# Week-10

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① Referred notes mentioned in "Reading".

Take $y = Px$   $A$ is the normalized Adj. matrix

Split $y = y^{\parallel} + y^{\perp}$

where $y^{\parallel}$ is parallel to $u$ and

$y^{\perp}$ is perpendicular to $u$.

$Ay^{\parallel} = y^{\parallel}$ ($\alpha \mathbb{1}$ is an eigenvector where $\alpha$ is ~~sca~~ scale $\neq 0$)

$$\|PAy\|_2 = \|PA(y^{\parallel} + y^{\perp})\|_2$$

From triangle inequality,

$$\|PAy\|_2 \leq \|PAy^{\parallel}\|_2 + \|PAy^{\perp}\|_2$$

$$= \|Py^{\parallel}\|_2 + \|PAy^{\perp}\|_2$$

$$\|Ay^{\perp}\|_2 \leq w_G \|y^{\perp}\|_2 \leq w_G \|y\|_2$$

$y = Px$ (~~Components for~~ $|B|$ components are atmost non-zero)

$$\|y\|_2 \leq \|x\|_2$$

Since $\|PAy^{\perp}\|_2 \leq \|Ay^{\perp}\|_2$

$$\|Ay^{\perp}\|_2 \leq w_G \|x\|_2 \rightarrow \|PAy^{\perp}\|_2 \leq w_G \|x\|_2$$

For $y^{\parallel}$ term,

$$y^{\parallel} = \left(\frac{y \cdot u}{\|u\|_2^2}\right) u \Rightarrow y^{\parallel} = \left(\sum_i y_i\right) u$$

$y = Px$ and so $y$ has support $|B| \leq \delta |N|$.

$$\|y^{\parallel}\|_2^2 = \sum_{i=1}^{n} \frac{\left(\sum_{i=1} y_i\right)^2}{N^2}$$

$$\|y''\|_2^2 = \frac{\left(\sum_i y_i\right)^2}{N}$$

Take $k$ such that $k = \lfloor \delta N \rfloor$

where $k$ is the no. of non-zero components in $y'$ values

$$\|y''\|_2^2 = \frac{\left(\sum_{i=1}^{k} y_i\right)^2}{N}$$

$$\leq \frac{k\left(\sum_i^{k} y_i^2\right)}{N} \quad \text{(Used Cauchy-Schwartz)}$$

$$\leq \frac{\delta N\left(\sum_i y_i^2\right)}{N} \leq \delta \|y\|_2^2 \quad \text{——} \ (1)$$

Note that
$$Py'' = \begin{pmatrix} \sum_i \frac{y_i}{N} \\ \vdots \\ \sum_i \frac{y_i}{N} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} |B| \text{ vertices present in } B \\ \\ \\ \end{matrix}$$

$$\|Py''\|_2^2 = \sum_{j=1}^{k} \frac{\left(\sum_i y_i\right)^2}{N^2}$$

$$\leq \frac{\delta N \left(\sum_i y_i\right)^2}{N^2}$$

$$= \frac{\delta\left(\sum_i y_i\right)^2}{N}$$

$$= \delta \|y''\|_2^2 \quad \text{——} \ (2).$$

From (1) and (2), we have

$$\|Py''\|_2^2 \leq \delta \|y''\|_2^2 \leq \delta^2 \|y'\|_2^2 \leq \delta^2 \|x\|_2^2$$

$$\therefore \|PAy\|_2^2 = \|(PAP)x\|_2^2 \leq \delta^2 \|x\|_2^2 + \omega_G^2 \|x\|_2^2$$

Hence we get $\|PAPx\|_2^2 \leq (\delta^2 + \omega_G^2)\|x\|_2^2$

where $A$ is normalized adj matrix