Teaching Probability Theory in the Inverted Style

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Overview

1. Motivation
2. Design
3. Experience

Learning Probability Theory: Why and How

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Goals (why):

- Prepare students for subsequent courses, especially mathematical statistics and statistical modeling
- Provide tools for codifying uncertainty in and apply probability models to relevant real-world phenomenon
- Build a critical and conscientious community of scholars fluent in language of probability

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Objectives (how):

- Gain proficiency with fundamental notation, definitions, theorems and calculations
- Cultivate familiarity with software for simulation, approximation and computation
- Practice verbal and written communication using formal and informal language
- Develop probabilistic intuition and problem-solving acumen

Teaching Styles: Traditional and Inverted

In the traditional (lecture) style, the instructor often...

- Delivers a lecture to audience of students for most of class time
- Assigns problems and exercises to be completed outside of class
- Provides feedback via corrections or assignment scores

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In the inverted (flipped) style, students often...

- Read a textbook section or watch a video before class
- Spend most of class time collaborating on problems or presentations
- Gain feedback from peers and the instructor during class

Teaching Styles: Course Objectives

Recalling my course objectives. . .

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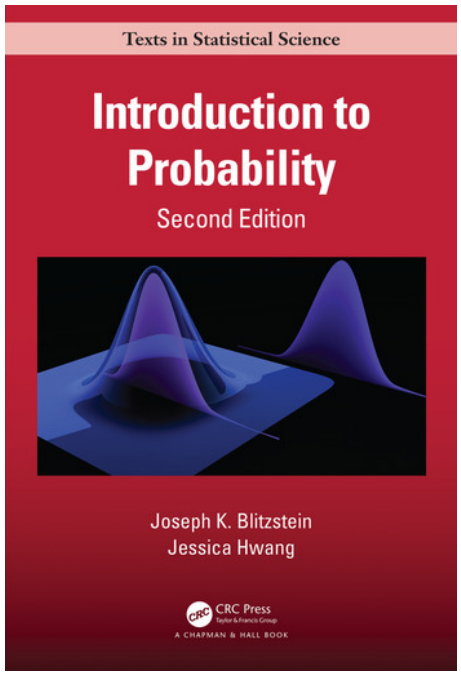
- Pre-class textbook reading / lecture video
  - Choice of which medium to engage with
  - Includes short set of reflection questions, due an hour before class
  - Also includes a list of objectives and key ideas for the day
- Active learning class session
  - Collaborative group work on 2 - 3 problems
  - Often includes a "warm-up" calculation exercise
  - Focuses on one or more open-ended activity or problem
  - Sometimes prefaced by short mini-lecture

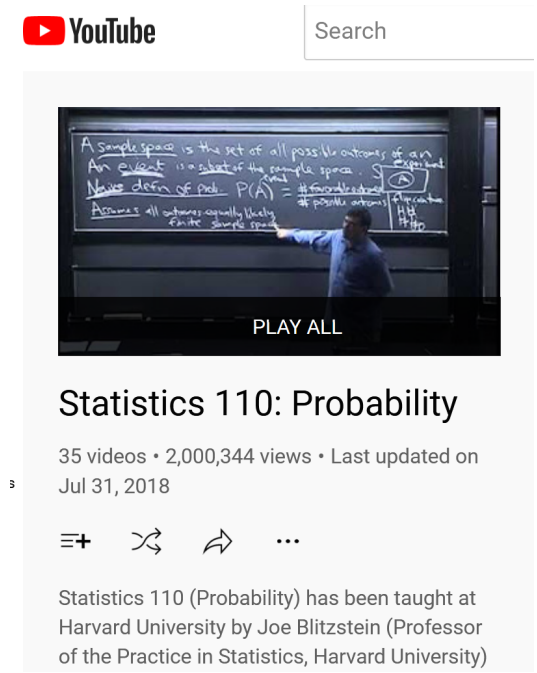
Textbook and Video Lectures

Textbook readings are primarily from J. Blitzstein and J. Hwang's *Introduction to Probability* 2e (2019)



http://probabilitybook.net/

Lecture videos are recorded from Blitzstein's Stat 110 course at Harvard, available on YouTube and EdX



https://projects.iq.harvard.edu/stat110/youtube

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1. **Reading** Read sections 2.5 and 2.6 in *Introduction to Probability*; OR
2. **Videos** Watch Lecture 5: Conditioning Continued on the Stat 110 YouTube channel
3. **Objectives**
   - State the set-theoretic definition of independence, and explain in everyday language what independence means.
   - Discuss the relationship between independence and conditional independence. Provide examples of events which are independent, but not conditionally independent, and vice versa.
4. **Questions**
   - Is it possible for an event to be independent of itself? Why or why not?
   - A weather forecaster claims that the event "it rains tomorrow" is conditionally independent of the event "it rained yesterday", given the event "it rained today." Explain what this means in everyday language. Discuss why this **does not** mean that tomorrow's weather is independent of yesterday's.
5. **Reflection** What is one topic you would like further clarification about, or one question you have based on the reading / video?

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1. What are some consequences to receiving a positive diagnosis when a community member is actually COVID-19 negative? Conversely, what are some consequences to receiving a negative diagnosis when a community member is actually COVID-19 positive?
2. Assume a prevalence of $p = 0.05\%$. How many positive test results would you anticipate? Reed reported 35 positive results during this period. What might this suggest about the true prevalence rate, sensitivity, and/or specificity?
3. Suppose a Reed community member has a positive test result. Find a formula (in terms of p) for the posterior probability that that the community member has COVID-19. Then evaluate for $p = 0.01\%, .05\%, .1\%, .5\%$ (a plausible range of values for the prevalence, based on existing data). For which values would you be comfortable concluding the community member has COVID-19?
4. Conversely, suppose the community member receives a negative test. Express the posterior probability that the community member does not have COVID-19 in terms of p. What do you think is an acceptable threshold to conclude that the individual does not have COVID? Evaluate for $p = 0.01\%, .05\%, .1\%, .5\%$.
5. What is the moral of the story? Would you recommend that all Reed community members undergo weekly COVID-19 surveillance testing?
6. Calculating posterior probabilities of infection is only possible if we know the prevalence p. But what are some fundamental challenges to obtaining a good estimate for p?

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3. Minimize calculus- and algebra-style computation problems.

Challenges: Managing Group Dynamics

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2. Explicit discussion and review of community conduct at start and throughout course
3. Visible, but supportive faculty intervention during class time

Contact and Additional Information

Thank you!

email

wellsjon@grinnell.edu

GitHub

ChalkboardSonata

Slides and Additional Resources

https://github.com/ChalkboardSonata/jsm22_prob

End

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- Sections usually have 20 - 25 students, meeting 50 minutes 3 times each week for 13 weeks
- Students are primarily juniors and seniors from Math, Math-Stat, Computer Science, Physics and Economics majors
- The course has several math prerequisites:
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At other colleges and universities. . .

- Student demographics and stats program curriculum might change depth and content of course
- But inverted teaching style still provides benefits in achieving course objectives