Assignment 10

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Outline

1. Question
2. Solution

Question

Excercise 6 Quetion 12

X and Y are independent uniformly distributed random variables on (0,1). Find the joint p.d.f of $X + Y$ and $X - Y$.

Definitions

$X \sim (0,1)$, $Y \sim (0,1)$
Let random variables U and V be defined as follows:

$$U = X + Y \tag{1}$$
$$V = X - Y \tag{2}$$
$$0 < |V| \leq U < 2 \tag{3}$$

Doing inverse transformation, we get

$$x_1 = \frac{U+V}{2} \tag{4}$$
$$y_1 = \frac{U-V}{2} \tag{5}$$

Jacobian Transformation

Let $g(x, y) = U$ and $h(x, y) = V$

$$J(x_i, y_i) = \begin{vmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \end{vmatrix} \tag{6}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \tag{7}$$

$$= 2 \tag{8}$$

Joint p.d.f

$$f_{UV}(u,v) = \sum_i \frac{1}{|J(x_i,y_i)|} f_{XY}(x_i,y_i) \tag{9}$$

$$= \frac{1}{|J(x_i,y_i)|} \sum_i f_X(x_i) f_Y(y_i) \tag{10}$$

$$= \frac{1}{2} \tag{11}$$

$$\therefore f_{UV}(u,v) = \frac{1}{2}, 0 < |V| \leq U < 2 \tag{12}$$