Assignment 9

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Outline

Question

CBSE 12 13.5 Q 1

A die is thrown 6 times. If ‘getting an odd number’ is a success, what is the probability of

1. 5 successes ?
2. at least 5 successes ?
3. at most 5 successes ?

Definitions

Let $X \in \{0, 1, 2, 3, 4, 5, 6\}$ be a random variable denoting the number of successes (getting an odd number) in an experiment of 6 trials.

Event	Probability
Getting an odd no. (Success)	$p = \frac{1}{2}$
Getting an even no. (Failure)	$q = \frac{1}{2}$

Table: Outcomes of the Experiment

Throwing a die is a Bernoulli trail. So, X has a binomial distribution

$$\Pr(X = x) = \frac{n!}{x!(n-x)!}(1-p)^{n-x}\,p^{x} \tag{1}$$

5 successes

$$\Pr(X = 5) = \frac{6!}{5!(6-5)!} p^5 q^1 \tag{2}$$

$$= 6 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^1 \tag{3}$$

$$= 6 \times \frac{1}{32} \times \frac{1}{2} \tag{4}$$

$$= \frac{3}{32} \tag{5}$$

at least 5 successes

$$\Pr(X \geq 5) = \Pr(X = 5) + \Pr(X = 6) \tag{6}$$

$$= \frac{6!}{5!(6-5)!} p^5 q^1 + \frac{6!}{6!(6-6)!} p^5 q^0 \tag{7}$$

$$= 6 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^1 + 1 \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^0 \tag{8}$$

$$= \frac{6}{64} + \frac{1}{64} \tag{9}$$

$$= \frac{7}{64} \tag{10}$$

at most 5 successes

$$\Pr(X \leq 5) = 1 - \Pr(X = 6) \tag{11}$$

$$= 1 - \frac{6!}{6!(6-6)!} p^6 q^0 \tag{12}$$

$$= 1 - 1 \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^0 \tag{13}$$

$$= 1 - \frac{1}{64} \tag{14}$$

$$= \frac{63}{64} \tag{15}$$

PMF Graph

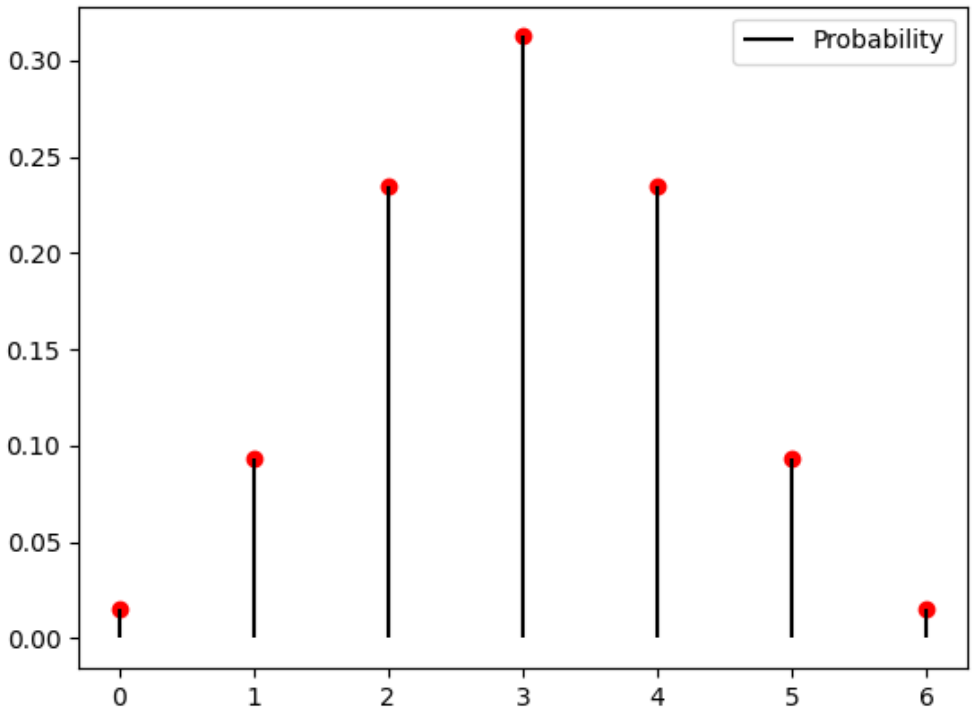

Figure: Probability Mass Function

CDF Graph

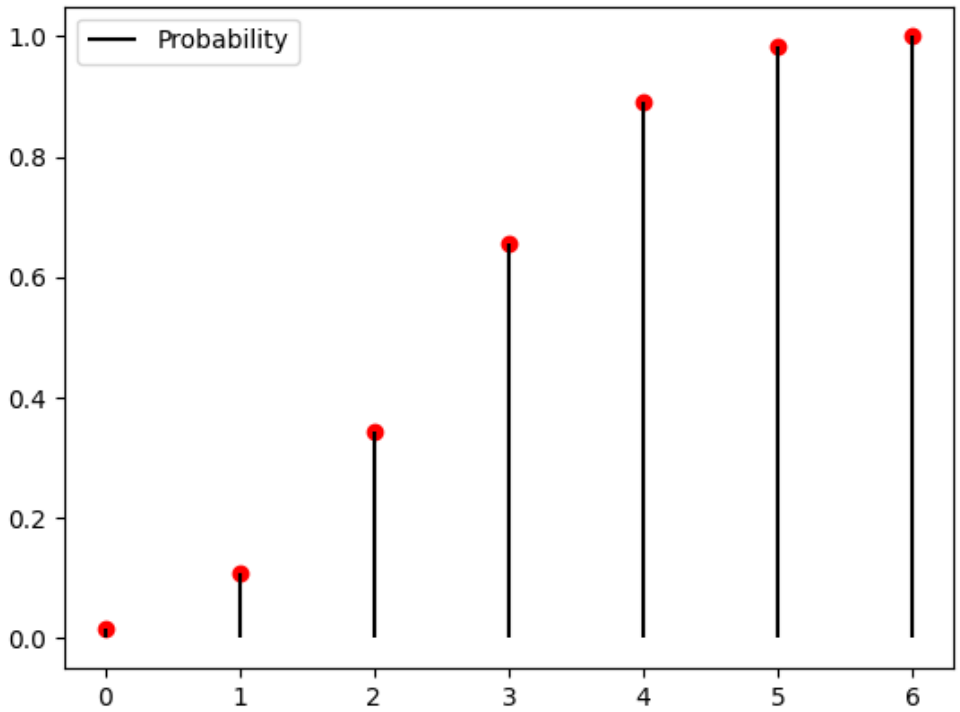

Figure: Cummulative Distribution Function