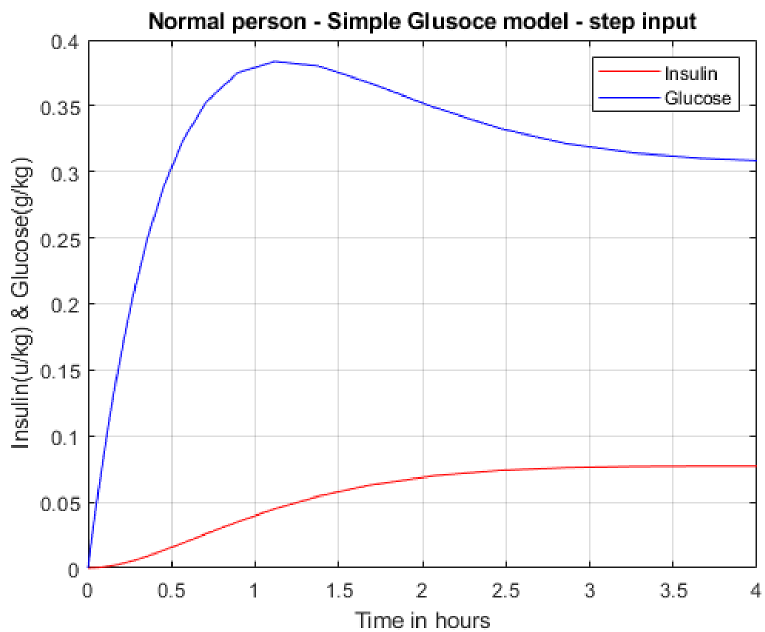
190501V - Ranathunga R.A.C.D.

BM2101 - Analysis of Physiological Systems – Assignment 04

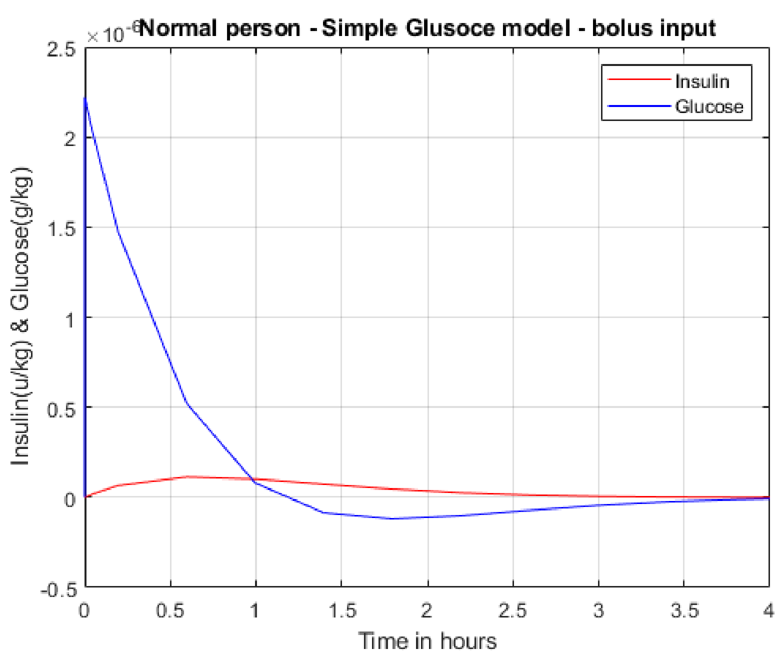
Compartment Modelling

Part 01 - Question 01

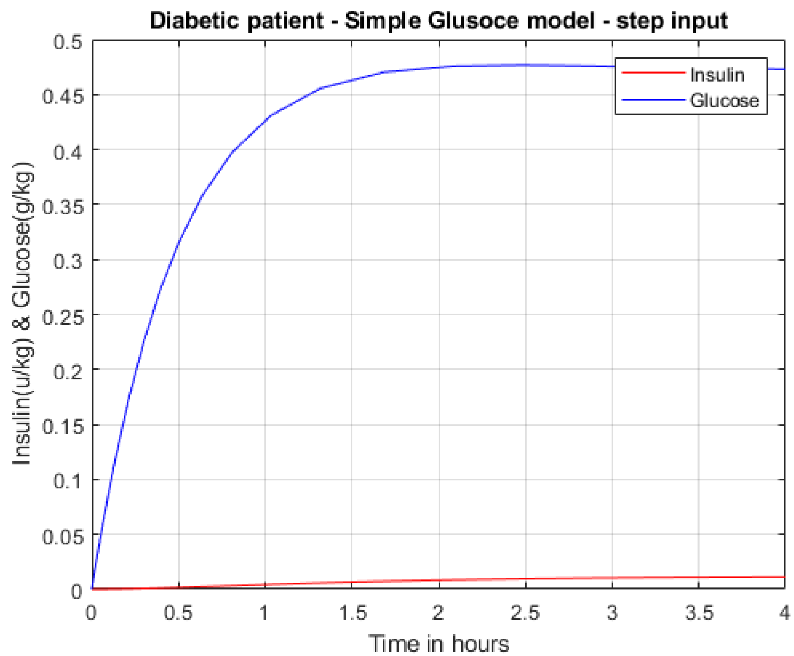
1) Step input to a normal person



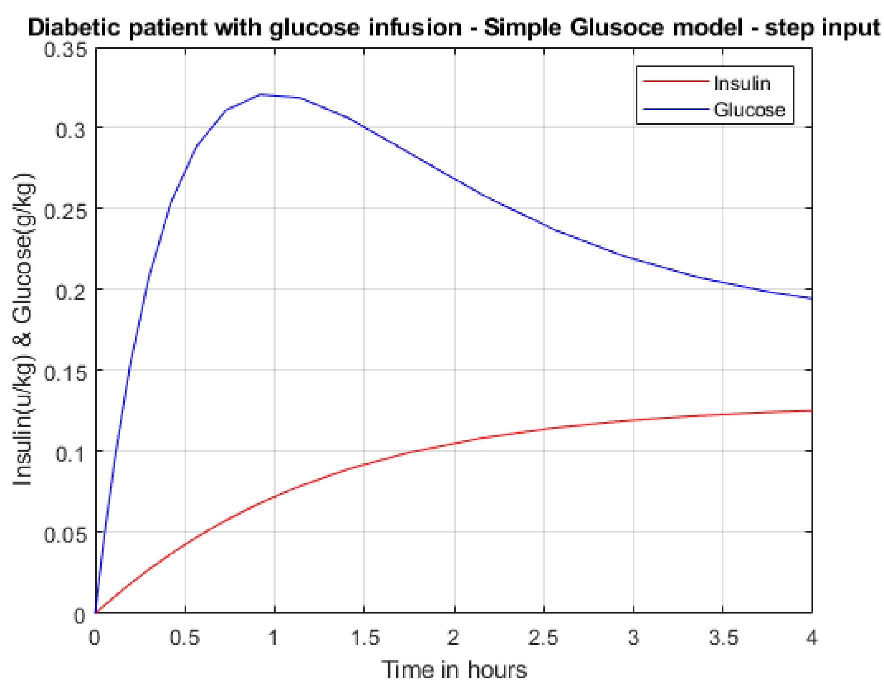
2) Bolus input to a normal person



3) Step input to a diabetic patient

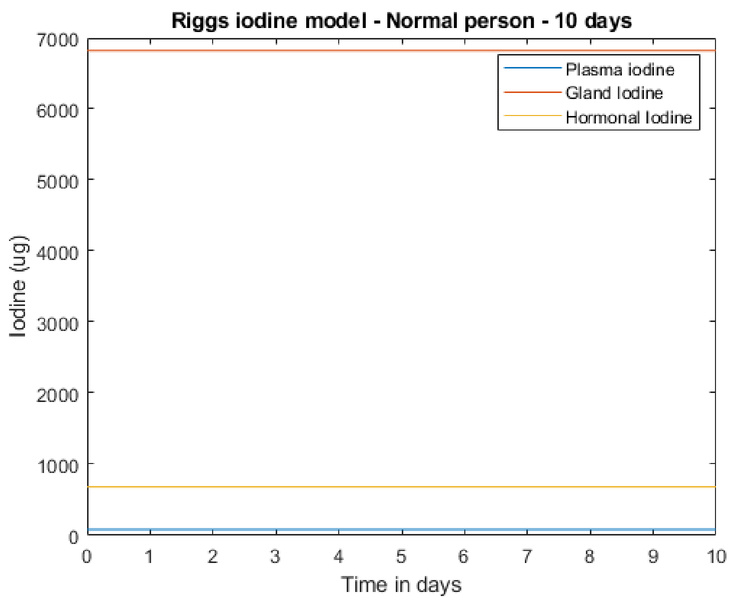


4) Step input to a diabetic patient with glucose infusion

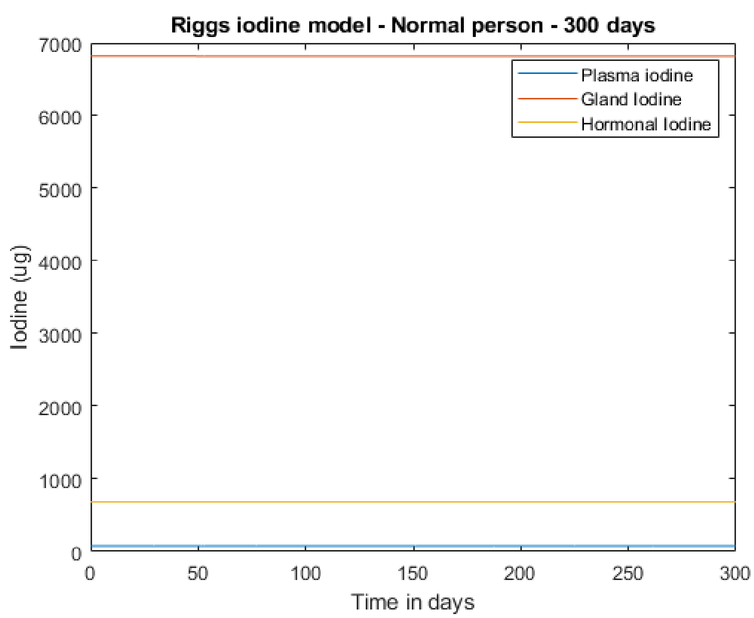


Question 02

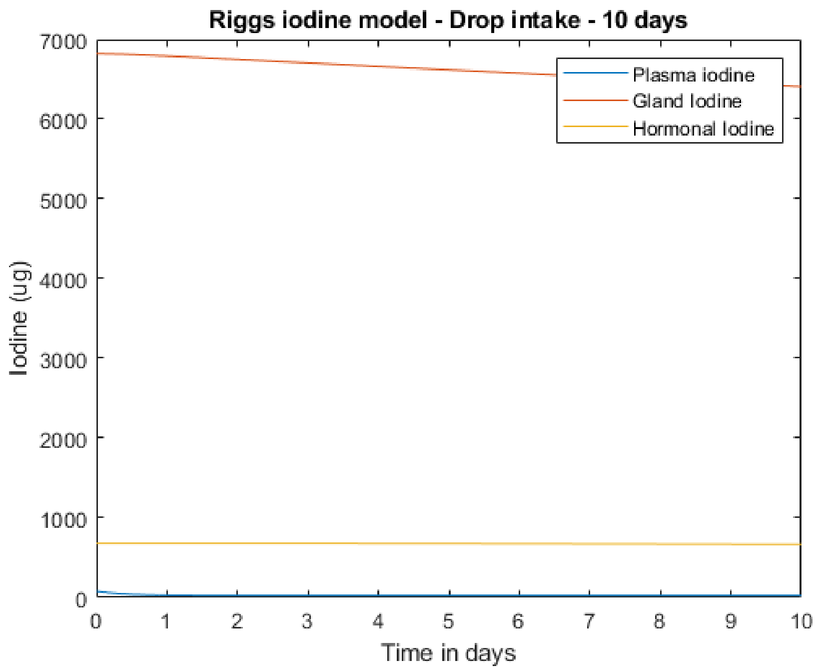
1) Iodine intake for normal condition – 10 days



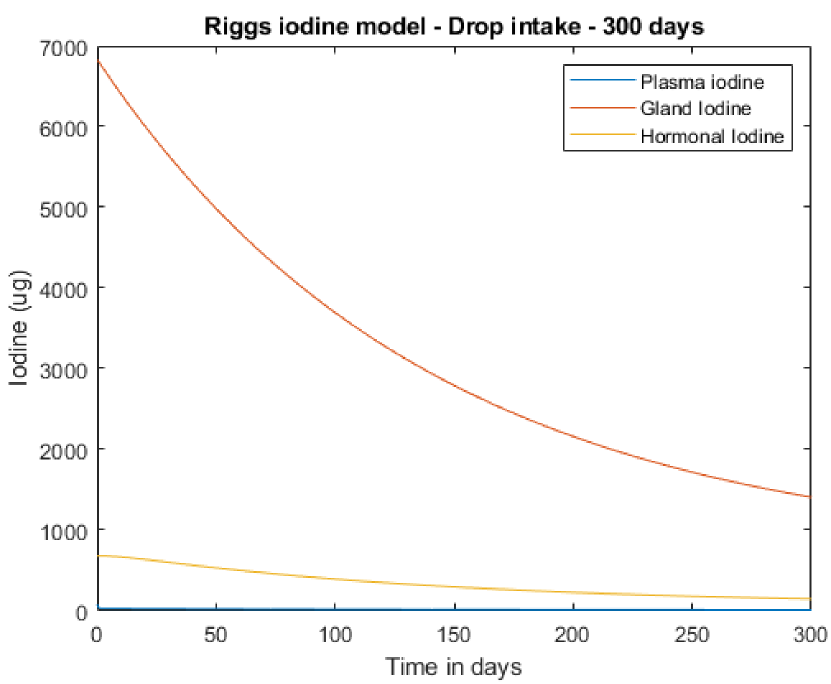
2) Iodine intake for normal condition – 300 days



3) Drop in intake – 10 days

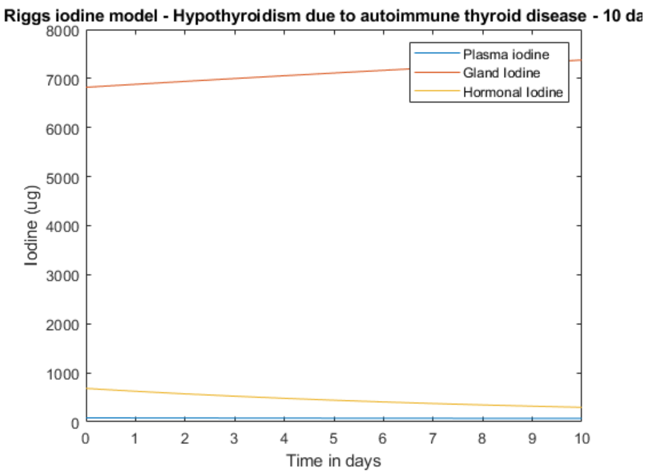


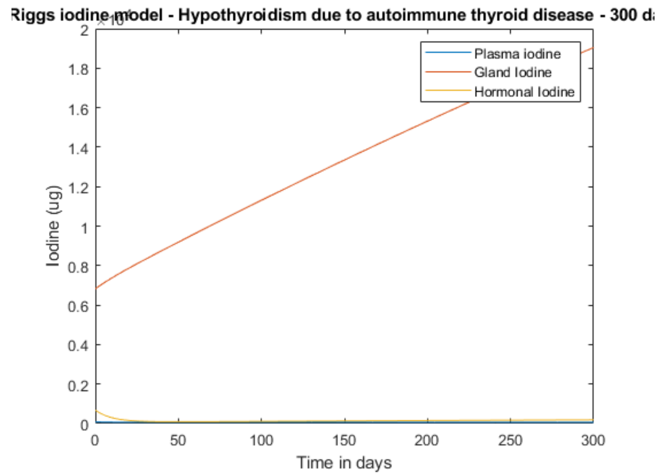
4) Drop in intake – 300 days



a) Hyperthyroidism due to autoimmune thyroid disease.

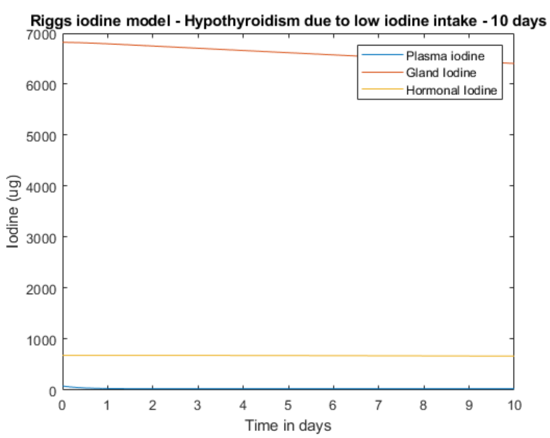
Hypothyroidism is a disease where the thyroid gland can't produce and release enough thyroid hormone to the blood. When it happens due to autoimmune thyroid disease, the magnitudes of the coefficients related to gland iodine should be reduced.

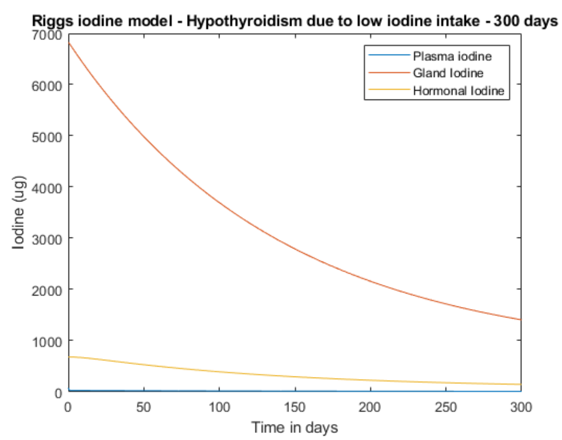




b) Hypothyroidism due to low iodine intake.

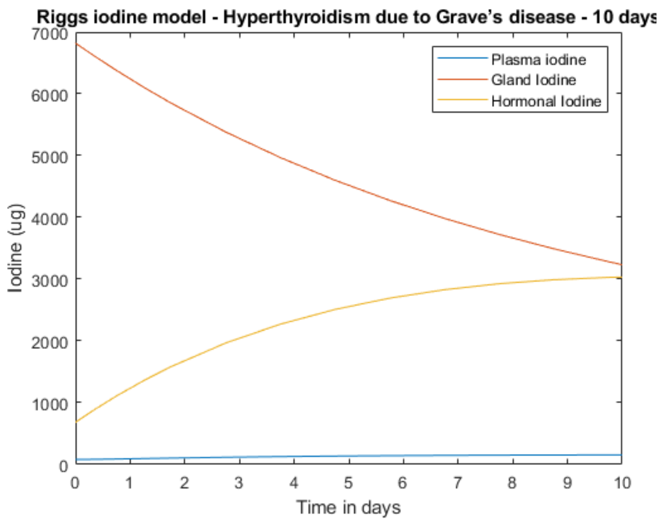
Hypothyroidism can happen also due to low Iodine intake. Then we can change the vector related to intake of iodine.

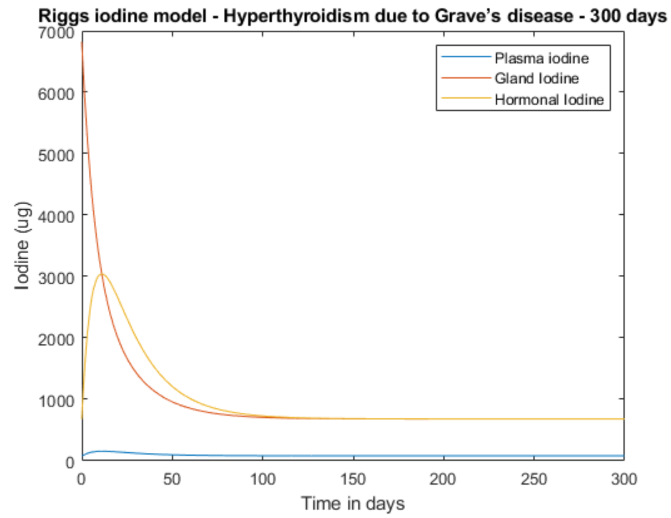




c) Hyperthyroidism due to Grave's disease

Hyperthyroidism is a disease that caused by the over production of thyroxine hormone by thyroid gland. Grave's disease leads to the above mentioned over production of hormones. It is an immune system disorder. Hence the coefficients related to gland iodine should increase.



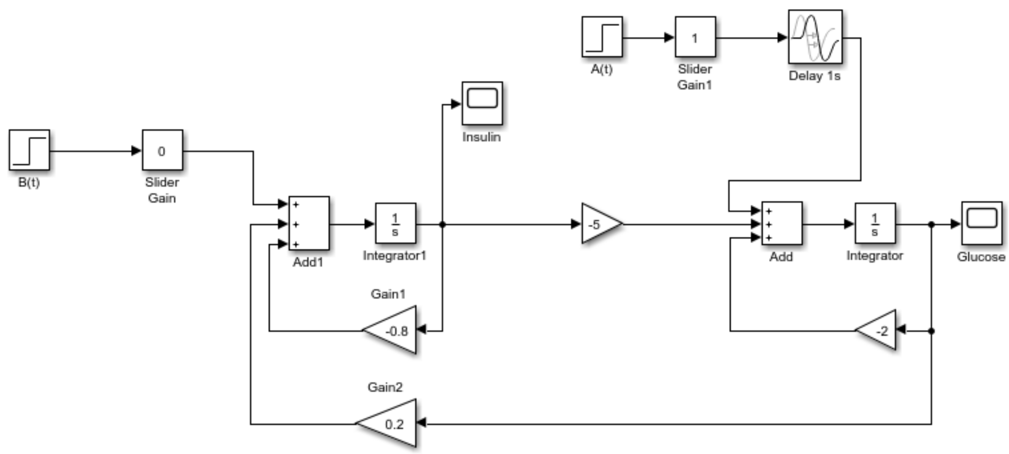


d) When a person has a goiter, his/her thyroid gland grows larger. Lack of iodine in the diet is the main reason for goiter. Few of common causes for goiters are Hashimoto's disease, Grave's disease, Thyroid nodules. We can reduce the iodine input in the riggs model. Also, goiters can lead to either hyperthyroidism or hypothyroidism. Therefore, we cannot exactly say what should we do for the coefficients related to gland iodine.

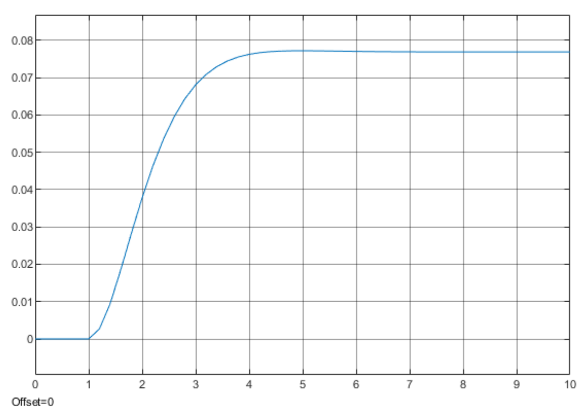
Not all thyroid tumors are cancerous. Only a small number of types of thyroid tumors are cancerous. The reason behind these cancers can be Iodine deficiency. It can cause either hyperthyroidism or hypothyroidism. Hence, we can use the riggs model with low iodine input.

Part 02 - Question 01

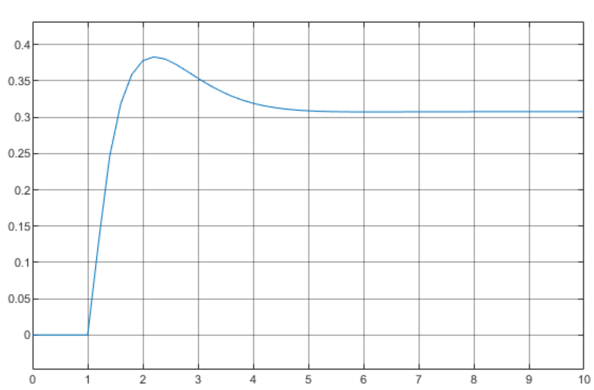
Model 01



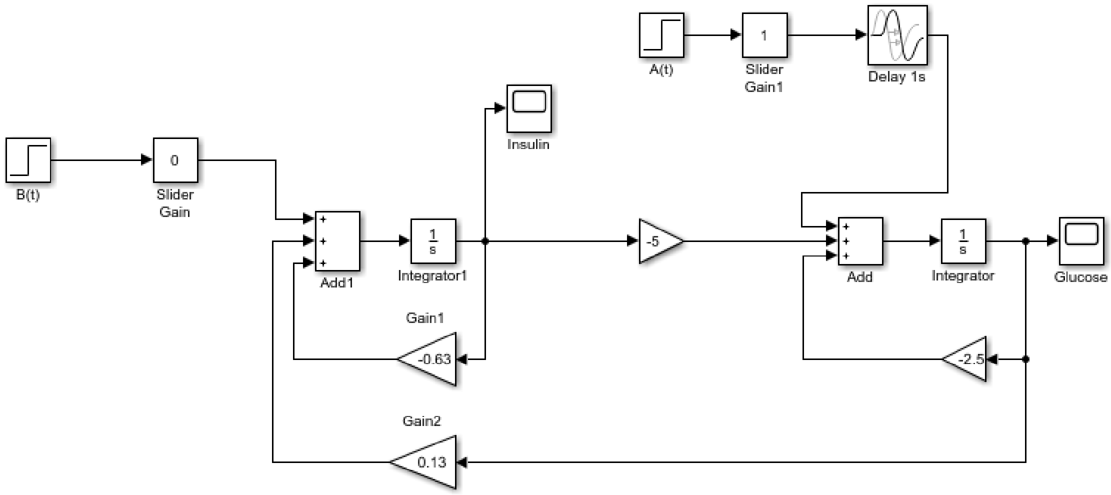
Insulin graph



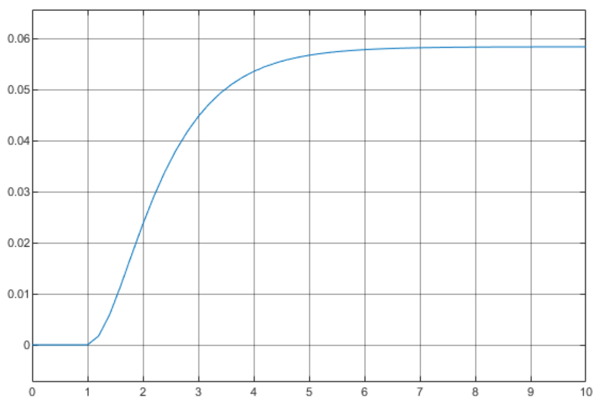
Glucose graph



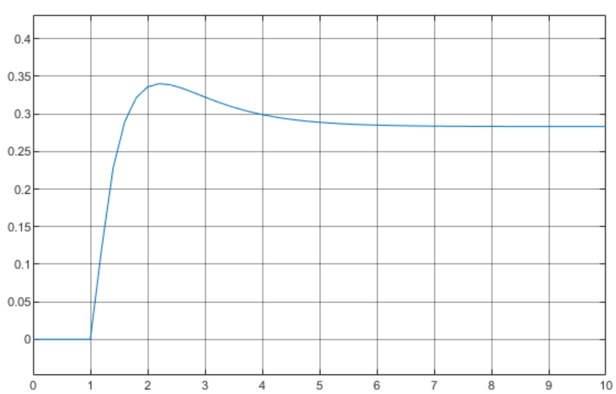
<u>Model 02</u>



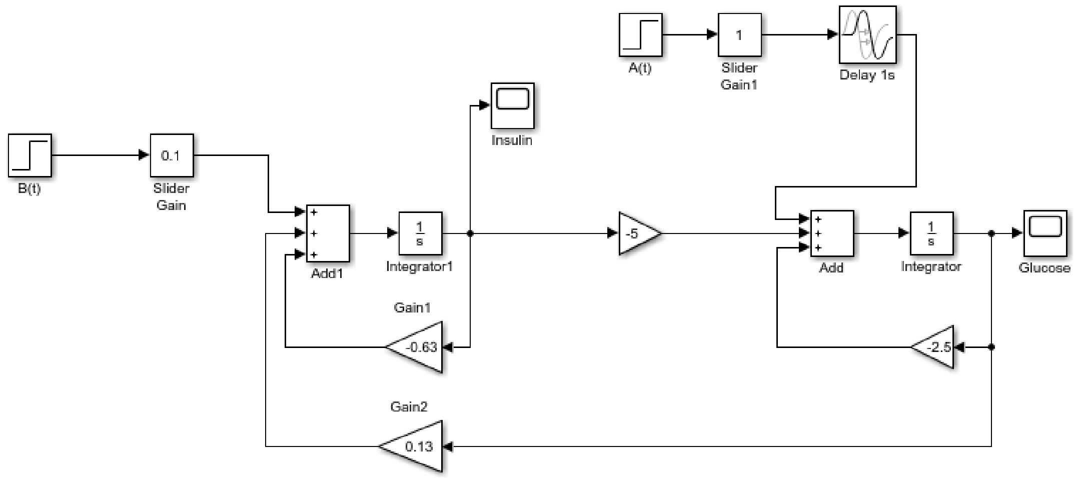
Insulin graph



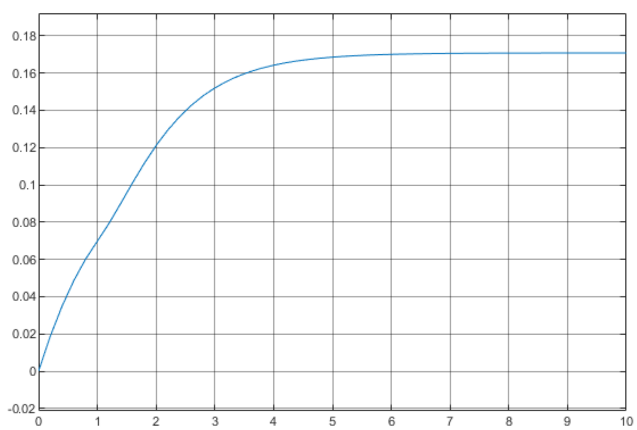
Glucose graph



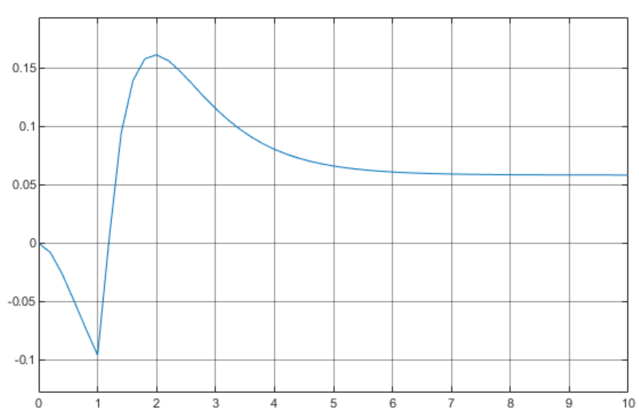
Model 03



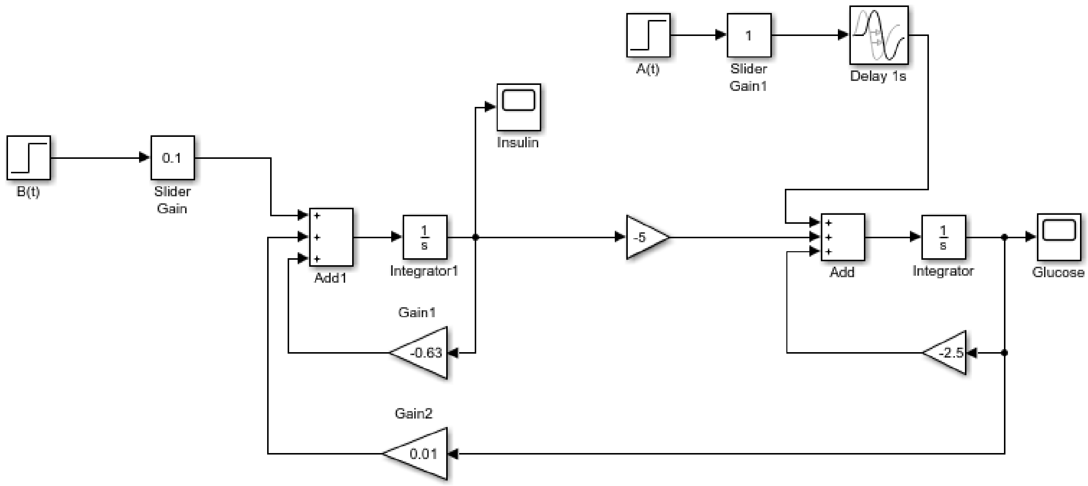
Insulin graph



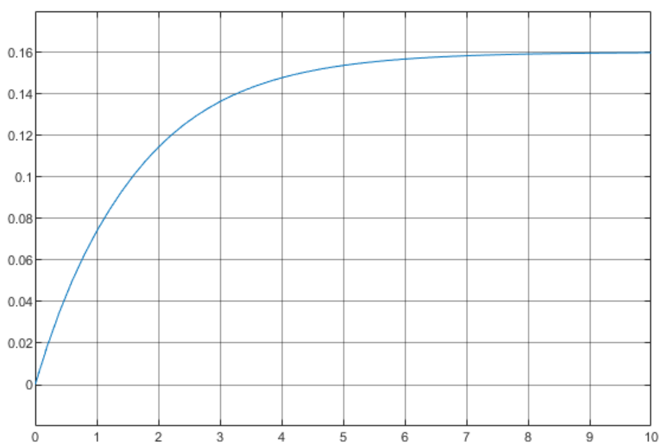
Glucose graph



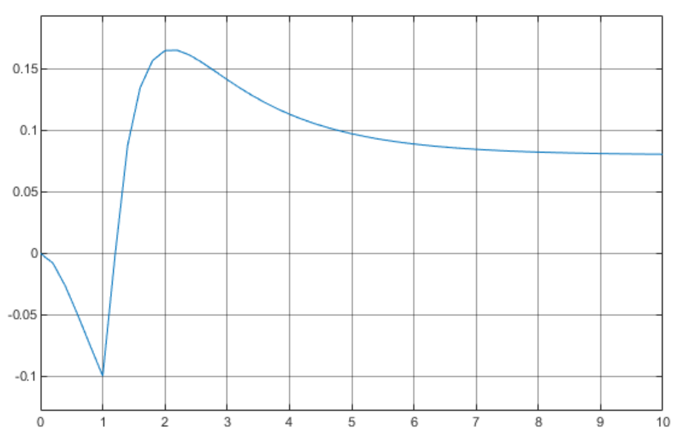
Model 04



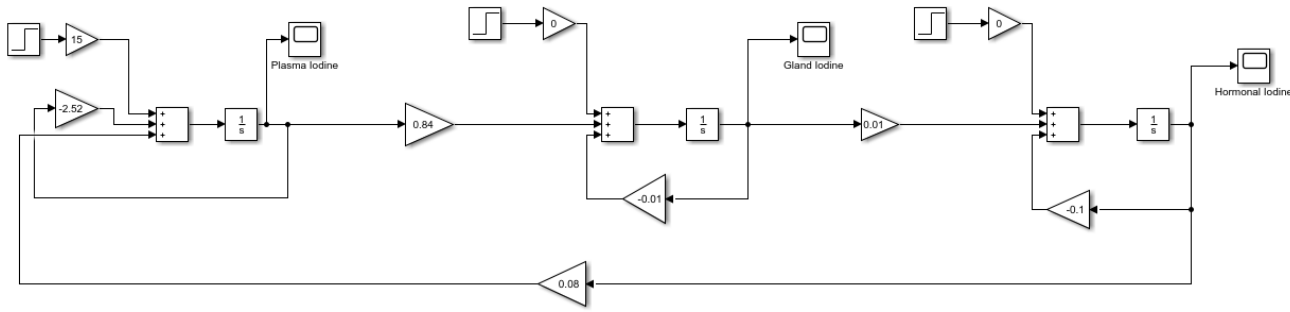
Insulin graph



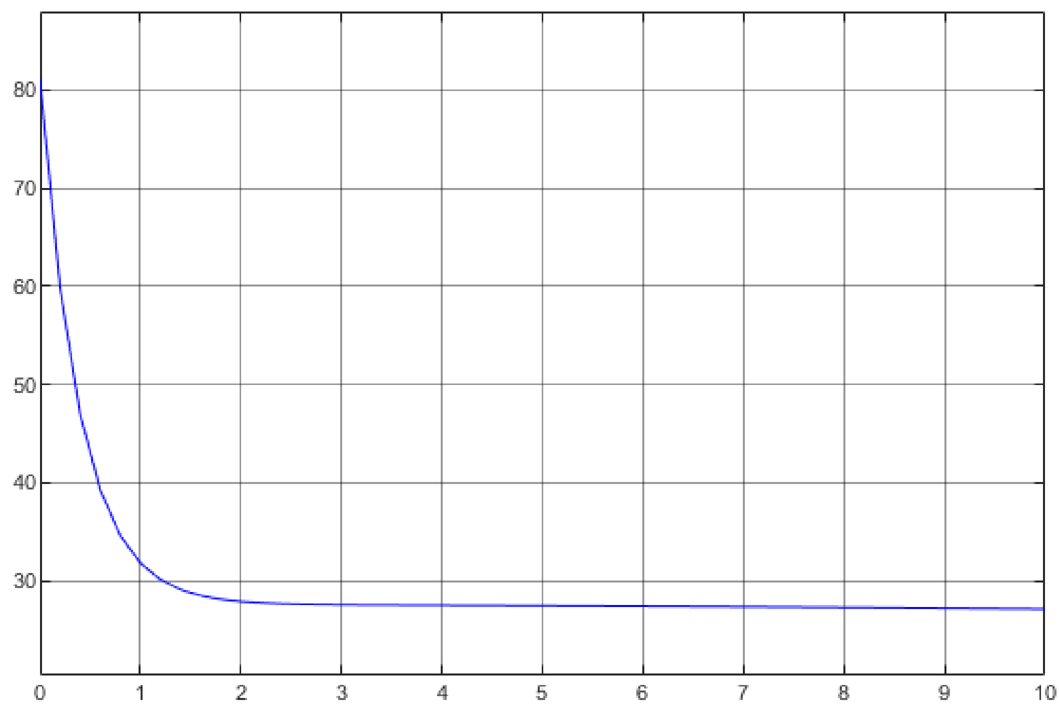
Glucose graph



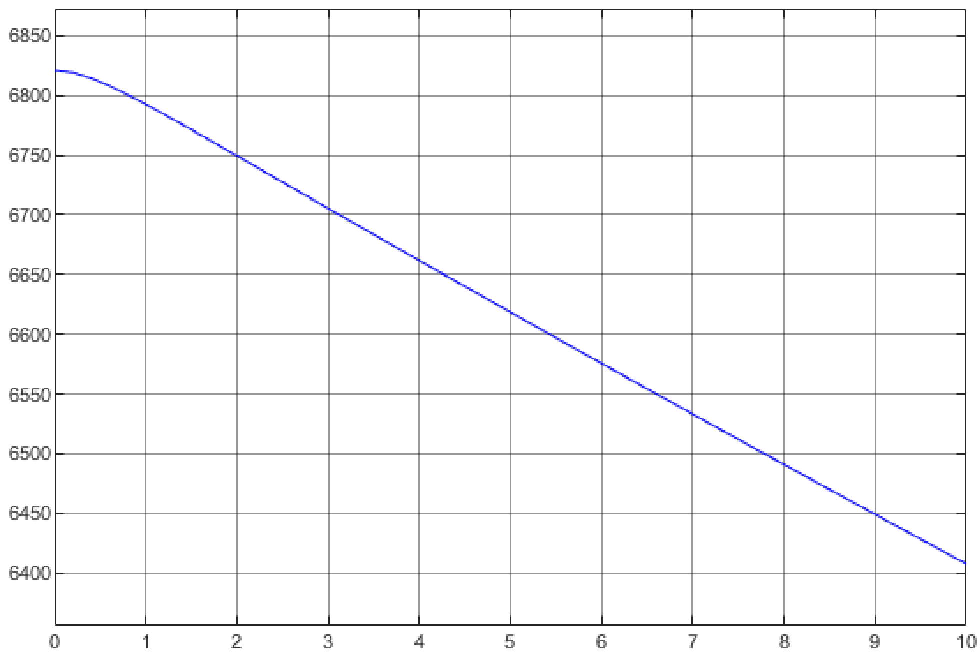
Part 02 - Question 02



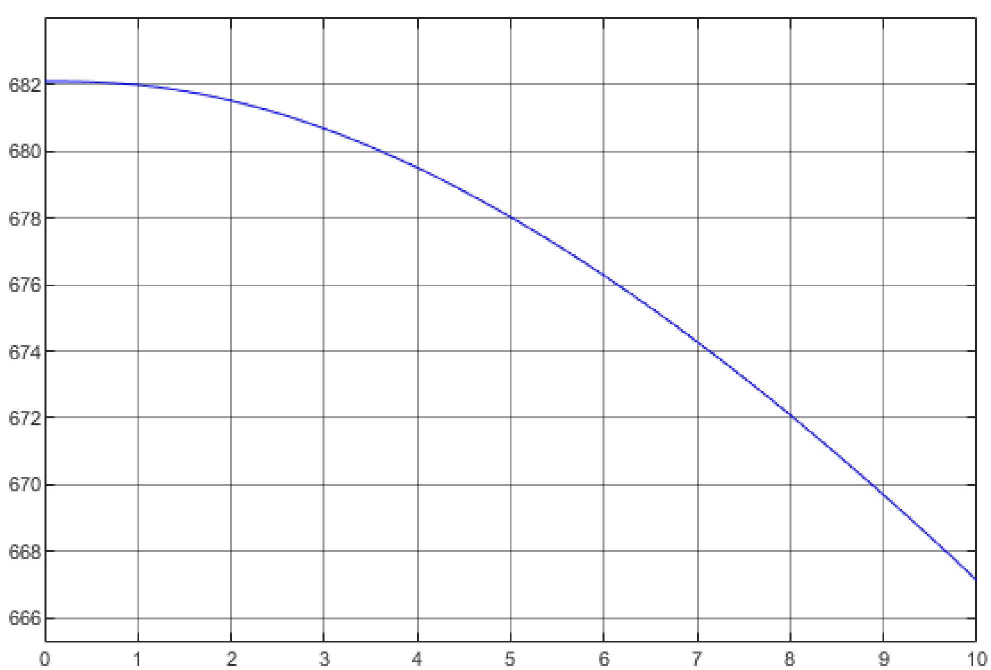
Plasma Iodine graph



Gland Iodine graph



Hormonal Iodine graph



Part 3

Question 9

$$\frac{d^2g}{dt^2} + (K_1+K_4)\frac{dg}{dt} + (K_1K_4 + K_3K_6)g = K_1a + a\frac{du}{dt}$$

Here $K_1 = 0.8h^{-1}$ $\quad K_4 = 2h^{-1}$ $\quad a = 1g/L/h$

$K_3 = 0.2\ IU/h/g$ $\quad K_6 = 5\ g/h/IU$

for any $t>0$ we have $\frac{du}{dt} = 0$

hence, $\frac{d^2g}{dt^2} + (K_1+K_4)\frac{dg}{dt} + (K_1K_4 + K_3K_6)g = K_1a$

$$\frac{d^2g}{dt^2} + 2.8\frac{dg}{dt} + 2.6g = 0.8$$

for the differential equation, we have complimentary solution and particular solution.

$\therefore\ g(t) = g_c(t) + g_p(t)$

lets find the complementary solution,

then, $\frac{d^2g_c(t)}{dt^2} + 2.8\frac{dg_c(t)}{dt} + 2.6g_c(t) = 0$

Take $g_c(t) = Ae^{rt}$

Then, $\frac{dg_c(t)}{dt} = Are^{rt}$ and $\frac{d^2g_c(t)}{dt^2} = Ar^2e^{rt}$

$\therefore\ Ar^2e^{rt} + 2.8Are^{rt} + 2.6Ae^{rt} = 0$

$r^2 + 2.8r + 2.6 = 0$

$r = \frac{-2.8 \pm \sqrt{2.8^2 - 4\times 2.6}}{2} = -1.4 \pm j0.8$

$\therefore\ g_c(t) = A_1e^{(-1.4+j0.8)t} + A_2e^{(-1.4-j0.8)t}$

$= (A_1e^{j0.8t} + A_2e^{-j0.8t})e^{-1.4t}$

$= \{A_1[\cos(0.8t) + j\sin(0.8t)] + A_2[\cos(0.8t) - j\sin(0.8t)]\}e^{-1.4t}$

$= \{(A_1+A_2)\cos(0.8t) + [(A_1-A_2)j]\sin(0.8t)\}e^{-1.4t}$

$\therefore\ g_c(t) = [K_1\cos(0.8t) + K_2\sin(0.8t)]e^{-1.4t}$ complementary solution.

let's find the particular solution,

$$\frac{d^2 g_p(t)}{dt^2} + 2.8 \frac{dg_p(t)}{dt} + 2.6\, g_p(t) = 0.8$$

when $t \to \infty$, $\frac{d^2 g_p(t)}{dt^2} = \frac{dg_p(t)}{dt} = 0$

$\therefore 2.6\, g_p(t) = 0.8$ $\therefore g_p(t) = \frac{4}{13}$ ← particular solution.

$\therefore g(t) = g_c(t) + g_p(t)$

$$g(t) = \left[K_1 \cos(0.8t) + K_2 \sin(0.8t)\right] e^{-1.4t} + \frac{4}{13}$$

considering boundary conditions,

$g(0) = 0$

$\therefore K_1 e^0 + \frac{4}{13} = 0$

$\therefore K_1 = \frac{-4}{13}$

$g'(0) = 1$

$\left[K_1 \cos(0.8t) + K_2 \sin(0.8t)\right] e^{-1.4t} (-1.4)$

$+ \left[-0.8 K_1 \sin(0.8t) + 0.8 K_2 \cos(0.8t)\right] e^{-1.4t} = 1$

$-1.4 \times K_1 + 0.8 K_2 = 1$

$\cancel{K_2 = 1 + 1.4 \times \frac{-4}{13} \times \frac{1}{0.8} = \frac{-5.6}{13 \times 0.8} =}$

$$K_2 = \frac{1 + 1.4 \times \frac{-4}{13}}{0.8} = \frac{13 - 5.6}{0.8 \times 13} = \frac{7.4}{0.8 \times 13}$$

$\therefore K_2 = \frac{37}{52}$

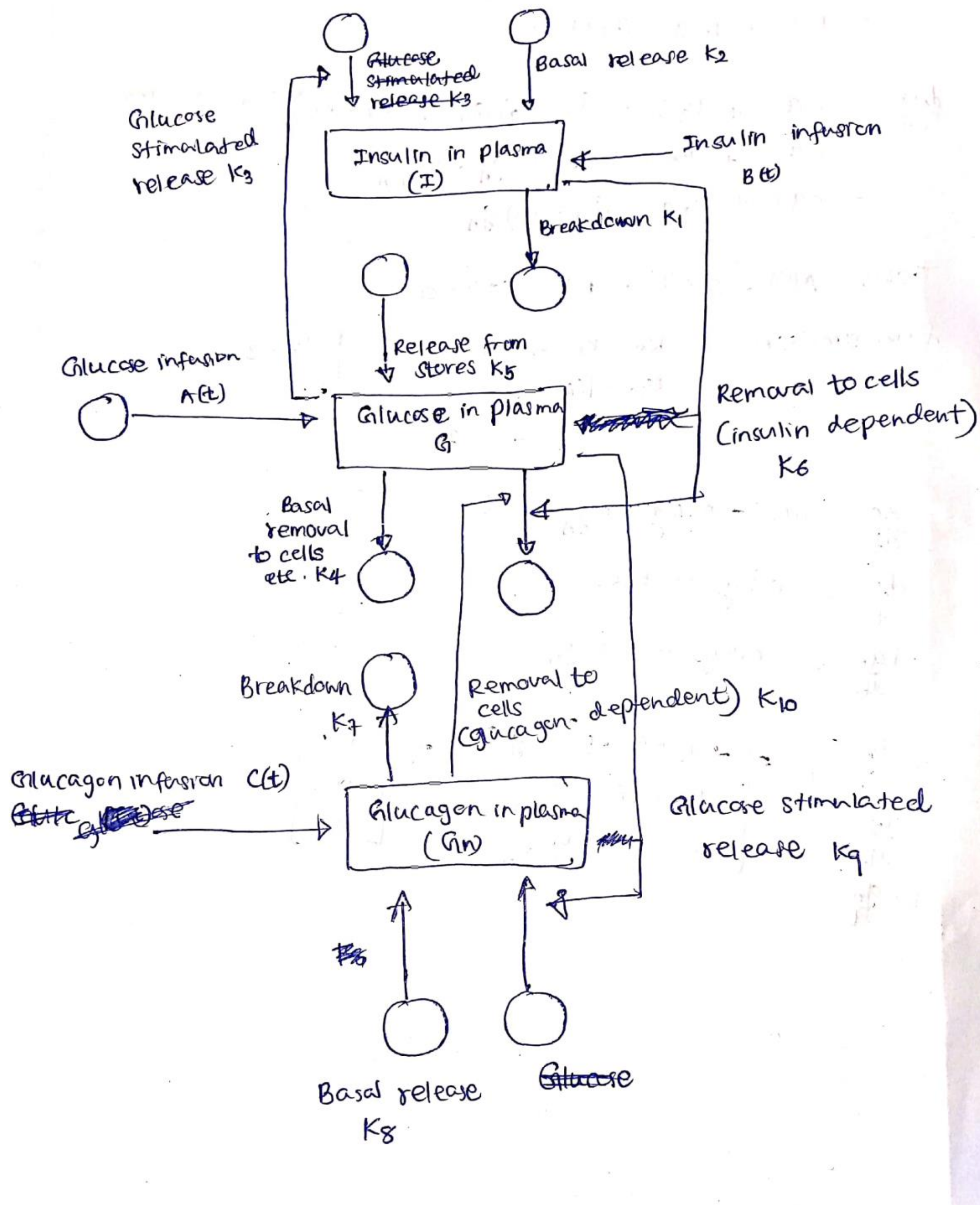
$$\therefore g(t) = \left[\frac{-4}{13} \cos(0.8t) + \frac{37}{52} \sin(0.8t)\right] e^{-1.4t} + \frac{4}{13}$$

since, $\frac{dg(t)}{dt} = -K_4 g(t) - K_6 i(t) + A(t)$

$$\left[\frac{-4}{13} \cdot \sin(0.8t) \times -0.8 + \frac{37}{52} \cos(0.8t) \times 0.8\right] e^{-1.4t} + (-1.4)\left[\frac{-4}{13} \cos(0.8t) + \frac{37}{52} \sin(0.8t)\right] e^{-1.4t}$$

$$= -2\, g(t) - 5\, i(t) + 1$$

$$\therefore i(t) = \left[\frac{-1}{13} \cos(0.8t) - \frac{7}{52} \sin(0.8t)\right] e^{-1.4t} + \frac{1}{13}$$


~~Glucose stimulated release K3~~
Basal release K2
Glucose stimulated release K3
Insulin in plasma (I)
Insulin infusion B(t)
Breakdown K1
Release from stores K5
Glucose infusion A(t)
Glucose in plasma G
Removal to cells (insulin dependent) K6
Basal removal to cells etc. K4
Breakdown K7
Removal to cells (glucagon-dependent) K10
Glucagon infusion C(t)
Glucagon in plasma (Gn)
Glucose stimulated release K9
Basal release K8
~~Glucose~~

$$\frac{dG}{dt} = A(t) + K_5 - K_3 G - K_4 G - K_6 I - K_9 G - K_{10} G_n$$

$$\frac{dI}{dt} = B(t) - K_1 I - K_6 I + K_2 + K_3 G$$

$$\frac{dG_n}{dt} = C(t) - K_7 G_n - K_{10} G_n + K_9 G + K_8$$

$$\cancel{\frac{dG}{dt} = A(t) - (K_3+K_4) G}$$

$$\frac{dG}{dt} = A(t) - (K_3+K_4+K_9) G - K_6 I - K_{10} G_n + K_5$$

$$\frac{dI}{dt} = B(t) + K_3 G - (K_1+K_6) I + K_2$$

$$\frac{dG_n}{dt} = C(t) + K_9 G - (K_7+K_{10}) G_n + K_8$$

Considering equilibrium state,

$$\frac{dG}{dt} = 0 \rightarrow K_5 = (K_3+K_4+K_9)G_0 + K_6 I_0 + K_{10} G_{n0}$$

$$\frac{dI}{dt} = 0 \rightarrow K_2 = K_1 I_0 + K_6 I_0 - K_3 G_0$$

$$\frac{dG_n}{dt} = 0 \rightarrow K_8 = (K_7+K_{10}) G_{n0} - K_9 G_0$$

$$\therefore \frac{dG}{dt} = A(t) + (K_3+K_4+K_9)G_0 + K_6 I_0 + K_{10} G_{n0} - K_3 G - K_4 G - K_6 I - K_9 G - K_{10} G_n$$

$$= A(t) + K_3(G_0 - G) + K_4(G_0 - G) + K_9(G_0 - G) + K_{10}(G_{n0} - G_n) + K_6(I_0 - I)$$

substituting $i = I - I_0$, $g = G - G_0$ and $g_n = G_n - G_{n0}$

$$\frac{dG}{dt} = A(t) - K_3 g - K_4 g - K_9 g - K_{10} g_n - K_6 i$$

$$\frac{dG}{dt} = A(t) - (K_3+K_4+K_9) g - K_{10} g_n - K_6 i$$

$$\frac{dI}{dt} = B(t) + K_3 G - \cancel{K} K_1 I - K_6 I + K_1 I_0 + K_6 I_0 - K_3 G_0$$

$$= B(t) + K_3 g - K_1 i - K_6 i$$

$$= B(t) + K_3 g - (K_1 + K_6) i$$

$$\frac{dG_n}{dt} = C(t) + K_9 G - (K_7 + K_{10}) G_n + (K_7 + K_{10}) G_{n0} - K_9 G_{n0}$$

$$= C(t) + \cancel{K_9} K_9 g - K_7 g_n - K_{10} g_n$$

$$= C(t) + K_9 g - (K_7 + K_{10}) g_n$$

Take $A(t) = a.u(t)$, $B(t) = C(t) = 0$

from similarities, $K_7 = K_1 = 0.8$ | $K_4 = 2$

$K_{10} = K_6 = 5$

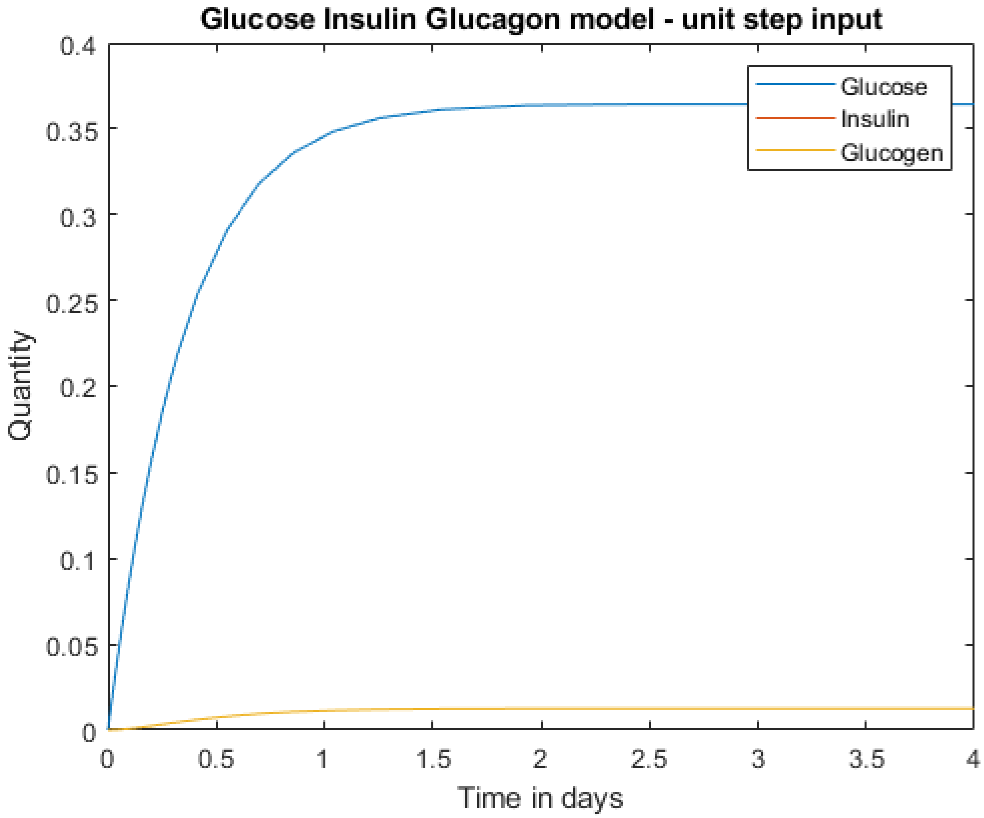
$K_9 = K_3 = 0.2$

$$\frac{dg}{dt} = u(t) - 2.4g - 5g_n - 5i$$

$$\frac{di}{dt} = 0.2g - 5.8i$$

$$\frac{dg_n}{dt} = 0.2g - 5.8g_n$$

$$\begin{bmatrix} \frac{dg}{dt} \\ \frac{di}{dt} \\ \frac{dg_n}{dt} \end{bmatrix} = \begin{bmatrix} -2.4 & -5 & -5 \\ 0.2 & -5.8 & 0 \\ 0.2 & 0 & -5.8 \end{bmatrix} \begin{bmatrix} g \\ i \\ g_n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$


Glucose Insulin Glucagon model - unit step input
Quantity
Time in days
Glucose
Insulin
Glucogen
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3
3.5
4