20.8　证明：

$P_1 = P \to (\neg q \to p)$ A_1

$P_2 = \neg q \to p$ $\left(P_1 = P_2 \to P, \text{MP 规则}\right)$

$P_3 = \neg q \to \neg P = \neg q \to (P \to F)$ $(\neg P = P \to F)$

$P_4 = \left(\neg q \to (P \to F)\right) \to ((\neg q \to P) \to (\neg q \to F))$

$P_5 = (\neg q \to p) \to (\neg q \to F)$ (MP 规则)

$P_6 = \neg q \to F = \neg\neg q$ $(P_5 = P_2 \to P_6 \quad \text{MP})$

$P_7 = \neg\neg q \to q$ (A_3)

$\therefore P_8 = q$ $\left(\text{MP 规则}\right)$ $\therefore \{p, \neg q \to, \neg P\} \vdash q$

20.10　证明：

$P_1 = P \to \left((P \to \neg\neg P) \to P\right)$ A_1

$P_2 = \left(P \to \left((P \to \neg\neg P) \to P\right)\right) \to ((P \to (P \to \neg\neg P)) \to (P \to P)$ A_2

$P_3 = \left(P \to (P \to \neg\neg P)\right) \to (P \to P)$

$P_4 = P \to (P \to \neg\neg P)$

$P_5 = P \to P$

$P_6 = \neg\neg P \to P$

$P_7 = P \to \neg\neg P$

所以有$\vdash P \to \neg\neg P$ 成立

20.18　由完备性定理 $A \vDash P$ 则在 $\text{Prop}(x)$中有 $A \vdash p$

$\therefore$ 存在有限序列 $P_1, P_2 \ldots\ldots.. P_n$ $P_i \in P(x)$ $P_n = P$

且对每一个 i $P_i \in A \cup A$ 或存在 $j, k (j, k < i)$有$P_R = \left(P_j \to P_i\right)$

$\therefore$ 记$A' = \{P_1, P_2 \ldots P_{n-1}\}$, 则$A' \sqsubseteq A$

且显然有$A' \vdash P$

又有可靠性定理$A' \Vdash P$　故得证