

# A transformer design optimisation tool for oil immersed distribution transformers

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**Abstract**—This paper presents a methodology to design hermetically sealed shell type oil immersed distribution transformers with minimum evaluated cost according to the IEC standards. Distribution transformer model and design constraints are implemented as a user friendly software using MATLAB. An objective function for total evaluated cost is optimized subjected to ten constraints according to IEC 60076 in addition to geometrical constraints. A design example on a 630kVA transformer is presented for illustration.

**Index Terms**— Cost function, Design optimization, Dielectric losses, Eddy currents, Hysteresis, Impedance, Inductance, MATLAB, Transformers, Transformer cores.

## I. INTRODUCTION

Distribution transformers play a vital role in electricity distribution networks. Transformer losses contribute to a significant portion of the total system losses and therefore keeping transformer losses under control is a target of network operators. A transformer designed with better material, larger cross sections in conductors and the core will have lower losses but the cost of the transformer will be high. On the other hand a cheap transformer can be manufactured with average material and with minimum conductor and core cross sections but will have higher no-load losses and higher load losses. A transformer having higher losses will have higher lifetime cost. Therefore a compromise has to be made and optimum transformer parameters have to be used for the production of the transformer. By hand calculations this is an impossible task and since the invention of the digital computers, transformer design optimisation tools have been developed by various manufacturers and such tools have been the design monopoly of them. The first transformer design optimisation with the help of a digital computer was implemented in 1955. Since 1955 there have been huge revolutions in the transformer industry. In early days designers optimized its design based on trial and error. Based on those results finally most of them derived their own methodologies to optimize the transformer designs.

P.S.Georgilakis [12] presents an innovative method, combining genetic algorithm and finite element method for the solution of transformer manufacturing cost minimization problem. H.Malik, and R.K.Jarial, [13] present a new method to estimate the weight and cost of main materials for

transformers by use of fuzzy logic technique. H.Malik, et al [14] present methodology to minimize the total mass of the core and wire material by satisfying constraints imposed by international standards and transformer user specification. L.D.S.Coelho, et al [15] present methodology to solve transformer design optimization problem using the differential evolution (DE) algorithm.

This paper proposes a design methodology which has capability to minimize the total lifetime cost (objective function) while meeting the conditions laid out by constraints for three-phase hermetically sealed oil immersed distribution transformers with capacities from 50kVA to 2500kVA. The objective of transformer design optimization (TDO) is to design the transformer so as to minimize the transformer manufacturing cost, cost of no-load loss and cost of full-load loss subject to constraints according to the international standards and transformer user specification. It is also necessary to consider the manufacturing limitations and to maintain the elegant appearance of the transformer. Following constraints are considered in the distribution transformer optimisation process; flux density constraint, induced voltage constraints, impulse voltage constraints, impedance voltage constraint, no-load loss constraint, full-load loss constraint and total loss constraint. In this optimization methodology 8 major design variables are considered for optimisation.

Most of the equations involved in the transformer manufacturing process are non-linear multivariable functions and it is difficult to solve these equations using conventional methods and have to follow the advance methods like Sequential Quadratic programming method (SQP). SQP has capability to solve non-linear multivariable functions under constraints on the variables.

The proposed method uses SQP method and gives the global minimum for objective function while satisfying the constraints. The presented system is executed and formed by a user interface with 3D visualization of the optimized transformer developed in MATLAB.

## II. METHODOLOGY

Table 1 defines the variables used in subsequent equations.

### A. Formulation of the objective function.

The proposed transformer design methodology has ability to design three phase hermetically sealed shell type oil immersed distribution transformers with following specifications.

kVA rating: 50kVA-2500kVA  
 Primary voltages: 33kV/11kV  
 Secondary voltage: 400V

TABLE I  
 DEFINITIONS OF THE DESIGN VARIABLES

Symbol	Quantity	Unit
TEC	Total Evaluated Cost	USD
$C_{TM}$	Cost of Transformer Manufacturing	USD
A	Cost of No load Loss	W/USD
NLL	No Load Loss	W
C	Cost of load Loss	W/USD
LL	Load Loss	W
$C_{MM}$	Cost of Main Material	USD
$C_{RM}$	Cost of Remaining Material	USD
$C_{Lab}$	Cost of Labour	USD
f	Frequency	Hz
CSF	Core Stacking Factor	
$V_{LV}$	Voltage in secondary	V
$n_{LV}$	Turns in secondary	
$\rho_{Core}$	Density of core material	kg/mm <sup>3</sup>
J	Current density in foil	A/mm <sup>2</sup>
$D_{HV}$	Primary conductor diameter	mm
$H_p$	Pressboard height	mm
S	Transformer capacity	kVA
$L_{LV}$	LV conductor length	m
$\rho_{cu}$	Cu density	kg/mm <sup>3</sup>
$L_{HV}$	HV conductor length	m
$W_{LV}$	LV copper mass	kg
$W_{HV}$	LV copper mass	kg
$W_{IP}$	Insulation paper mass	kg
$W_{DS}$	Duct strip mass	kg
$W_{Core}$	Core steel mass	kg
$t_{LV,DDP}$	LV diamond dot paper thickness	mm
$t_{HV,DDP}$	HV diamond dot paper thickness	mm
$L_{LV,DDP}$	LV diamond dot paper length	mm
$L_{HV,DDP}$	HV diamond dot paper length	mm
$\rho_{DDP}$	Diamond dot paper density	kg/mm <sup>3</sup>
$T_L$	Tank length	mm
$T_H$	Tank height	mm
$T_W$	Tank width	mm
$T_t$	Tank thickness	mm
$\rho_t$	Tank steel density	kg/mm <sup>3</sup>
$P_o$	Mineral oil density	kg/mm <sup>3</sup>
$LL_{LV}$	Secondary load loss	W
$LL_{HV}$	Primary load loss	W
$NLL_G$	Guaranteed No-load loss	W
$LL_G$	Guaranteed load loss	W
TL	Total loss	W
$TL_G$	Guaranteed total loss	W
$U_G$	Guaranteed impedance voltage	
$VPT_{LV}$	LV voltage per turn	V
$VPL_{HV}$	HV voltage per layer	V
$In_{LV,max}$	Maximum LV induced voltage	V
$In_{HV,max}$	Maximum HV induced voltage	V
$BIL_{LV}$	LV basic insulation level	V
$BIL_{HV}$	HV basic insulation level	V
$T_{LV}$	Total LV turns	
$T_{HV}$	Total HV layers	
$Im_{LV,max}$	LV maximum impulse voltage	V
$Im_{HV,max}$	HV maximum impulse voltage	V
USD	United State Dollar	

Transformers are designed at the minimum evaluated cost. Formula for the objective function is given below.

$$TEC = C_{TM} + A(NLL) + C(LL) \quad (1)$$

$$C_{TM} = C_{MM} + C_{RM} + C_{Lab} \quad (2)$$

Low voltage winding's copper foil, high voltage winding's copper wire, core magnetic material, insulation paper, insulation duct strips, mineral oil and tank steel sheets are considered as main materials which heavily affect the cost of the transformer.

$C_{RM}$  and  $C_{Lab}$  are consider as constants for design variables. Therefore equations are derived to calculate main materials costs, NLL and LL using the following eight design variables. The variable are T(second step width), W(second step build), H(first step width), n(LV turns), B(flux density),  $T_{IV}$ (LV foil thickness), J(current density) and  $d_{HV}$ (HV wire diameter). Following assumptions are made for the derivations of the equations. The magnetic flux density is constant throughout the core. Low voltage (LV) and high voltage (HV) windings are oval shaped, Material waste costs are ignored. High-voltage winding is manufactured with copper round conductor, while low-voltage winding is manufactured with copper foil.

### 1) Mass of core

Core cross-section area of the core for star connected secondary is calculated using basic equation [1]

$$A_{eff} = \frac{130034 \cdot V_{LV}}{f \cdot n_{LV} \cdot B \cdot CSF} \quad (3)$$

Mass of the core is calculated as follows [1],

$$W_{core} = (3 \cdot D + 4 \cdot H) \cdot A_{eff} \cdot \rho_{core} \quad (4)$$

The dimensions of the core are shown in figure 1.

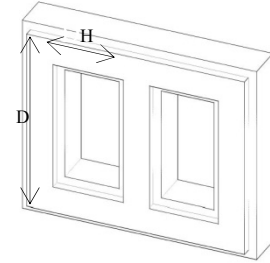


Fig. 1. Dimensions of the Transformer core

Shape of the winding is assumed as elliptical and parameters of each ellipse are calculated using dimensions of the core, thickness of the pressboard, thickness of a single LV foil, thickness of a single HV layer, Thickness of LV cooling duct, Thickness of HV cooling duct, thickness of LV paper insulation, thickness of HV paper insulation and thickness of LV-HV barrier. Circumference of each ellipse is calculated using each ellipse's parameters.

By adding corresponding circumferences, equations are derived for the length of LV copper foil, HV conductor length, LV insulation paper length, HV insulation paper length, LV cooling duct, HV cooling duct, LV-HV barrier and press

board. Figure 2 shows the cross sectional view of the transformer core for an oval shaped winding.

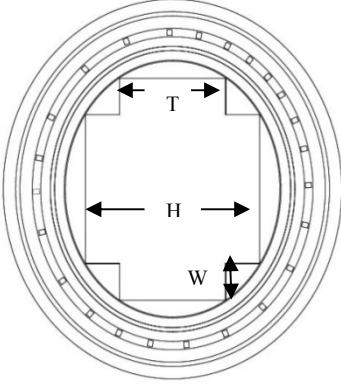


Fig. 2. Cross sectional view of the transformer core

## 2) Conductor mass of windings

Winding conductor masses are derived from [1],

$$W_{LV} = \frac{S \cdot 10^3 \cdot L_{LV} \cdot \rho_{CU}}{J \cdot V_{LV} \cdot \sqrt{3}} \quad (5)$$

$$W_{HV} = \frac{\pi \cdot d_{HV}^2 \cdot L_{HV} \cdot \rho_{CU}}{4} \quad (6)$$

## 3) Insulation paper mass

Insulation paper mass ( $W_{IP}$ ) is derived by adding LV ( $W_{IP, LV}$ ) and HV ( $W_{IP, HV}$ ) diamond dot paper weights.

$$W_{IP} = W_{IP, LV} + W_{IP, HV} = H_p \cdot (t_{LV, DDP} \cdot L_{LV, DDP} + t_{HV, DDP} \cdot L_{HV, DDP}) \cdot \rho_{DDP} \quad (7)$$

## 4) Insulation duct strips mass

Insulation duct strip mass ( $W_{DS}$ ) is derived by adding cooling duct strips weights in LV ( $W_{C, LV}$ ) and HV ( $W_{C, HV}$ ) to the LV-HV ( $W_{C, HV-LV}$ ) barrier duct strip weight.

$$W_{DS} = W_{C, LV} + W_{C, HV} + W_{C, HV-LV} \quad (8)$$

## 5) Tank steel sheet mass

Dimensions of the steel tank are calculated using the dimensions of core, winding and clearances of HV to tank from length side, HV to tank from width side, yoke to tank bottom, top to tap changer base, tap changer to tank cover and height of tap changer.

Mass of the tank ( $W_T$ ) is derived by [1],

$$W_T = 2 \cdot (T_L \cdot T_W + T_L \cdot T_H + T_H \cdot T_W) \cdot T_T \cdot \rho_T \quad (9)$$

## 6) Mineral oil mass

Mineral oil volume ( $V_O$ ) is derived by subtracting all parts inside the tank from the volume of the tank. So mass of the oil ( $W_O$ ) is derived by,

$$W_O = V_O \cdot \rho_O \quad (10)$$

## 7) No-Load loss

No-Load loss of a transformer include core loss, dielectric loss, conductor loss in the winding due to excitation current and conductor loss due to circulating current in parallel windings (IEEE 2002).

Transformer specific no-load loss (W/kg) for a given magnetic flux density can be obtained from the no-load loss curves given by magnetic material manufacturers.

Specific no-load loss ( $W_{NLL}$ ) curve for 0.23mm thick Nippon silicon steel sheets at 50Hz is given by,

$$W_{NLL} = 0.8398 \cdot B^3 - 2.254 \cdot B^2 + 2.407 \cdot B - 0.6884 \quad (11)$$

$$NLL = W_{NLL} \cdot W_{core} \quad (12)$$

## 8) Load loss

Load losses are incident to the carrying of a specific load. Load losses include  $I^2R$  losses in current carrying parts (winding, leads, busbars, and bushings), eddy current losses in conductors and stray losses.

$$LL_{LV} = \frac{R_{CU} \cdot L_{LV} \cdot J \cdot S \cdot 10^3}{V_{LV} \cdot \sqrt{3}} \quad (13)$$

$$LL_{HV} = \frac{4 \cdot R_{CU} \cdot L_{HV} \cdot S \cdot 10^3}{\pi \cdot d_{HV}^2 \cdot V_{LV} \cdot \sqrt{3}} \quad (14)$$

$$LL = LL_{LV} + LL_{HV} \quad (15)$$

## B. Design Constraints formulation methodology

### 1) No-load loss/Load loss/Total loss

$$\begin{aligned} NLL &< 1.5 \times NLL_G \\ LL &< 1.5 \times LL_G \\ TL &< 1.1 \times TL_G \end{aligned}$$

Guaranteed loss values for a standard kVA rating are assigned in CENELEC 1992 and design tolerances according to the IEC 60076-1.

### 2) Short-Circuit impedance

Inductive part of short circuit impedance is calculated using stored energy method [4]. Resistive part is calculated using load loss [1] and total impedance ( $U_K$ ) which should satisfy following condition.

$$0.9 \times U_G < U_K < 1.1 \times U_G$$

### 3) Induced voltages in LV and HV

The thickness of insulation papers between the winding layers must withstand the induced voltage test[1].

$$4.VPT_{LV} < In_{LV,max}$$

$$4.VPL_{HV} < In_{HV,max}$$

### 4) Impulse voltages in LV and HV

The thickness of insulation papers between the winding layers must withstand the impulse voltage test[1].

$$\frac{2.BIL_{LV}}{T_{LV}} < Im_{LV,max}$$

$$\frac{2.BIL_{HV}}{T_{HV}} < Im_{HV,max}$$

### 5) No-load current

No-load current is equal to the vectorial sum of the magnetising current ( $I_m$ ) and hysteresis and eddy current ( $I_{h+e}$ ). Transformer specific magnetising power (VA/kg) for a given induction can be obtained from the curves given by magnetic material manufacturers. Hysteresis and eddy current component is obtained through the no-load loss. In the design no-load current is constrained to 2% of full load current at 112.5% rated voltage.

### 6) Efficiency and voltage regulation

$\eta$  (Efficiency) is computed for full-load at unity power factor by[1]:

$$\eta = \frac{S \times 100\%}{S + LL + NLL} \quad (16)$$

The Voltage regulation ( $\Delta V$ ) is computed for full-load at unity power factor by[1]:

$$\Delta V = \left( e_r + \frac{e_x^2}{2} \right) \times 100\% \quad (17)$$

Where,

$$e_r = \frac{LL}{S} \text{ and } e_x = \sqrt{U_K^2 - e_r^2}$$

Efficiency of the designs is higher than 98% and voltage regulation is lower than 4%.

### 7) Maximum flux density

The maximum flux density of the core is required to be smaller than the saturated flux density.

## C. Transformer design optimization(TDO) methodology

Conventional transformer design optimization method is known as heuristic technique that assigns many alternative values to the design variables so as to generate a large number of alternative designs and finally to select the design that satisfies all the problem constraints with minimum evaluated cost.

This is not very accurate, efficient and quick for multivariable non-linear objective function with multivariable non-linear constraints. So in this study, mathematical optimization method, Sequential Quadratic Programming is suggested as it is a powerful technique for solving non-linear constrained optimization problem which is implemented using MATLAB.

The principal idea of sequential quadratic programming (SQP) is the formulation of a quadratic programming sub problem for the non-linear objective function based on a quadratic approximation and by linearizing the nonlinear constraints through its partial derivatives. This sub problem find better approximation to the design variables ( $x^{k+1}$ ) than current approximation ( $x^k$ ) and converges it to an optimal solution through iterative sequence.

MATLAB Optimization tool provides built-in function called 'Fmincon' where SQP algorithm can be easily implemented. In this design optimization method, aim is to minimize the objective function that gives total evaluated cost. Figure 3 shows the flow diagram of the design optimization criteria. In this optimization criterion, after selecting the transformer capacity, frequency, LV and HV voltages and insulation levels, objective function runs with the "Fmincon". Then it gives the optimum major design variable values which gives lowest Total Owning Cost (TOC). Since these variable values do not exactly match with the standard materials which are available in the market, it is necessary to match the optimum variable values with the available values to find a large number of combinations for the major variables and then those combinations are checked with the design constraints. Combinations which satisfy all of the constraints are used to calculate the TOC and finally the solution with the lowest TOC is selected for the design of the transformer.

## III. DESIGN EXAMPLE: 630KVA TRANSFORMER

Table 2 shows the optimum values of the major design variables for a 630kVA, 33kV/400V transformer. Table 3 shows the cost of the transformer main materials, Cost of the no-load loss and Cost of the full-load loss in optimum transformer design. Total owning cost of this transformer is USD 21105.80. For this design following constraints are imposed, Guaranteed Load Loss ( $LL_G$ ) < 8100W, Guaranteed NO Load Loss ( $NLL_G$ ) < 1290W, Guaranteed Total Loss ( $TL_G$ ) < 6886W,  $4\% < U_K < 5\%$ ,  $In_{LV,max} = 10kV$ ,  $In_{HV,max} = 30kV$ ,  $Im_{LV,max} = 23.5kV$ ,  $Im_{HV,max} = 50kV$ ,  $B < 1.6T$ . Under the above constraints design software gives the optimum transformer design output data which results in the lowest TOC. Table 4 shows the other parameters that designer should be concerned in any design. Due to the 3-D visualization capability that software it gives the optimum transformer design in 3-D. Designer can a clear idea about what does it

look like and whether any changes in the design are required. Figure 4 and Figure 5 show the 3-D view of the optimum transformer for 630kVA.

TABLE II  
MAJOR DESIGN VARIABLE VALUES IN OPTIMUM DESIGN

Variable	Values
T(mm)	117.00
W(mm)	40.23
H(mm)	191.00
$n_{LV}$	19.00
B(T)	1.45
$T_{lv}$ (mm)	0.90
J(A)	2.10
$d_{HV}$ (mm)	2.00

TABLE III  
630KVA, 33KV/415V TRANSFORMER DESIGN AT 50 HZ

Item	unit cost (kg/USD)/ (W/USD)	Amount	Cost(USD)
Core material	2.50	1136.2kg	2,840.0
LV copper	8.50	188.17 kg	1,599.5
HV copper	8.50	262.2 kg	2,228.7
Insulation	4.50	43.32 kg	194.9
Mineral oil	1.72	537.4 kg	924.3
No load loss	7.58	712.03 W	5,397.2
Full load loss(@ 75°C)	1.52	5211.32 W	7,921.2
Total owning cost(USD)			21105.8

TABLE IV  
IMPORTANT DESIGN PARAMETERS

Efficiency(%) -full load	98.80
Impedance Voltage (%)	4.66
Voltage Regulation (%)	3.40
No load Current (%)	0.87

#### IV. CONCLUSION

Most of the TDO methodologies concern on minimization of transformer main material cost under constraints. It's the conventional method where customer has to evaluate cost of losses. Summation of it with initial buying cost is the cost, customer has to pay throughout the life time which is not optimized. In this study we have developed an algorithm and software to design oil immersed distribution transformers with minimize total evaluated cost. So using this tool transformer manufactures able to design distribution transformers with the cost satisfying manufacture as well as customer.

Due to the transformer manufactures concern about to reduce the total manufacturing cost of transformer. So most of the designs come up with higher flux densities and current densities to reduce material cost. Considering life time of the transformer it is not an economical to customer to buy low cost transformer with high loss. So minimization of total evaluated cost is the most suitable criteria for design transformers.

The study was based on hermetically sealed type oil immersed three phase distribution transformer design optimization method. The proposed method has many advantages since transformer manufacturers can design the transformer with lowest evaluated cost while enhancing the life time of the transformer by adhering to relevant transformer standards. In this paper a methodology is described to optimize the total evaluated cost function restricted by a set of constraints. The main advantage of the software package are, i) Even relatively less experienced engineers can easily handle, ii) The process uses Matlab, ensuring convergence to global optimum iii) Program contains tools for the 3D visualization of the optimum design iv) Program facilitates updating of transformer materials v) User friendly input/output interface.

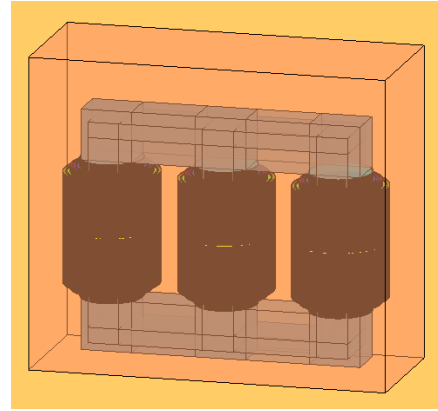


Fig. 4. 3-D view of the optimum transformer for 630kVA, 33kV/415V

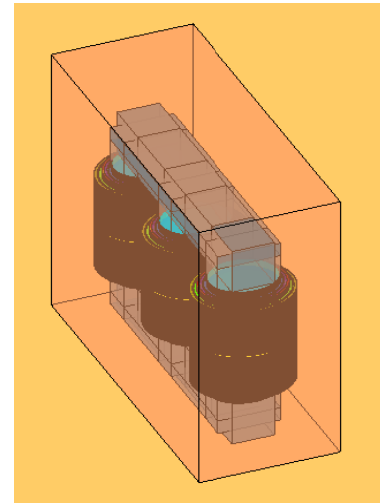


Fig. 5. 3-D view of the optimum transformer for 630kVA, 33kV/415V

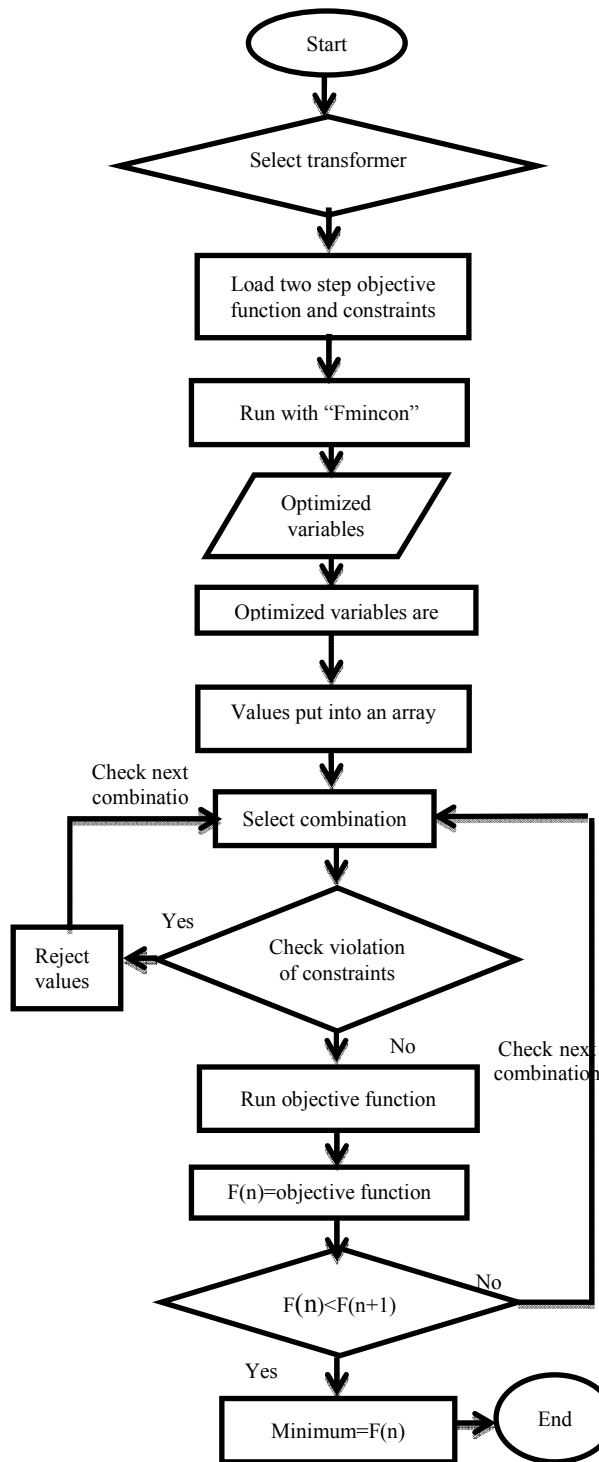


Fig. 3. Simplified flow diagram for transformer design optimization method

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