

Programming Assignment 2: Linear Simultaneous Equations and Eigenvalues

1. Write a computer program for solving a system of linear equations $Ax = b$. The program should have the following features:

Input: The program should read the following inputs from a text file – (i) the number of equations (n), (ii) elements of the augmented matrix. [An example of input data file is given].

Options: The user should have the option of selecting one of the following methods–

- a. Gauss elimination (GE; without pivoting)
- b. GE (with pivoting)
- c. LU decomposition by Doolittle method (without pivoting)
- d. LU decomposition by Crout method (without pivoting)
- e. Cholesky decomposition (for symmetric positive definite matrix)

Output: The output from the program should be written in a text file. This file should contain the following results for different methods–

- a. GE: the unknowns, x
- b. LU by Doolittle: the unknowns, x , and the elements of L and U
- c. LU by Crout method: the unknowns, x , and the elements of L and U
- d. Cholesky decomposition: the unknowns, x , and the elements of L

Test data:

$$4x_1 + 2x_2 = 10$$

$$2x_1 + 4x_2 + x_3 = 11.5$$

$$x_2 + 5x_3 = 5$$

Sample input file

```
3
4 2 0 10
2 4 1 11.5
0 1 5 5
```

Sample output file

Crout method

```
x
1.5
2.0
0.5
```

L

| | | |
|-----|-----|-------|
| 4.0 | 0.0 | 0.0 |
| 2.0 | 3.0 | 0.0 |
| 0.0 | 1.0 | 4.667 |

U

| | | |
|-----|-----|--------|
| 1.0 | 0.5 | 0.0 |
| 0.0 | 1.0 | 0.3333 |
| 0.0 | 0.0 | 1.0 |

2. Write a computer program for finding eigenvalues of a matrix A . The program should have the following features:

Input: The program should read - (i) the size of the matrix A and its elements, (ii) maximum iterations, (iii) threshold on approximate relative error, and (iv) for inverse power method with shift, the scalar value to which the eigenvalue should be closest.

Options: The user should have the option of selecting one of the following methods–

- Power method (to find the eigenvalue having the maximum magnitude and the corresponding eigenvector)
- Inverse power method (to find the eigenvalue having the minimum magnitude and the corresponding eigenvector)
- Inverse power method with shift (to find the eigenvalue closest to a given value and corresponding eigenvectors)
- QR method (to find all eigenvalues of a matrix)

Output: The output from the program should be written in a text file. This file should contain eigenvalues and the corresponding normalized eigenvectors (unit length) for options (a) to (c), and only eigenvalues for option (d). Number of iterations required should also be written as an output, *along with the estimate of eigenvalue(s) at each iteration* (note that the iterative sequence is not shown in the sample output).

Note: Use a column vector of appropriate size with each element unity as the guess vector for starting the power methods.

Test Case:

$$A = \begin{bmatrix} 8 & -1 & -1 \\ -1 & 4 & -2 \\ -1 & -2 & 10 \end{bmatrix}$$

Maximum iterations: 50

Maximum relative approximate error: 0.001%

Find Eigenvalue closest to: 8

Sample input file

```
3
8.0  -1.0  -1.0
-1.0  4.0  -2.0
-1.0  -2.0  10.0
100
0.001
8.0
```

Sample output files

1. Power method

```
Eigenvalue
10.7787
```

```
Eigenvector
-0.2509
-0.2397
0.9379
```

```
Iterations
30
```

2. Inverse power method

```
Eigenvalue
3.0749
```

```
Eigenvector
0.2482
0.9205
0.3017
```

```
Iterations
12
```

3. Inverse power method with shift

Eigenvalue

8.1461

Eigenvector

0.9356

-0.3085

0.1718

Iterations

6

4. QR method

Eigenvalues

10.7788

8.1462

3.0749

Iterations

23