Assignment 7

Challa Akshay Santoshi - CS21BTECH11012

CBSE Class 12 Probability

Excercise: 13.2 Question: 15

One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

1) E: the card drawn is a spade
   F: the card drawn is an ace
2) E: the card drawn is black
   F: the card drawn is a king
3) E: the card drawn is a king or queen
   F: the card drawn is a queen or jack

Solution:

Let $X \in \{0, 1, 2, 3\}$ be a random variable representing different suits in a deck of cards, that is, clubs, diamonds, hearts and spades.
Let $Y \in \{0, 1, 2, 3\}$ be a random variable representing the cards, King, Queen, Jack and Ace.
Clubs and Hearts are black coloured cards.

Description	Random Variable	Probability
Getting a king	Y = 0	$\frac{1}{13}$
Getting a queen	Y = 1	$\frac{1}{13}$
Getting a jack	Y = 2	$\frac{1}{13}$
Getting a club	X = 0	$\frac{1}{4}$
Getting a diamond	X = 1	$\frac{1}{4}$
Getting a heart	X = 2	$\frac{1}{4}$
Getting a spade	X = 3	$\frac{1}{4}$

TABLE 3: Probable Events Representation

Two events E and F are said to be independent if

$$\Pr(EF) = \Pr(E) \times \Pr(F) \tag{0.0.1}$$

1) E: the card drawn is a spade
F: the card drawn is an ace

$$\Pr(E) = \Pr(X = 3) = \frac{1}{4} \tag{0.0.2}$$

$$\Pr(F) = \Pr(Y = 3) = \frac{1}{13} \tag{0.0.3}$$

$$\Pr(EF) = \Pr(Y = 3|X = 3) = \frac{1}{52} \tag{0.0.4}$$

$$\Pr(E) \times \Pr(F) = \frac{1}{52} \tag{0.0.5}$$

$$\Pr(EF) = \Pr(E) \times \Pr(F) \tag{0.0.6}$$

E and F are independent events.

2) E: the card drawn is black

$$\Pr(E) = \Pr(X = 0) + \Pr(X = 2) \tag{0.0.7}$$

$$= \frac{1}{4} + \frac{1}{4} \tag{0.0.8}$$

$$= \frac{1}{2} \tag{0.0.9}$$

F: the card drawn is a king

$$\Pr(F) = \Pr(Y = 0) = \frac{1}{13} \tag{0.0.10}$$

To get a king which is a club or a heart:

$$\Pr(EF) = \Pr(Y = 0|X = 0) + \Pr(Y = 0|X = 2) \tag{0.0.11}$$

$$= \frac{1}{52} + \frac{1}{52} \tag{0.0.12}$$

$$= \frac{1}{26} \tag{0.0.13}$$

To check independency of events:

$$\Pr(E) \times \Pr(F) = \frac{1}{26} \tag{0.0.14}$$

$$\Pr(EF) = \Pr(E) \times \Pr(F) \tag{0.0.15}$$

E and F are independent events.

3) E: the card drawn is a king or queen

$$\Pr(E) = \Pr(Y=0) + \Pr(Y=1) \tag{0.0.16}$$
$$= \frac{1}{13} + \frac{1}{13} \tag{0.0.17}$$
$$= \frac{2}{13} \tag{0.0.18}$$

F: the card drawn is a queen or jack

$$\Pr(F) = \Pr(Y=1) + \Pr(Y=2) \tag{0.0.19}$$
$$= \frac{1}{13} + \frac{1}{13} \tag{0.0.20}$$
$$= \frac{2}{13} \tag{0.0.21}$$

To check independency of events:

$$\Pr(EF) = \Pr(Y=1) = \frac{1}{13} \tag{0.0.22}$$
$$\Pr(E) \times \Pr(F) = \frac{4}{169} \tag{0.0.23}$$
$$\Pr(EF) \neq \Pr(E) \times \Pr(F) \tag{0.0.24}$$

E and F are not independent events.