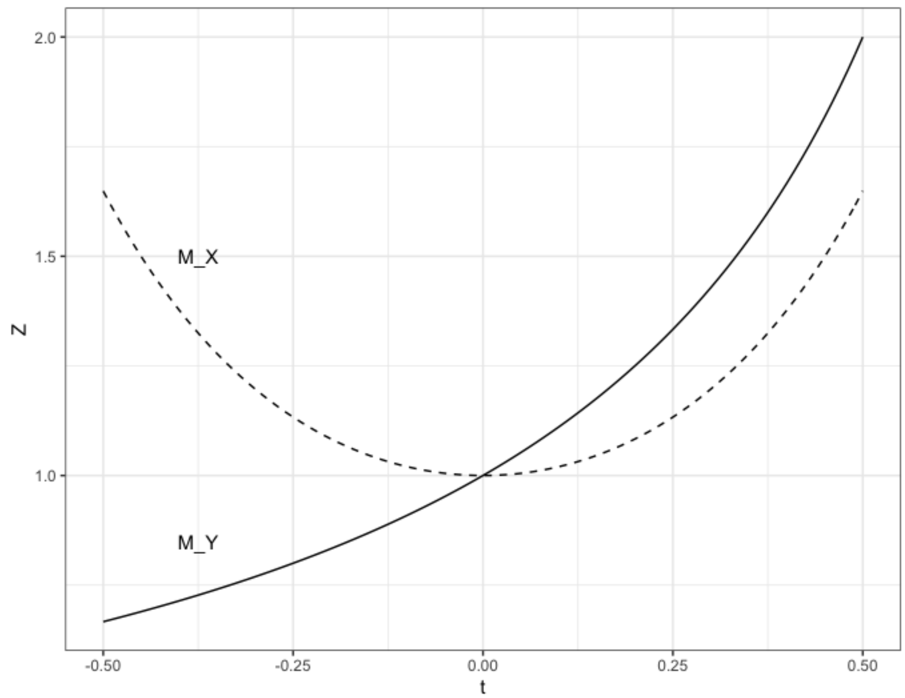
Moment Generating Functions

1. The graphs of the MGFs for two random variable X and Y are shown below. Based on the graph, which variable has the larger mean? Which has the larger variance? Is either variable symmetric? Carefully explain how you know.



2. Let $X \sim \text{DUnif}(\{-1, 1\})$ (i.e. $P(X = 1) = \frac{1}{2}$ and $P(X = -1) = \frac{1}{2}$). Find the MGF of X, then use the Taylor series expansion for the MGF to find **all** moments of X.

3. (*) Suppose X is a random variable with MGF $M(t)$. The *cumulant generating function* for X is defined to be $g(t) = \log M(t)$. Expanding $g(t)$ as a Taylor series,

$$g(t) = \sum_{j=1}^{\infty} \frac{c_j}{j!} t^j$$

and the coefficient c_j is called the jth *cumulant* of X.

   (a) How do they 1st and 2nd cumulants of X relate to the moments of X?

   (b) Suppose $X \sim \text{Pois}(\lambda)$ and $Y \sim N(\mu, \sigma^2)$. Find formulas for the cumulant generating functions for X and Y, and then compute then nth cumulants of each variable.