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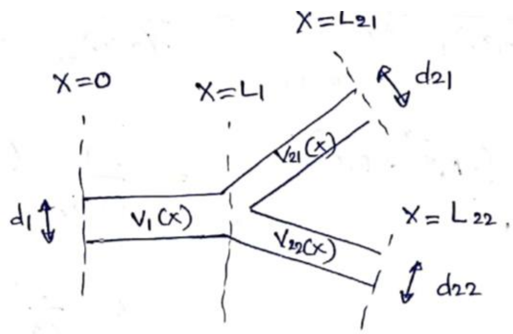
BM2101 - Analysis of Physiological Systems – Assignment 02

Branched Cylinders: Dendritic Tree Approximations

Question 01

$$V_1(x) = A_1 e^{-x} + B_1 e^{x}$$
$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x}$$
$$V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x}$$



from boundary conditions,

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -(r_i\lambda_c)_1 I_{app}$$

$$\left(-A_1 e^{-x} + B_1 e^{x}\right)_{x=0} = -(r_i\lambda_c)_1 I_{app}$$

$$-A_1 + B_1 = -(r_i\lambda_c)_1 I_{app}$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \quad \text{--- (1)}$$

$$V_{21}(L_{21}) = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad \text{--- (2)}$$

$$V_{22}(L_{22}) = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad \text{--- (3)}$$

using continuity of V at node

$$V_1(L_1) = V_{21}(L_1)$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0 \quad \text{--- (4)}$$

$$V_{21}(L_1) = V_{22}(L_1)$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0 \quad \text{--- (5)}$$

using current conversion at node,

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1}{dx}\bigg|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dx}\bigg|_{x=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dx}\bigg|_{x=L_1}$$

$$\frac{-1}{(r_i\lambda_c)_1}\left(-A_1e^{-x}+B_1e^{x}\right)\bigg|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21}e^{-x}+B_{21}e^{x}\right)\bigg|_{x=L_{21}}$$

$$+\frac{-1}{(r_i\lambda_c)_{22}}\left(-A_{22}e^{-x}+B_{22}e^{x}\right)\bigg|_{x=L_{22}}$$

$$\frac{-1}{(r_i\lambda_c)_1}\left(-A_1e^{-L_1}+B_1e^{L_1}\right) = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21}e^{-L_{21}}+B_{21}e^{L_{21}}\right) - \frac{1}{(r_i\lambda_c)_{22}}\left(-A_{22}e^{-L_{22}}+B_{22}e^{L_{22}}\right)$$

$$\frac{A_{21}e^{-L_{21}}-B_{21}e^{L_{21}}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_{22}}-B_{22}e^{L_{22}}}{(r_i\lambda_c)_{22}} - \frac{A_1e^{-L_1}+B_1e^{L_1}}{(r_i\lambda_c)_1} = 0 \quad (B)$$

Question 02

$$Ax = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix}$$

$$= \begin{bmatrix} A_1 - B_1 \\ A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} \\ A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} \\ A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} + B_{22}e^{L_{22}} \\ A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} \\ \frac{-A_1e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} \end{bmatrix}$$

$$\text{since}\quad b = \begin{bmatrix} (r_i\lambda_c)_1 I_{pp} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 - B_1 = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0$$

$$\frac{-A_1 e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21} e^{-L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

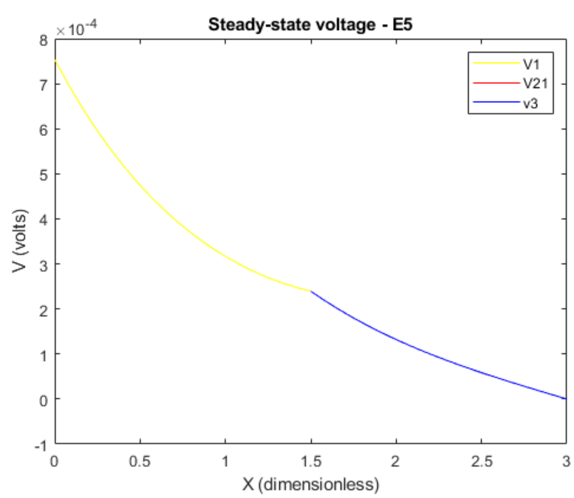
Question 03

```
cable;
x = A\b;
A1 = x(1)
A2 = x(2)
A21 = x(3)
B21 = x(4)
A22 = x(5)
B22 = x(6)
```

```
A1 = 7.3698e-04
A2 = 1.6723e-05
A21 = 0.0011
B21 = -2.7987e-06
A22 = 0.0011
B22 = -2.7987e-06
```

Question 04

```
figure;
y1 = linspace (0,l1,20);
y21 = linspace (l1,l21,20);
y22 = linspace (l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','v3'})

xlim('auto')
ylim('auto')
```

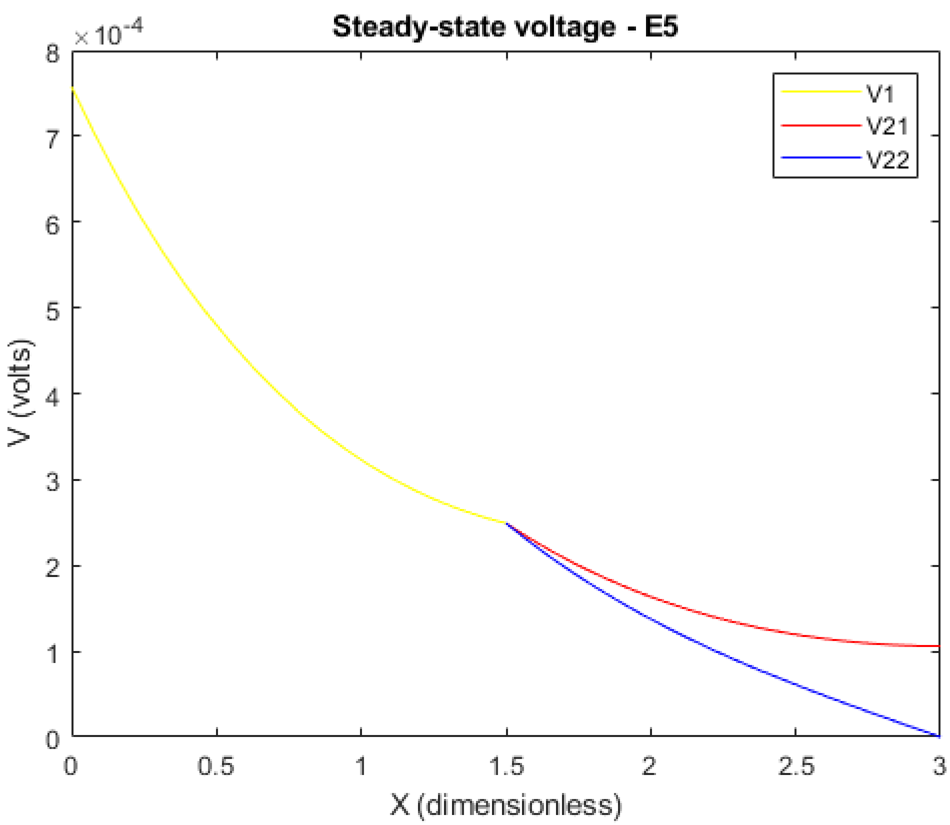


The steady state voltage profiles of the two daughter branches are same. The dimension changes don't matter. When we plot them, they overlap.

Question 05

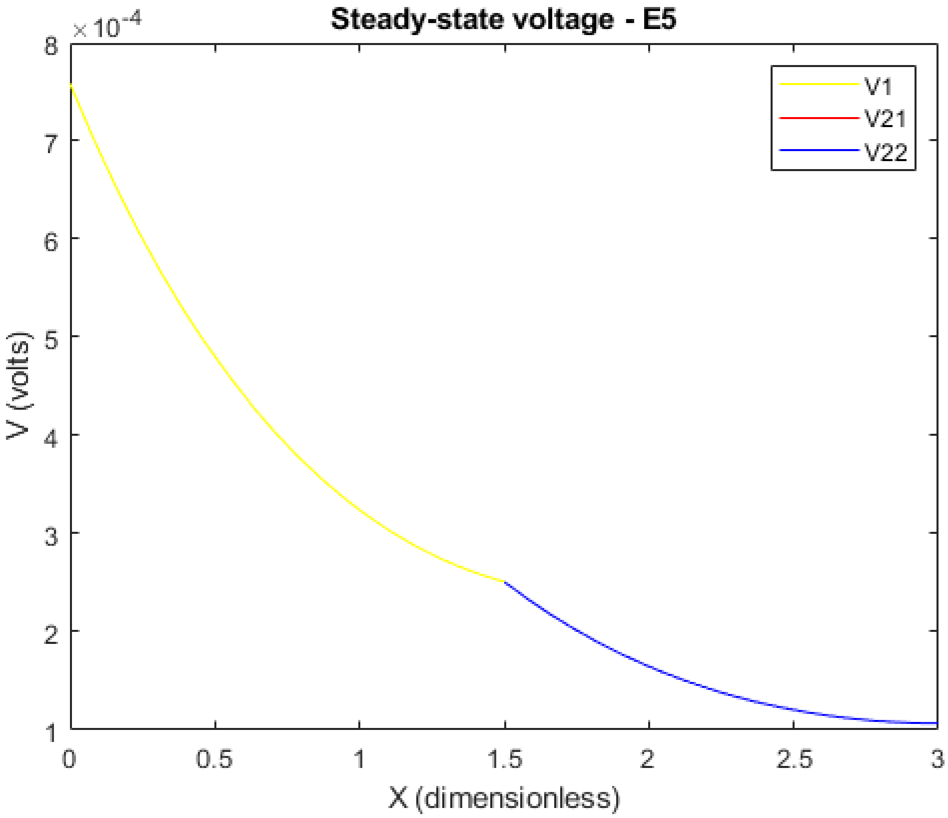
part (a)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
x=A\b;
v1 = x(1)*exp(-y1) + x(2)*exp(y1);

v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','V22'})
```



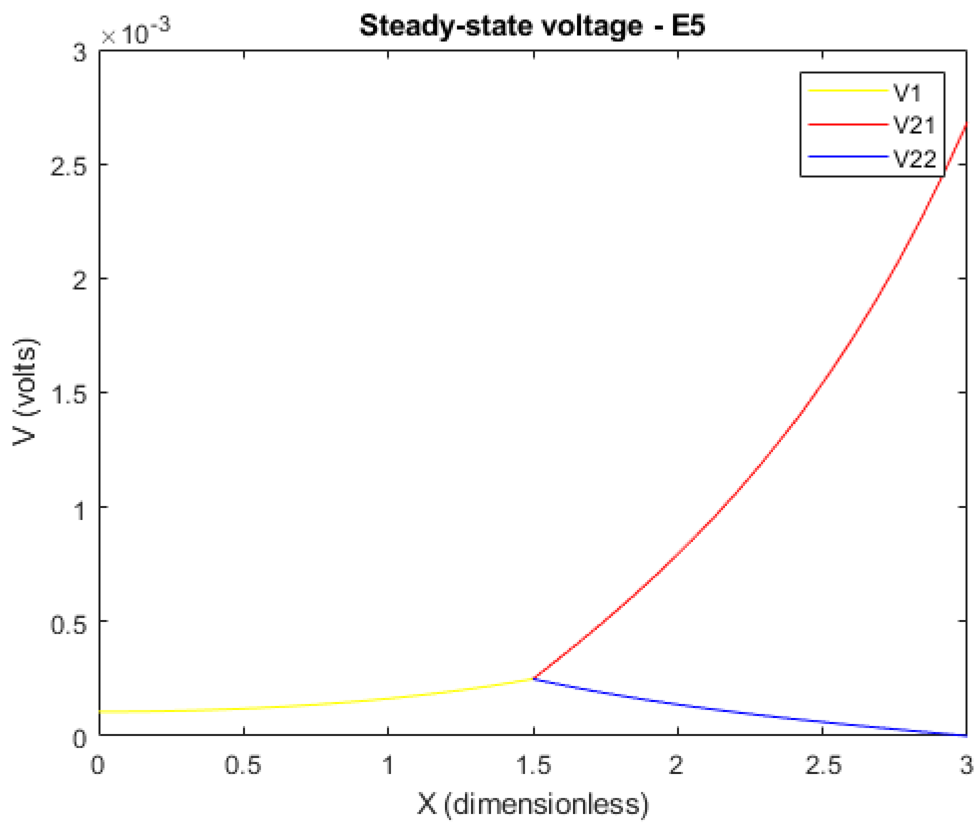
part(b)

```
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A\b;
y1 = linspace (0,l1,20);
y21 = linspace (l1,l21,20);
y22 = linspace (l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);

v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','V22'})
```



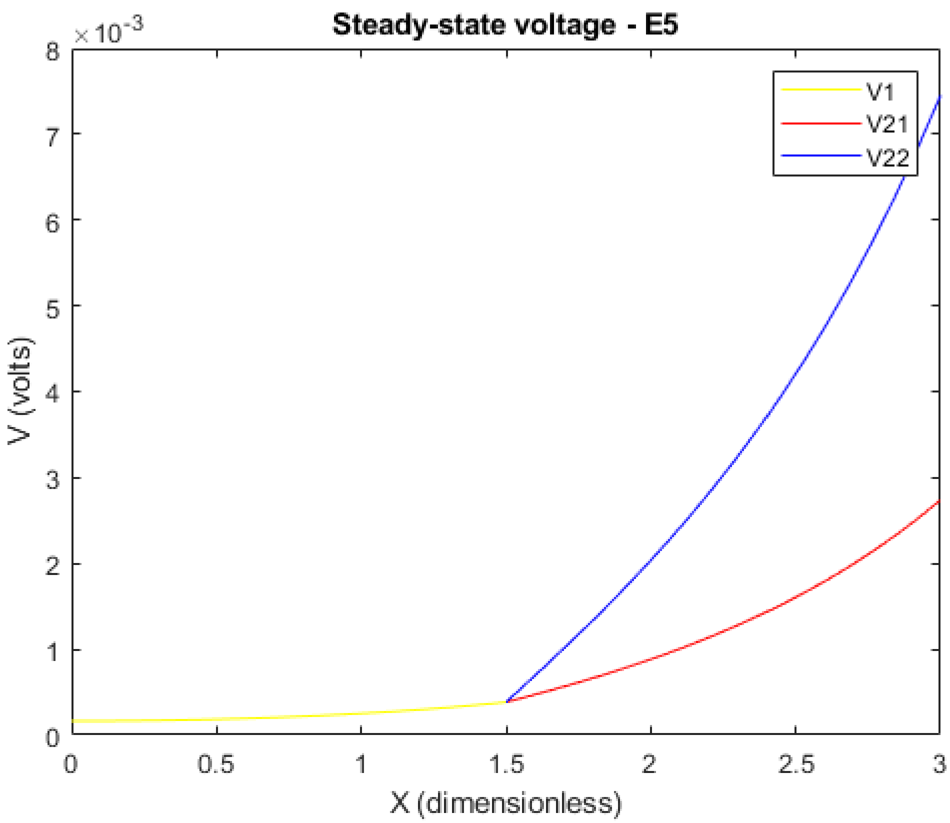
part (c)

```
b(1) = 0;
b(2) = rl21*iapp;
A(3,:) = [0 0 0 0 exp(-l22) exp(l22)];
x=A\b;

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','V22'})
xlim("Auto")
ylim("Auto")
```



part (d)

```
b(3) = rl22*iapp;
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A\b;

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','V22'})
```



In part c: the branch L21 is at positive potential while L22 is at 0 potential. Also the gradient of L22 is negative while other two are positive. Current is coming from L21 and it divides at the node. Then the current flow through both parent and L22 branch.

In part D: Both daughter branches are at positive potential. They have a positive gradient. Current comes through the two daughter branches and then summed up at the node. Summation goes through the parent branch.

Question 06

Question 06

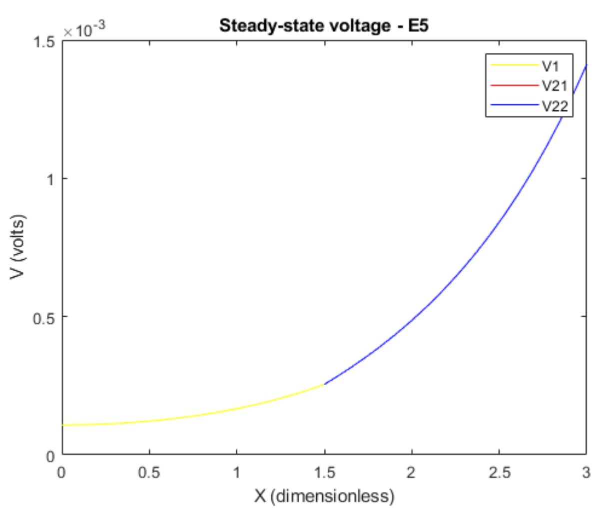
```
d1 = 75e-4;    % cm
% d21 = 30e-4;    % cm
% d22 = 15e-4;    % cm
d21 = 47.2470e-4;       % E9 cm
d22 = d21;               % E9 cm

l1 = 1.5;   % dimensionless
l21 = 3.0;   % dimensionless
l22 = 3.0;   % dimensionless

% Electrical properties of compartments

Rm = 6e3;   % Ohms cm^2
Rc = 90;   % Ohms cm
Rs = 1e6;   % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);  % Ohms
rl21 = c1*d21^(-3/2);  % Ohms
```

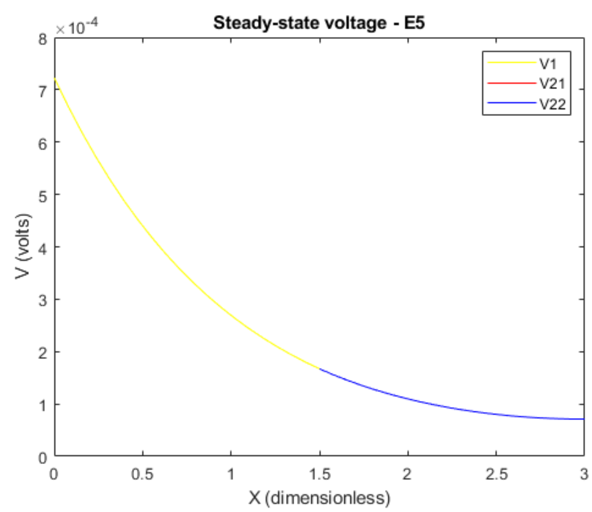
```
rl22 = c1*d22^(-3/2);  % Ohms
% Applied current

iapp = 1e-9;  % Amps

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22 -rl1*exp(-l1)/rl2
```

```
b(1) = rl1*iapp;
b(2) = 0;
b(3) = 0;
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A\b;
y1 = linspace (0,l1,20);
y21 = linspace (l1,l21,20);
y22 = linspace (l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend({'V1','V21','V22'})
```

```
b(1) = 0;
b(2) = rl21*iapp;
b(3) = rl22*iapp;
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A\b;
y1 = linspace (0,l1,20);
y21 = linspace (l1,l21,20);
y22 = linspace (l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot (y1,v1, 'y-',y21, v21, 'r-',y22, v22, 'b-')
xlabel('X (dimensionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
```

```
legend({'V1','V21','V22'})
```





The voltage variation through the parent and daughter branches looks similar. The reason for this should lie within the resistance of those branches. The parent branch and the two daughter branches together have the same resistivity.

Since the diameters and the lengths of the daughter branches are equal, those two has the same resistance. Hence, both has the same voltage variation.