An Efficient Distributed Data Correspondence Scheme for Multi-Robot Relative Localization

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***Abstract*—This research addresses the problem of relative localization within a robot network possessing relative measurements between robots. The problem of correspondence is inherent to most multi-robot relative sensing methods, such as LiDAR, RADAR and vision based solutions. Multi-sensor multi-target tracking approaches addresses the problem of correspondence, when good *prioris* for initial poses of the sensing platforms are assumed. However the multi-robot relative localization problem differs from the classical multi-target tracking scenario due to; a) the unavailability of initial poses of sensing platforms, b) the existence of mutual measurements between the sensing platforms, and c) the measurement set being mixed with both known and unknown correspondences. To address these specific characteristics of multi-robot systems, this study proposes a distributed data correspondence architecture which performs multi-hypothesis estimation of the robot states. The proposed architecture is implemented on a multi-robot relative sensor configuration which possess range measurements with known data correspondence and bearing measurements with unknown data correspondence. The proposed distributed multi-robot localization method is capable of addressing measurement correspondence, noise, and measurement clutter effectively, while possessing inherent initialization and recovery capability from unknown poses.**

I. Introduction

Multi-robot systems are preferred over single robot configurations, for its robustness to failure, wide coverage, better resource utilization, and cooperative capabilities in mission execution. Relative localization is the process of estimating the formation of a robot team, by using the inter-robot relative measurements (IRRM) which are communicated within the team. IRRM sensors often used in robotic applications include; LiDAR, RADAR, IR received signal strength sensors which measure both relative range and bearing of robots [1], [2]; vision, Infrared search and track sensors which measure only the bearing of robots [1]; or Ultrasonic, RF range measurement methods which measure only the relative range between robots.

Prior to tracking or localization of robots, most sensor solutions presented above require to perform two main tasks. First is *data association*, where the measurements that have unknown origins are associated to corresponding robots of the team. Second is *sensor registration*, where the communicated beliefs from robots are transformed to a common reference coordinate frame for filtering purposes. Then the IRRMs are used for pose estimation of robots, which is traditionally performed by a centralized Extended Kalman Filter (EKF). However, due to communication and computation constraints in real world multi-robot networks, it is practical to use *distributed filtering* as efficient means of solving the localization problem.

Methods available in target tracking literature are well suited to address the multi-robot localization problem. Data association methods available for the design include, nearest neighbor, joint probabilistic data association (JPDA), and multi-hypothesis tracking (MHT) [1], [3], [4]. Standard sensor registration methods available for the design include, maximum likelihood batch estimators, joint registration and track to track association approaches, and pseudo measurements [1]. State de-correlation based distributed fusion architectures are used in multi-sensor tracking application for consistent fusion of tracks [5], [6].

However these methods are not directly applicable to the multi-robot localization problem due to three unique characteristics inherent to multi-robot systems. First, is the availability of *mixed measurements*, where only a subset of IRRMs are of unknown correspondence and some measurements such as platform velocities are of known correspondence. Second, is the availability of *mutual measurements*. Mutual measurements are measurements that are made between two platforms relative to one another. Third, is *unknown initialization*, which is not the typical case considered in classical target tracking where sufficient knowledge of the sensing platforms' locations are available as *a priori*.

In previous work we have investigated the viability of vision based bearing, and Ultrasonic based range IRRM combinations for multi-robot localization purposes [7], [8], [9]. Recent work attempts to integrate laser scanners and image sensors so that the robots are capable of generating estimates of the surrounding moving objects and neighboring robots. To address this scenario we have developed a novel distributed multi-robot localization framework which effectively addresses the problems related to communication constraints by using a covariance intersection based fusion module. The strategy allows communication of estimates at low predefined rates within the network as reported in authors' recent work [10]. Since laser scanners and image sensors are notorious for providing unknown and false correspondences, this paper reports the necessary preliminary work for addressing unknown correspondence of IRRMs under a distributed framework.

In this study we design a multi hypothesis tracker using a novel architecture to exploit specifically the availability of mutual measurements and measurements with mixed data correspondence. The specific class of sensors addressed is range with known data correspondence and bearing with unknown correspondence (presented in our previous paper [7]). However, the proposed multi-robot tracking framework is formulated to support a general class of sensors having mixed data correspondence. The main contributions of this paper are; 1) A Gaussian sum filtering (GSF), based novel fusion architecture for multi-robot relative localization where measurements with known correspondence, unknown correspondence, and mutual measurements of sensing agents are present. 2) A multi hypothesis sensor registration scheme which supports data association and localization in a distributed computing framework.

II. Proposed Approach

A. *Problem formulation*

A team of n robots denoted by the set $\mathcal{V} = \{1, 2, ..i, ..j, ..l, ..n\}$, consists heterogenous agents which possess varying sensing capabilities. The sensors of the robots acquire absolute measurements, and relative measurements. Each robot i takes measurements of its platform velocities $\mathbf{u}_k^{(i)}$. Additionally, each robot i takes absolute reference measurements $\mathbf{z}_k^{(i)}$, of quantities which acts as common reference to all robots (gravity, magnetic north, barometric height or depth, horizon etc.). The single superscript (i) is used to denote that the vector quantity is an absolute quantity (measured relative to an inertial frame) and it is expressed in the body fixed frame of i. The relative measurements acquired by robots include the IRRMs $\mathbf{z}_k^{(i,j)}$. The double superscripts (i, j) are used to denote that the vector quantity of robot j is measured relative to robot i, and it is expressed in body fixed frame of i. For clarity of presentation, the subscript k which denotes the time index is only used where necessary.

The IRRMs that occur in the team can be viewed as a directed and connected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. The nodes $v = i \in \mathcal{V}$ represents a robot i in the team and edges $e = (i, j) \in \mathcal{E}$ represents existence of an inter-robot measurement $\mathbf{z}^{(i,j)}$ of robot j with respect to a robot i and vice versa. The nodes $\mathcal{V}^+ \subseteq \mathcal{V}$ identify the robots with relative sensing capability, which takes a relative noisy measurement set Z of the states of the robots in the network with a detection probability of P_D. Additionally, each absolute reference measurement $\mathbf{z}^{(i)}$ is expressed as a relative measurement using a pose composition operator $\ominus$. i.e., $\mathbf{z}^{(i,j)} = \mathbf{z}^{(j)} \ominus \mathbf{z}^{(i)}$. Some or all relative measurements are of unknown origin (correspondence), thus a relative measurement set Z made by robot i comprises measurement vectors with known correspondence $\mathbf{z}^{(i,j)}$, and ones with unknown correspondence $\mathbf{z}_{1:m}^{(i,*)}$. The index $m \in \mathbb{N}$ denotes the number of measurements which are acquired with unknown correspondence. We refer such a measurement set as "mixed". An association event $A_r^{(i,j)}$ $r = 0, ..m$ maps a robot j in the team to an indexed measurement vector $\mathbf{z}_r^{(i,*)} \in Z^{(i,*)}$ in the measurement set. The association event $A_0^{(i,j)}$ denotes that robot j does not associate to any measurement (the measurements are false alarms). The set Z is further corrupted by measurement clutter uniformly distributed over the measurement space with clutter rate following a Poisson clutter model of spatial density λ.

The objective of the multi-robot localization system is to estimate the belief of state $\mathbf{x}_k^{(i,j)}$ of each robot j at time k, relative to a selected reference node i using all available information. Since this is a distributed design each robot $l \in \mathcal{V}^+$ creates beliefs of all neighboring robots in the network. These beliefs are communicated in the form of *equivalent measurements* for fusion purposes. Therefore, the available information for the estimation task can be partitioned as follows:

1) *Velocity measurements* ($U_k^{i\wedge j}$) - The set containing the velocity measurements $\mathbf{u}^{(i)}$ of robot i, and the velocity measurements $\mathbf{u}^{(j)}$ of robot j.
2) *Direct measurements* (Z_k^{ij})-The set containing relative measurements of robot j relative to robot i.

$$Z^{ij} := \left[\bigcup_{\forall r=1:m} \left(z^{(i,j)}, z_r^{(i,*)} \right) \right] \tag{1}$$

3) *Mutual equivalent measurements* (${}^{eq}Z_k^{ji}$)-The set containing equivalent measurement of i relative to j.

$${}^{eq}Z_k^{ji} := p(\mathbf{x}^{(j,i)} | Z_k^{ji}, U_k^{j\wedge i}, I_{1:k-1}) \tag{2}$$

4) *Indirect equivalent measurements* (${}^{eq}Z_k^{lj}$)- Consider a robot $l (\in \mathcal{V}^+) \neq i \vee j$. Thus ${}^{eq}Z_k^{lj}$ is the set containing equivalent measurement of j relative to l.

$${}^{eq}Z_k^{lj} := p(\mathbf{x}^{(l,j)} | Z_k^{lj}, U_k^{l\wedge j}, I_{1:k-1}) \tag{3}$$

Using all these measurements a Bayesian formulation of this problem can be formed as follows.

$$p(\mathbf{x}_k^{(i,j)} | U_k^i,\ Z_k^{ij},\ U_k^j,\ Z_k^{ji},\ U_k^l,\ Z_k^{lj},\ I_{1:k-1})_{\forall (l \in \mathcal{V}^+) \neq i \vee j} \tag{4}$$

A sequential Monte Carlo or an extended Kalman filter implementation is not straight forward due to data correspondence issues, measurement correlation, and sensor registration issues in the considered system.

B. *Distributed multi-robot localization framework*

The structure of the multi-robot tracking architecture is as follows. Each robot $i \in \mathcal{V}^+$ implements a sensor fusion module $FUS^{(i)}$. The "fuser" estimates the state of each robot (also termed as a target) $j \in \mathcal{V} : j \neq i$. In order to propagate the multiple hypothesis of the robots' states, it is assumed that the prior distribution of robot j ,$p(\mathbf{x}_k^{(i,j)} | I_{1:k-1})$, can be approximated by a mixture of Gaussians. Therefore, each fuser implements a Gaussian sum filter $GSF^{(i,j)} \in FUS^{(i)}$ for each robot j. The $GSF^{(i,j)}$ recursively estimates the state $\mathbf{x}^{(i,j)}$ relative to an anchor node i. A Gaussian mixture of p components is assumed with each Gaussian component identified as a "track" ${}^hTr^{(i,j)} \in GSF^{(i,j)}$, indexed with h, and parameterized with mean ${}^h\mathbf{m}^{(i,j)}$, covariance ${}^h\mathbf{P}^{(i,j)}$, mixture weight ${}^h\mathbf{w}^{(i,j)}$, and track association event ${}^hA_r^{(i,j)}$.

The flow of the proposed architecture is illustrated in Fig. 1. A fuser $FUS^{(i)}$ first performs a prediction operation which incorporates the velocity measurements $U^{i\wedge j}$ to the current belief. The correction operation which is also termed as direct fusion updates the Gaussian mixture $GSF^{(i,j)} \in FUS^{(i)}$

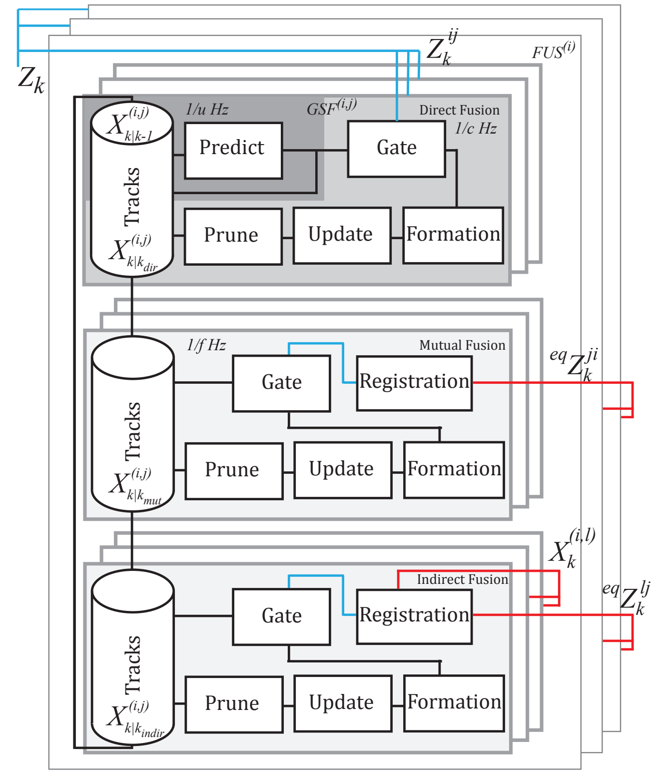

Fig. 1. Proposed data fusion architecture for multi-robot relative localization with mixed and mutual measurements

using the direct measurement set Z^{ij}. Then the target tracks of $FUS^{(j)} : j \neq i \in \mathcal{V}^+$ are communicated as mutual equivalent measurements $^{eq}Z^{ji}$ for the mutual fusion step. Finally, the target tracks of all fusers $FUS^{(l)} : l \neq i \wedge j \in \mathcal{V}^+$ are communicated as indirect equivalent measurements $^{eq}Z^{lj}$ for indirect fusion, along with the tracks $Tr^{(i,l)} \in GSF^{(i,l)}$ for the purposes of sensor registration. The multi-robot tracking filter updates at $1/u$ Hz, while the direct fusion runs at $1/c$ Hz. The mutual and the indirect fusions occurs at $1/f$ Hz, where $f > c \geq u$. This captures the different frequencies for filter update, filter correction, and inter-robot communication respectively.

C. *The multi-robot system*

The multi robot system considered in this study is a ground aerial system operating in 2.5D. The state of the system $\mathbf{x}^{(i,j)}$ is parameterized by the position $x,\ y,\ z$, and orientation θ of a robot j relative to a reference robot i. The process model for this system can be expressed by (5)

$$\dot{\mathbf{x}} = f(\mathbf{x})$$
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v_x^{(j)} \cos\theta - v_y^{(j)} \sin\theta - v_x^{(i)} + \omega_z^{(i)} y \\ v_y^{(j)} \cos\theta + v_x^{(j)} \sin\theta - v_y^{(i)} - \omega_z^{(i)} x \\ v_z^{(j)} - v_z^{(i)} \\ \omega_z^{(j)} - \omega_z^{(i)} \end{bmatrix} + \boldsymbol{\epsilon} \tag{5}$$

where $u^{(i)} = [v_x^{(i)},\ v_y^{(i)},\ v_z^{(i)},\ \omega_z^{(i)}]^T$ and $u^{(j)} = [v_x^{(j)},\ v_y^{(j)},\ v_z^{(j)},\ \omega_z^{(j)}]^T$ denotes the platform velocity measurements of robots i and j respectively. The process noise is denoted by $\boldsymbol{\epsilon}$ which is assumed to be drawn from a zero mean Gaussian distribution with covariance $\mathbf{Q}$.

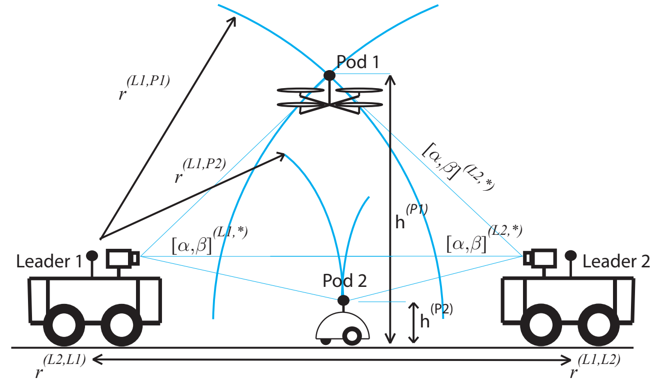

Fig. 2. The multi-robot system considered in the study. A heterogenous multi-robot team possessing relative range measurement capability with known correspondence and relative bearing measurement capability with unknown correspondence [7].

The measurement model includes an absolute measurement h which is transformed to a relative measurement, the relative range $r^{(i,j)}$, and the relative bearing (azimuth, elevation) $(\alpha^{(i,*)}, \beta^{(i,*)})$. The relative bearing is of unknown data correspondence. The measurement model for the discussed sensor set is given by (6), where the vector $\boldsymbol{\nu}$ denotes the measurement noise which is assumed to be drawn from a zero mean Gaussian distribution with covariance $\mathbf{R}$.

$$\mathbf{z} = g(\mathbf{x})$$
$$\begin{bmatrix} h \\ \alpha \\ \beta \\ r \end{bmatrix} = \begin{bmatrix} z \\ Tan^{-1}(\frac{y}{x}) \\ Tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right) \\ \sqrt{x^2+y^2+z^2} \end{bmatrix} + \begin{bmatrix} \nu_h \\ \nu_\alpha \\ \nu_\beta \\ \nu_r \end{bmatrix} \tag{6}$$

D. *Distributed multi-robot localization modules*

The distributed estimation framework in Fig. 1 is implemented for the multi-robot system presented above. Data correspondence in this architecture occurs implicitly by a likelihood based weighting of the Gaussian mixture components. The filters start with tracks representing all possible associations. Within a number of iterations, the hypothesis with most probable association gains the dominant mixture weights. The mixtures with low weights are removed by a pruning operation. The tracks of the Gaussian mixture are dynamic which gets re-initialized and removed as new measurements enters the system. Here we discuss each module presented in Fig. 1 in detail.

*1) **Prediction**:* The prediction step performs standard linear Gaussian propagation of the Gaussian mixture by incorporating the velocity measurements $U_k^{i \wedge j}$ to the current belief. Standard Kalman mean and covariance propagation equations are utilized for this purpose.

*2) **Gating**:* Measurement gating process discards measurements not in the selected confidence ellipse of a track. The standard innovation is found for each measurement and filtered if it exceeds a predefined confidence limit P_G (*probability of gating*). The chi-square distribution is used to calculate the corresponding threshold value. The gated measurements Z^{ij}_{gate}, and the out of gate measurements Z^{ij}_{init} are presented to the track formation step.

3) ***Formation****:* Three types of tracks are propagated forward with the corresponding weight updates:

- *False alarms* $Tr^{(i,j)}_{FA}$: The false alarm tracks account for the $A^{(i,j)}_0$ measurement association of all tracks, which are propagated with a probability of ${}^{h}w^{(i,j)}(1-P_D P_G)$.
- *New tracks* $Tr^{(i,j)}_{NT}$: Each out of gate measurement in Z^{ij}_{init} is initialized as a new track for robot j, with a probability of P_B. In standard MHT filters all measurements Z^{ij} are initialized as new target tracks. However, in the current formulation all tracks in a GSF corresponds to the same target robot j rather than separate targets. Therefore, formulation of tracks from in gate measurements Z^{ij}_{gate} is not performed since it results in unnecessary duplication of possible hypotheses for robot j.
- *Detected tracks* $Tr^{(i,j)}_{DET}$: All in gate tracks are propagated with a weight ${}^{h}w^{(i,j)}P_D$, which accounts for all remaining measurement associations $A^{(i,j)}_{1:m}$.

4) ***Update- Direct fusion****:* This step incorporates the direct measurements Z^{ij} to the current belief. The measurement update is performed using standard Kalman correction equations, where $\mathbf{G}, \mathbf{R}, \mathbf{S}$ and $\mathbf{K}$ are the measurement matrix, measurement noise, innovation covariance, and Kalman gain in standard definition. The weight of the corrected mixture components ${}^{h}\mathbf{w}^{(i,j)}$ are updated as follows.

$$
{}^{h}w^{(i,j)}_{k|k} = {}^{h}w^{(i,j)}_{k|k-u} . \mathcal{N}(\mathbf{z}^{(i,j)}_{k} | {}^{h}x^{(i,j)}_{k|k-u},\ {}^{h}P^{(i,j)}_{k|k-u}) \tag{7}
$$

5) ***Pruning****:* Three types of pruning operations are performed to limit exponential growth of hypotheses.

- Low weight track deletion: All tracks with weights which fall below a predefined threshold are deleted.
- Track combining: The Mahalanobis distance of tracks are checked and combined if its within a predefined threshold.
- N-scan pruning: Tracks created from the same ancestor track at the $k-N$th time step are deleted if the sum of the weights of the track family falls below a predefined threshold.

6) ***Broadcast****:* An equivalent measurement set is communicated by each fuser robot to all other fusers in the network to perform distributed fusion of the tracks. To support measurement de-correlation the covariance at the pervious fusion step propagated to current time step $P^{(i,j)}_{k|k-f/u}$, is also communicated.

$$
{}^{eq}Z^{ij} = \bigcup_{\forall h \in\ {}^{h}Tr^{(i,j)}_{k|k}} \{ {}^{h}\mathbf{m}^{(i,j)}_{k|k},\ {}^{h}\mathbf{P}^{(i,j)}_{k|k},\ {}^{h}\mathbf{P}^{(i,j)}_{k|k-\frac{f}{u}},\ {}^{h}w^{(i,j)}_{k|k} \} \tag{8}
$$

7) ***Mutual fusion****:* This update step incorporates the mutual measurement set ${}^{eq}Z^{ji}$ to the current belief. ${}^{eq}Z^{ji}$ is formed from all the tracks of $GSF^{(j,i)}$ of the fuser robot $FUS^{(j)}$. The composition operator $\ominus$ is used to transform each mutual measurement ${}^{eq,h}Z^{ji}$ to the coordinate frame of the fuser i.

$$
{}^{i}({}^{eq}Z^{ji}) = \ominus\ {}^{eq}Z^{ji} := \begin{bmatrix} -x^{(j,i)}\cos(\theta^{(j,i)}) - y^{(j,i)}\sin(\theta^{(j,i)}) \\ x^{(j,i)}\sin(\theta^{(j,i)}) - y^{(j,i)}\cos(\theta^{(j,i)}) \\ -z^{(j,i)} \\ -\theta^{(j,i)} \end{bmatrix} \tag{9}
$$

Then all measurements are updated with the local tracks of fuser i similar to the approach followed in direct fusion. Gating, formation, update, and pruning steps are performed considering the set ${}^{i}({}^{eq}Z^{ji}) = \bigcup_{\forall h} {}^{eq,h}Z^{ji}$ as the measurements. Approximate measurement de-correlation was performed as given in (10), in order to generate consistent estimates for distributed broadcast fusion [5].

$$
\begin{aligned}
&\left({}^{h}P^{(i,j)}_{k|k_{mut}}\right)^{-1} = \left({}^{h}P^{(i,j)}_{k|k}\right)^{-1} + {}^{i}\left({}^{h'}P^{(j,i)}_{k|k}\right)^{-1} - {}^{i}\left({}^{h'}P^{(j,i)}_{k|k-f}\right)^{-1} \\
&\left({}^{h}P^{(i,j)}_{k|k_{mut}}\right)^{-1} {}^{h}x^{(i,j)}_{k|k_{mut}} = \left({}^{h}P^{(i,j)}_{k|k}\right)^{-1} {}^{h}x^{(j,i)}_{k|k} + \\
&{}^{i}\left({}^{h'}P^{(j,i)}_{k|k}\right)^{-1} {}^{h}x^{(j,i)}_{k|k} - {}^{i}\left({}^{h'}P^{(j,i)}_{k|k-f}\right)^{-1} {}^{h}x^{(j,i)}_{k|k-f} \\
&{}^{h}w^{(i,j)}_{k|k_{mut}} = {}^{h}w^{(i,j)}_{k|k}\ {}^{h}w^{(j,i)}_{k|k} . \mathcal{N}({}^{i}({}^{h}x^{(j,i)}_{k|k}) | {}^{h}x^{(i,j)}_{k|k},\ {}^{h}P^{(i,j)}_{k|k} + {}^{h}P^{(j,i)}_{k|k})
\end{aligned} \tag{10}
$$

8) ***Indirect fusion****:* This step effectively incorporates the information in set $\bigcup_{\forall l \in \mathcal{V}^{+}: l \neq i \vee j} Z^{lj}$ to the current belief. Sensor registration is performed using a pose composition operator $\oplus$. This transformation also updates the mixture weight of the measurement.

$$
\begin{aligned}
&{}^{i}({}^{eq}Z^{lj}) = {}^{eq}Z^{il} \oplus {}^{eq}Z^{lj} := \\
&\begin{bmatrix} x^{(i,l)} + x^{(l,j)}\cos(\theta^{(i,l)}) - y^{(l,j)}\sin(\theta^{(i,l)}) \\ y^{(i,l)} + x^{(l,j)}\sin(\theta^{(i,l)}) + y^{(l,j)}\cos(\theta^{(i,l)}) \\ z^{(i,l)} + z^{(l,j)} \\ \theta^{(i,l)} + \theta^{(l,j)} \end{bmatrix} \\
&{}^{h}w^{(l,j)}_{k|k} \in {}^{i}({}^{eq,h}Z^{lj}) = {}^{h'}w^{(i,j)}_{k|k}\ {}^{h}w^{(l,j)}_{k|k}
\end{aligned} \tag{11}
$$

State fusion was performed using approximate state de-correlation methods similar to the mutual fusion step.

$$
\begin{aligned}
&\left({}^{h}P^{(i,j)}_{k|k_{ind}}\right)^{-1} = \left({}^{h}P^{(i,j)}_{k|k}\right)^{-1} + \sum^{\forall l} {}^{i}\left({}^{h'}P^{(l,j)}_{k|k}\right)^{-1} - {}^{i}\left({}^{h'}P^{(l,j)}_{k|k-f}\right)^{-1} \\
&\left({}^{h}P^{(i,j)}_{k|k_{ind}}\right)^{-1} {}^{h}x^{(i,j)}_{k|k_{mut}} = \left({}^{h}P^{(i,j)}_{k|k}\right)^{-1} {}^{h}x^{(i,j)}_{k|k} + \\
&\sum^{\forall l} {}^{i}\left({}^{h'}P^{(l,j)}_{k|k}\right)^{-1} {}^{h}x^{(l,j)}_{k|k} - {}^{i}\left({}^{h'}P^{(l,j)}_{k|k-f}\right)^{-1} {}^{h}x^{(i,j)}_{k|k-f}
\end{aligned} \tag{12}
$$

III. Results

A group with two leaders and three pods were simulated on a path presented in Fig. 4, with a 4DOF kinematic stochastic model with velocity variance of $0.05(\mathrm{m/s})^2$, and angular velocity variance of $0.01(\mathrm{rad/s})^2$. Measurement simulation was performed with the presented measurement model with variance parameters $[0.0036\ \mathrm{rad}^2,\ 0.0036\ \mathrm{rad}^2\ 5\ \mathrm{mm}^2,\ 8\ \mathrm{mm}^2]$ for $[\nu_\alpha,\ \nu_\beta,\ \nu_h,\ \nu_r]$. The values correspond to the hardware

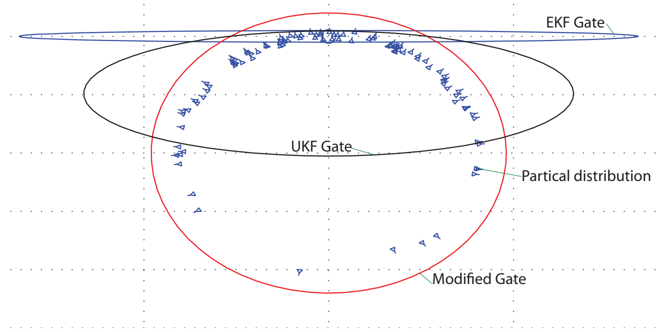

Fig. 3. Illustration of linearized (EKF), sigma point (UKF) and modified gates for the purposes of data association. The modified gate is used in the NMHT formulation to capture the uniform initial estimates of robot orientation.

setup presented in [7]. Measurement detection and clutter was simulated with detection probability $P_D = 0.95$, and uniform random clutter with poisson clutter spatial density $\lambda = 10^{-7}$, for IRRMs. A stochastic simulation of this multi-robot system was performed using Matlab while fully incorporating system noise and outliers, to capture an actual scenario. The proposed multi-robot localization framework was implemented with birth track probability $P_B = 0.5$, and gate probability $P_G = 0.95$. The filter was executed at $20Hz$, with direct fusion running at $10Hz$, and mutual/indirect fusions running at $1Hz$, to simulate practical situations.

Equation (13) was used for initialization of new tracks in the setup. The inherent initialization in the formation step of the algorithm handles both unknown initialization and kidnapped robot scenarios.

$$\begin{aligned} &\text{given}: \ \mathbf{z} = [r,\ \alpha,\ \beta,\ h] \\ &\mathbf{m} = [r\cos(\beta)\cos(\alpha),\ r\cos(\beta)\sin(\alpha),\ r\sin(\beta),\ 0] \\ &\mathbf{P} = \mathrm{diag}([0.1,\ 0.1,\ 0.05,\ 4\pi]) \\ &w = P_B\ \mathcal{N}(h|r\sin(\beta), P(3,3)) \end{aligned} \tag{13}$$

It should be noted that robot orientation cannot be initialized from measurements in the considered system, thus initial θ is set to mean 0 with variance 4π to approximate a uniform distribution. For problems where both pose and orientation are unobservable by a single robot as in the case of camera only multi-robot tracking problems, the generation of the initial hypothesis set should be performed using multiple robots, but initialization for these cases are not considered in this paper.

Due to the linear approximation of the noise distribution in EKF, the use of the orientation initialization scheme presented above develops problems in the gating step. The issue is illustrated in Fig. 3 where correct IRRM's would be discarded and re-initialized due to the occurrence outside the linear approximation ellipse. A modified gate as presented in figure is used for gating purposes, only for tracks with high orientation covariance. It should be noted that a well designed UKF implementation would address the problem with no modifications necessary for gating as illustrated in the figure.

Since the setup has 3 separate sensors for IRRM's, the gating, formation and update steps are implemented sequentially considering each sensor. This is because in presence of clutter of only one sensor, the whole measurement vector containing all sensor measurements would be discarded as clutter, although some sensors might be giving reliable measurements.

A. Simulation results

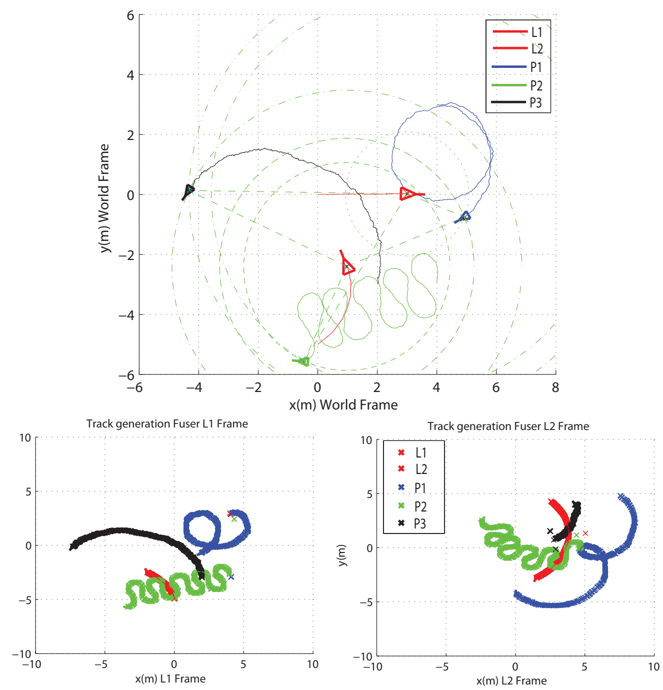

Fig. 4. The simulation setup in world coordinate system (top). The relative localization results generated by the proposed scheme for robot L1 frame (bottom left) and robot L2 frame (bottom right)

The proposed design was used to localize the presented scenario in Fig. 4. A result set generated with respect to each leader in the team is presented in the same figure, where successful tracking of all robots were obtained by both leaders in the team.

25 monte carlo average results were obtained for the given scenario on track generation and tracking error (Fig. 5). Time history of the average number of confirmed tracks of each robot j, is presented with the RMS error of the position and the orientation of the dominant hypothesis of each $GSF^{(i,j)}$. The results validates the capability of the proposed architecture

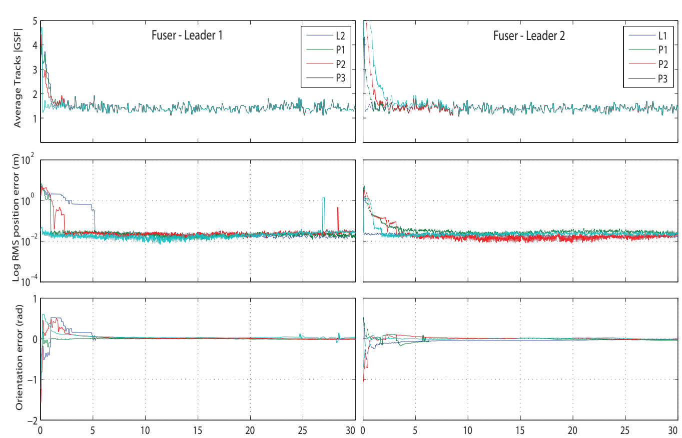

Fig. 5. Average results from 25 Monte Carlo runs. Hypothesis generation by the robots and log RMS position tracking accuracy and the orientation tracking accuracy for both robots L1 and L2

for multi-robot relative localization with mutual measurements and mixed data association. In all executed Monte-Carlo runs the proposed multi-robot relative localization method was able to successfully initialize the robots in the network and estimate the robots' pose while solving for unknown correspondence.

IV. Conclusion

This paper presented a multi-robot relative localization framework for handling mixed data correspondence and mutual measurements. The proposed approach performs distributed multi-hypothesis estimation of the robots in a network with respect to a selected reference platform. A standard multi-hypothesis tracker was modified to a set of Gaussian sum filters for each target robot and implemented in a distributed fashion for multi-robot tracking purposes. The design was applied to a class of relative localization sensors presented in [7].

Monte Carlo simulations were performed to evaluate the proposed approach which exhibits robust initialization capability, and data association capability of the design. Future work aims to apply the design to other classes of relative localization sensor configurations along with experimental evaluation of the proposed approach.

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