# Mathematical Analysis of Startup Mark in Elastic Tape on a Narrow Fabric Loom

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Abstract— Start-up marks are considered as one of the major defect in woven fabric, it occurs when the loom is restarted after loom stoppage due to various reasons. This defect is more prominent in high speed weaving looms especially with elastic warp yarns and is caused to deteriorate the fabric quality. This research paper deals with the mathematical analysis of the start-up mark generation in elastic tape on narrow fabric loom. Mathematical analysis was carried out for both continuous operation condition and under stopping condition of the narrow fabric loom. A system simulation was carried out using the model developed and compared with the experimental results to show the accuracy of the model.

Keywords— Start-up marks, Mathematical model, Tension variation, Narrow fabric loom, Elastic tapes

## I. INTRODUCTION

The elastic manufacturing has a higher demand in both local and international apparel business. Even though this is a profit making business, the full potential of productivity could not be met due to several fabric defects. The startup marks in narrow fabrics is a critical issue for the elastic tapes. There are variety of newly developed looms which has high performances in terms of various patterns and width at high speed but the stoppages due to power failures and yarn breaks are unavoidable. Tension variation of warp yarns when a loom is restarted after every stoppage is quite significant. As a result there is a variation in pick density as soon as machine starts which is considering as startup mark.

Startup mark causes to create variations in colour during dyeing and poor appearance in elastic tapes. In terms of time, energy and waste, startup marks are barrier to the business growth of elastic manufacturers. Therefore this can be identified as a global issue.

## A. Literature Review

Investigation of start-up marks were initially tried out by Greenwood and Cowhig published a series of research papers. A theoretical analysis of the factors governing pick was a part of the investigation about the start-up marks in their first paper. Mathematical relationships between beat-up forces, cloth fell position, take-up rate and pick space were derived

and analyzed in that paper [6]. Subsequently, this analysis was extended to explain the various causes of irregular pick spacing with respect to warp and fabric relaxing during the machine stoppage in the second paper. The effect of the length of warp and the fabric as well as the type of let-off motion contributed to the severity of start-up marks flaw were examined. Third paper was dedicated to an experimental investigation of loom and warp and indicated that effect of loom speed was so small for slight variations in it during running or after staring up did not cause any appreciable irregularity in the pick spacing. Hence it was concluded that the main cause of start-up marks is the relaxations of warp and cloth

The variation of warp tension during initial picks after restarting the narrow fabric weaving machine from its steady state running tension is the critical cause for start-up marks. Further author observed that the variation in warp tension was related to the inertia moment of the oscillating backrest [7]. Thin start-up marks are a result of lower warp tension while thick start –up marks are a result of higher tension during restarting in comparison to steady state running tension.[2] This argument can be accepted according to the practical experience.

The picks of synthetic yarn at cloth fell slip back when the reed move back after beating up causing non-uniform pick spacing at the cloth fell. The backward movement of the picks at the cloth fell may occur due to both slipping and rolling back. Rolling back is easier than slipping back as it requires less energy. The release of energy stored in a twisted yarn promotes rolling back of inserted weft [3].

An image processing technique was used to study start-up marks. Tendency towards formation of start-up marks increases with fabric tightness and other factors such as stoppage time, stop position, warp let-off control and back rest damping was their findings [8].

Author emphasized that any disturbance of the cloth fell position is responsible for the variation in pick-spacing. [4] When these pick spacing's are severe enough, it will lead to a fault in the fabric. Most of the variations in pick spacing are caused by displacement of the cloth fell. The main sources of

variations in pick spacing were identified as take-up motion, let-off motion and loom stoppage [9]. Any variation in the rate of take-up must be enough to make a change in pick spacing due to the change in cloth-fell position. Most of these types of variations are occurred due to mechanical issues such as eccentric or eccentrically mounted rollers or gear wheels and damaged or incorrectly shaped teeth of gears. Even though take – up motion was emphasized as a key factor responsible for start- up marks, the research works out so far revealed that the improper functions of let-off motion are caused to the variations in warp tension and it will lead to the variations in the cloth-fell position and pick-spacing [1].

This research was carried out on a narrow fabric weaving loom with a negative friction type let- off motion. Having negative friction type let-off motion not allow to float backrest and beam. When the backrest is fixed warp length and the fabric length remain constant [10]. Therefore during the loom stoppage the warp and the fabric tension will tend to reduce due to relaxation of warp sheet and fabric, and some movement of the cloth fell also. The directions of the displacement of the fabric cloth fell depend on relative change in the elastic module of the warp sheet and the fabric [5].

## B. Overview

Most of the work reviewed and described above was useful in explaining how start-up marks are created and what factors are responsible in creating start – up marks. Those works were based on either with mathematical models, experimental models or laboratory experiments. For those experiments, only shuttle looms were used. In this paper authors attempt to develop a mathematical model for tension variation during weaving and in the transient time of stopping of a narrow fabric loom which weaves an elastic tape. Further accuracy of the model developed was illustrated through the experimental data of the narrow fabric loom.

## II. MATHEMATICAL MODEL

# A. Narrow Fabric Loom Setup

Experimental narrow fabric elastic loom consist with four weaver's beam with negative friction let-off mechanism. Warp yarn which is unwound from the weaver's beam is gone through the two sets of back rests. Before enter to the weaving zone it is gone through a separator to separate all warp yarns into four sections which is woven into four elastic tapes. After the weaving zone, woven elastic tape is pulled out by the take-up roller as shown in Fig.1.

The experimental set up is a loom with negative let-off mechanism. In a negative let-off mechanism driving force against friction is provided by the tension on warp yarn. The tension of warp is regulated by the friction between rope and beam flange as given in Fig.2.

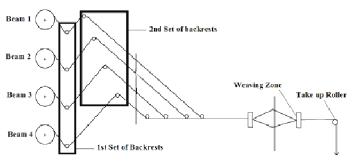


Fig.1. Narrow Fabric Loom

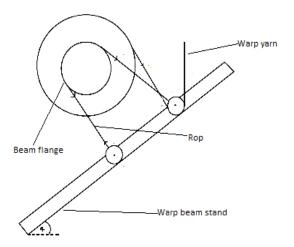


Fig.2. Warp Tension Mechanism

Because of negative let off mechanism, at the stoppage, weaver's beam was unable to stop with the machine head. Therefore an additional rotation of weaver's beam was observed. With this additional rotation, tension drop is experienced in warp yarn and at the restart position pick density variation can be observed in elastic tape. It is identified as a start-up mark.

### B. Development of the Mathematical Model

In the literature, it was found that the inertia of negatively driven weavers beam cause to continue warp feeding even after the loom head stops. Due to negative feeding of the weavers beam, the torque due to yarn tension is counter balanced by the frictional torque and the inertia torque required rotating the beam. In stoppage of the beam, the inertia torque is no more persists and due to the imbalance of inertia torque weavers beam rotates further. In addition, inertia of the rotating weavers beam supports for this further rotation and thus reduces the yarn tension. The excess yarn length driven out by this motion, directly leads the tension changes in warp yarns and eventually low tension creates stop mark once the loom start running. This mathematical model quantitatively explains the factors which influences the excess yarn length during the very first pick insertion when the machine starts.

# C. Mathematical Analysis of Continoues Operation

In continuous operation, mass conservation law is applied to the control region and yields the following equation

$$\frac{dm_c}{dt} = \frac{dm_{in}}{dt} - \frac{dm_{out}}{dt} \tag{1}$$

where  $\frac{dm_c}{dt}$ ,  $\frac{dm_{in}}{dt}$  and  $\frac{dm_{out}}{dt}$  represent rate of change of mass in control region, the mass inflow rate and the mass outflow rate of the yarn to the weaving zone respectively. The rate of change of mass in control region can be express in terms of length change and mathematically given by

$$\frac{dm_c}{dt} = \rho_o A \frac{dl_o}{dt} \tag{2}$$

 $\frac{dm_c}{dt} = \rho_o A \frac{dl_o}{dt}$  (2) where A is the cross section of the unstrained yarn,  $\rho_o$  is the linear density of the unstrained yarn and  $l_0$  is the unstrained length.

When the unstrained length is expressed in terms of strained length in continuous operation

$$l_0 = \frac{l}{(1+\varepsilon_C)} \tag{3}$$

where  $\varepsilon_c$  - Strain of the yarn in continuous operation and l is the strained length as refers to Fig.3.

By differentiating equation (3) yields

$$\frac{dl_0}{dt} = \frac{1}{(1+\varepsilon_c)^2} \left[ (1+\varepsilon_c) \frac{dl}{dt} - \frac{d\varepsilon_c}{dt} \right] \tag{4}$$

$$\frac{dl_0}{dt} = (2\varepsilon_c - 1)l\frac{d\varepsilon_c}{dt} + (1 - \varepsilon_c)\frac{dl}{dt}$$
 (5)

So the rate of change of mass in control region is equal to

$$\frac{dm_c}{dt} = \rho_o A l(2\varepsilon_c - 1) \frac{d\varepsilon_c}{dt} + \rho_o A (1 - \varepsilon_c) \frac{dl}{dt}$$
 (6)  
At the cloth take-up position strained advance length can

be expressed as

$$l_{f0} = \frac{l_f}{(1 + \varepsilon_f)} \tag{7}$$

where  $l_{f0}$  is the unstrained length advance per weft,  $l_f$  is the strained length advance per weft and  $\mathcal{E}_f$  is the strain of the yarn at take up. Note that  $l_{fo}$  and  $l_f$  are the crimped lengths.

By differentiation of the unstrained length advancement per weft and first order approximation the following equation can be obtained.

$$\frac{dl_{f0}}{dt} = (2\varepsilon_f - 1)l_f \frac{d\varepsilon_f}{dt} + (1 - \varepsilon_f) \frac{dl_f}{dt}$$
 (8)

$$\frac{dm_{out}}{dt} = \rho A' v_f \tag{9}$$

Because of equal density in straining and unstaring conditions,

$$\frac{m'}{l_{f0}A} = \rho_o$$

$$\frac{m'}{l_{f}A'} = \rho_o$$

$$\frac{dm_{out}}{dt} = \rho_o A \frac{v_f}{(1+\varepsilon_f)} \tag{10}$$

 $\frac{dm_{out}}{dt}=\rho_oA\frac{v_f}{(1+\varepsilon_f)}$  where  $v_f$  - Speed of the take up roller.

At the warp unwinding position, warp yarns are released from the weaver's beam and entered to the control region.

Relaxed length (l<sub>r</sub>) of yarn out from the weaver's beam is equal to

$$l_r = \frac{R\omega}{1+\varepsilon_i} \tag{11}$$

where R is the radius of the weaver's beam,  $\omega$  is the rotational speed of the weaver's beam and  $\mathcal{E}_i$ - is the strain of the in yarn.

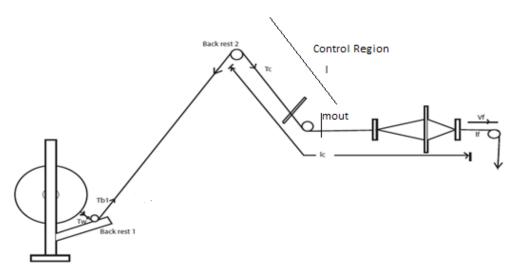


Fig.3. Warp yarn path

Hence the rate of yarn enter to the control region in continues operation can be given as,

$$\frac{dm_{in}}{dt} = \frac{R\omega}{1+\varepsilon_i} \rho_o A \tag{12}$$

During the constant operation no accumulation of yarn in control region and it is mathematically given by

$$\frac{dm_c}{dt} = 0 \tag{13}$$

So the equation (1) is deduced to

$$\frac{dm_{in}}{dt} = \frac{dm_{out}}{dt} \tag{14}$$

In constant operation, At the point of power failure no yarn takes out from the control region and output rate of change of mass is equal to zero.

$$\frac{dm_{out}}{dt} = 0 \tag{15}$$

So the equation (1) becomes  $\frac{dm_c}{dt} = \frac{dm_{in}}{dt}$ 

$$\frac{dm_c}{dt} = \frac{dm_{in}}{dt} \tag{16}$$

From equation (10) and (12) the following equation could be derived.

$$\frac{R\omega}{1+\varepsilon_i}\rho_o A = \rho_o A \frac{v_f}{(1+\varepsilon_f)} \tag{17}$$

$$\frac{R\omega(1+\varepsilon_f)}{1+\varepsilon_i} = v_f \tag{18}$$

## D. Mathematical Analysis of Stopping Condition

In negative let-off the rotational movement of weaver's beam is unable to stop though the loom stops and therefore beam is further rotated until its moment in balanced with torque created by the frictional force.

In this situation kinetic energy of the beam and the work doing by the warp yarns are balanced with the frictional energy of the system.

As the frictional energy loss is considered, the energy conservation law can be applied as,

$$\frac{1}{2}I_{eq}\omega^2 + \theta RT_w = \theta RF_R \tag{19}$$

 $\theta R$  is the excessive length which enters to the control region because of the further rotation of the weaver's beam after machine head stoppage.  $F_R$  is the frictional forces in the weaver's beam,

 $I_{eq}$  is the equitant moment of inertia of the weaver's beam.

From equation (18) and (19) 
$$\theta R = I_{eq} \left(\frac{1+\varepsilon_i}{1+\varepsilon_f}\right)^2 \frac{v_f^2}{2R^2(F_R - T_W)} \tag{20}$$

Replacing tension values by excessive length which is entered to the control region can be express as follows.

$$\theta R = I_{eq} \left( \frac{AE + T_w}{AE + T_f} \right)^2 \frac{v_f^2}{2R^2 (F_R - T_w)}$$
 (21)

where E is the Young's modulus of the warp yarn.

### III. RESULTS

# A. Simulation

In the experimental setup  $I_{eq}$  the value of the weaver's beam can be expressed as,  $I_{eq} = \frac{1}{2} [2m_1r_1^2 + m_2r_2^2 +$  $m(R^2 + r_2^2)$  And substituting the following parameters yields  $I_{eq} = 2.4769 \text{kgm}^2$ 

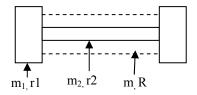


Fig.4. Moment of inertia or weaver's beam

 $E - 1.97 \times 10^5 \text{N/cm}^2$  $A - 1 \text{mm}^2$ 

Parameters of the experimental setup

 $v_f$  - 0.025 m/s

 $T_f$ - 70 cN/ warp

 $F_R - 183.34N$ 

No of warps in the weaver's beam - 240

Assuming above parameters are adequately invariable throughout the process, excessive length under different tensions  $(T_w)$  and different radius(R) were calculated from the developed mathematical model.

TABLE I. THEORETICAL VALUES FOR EXCESSIVE LENGTH

$T_w$ (cN)/warp	R (cm)	θR(mm)
52	24	0.219
55	20	0.363
60	18	0.592
62	12	1.524
65	6	1.938

# B. Experimental Value

Experimental data were collected related to the warp tension from the contact tension meter. Ten tension values were taken across the warp sheet and calculated the average tension value as the final tension value.

Radius of the beam was changed from time to time and therefore radius of the beam was measured in several instances.

In practical situation it is difficult to measure excessive length of the warp which is entered to the fabric formation zone. Defect size is depended on that excessive length is woven in the fabric. Because when the excessive length unwound from the weaver's beam, after weaving it should be in the fabric. Therefore in practical situation, using pick density variation of startup mark excessive length for each case was measured.

TABLE II.	PRACTICAL VALUES FOR EXC	ESSIVE LENGTH

$T_w$ (cN)/warp	R (cm)	θR (mm)
52	24	0.2
55	20	0.4
60	18	0.7
62	12	1.6
65	6	2.1

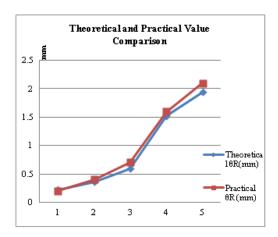


Fig.5. Comparison of theoretical and practical values

## IV. DISCUSSION

When compare theoretical values with practical values, practical values were slightly higher than the theoretical value. Practical values were measured using the startup mark which created on the elastic tape in particular instance by assuming excessive length which enters to the weaving zone should be woven in the fabric. But because of the tension applied by the take-up roller can pull more amount of yarn from the weaving zone than unwound excessive length during the stoppage. Therefore practical values are always above the theoretical values. Further, constant frictional forces were assumed in the theoretical model but in practical situation this frictional forces can be varied. And hence experimental values can be different from that of theoretical.

When measuring practical values for excessive length pick density variation in startup mark was used. But in this case if it is possible to measure excessive length which is entered to the weaving zone it would be more accurate because when excessive length measure at the weaving zone it is not subjected to the frictional forces and force applied by the take up roller — At the stage of mathematical model development cross section variation due to strain was neglected by

assuming it is negligible. But there can be some influence in tension variation of the warp yarn.

It is mentioned that in literatures about the short term variations, medium term variations and long term variations in the weaving process. Here short term and long term variation were considered but variations occur due to changes in the coefficient of friction between the faces of ruffles and ropes or chains (Medium term variations) were neglected. But it also can be affected to the startup mark.

If it is possible to consider all neglected parameters when developing a mathematical model, exact amount of excessive warp yarn length can be calculated with greater accuracy.

# V. CONCLUSION

This research mathematically analyzes the formation of startup mark on a narrow fabric elastic loom. Here a mathematical model was developed for the excessive length which is entered to the control region once the loom is stopped and before stopping the weavers beam.

Developed model was used to calculate excessive length which cause for startup mark and compared it with the practical value. Though the practical values are slightly higher than the theoretical values, both follow the same pattern with closer proximity. Therefore the developed mathematical model can be used to design a tension control system in order to eliminate startup mark defect. So the mathematical model developed has a greater importance and high practical implications.

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