CS5110 - Assignment 1

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1. Let *DOUBLE-SAT* be the language consisting of all Boolean formulas that have at least two distinct satisfying assignments. Show that *DOUBLE-SAT* is NP-complete.

2. A Boolean formula is in DNF form (Disjunctive normal form) if it is an OR of clauses: $C_1 \vee C_2 \vee \ldots \vee C_m$, where each clause C_j is an AND of literals.

   Let *DNF-SAT* be language consisting of Boolean formulas $\langle \phi \rangle$ that are in DNF form and are satisfiable. In other words, the goal is to decide if a given formula in DNF form is satisfiable. Is *DNF-SAT* in P? Is it in NP? Is it NP-complete?

3. A Boolean formula is in 2-CNF form if it is an AND of clauses: $C_1 \wedge C_2 \wedge \ldots \wedge C_m$, where each clause C_j is an OR of **exactly** two literals.

   Let *2-SAT* or *2-CNF-SAT* be the language consisting of satisfiable Boolean formulas that are in 2-CNF form. Is *2-CNF* in P? Is it in NP? Is it NP-complete?

4. If P = NP, which are the languages that are NP-Complete?

5. Show that if P = NP, there is a polynomial time algorithm to find a satisfying assignment to a *3-SAT* formula if such an assignment exists.

6. Show that $A \leq_P B$ and $B \leq_P C \Rightarrow A \leq_P C$.

7. Show that a language L is co-NP complete if and only if $\overline{L}$ is NP-complete.

8. Show that NP $\neq$ co-NP $\implies$ P $\neq$ NP.

9. The language *EXACT-CLIQUE* consists of all $\langle G, k \rangle$ where G is an undirected graph and k is a natural number such that the largest clique in G is of size exactly k. Show that *EXACT-CLIQUE* $\in \Sigma_2 \cap \Pi_2$.