St Petersburg Paradox Redux

The Original St. Petersburg Paradox: Suppose a wealthy stranger offers to play the following game: You flip a fair coin until it lands heads for the first time, and you will receive 2^n dollars, where n is the number of flips performed. What is the expected payout for this game? What is the expected playtime for this game?

1. In practical terms, what does it mean to say that expected value of a random variable is infinite? *Hint: consider the arithmetic mean of a large number of iid variables.*
2. Suppose instead of giving you 2^n dollars if the first heads happens on the nth flip, the stranger gives you r^n dollars, where r is a non-negative real number. Is there a value of r for which the game has a finite expected value?
3. Let x be the expected value of the game in the previous part. Is there a value of r for which you would actually be willing to pay x to play this game?
4. Suppose instead of using a fair coin, the stranger uses a biased coin which comes up heads with probability p. What is the expected value of the game? In what ways does the answer depend on p?
5. Suppose the stranger rolls a fair 6-sided die until either a 1 or a 6 is rolled. On a 6, the stranger gives you 2^n dollars, and on a 1, the stranger gives you nothing. What is the expected value of the game?
6. Now suppose the stranger again rolls the fair 6-sided die until either a 1 or a 6 is rolled. On a 6, the stranger gives you 2^n dollars, where n is the number of rolls, and on a 1, you must give the stranger 2^n dollars. What is the 'expectation' of this game? Does the answer change if the stranger gives you 2^n dollars on 6, but you must give the stranger 3^n dollars on a 1?