Assignment 12

Challa Akshay Santoshi - CS21BTECH11012

June 14, 2022

Outline

Question

Excercise 9 Quetion 18

Show that if $R_{XX}(t_1, t_2) = q(t_1)\delta(t_1 - t_2)$ and $y(t) = x(t) * h(t)$, then $E\{x(t)y(t)\} = h(0)q(t)$.

Definitions

The autocorrelation $R(t_1, t_2)$ of x(t) is the expected value of the product $x(t_1)x(t_2)$:

$$R_{XX}(t_1, t_2) = E\{x(t_1)x(t_2)\} \tag{1}$$

Given that y(t) is the convolution of x(t) and h(t)

$$y(t) = x(t) * h(t) \tag{2}$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\alpha)h(\alpha)d\alpha \tag{3}$$

Dirac delta distribution is defined as follows

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Proof

$$E\{x(t)y(t)\} = E\{x(t)\int_{-\infty}^{\infty} x(t-\alpha)h(\alpha)d\alpha\} \tag{5}$$

$$= \int_{-\infty}^{\infty} E\{x(t)x(t-\alpha)\}h(\alpha)d\alpha \tag{6}$$

$$= \int_{-\infty}^{\infty} R_{XX}(t, t-\alpha)h(\alpha)d\alpha \tag{7}$$

$$= \int_{-\infty}^{\infty} q(t)\delta(\alpha)h(\alpha)d\alpha \tag{8}$$

Proof

$$E\{x(t)y(t)\} = q(t)\int_{-\infty}^{\infty}\delta(\alpha)h(\alpha)d\alpha \tag{9}$$

Using the definition of $\delta(x)$, we get

$$E\{x(t)y(t)\} = q(t)h(0) \tag{10}$$

Hence proved.