ANSWER SCRIPT

213

UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

Academic Year 2017/2018 – Second Year Examination – Semester I – 2019

SCS 2206 – Mathematical Methods II

TWO (2) HOURS

To be completed by the candidate

Examination Index No: ……………………………………………………

Important Instructions to candidates:

1. The medium of instruction and question is **English**.
2. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
3. Note that questions appear on both sides of the paper. If a page is not printed, please inform the supervisor immediately.
4. Write your index number on each and every page of the answer script.
5. This paper has **04** questions and **02** pages (Excluding the coverage).
6. Answer **ALL** questions. All questions carry equal marks (25 marks).
7. Any electronic device capable of storing and retrieving text including electronic dictionaries and mobile phones are **not allowed**.
8. **Non-Programmable** calculators are **allowed**.

For Examiner's use only	
Question No	**Marks**
1	
2	
3	
4	
Total	

1. (a) i. Let $\{x_n\}$ be a sequence of real numbers. What is meant by the term " $\{x_n\}$ *is Cauchy* "?

ii. Suppose $\lim_n x_n = \sqrt{\pi + e}$ and $|x_n - x_m| < 10^{-5}$ for every $n, m \geq 2019$. If x_{2019} is used to approximate $\sqrt{e + \pi}$, what is the *guaranteed minimum error*?

(8 points)

(b) Suppose the Bisection method is used to determine an approximate solution in the interval $(-1, 1,)$ of the equation

$$\cos(3\pi x) - 3x^3 - 1.5 = 0$$

i. Show that the equation has a solution in the interval $(-1, 1)$.

ii. Determine the approximate solution obtained by the third iterate.

iii. Find the number of iterates required to achieve an accuracy of 10^{-4}.

(9 points)

(c) Find an appropriate iterative formula for a sequence $\{x_n\}$ to approximate $\sqrt{15}$ using Newton-Rapson method with initial guess $x_0 = 4$.

(8 points)

2. (a) Consider the function f, given in the following tabular form.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	2	4	5	7	10	8	4	1	-2	-1	0

Estimate $\int_0^1 f(x)dx$ using,

i. Mid-point rule with 5 sub-intervals.

ii. Simpson's rule with 5 sub-intervals.

(10 points)

(b) Suppose the mid-point method, trapezoidal method and Simpson's method were used to compute the integral $\int_0^1 (3x+1)\, dx$ with n-subdivisions. Which method/ methods would give the exact answer irrespective of the value of n? Give reasons to justify your answer.

(6 points)

(c) Consider the initial value problem,

$$y' = t^2 + ye^t,\ t > 0,\ \ y(0) = 1.$$

Approximate y in the interval $[0, 1]$ using fourth order Runge Kutta method with three sub-intervals.

(9 points)

3. (a) Consider the set G given by,

$$G = \{([x]_8, [y]_8) : x, y \in \mathbb{Z}, 0 \leq x \leq 7, \text{ and } 0 \leq y \leq 7\}.$$

Let the binary operation $*$ on G be defined by

$$([s]_8, [t]_8) * ([u]_8, [v]_8) = ([s + u]_8, [tv]_8)$$

i. Compute $([1]_8, [2]_8) * ([4]_8, [5]_8)$.

ii. Show that $(G, *)$ has an identity element.

iii. Is $(G, *)$ a group? Give reasons to justify your answer.

(12 points)

(b) Let $(G, \star)$ be a group. Define what is meant by the term " *G-is cyclic* ".

i. Show that $(\mathbb{Z}_m, +_m)$ is cyclic and $[x]_m$ is a generator for $(\mathbb{Z}_m, +_m)$ if and only if $[1] \in \langle [x]_m \rangle$.

ii. Determine all the generators of $(\mathbb{Z}_{15}, +_{15})$.

iii. Determine with reasoning the number of generators of the group $(\mathbb{Z}_{2019}, +)$.

(13 points)

4. (a) State *Lagrange's Theorem.*

i. Determine all subgroups of $(\mathbb{Z}_{11}^{\star}, \times_{11})$.

ii. Let $(G, *)$ be group with $|G| = 100$. Suppose $H, K \leq G$ with $H \neq K$ but $|H| = |K| = 50$. Find the least possible value of $|H \cup K|$.

(10 points)

(b) Let $(R, +, \times)$ be ring. Define the term *a-is a zero divisor* in R.

i. Determine all zero divisors of the ring $(\mathbb{Z}_{20}, +_{20}, \times_{20})$.

ii. Find all solutions of the equation $5x - 3 = 7$ in $\mathbb{Z}_{20}$.

(8 points)

(c) Compute $\gcd(x^5 + x^2 + 2x + 1, x^3 + x^2 + x + 1)$ in the polynomial ring $\mathbb{Z}_4[x]$.

(7 points)