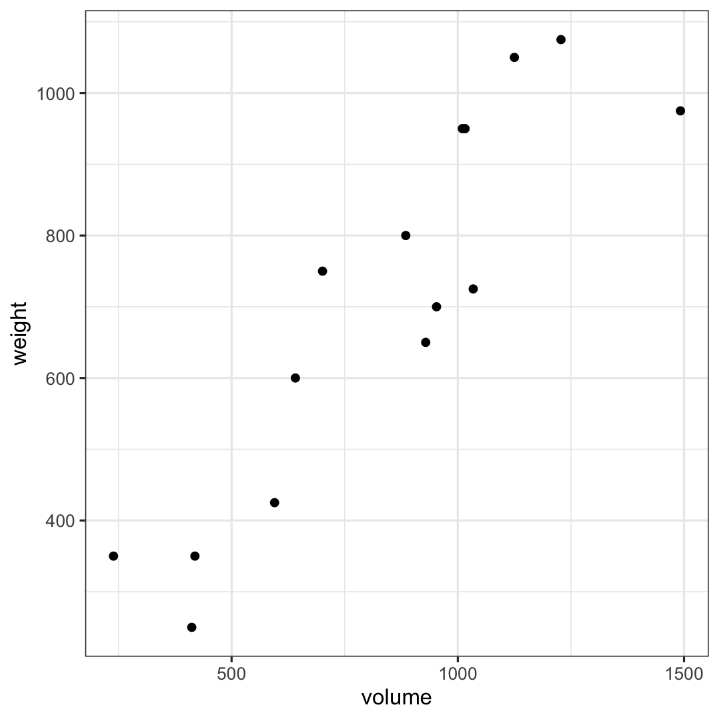
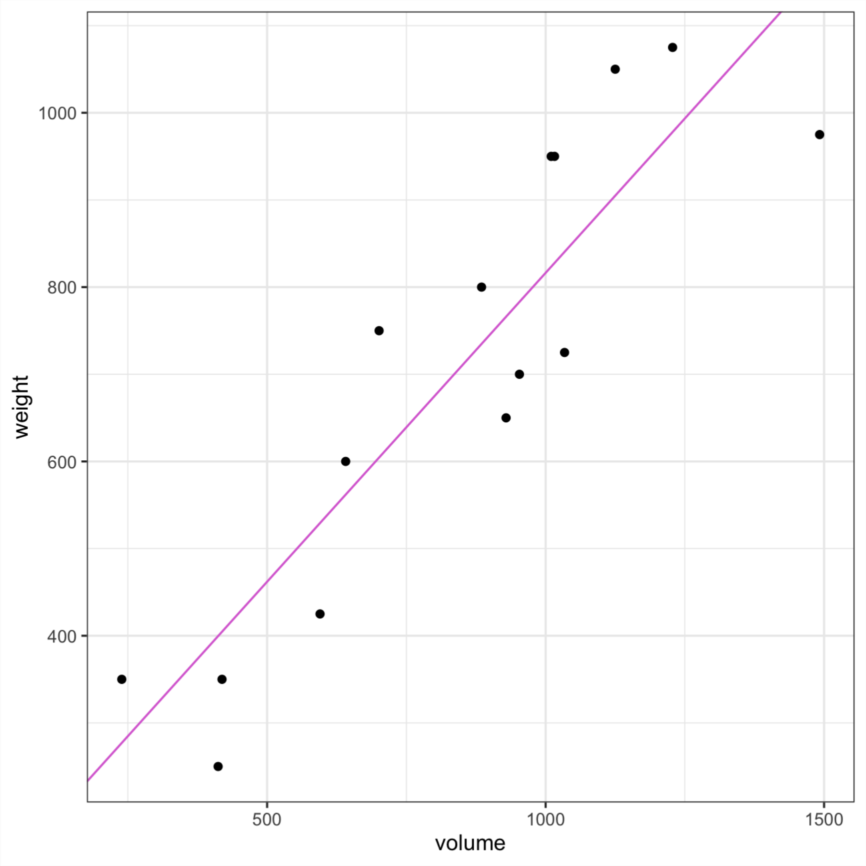
# Extending the Linear Model

# Example: shipping books

```
ggplot(books, aes(x = volume, y = weight)) +
  geom_point()
```



```r
ggplot(books, aes(x = volume, y = weight)) +
  geom_point() +
  geom_abline(intercept = m1$coef[1],
              slope = m1$coef[2], col = "orchid")
```



# Fitting the linear model

```
m1 <- lm(weight ~ volume, data = books)
```

```
summary(m1)
```

```
##
## Call:
## lm(formula = weight ~ volume, data = books)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -189.97 -109.86   38.08  109.73  145.57
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 107.67931   88.37758   1.218    0.245
## volume        0.70864    0.09746   7.271 6.26e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 123.9 on 13 degrees of freedom
## Multiple R-squared:  0.8026,    Adjusted R-squared:  0.7875
## F-statistic: 52.87 on 1 and 13 DF,  p-value: 6.262e-06
```

# Multiple Regression

Allows us create a model to explain one *numerical* variable, the response, as a linear function of many explanatory variables that can be both *numerical* and *categorical*.

We posit the true model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \epsilon; \quad \epsilon \sim N(0, \sigma^2)$$

# Estimating $\beta_0$, $\beta_1$ etc.

In least-squares regression, we're still finding the estimates that minimize the sum of squared residuals.

$$e_i = y_i - \hat{y}_i$$

$$RSS = \sum_{i=1}^{n} e_i^2$$

And yes, they have a closed-form solution.

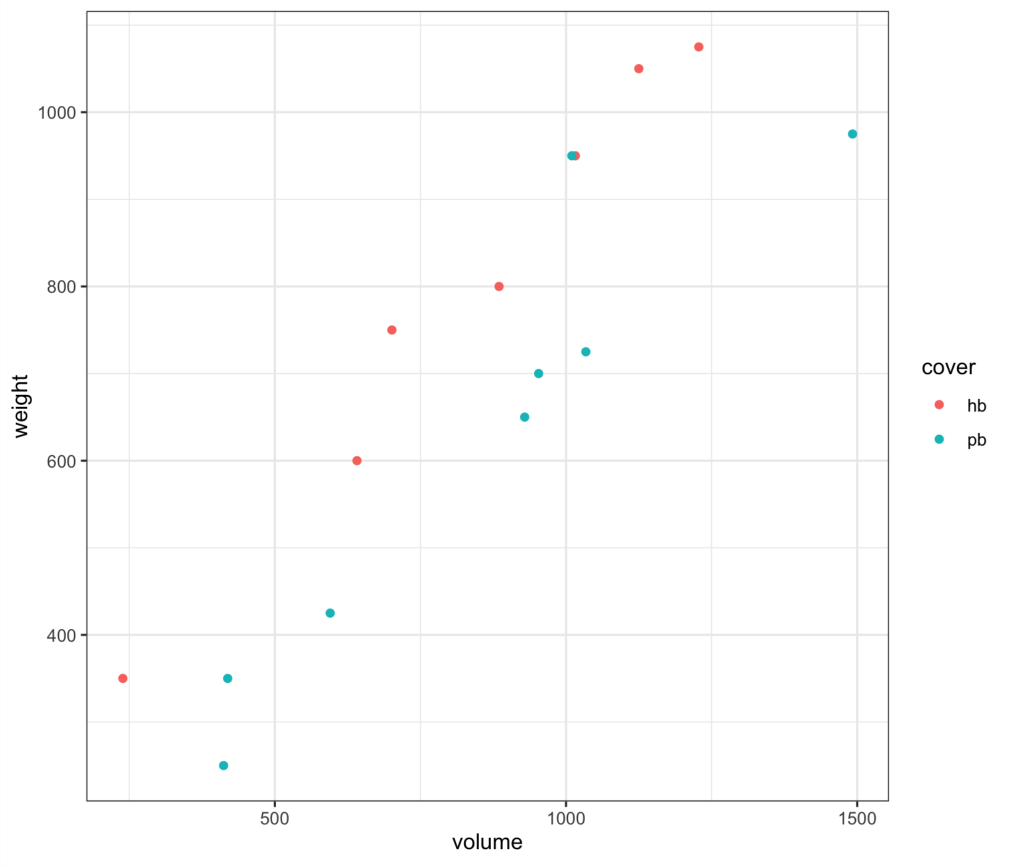
$$\hat{\beta} = (X'X)^{-1}X'Y$$

In R:

```
lm(Y ~ X1 + X2 + ... + Xp, data = mydata)
```

# Example: shipping books

```
ggplot(books, aes(x = volume,
                  y = weight,
                  color = cover)) +
  geom_point()
```



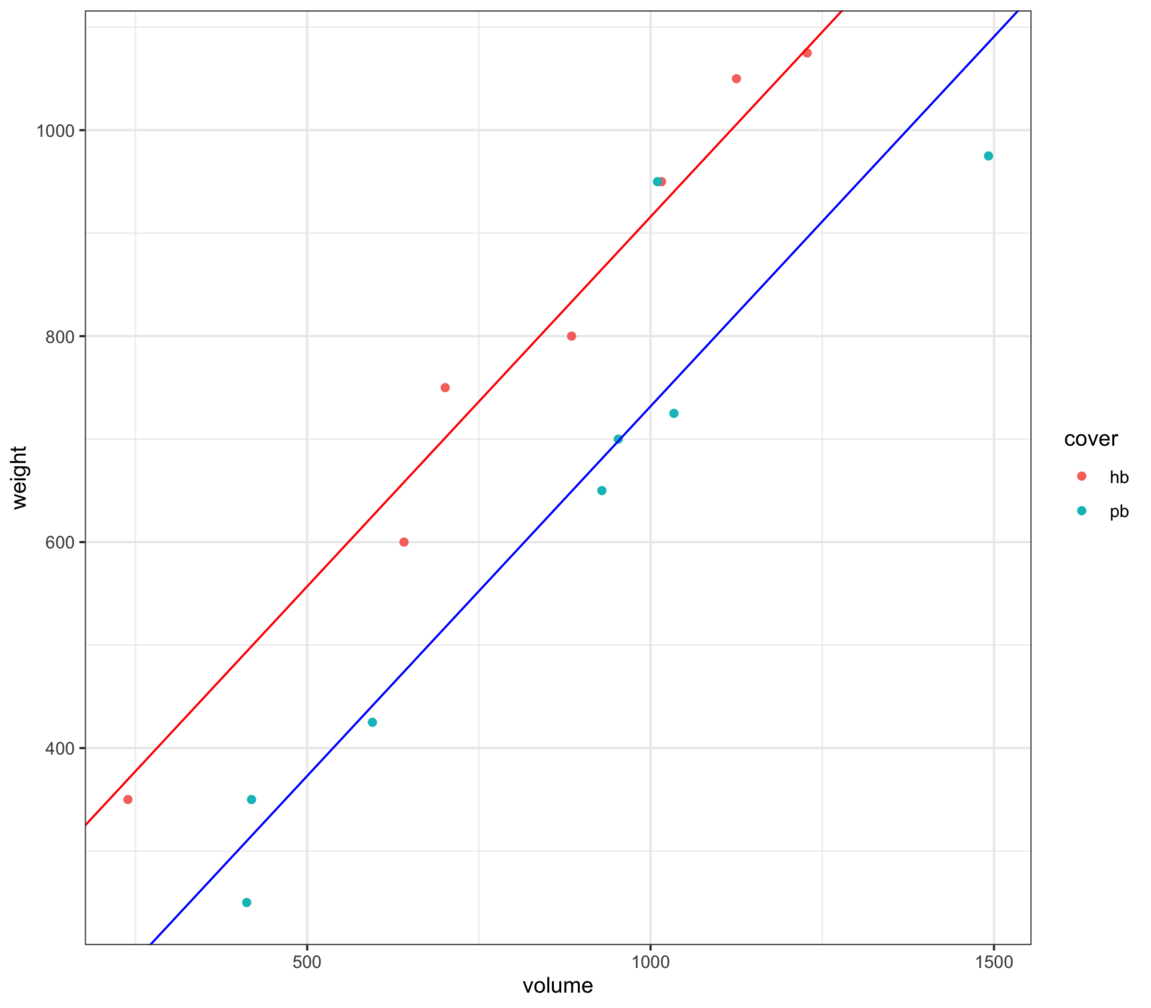
# Example: shipping books

```
m2 <- lm(weight ~ volume + cover, data = books)
```

```
summary(m2)
```

```
##
## Call:
## lm(formula = weight ~ volume + cover, data = books)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -110.10  -32.32  -16.10   28.93  210.95
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  197.96284   59.19274   3.344 0.005841 **
## volume         0.71795    0.06153  11.669  6.6e-08 ***
## coverpb     -184.04727   40.49420  -4.545 0.000672 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 78.2 on 12 degrees of freedom
## Multiple R-squared:  0.9275,    Adjusted R-squared:  0.9154
## F-statistic: 76.73 on 2 and 12 DF,  p-value: 1.455e-07
```

# Example: shipping books



## MLR slope interpretation

The slope corresponding to the dummy variable tell us:

- How much vertical separation there is between our lines
- How much `weight` is expected to increase if `cover` goes from 0 to 1 and `volume` is left unchanged.

Each $\hat{\beta}_i$ tells you how much you expect the $Y$ to change when you change the $X_i$, while **holding all other variables constant**.

# Activity

Load in the LA homes data set and fit the following model:

$$logprice \sim logsqft + bed + city$$

```
URL <- "http://andrewpbray.github.io/data/LA.csv"
LA <- read.csv(URL)
```

1. What is the geometry of this model?
2. What appears to be the reference level for `city`?
3. In the context of this problem, what is suggested by the *sign* of the coefficient for `bed`? Do this make sense to you?

```r
LA <- read.csv("http://andrewpbray.github.io/data/LA.csv")
LA <- mutate(LA, logprice = log(price), logsqft = log(sqft))
m1 <- lm(logprice ~ logsqft + bed + city, data = LA)
summary(m1)
```

```
##
## Call:
## lm(formula = logprice ~ logsqft + bed + city, data = LA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.26020 -0.24897 -0.01613  0.21804  1.37277
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         5.13068    0.21201  24.200   <2e-16 ***
## logsqft             1.20729    0.03036  39.769   <2e-16 ***
## bed                -0.03010    0.01284  -2.345   0.0191 *
## cityLong Beach     -0.88280    0.03467 -25.464   <2e-16 ***
## citySanta Monica -0.09416    0.04022  -2.341   0.0194 *
## cityWestwood       -0.46244    0.04876  -9.484   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3704 on 1588 degrees of freedom
## Multiple R-squared:  0.8742,    Adjusted R-squared:  0.8738
## F-statistic:  2206 on 5 and 1588 DF,  p-value: < 2.2e-16
```

# Interactions

Does the relationship between `logsqft` and `logprice` change depending on the `city`?

```
m2 <- lm(logprice ~ logsqft + bed + city + logsqft:city,
         data = LA)
```

```r
summary(m2)
```

```
##
## Call:
## lm(formula = logprice ~ logsqft + bed + city + logsqft:city,
##     data = LA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.30385 -0.23866 -0.01576  0.21562  1.36668
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               4.43151    0.38515  11.506  < 2e-16 ***
## logsqft                   1.29578    0.05019  25.820  < 2e-16 ***
## bed                      -0.03794    0.01296  -2.928 0.003460 **
## cityLong Beach           -0.53386    0.37968  -1.406 0.159902
## citySanta Monica          1.75128    0.47010   3.725 0.000202 ***
## cityWestwood              2.43192    0.90674   2.682 0.007394 **
## logsqft:cityLong Beach   -0.03663    0.04730  -0.774 0.438807
## logsqft:citySanta Monica -0.24345    0.06052  -4.022 6.03e-05 ***
## logsqft:cityWestwood     -0.38773    0.12251  -3.165 0.001581 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3676 on 1585 degrees of freedom
## Multiple R-squared:  0.8763,    Adjusted R-squared:  0.8757
```

# Interactions

Does the relationship between `logsqft` and `logprice` change depending on the number of `bed`?

```
m3 <- lm(logprice ~ logsqft + bed + logsqft:bed,
         data = LA)
```

```
summary(m3)
```

```
##
## Call:
## lm(formula = logprice ~ logsqft + bed + logsqft:bed, data = LA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.75668 -0.32825 -0.04576  0.31841  1.85602
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.803227   0.271328  10.331  < 2e-16 ***
## logsqft      1.487273   0.040007  37.175  < 2e-16 ***
## bed         -0.644164   0.067255  -9.578  < 2e-16 ***
## logsqft:bed  0.064093   0.008023   7.989 2.59e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4783 on 1590 degrees of freedom
## Multiple R-squared:  0.7899,    Adjusted R-squared:  0.7895
## F-statistic:  1992 on 3 and 1590 DF,  p-value: < 2.2e-16
```