Assignment 13

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Outline

Question

Excercise 11 Quetion 12

Show that, if the process $X(\omega)$ is white noise with zero mean and autocovariance $Q(u)\delta(u-v)$, then its inverse Fourier transform, x(t) is WSS with power spectrum $\frac{Q(\omega)}{2\pi}$.

Definitions

Inverse Fourier transform of $X(\omega)$ is given by

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \tag{1}$$

From the Fourier-inversion formula, it follows that

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega \tag{2}$$

The process x(t) is WSS iff $E\{X(\omega)\} = 0$ for $\omega \neq 0$ and

$$E\{X(u)X^*(v)\} = Q(u)\delta(u - v) \tag{3}$$

Proof

Let $x(t_1)$ and $x(t_2)$ be the inverse Fourier transform of X(u) and X(v) respectively.

$$E\{x(t_1)x^*(t_2)\} = E\{(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(u)e^{jut_1}du)(\frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(v)e^{-jvt_2}dv)\} \tag{4}$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\{X(u)X^*(v)\}e^{j(ut_1-vt_2)}dudv \tag{5}$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}(\int_{-\infty}^{\infty} Q(u)\delta(u-v)e^{j(ut_1-vt_2)}dv)du \tag{6}$$

Proof

$$E\{x(t_1)x^*(t_2)\} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} Q(u)e^{ju(t_1-t_2)}du \tag{7}$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty} Q(u)e^{ju\tau}du \tag{8}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{Q(u)}{2\pi}e^{ju\tau}du \tag{9}$$

$$R_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{Q(\omega)}{2\pi}e^{j\omega\tau}d\omega \tag{10}$$

Finding Power Spectrum

To find the Power Spectrum, we compare the equations given below,

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega \tag{11}$$

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{Q(\omega)}{2\pi} e^{j\omega\tau} d\omega \tag{12}$$

Therefore the Power Spectrum is

$$S_{xx}(\omega) = \frac{Q(\omega)}{2\pi} \tag{13}$$