CS5110 - Assignment 2

Subrahmanyam Kalyanasundaram

September 2024

1. Suppose $A \leq_L B$ using the reduction function f. Given w, an instance of A, what is an upper bound on $|f(w)|$ in terms of $|w|$?

2. Show that A_{NFA} is NL-complete.

$$A_{NFA} = \{\langle N, w\rangle |\ N \text{ is an NFA that accepts the string } w\}$$

3. Show that 2-SAT is NL-complete.

4. A *ladder* is a sequence of strings $s_1, s_2, \ldots, s_k$, wherein every string differs from the preceding one in exactly one character. For example, the following is a ladder of English words, starting with the word "head" and ending with the word "free".

   head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free.

   Let $LADDER_{DFA} = \{\langle M, s, t\rangle |\ M \text{ is a DFA and } L(M) \text{ contains a ladder of strings, starting with } s \text{ and ending with } t\}$. Show that $LADDER_{DFA}$ is in PSPACE.

5. If $A \in$P, then show that $\mathrm{P}^A =$P.

6. A directed graph is *strongly connected* if for every pair of vertices (u, v) there is a directed path from u to v in G. Show that the problem of deciding if a graph is strongly connected is NL-complete.

7. For a language $L \subseteq \{0,1\}^*$, and a function $f(n)$ (assuming $f(n)$ can be computed in time $O(f(n))$, let $L_f \subseteq \{0,1,\#\}^*$ denote the following language:

$$L_f := \{x\#^{f(|x|)} | x \in L\}$$

   - Suppose that that $L \in \mathrm{DTIME}(f(n))$. Then show that $L_f \in \mathrm{DTIME}(O(n))$.
   - Show that if $f(n)$ is a polynomial function, then $L \in$ P if and only if $L_f \in$ P.

- Show that P $\neq$ DSPACE($O(n)$).

  Hint: Assume an equality and arrive at a contradiction via suitable padding and the Space Hierarchy Theorem.

- Define the class NEXP as

$$\text{NEXP} := \bigcup_k \text{NTIME}\left(2^{n^k}\right)$$

  Prove that if P = NP then EXP = NEXP.

8. Show that if $\Sigma_k = \Pi_k$ for some k, then polynomial hierarchy collapses to Σ_k.

9. What is the count (number) of functions $f : \{0,1\}^n \to \{0,1\}$ that are both monotone and symmetric?