Assignment 11

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June 9, 2022

Outline

1. Question

2. Solution

Question

Excercise 7 Quetion 9

Show that if $X_i \geq 0$, $E(X_i^2) = M$ and $s = \sum_{i=1}^{n} X_i$, then $E(s^2) \leq ME(n^2)$.

Definitions

Given a discrete type random variable **n** taking the values 1,2,... and a sequence of random variables X_k independent of **n**, then the sum **s** is defined as

$$s = \sum_{k=1}^{n} X_k \tag{1}$$

Given that for any k,

$$E({X_k}^2) = M \tag{2}$$

Proof

$$(E(X_iX_j))^2 = (E(X_i)E(X_j))^2 \leq E({X_i}^2)E({X_j}^2) = M^2 \tag{3}$$

$$(E(X_iX_j))^2 \leq M^2 \implies E(X_iX_j) \leq M \tag{4}$$

$$E(s^2|\mathbf{n} = n) = E((\sum_{i=1}^{n} X_i)(\sum_{j=1}^{n} X_j)) \tag{5}$$

$$= E(\sum_{i=1}^{n}\sum_{j=1}^{n} X_iX_j) \tag{6}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} E(X_iX_j) \tag{7}$$

$$\leq \sum_{i=1}^{n}\sum_{j=1}^{n} M \tag{8}$$

Proof

$$E(s^2|\mathbf{n} = n) \leq n^2 M \tag{9}$$

We can write,

$$E(s^2) = E(E(s^2|\mathbf{n} = n)) \tag{10}$$

$$E(s^2|\mathbf{n} = n) \leq n^2 M \tag{11}$$

$$E(E(s^2|\mathbf{n} = n)) \leq E(n^2 M) \tag{12}$$

$$\implies E(s^2) \leq M E(n^2) \tag{13}$$

Hence proved.