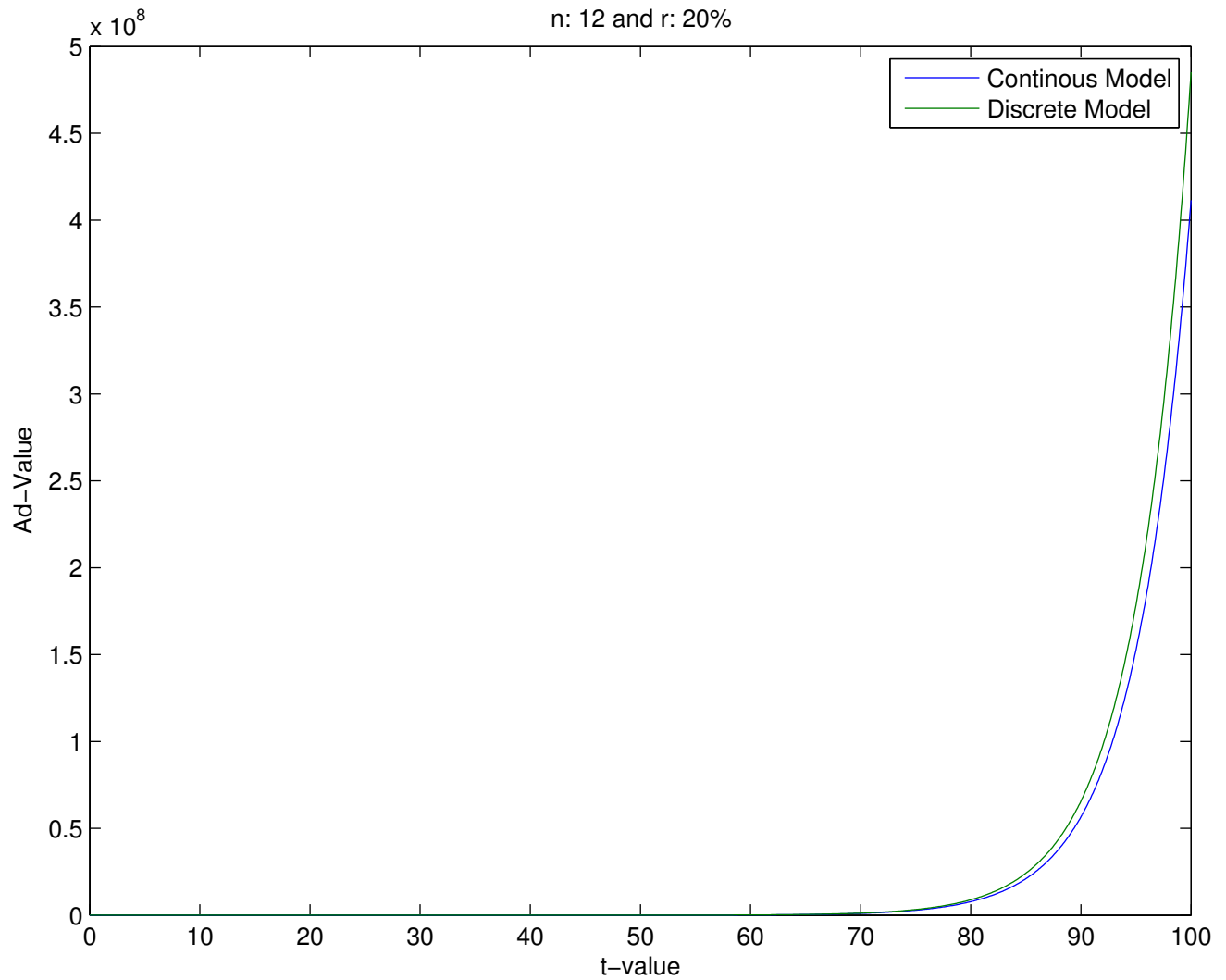


Figure 8: Monthly Compounding Rate of 20%

2.6 Accuracy of Discrete model compared to Continuous model

2.6.1 Qualitative Review

Review Graphs in detail here (behaviour of n and r)

2.6.2 Quantitative Review

In what year is the difference greater than a dollar?

Rate	Year
6.9%	107
12.99%	53
19.99%	33

Assumptions: $A_0 = 1$ and $n = 365$

Table 1: Year Discrete and Continuous Models differ by one dollar

Part 3

Characterization of Installment Debt Model

3.1 Equilibrium Solution

3.2 Derivative Behavior

3.3 Direction Field

3.4 Initial Value Problem

3.5 Behaviour of rate of payment over Time

3.5.1 Time to pay off debt at rate of payment

3.5.2 Payment needed to pay off debt in a given time

3.6 Accuracy of Installment Debt Model

Part 4

Installment Savings

4.1 Calculation

4.2 Initial Value Solution

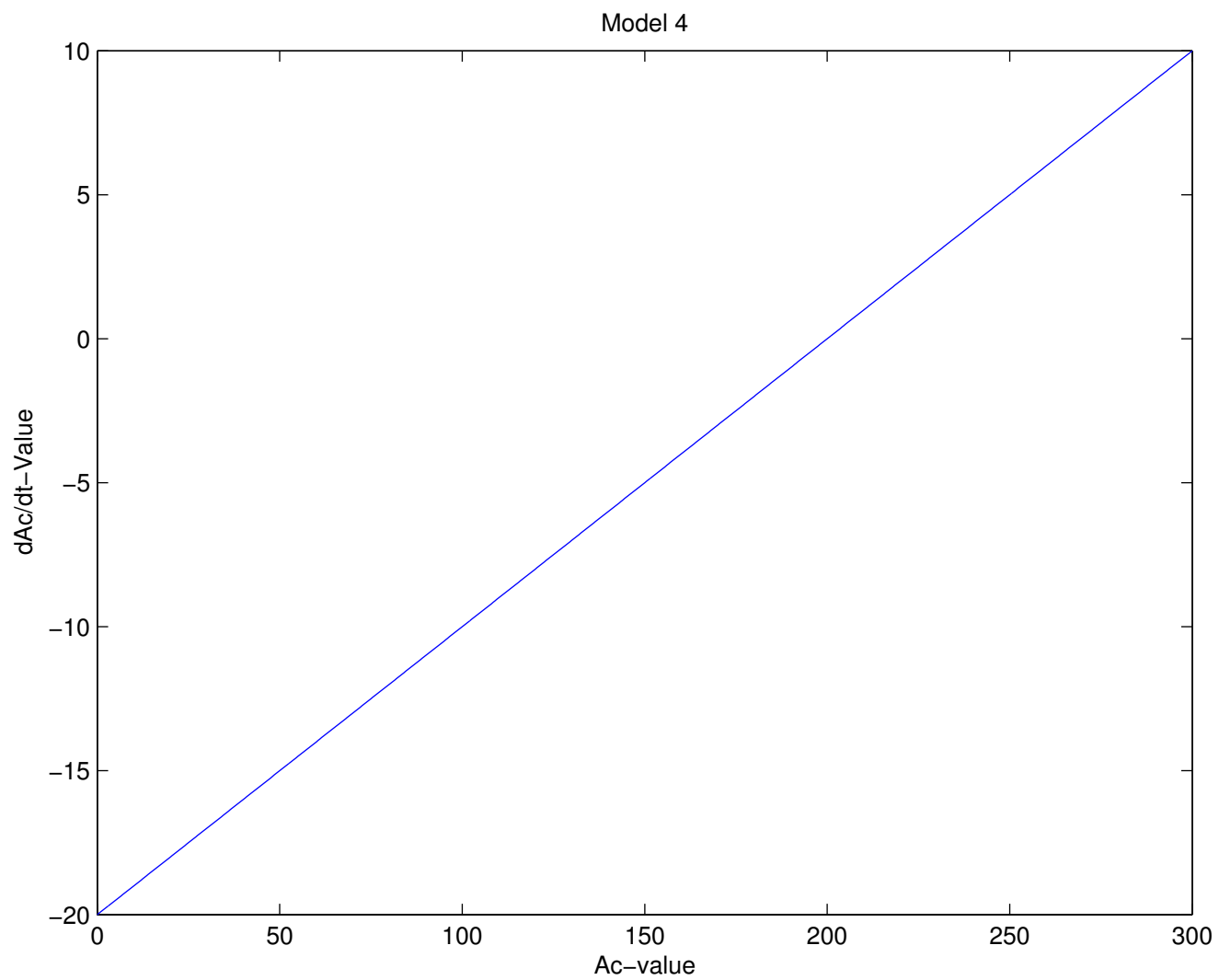


Figure 9: Derivative of Model 4 at account balance

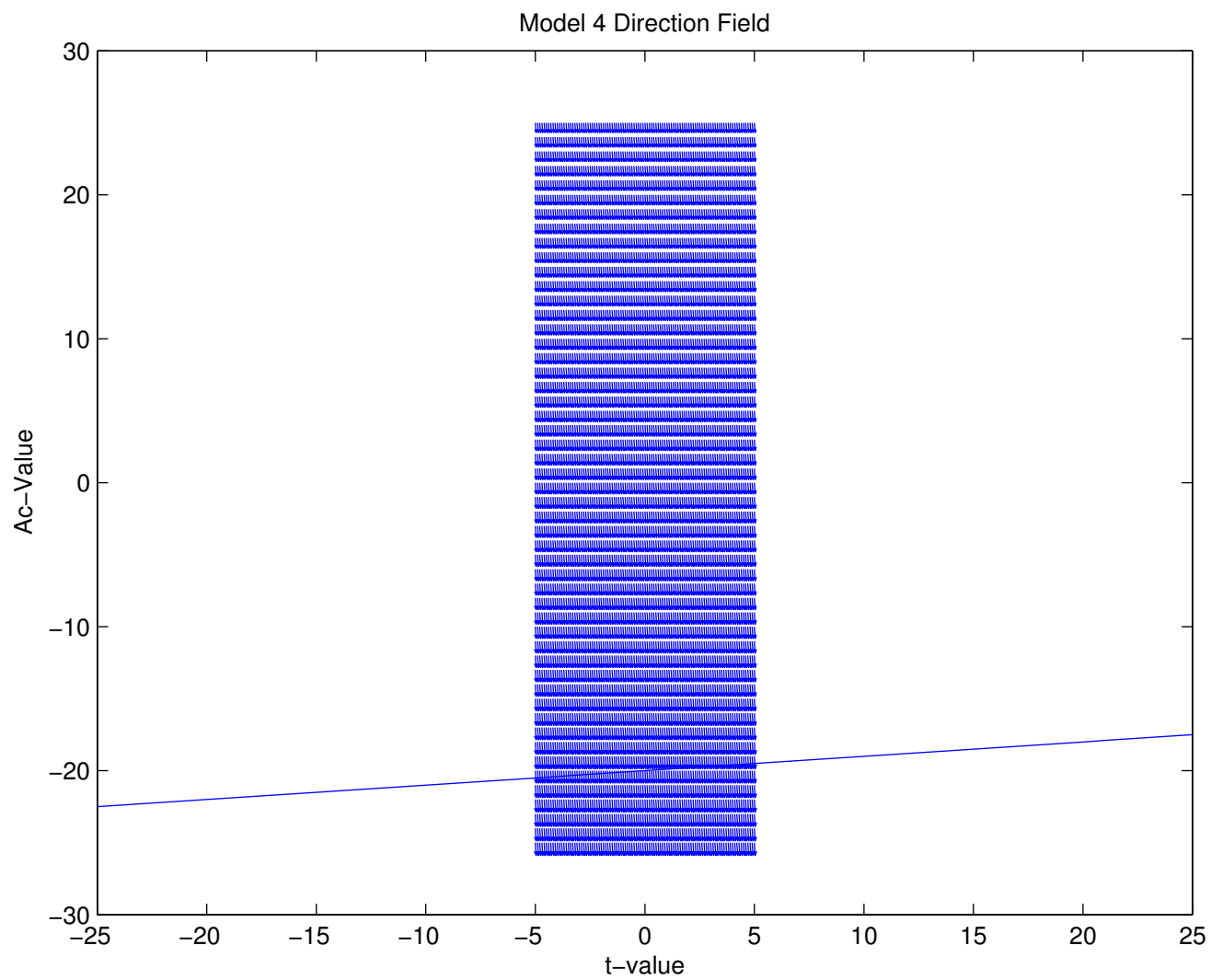


Figure 10: Direction Field of Model 4