# Class Activity 9-20

## Expectation

1. Consider a sequence of $n$ coin flips of a biased coin, with probability $p$ of landing heads. Let $X$ be the number of heads observed and let $Y$ be the number of tails observed. Compute $E[X+Y]$ in two ways:
   (a) Using linearity of expectation.
   (b) By thinking about what the "random" variable $X+Y$ represents.

2. We often abbreviate the expression $E[X^2]$ as $EX^2$. But care should be taken to distinguish this from $(E[X])^2$. Show that $E[X^2]$ and $(E[X])^2$ are not in general equal. Hint: Consider $X \sim \mathrm{DUnif}\{-1,1\}$.

3. (*) Player $A$ chooses a random integer between 1 and 100, with probability $p_j$ of choosing $j$ (for $j = 1, 2, \ldots, 100$). Player B guesses the number that player A picked, and receives from player A that amount of dollars if the guess is correct (i.e. if player A picks 10 and player B guesses correctly, then player B gets $10).
   (a) Suppose for this part that player B knows the values of $p_j$. What is player B's optimal strategy to maximize **expected** earnings?
   (b) Show that if both players choose their numbers so that the probability of picking $j$ is proportional to $1/j$, then neither player has an incentive to change strategies, assuming the opponent's strategy is fixed. (In game theory terminology, this says that we have found a *Nash equilibrium*).
   (c) Suppose player B follows the strategy outlined in the previous part. Find this player's expected earnings. Express your answer both as a sum of simple terms, and as a numerical approximation. Does the value depend on the strategy that player A uses?