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Colours of the Sun and Moon: the role of the optical air mass

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Abstract

A geometric model for the optical air mass of the atmosphere is developed. Using the model, simple formulae are derived for the optical thickness of light passing through (1) a molecular atmosphere, (2) an atmosphere with uniformly distributed tropospheric aerosols and (3) atmospheres with elevated aerosol layers. The formulae are used to model the spectra and perceived colours of the Sun and Moon.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The transmission of radiation through the atmosphere is a classical subject with many applications. Some problems such as determining the precise location of stars or the solar constant are now better handled from satellite, where atmospheric refraction, scattering and absorption are simply bypassed. But problems such as the colours of the Sun, Moon and sky, efficiency of using solar energy or remote sensing of oceanic temperature and atmospheric temperature and humidity profiles from satellite or radars require solving problems in radiation transmission through the atmosphere. The concept of the optical air mass (also called relative optical thickness), the ratio of the optical path of a slanted to a vertical light beam passing through the atmosphere, is central to solving such problems, provided the vertical optical thickness is known. The simplicity of obtaining relatively accurate approximations renders the relative optical thickness an important basic concept in physics education, which can be taught at the undergraduate level. Students can work out many of the problems presented in this paper by writing simple programs or by using an Excel spreadsheet available on the Internet [1].

2. Light transmission through the atmosphere

2.1. Bouguer's law and optical thickness

The fundamental law of radiation transmission through media, often called Bouguer's law, was first formulated by Bouguer in 1729, although sometimes attributed to Lambert and Beer. The law states that transmission of a pencil of monochromatic radiation with radiance $I(\lambda)$, at wavelength λ, decreases in proportion to its own magnitude and to the distance ds travelled. For a medium which consists of a single species which may scatter or absorb light, the Bouguer law can be written as [2]

$$\mathrm{d}I(\lambda) = -I(\lambda)n[\sigma_a(\lambda) + \sigma_s(\lambda)]\,\mathrm{d}s = -I(\lambda)n\sigma_e(\lambda)\,\mathrm{d}s. \tag{1}$$

Here n is the number density of scatterers or absorbers, σ_a is the absorption, σ_s is the scattering and σ_e is the extinction cross section per particle. Equation (1) is valid only when the contribution of multiply scattered light is neglected. The contribution of multiply scattered light increases both as the Sun approaches the horizon and for near horizontal light beams. Under these conditions, Bouguer's law is not an accurate approximation for sky light but, except under highly polluted conditions, it remains valid for sunlight, because solar radiance still far exceeds that of the background sky.

In clear air, air molecules and aerosol particles obstruct light. In a pure, dry molecular atmosphere, air density ρ is proportional to n. Scattering of light by molecules, which are tiny compared to the wavelengths of visible light, obeys Rayleigh's law and is proportional to λ^{-4} [3]. Light is also absorbed by gases such as nitrogen dioxide or ozone and both scattered and absorbed by aerosols. Thus, in general, $n\sigma_e(\lambda)$ must be replaced by $\sum n_i\sigma_{e,i}(\lambda)$.

Equation (1) is often expressed in terms of the optical thickness τ, the physical thickness in units of the mean free path of the light [4], which is dimensionless. The mean free path $1/(n\sigma_e(\lambda))$ is the mean distance a photon travels before it is scattered or absorbed. An increment of optical thickness dτ, along a path of length ds, is then given by

$$\mathrm{d}\tau = n\sigma_e\,\mathrm{d}s. \tag{2a}$$

Hence, the total optical thickness τ along a path s is given by the integral

$$\tau(\lambda) = \int n\sigma_e(\lambda)\,\mathrm{d}s. \tag{2b}$$

Using optical thickness, Bouguer's law can be integrated to give the simple exponential form

$$I(\lambda) = I_0(\lambda)\cdot \mathrm{e}^{-\tau(\lambda)}. \tag{3}$$

In a molecular atmosphere, the total number of particles encountered by a vertical sunbeam is proportional to the surface pressure. In that case, the vertical or normal optical thickness of the atmosphere is closely approximated by Rayleigh scattering [2–4], and given by

$$\tau_{N,\mathrm{Mol}}(\lambda) = \frac{K_{\mathrm{Mol}}}{\lambda^4}, \tag{4}$$

where $K_{\mathrm{Mol}} = 8.66 \times 10^{-27}$ m^{-4} is the scattering coefficient for the molecular atmosphere at standard mean sea level pressure ($p_0 = 1013.25$ hPa). The pronounced wavelength dependence and small magnitude of τ_{Mol} when the Sun is at the zenith are shown in figure 1 and account for the blue of the sky in clean air when the Sun is high in the sky.

2.2. The optical air mass

A vertical light beam (solar elevation angle 90° or zenith angle, $Z = 0°$) suffers the least attenuation because it takes the shortest possible path through the atmosphere. As Z increases,

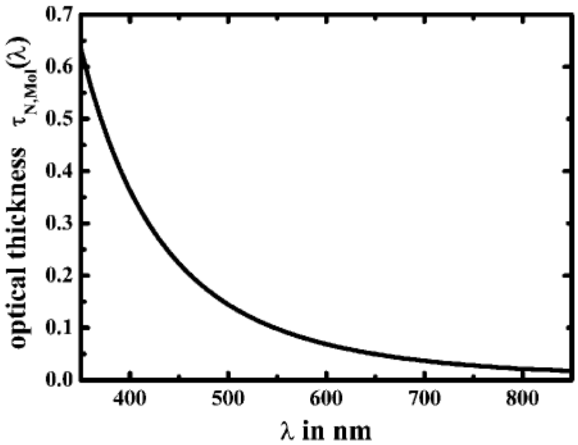


Figure 1. Normal optical thickness for a molecular atmosphere.

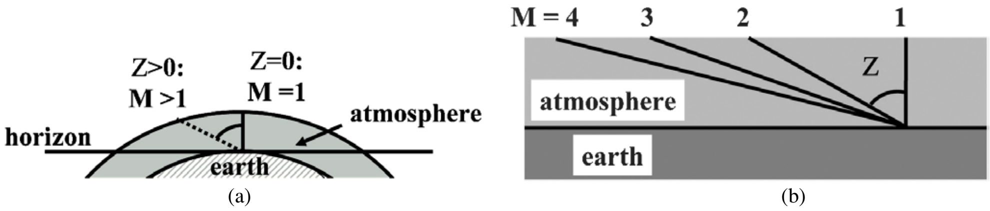


Figure 2. Models of the optical air mass. (a) The atmosphere is indicated by a layer, whose thickness is strongly exaggerated compared to the radius of the Earth! (b) The plane parallel atmosphere gives a simple approximation to the optical air mass except for Z near 90°.

figure 2 shows that a light beam penetrates more air to reach the ground; hence the transmitted light decreases.

Transmission of light beams that pass obliquely through the atmosphere can be compared to transmission of the vertical beam by use of the optical air mass, M, defined as the ratio of the optical thickness $\tau(\lambda, Z)$ of an oblique ray at zenith angle, Z, to the optical thickness $\tau_N(\lambda)$ of a vertical ray, i.e.,

$$\tau(\lambda, Z) = M(Z)\tau_N(\lambda). \tag{5}$$

For attenuation due to a single species, the transmitted light is then given in terms of the Bouguer law as

$$I(Z, \lambda) = I_0\,\mathrm{e}^{-M(Z)\cdot\tau_\mathrm{N}(\lambda)}. \tag{6}$$

Equations (5) and (6) show that the physical component of optical thickness τ contained in $\tau_N(\lambda)$ is separate from the geometrical factor. When the contributions of all atmospheric constituents are included, the total optical thickness is given by

$$\tau(Z, \lambda) = M_\mathrm{Mol}(Z)\cdot\tau_{N,\mathrm{Mol}}(\lambda) + M_\mathrm{Aer}(Z)\cdot\tau_{N,\mathrm{Aer}}(\lambda) + M_\mathrm{Lay}(Z)\cdot\tau_{N,\mathrm{Lay}}(\lambda), \tag{7}$$

where the subscripts Mol, Aer and Lay represent a molecular atmosphere, tropospheric aerosols and elevated aerosol layers, respectively. Expressions for M for each of these factors are given in sections 3–5.

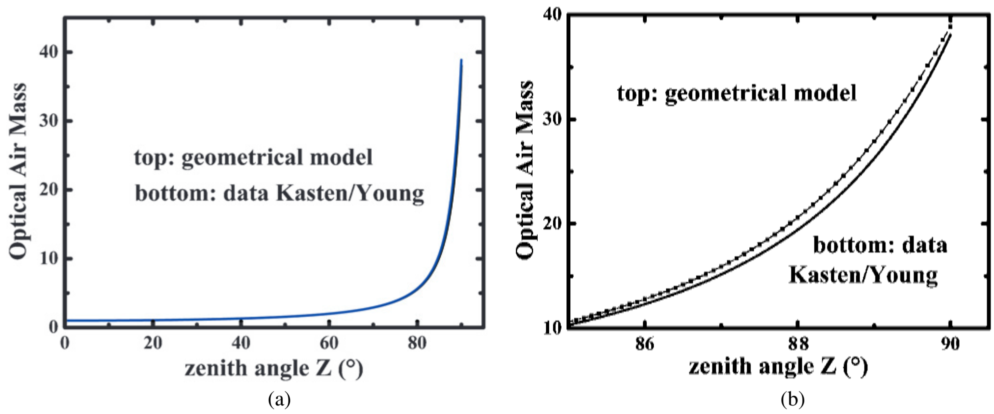


Figure 3. Comparison of Kasten and Young's [8] tabulated optical air mass values for the standard atmosphere as a function of zenith angle (lower curve) with values using the geometrical model of the homogeneous atmosphere (section 4, upper curve). Small deviations occur for $Z > 85°$. The optical air mass M is normalized to unity for $Z = 0$, i.e. the zenith direction.

3. Approximations of the optical air mass

Figure 2 suggests that the optical air mass is primarily geometric. For a flat Earth or plane parallel atmosphere in the absence of refraction, M is simply the secant of the zenith angle, Z, or

$$M(Z) = \sec(Z). \tag{8}$$

The plane parallel atmosphere produces accurate approximations for M for small values of Z but increasingly overestimates M as Z approaches $90°$ because the Earth's curvature shortens oblique paths through the atmosphere.

The dual impact of the Earth's curvature and refraction on the path of light through the atmosphere has long concerned astronomers. Astronomical refraction $R(Z)$ is a measure of the angular difference between the real and the apparent positions of a celestial object, when seen through the Earth's atmosphere [5]. In all that follows, Z is the apparent zenith angle as seen by a viewer. And because refracted rays in the atmosphere are longer than straight lines, the optical air mass and hence light transmission are affected by refraction. This was noted by Laplace in 1805, who mentioned in his famous work on celestial mechanics that 'the extinction of light of the heavenly bodies, in passing through the atmosphere, has so near a relation to the theory of refraction that we shall here take notice of it . . . ' [6]. He even gave a formula for the optical air mass that can be found in modern meteorological tables [7], although more accurate approximations [8, 9] have become the standard. As an example, figure 3 compares the simple geometrical model described in section 4 with the most widely used optical air mass as a function of Z for the ISO standard atmosphere, i.e. clean, dry air with $p_0 = 1013.25$ hPa, $T_0 = 288.15$, $g = 9.80665$ m s^{-2} and the tropospheric lapse rate (i.e. rate of temperature decreases with height, z), $\Gamma \equiv -\mathrm{d}T/\mathrm{d}z \approx 6.5$ K km^{-1}. Near the horizon, M approaches 40, i.e., the light must traverse 40 times more particles than a vertical beam.

Aerosols, ozone and other substances have vertical distributions that differ from the standard atmosphere and therefore have different values of the optical air mass. Much research on the optical air mass of aerosols came from attempts to determine the solar irradiance or related issues. The accuracy of these measurements was mainly limited by the rapid temporal changes of aerosol (and ozone) concentrations and heights. Most aerosols are confined to the

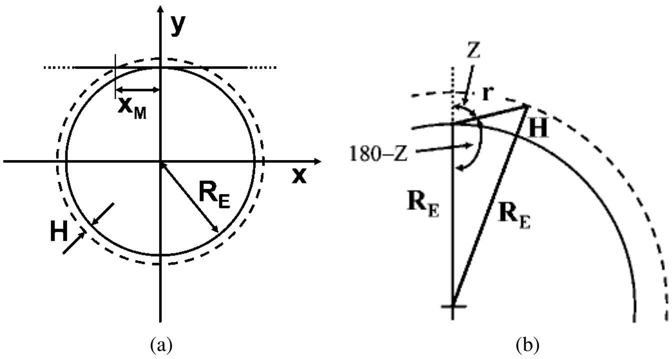


Figure 4. (a) Geometry for calculating the optical air mass for $Z = 90°$, i.e. at the horizon and (b) for arbitrary zenith angles Z.

lower troposphere (i.e., below 3 km), but after volcanic eruptions or large forest fires, aerosols reach the upper troposphere or stratosphere. Consequently, Young pointed out that 'it does not seem practical to devise wavelength dependent air mass tables, as they would depend on both the unknown amounts and vertical distributions of the variable components' [10].

Simple formulae for the optical air mass remain useful in problems where great precision is not needed, such as explaining the colours of the Sun or the Moon. The value of the optical air mass as both a working and pedagogical concept derives largely from the fact that refraction of light in the atmosphere is relatively small (except during mirages) and almost independent of wavelength (i.e. there is little dispersion). Whereas refraction reaches a maximum of 0.55° as $Z \to 90°$ in the standard atmosphere dispersion only amounts to about ±0.01°. For computing the colour of Sun, Moon or sky, these small deviations are negligible. This makes the optical air mass a particularly useful concept in physics teaching. Indeed, students are fascinated to discover how easy it is to calculate the colours of the Sun, Moon and sky.

4. Optical air mass of the homogeneous atmosphere

The homogeneous, i.e., uniform or constant density atmosphere provides the simplest approximation for the optical air mass. Because density is constant, light rays take straight paths through the homogeneous atmosphere and refraction takes place only at the top of the atmosphere. The optical air mass is then simply the ratio of the length of a slanted path to the thickness of the homogeneous atmosphere, $H = RT_0/(gM_{\text{mol}})$ with surface temperature T_0. For dry air ($M_{\text{mol}} = 28.97$ g Mol^{-1}), $g = 9.81$ m s^{-2} and $T_0 = 288$ K, $H \approx 8430$ m. Note that density in the homogeneous atmosphere is inversely proportional to T.

Figure 4(a) shows the geometry of how to compute M on a spherical Earth of radius R_E when the Sun appears at the horizon ($Z = 90°$) for an observer at $x = 0$ and $y = R_E$. The top of the atmosphere is then given by

$$x_M^2 + y^2 = x_M^2 + R_E^2 = (R_E + H)^2. \tag{9}$$

Solving for the ratio of the horizontal distance through the atmosphere x_M yields the expression for the optical air mass of a horizontal light beam in the homogeneous atmosphere:

$$M(90°) = \frac{x_M}{H} = \sqrt{1 + \frac{2R_E}{H}}. \tag{10}$$

Equation (10) shows that the smaller the ratio of scale height to the Earth's radius (and therefore the lower the temperature), the larger the optical air mass when $Z = 90°$. Thus, a

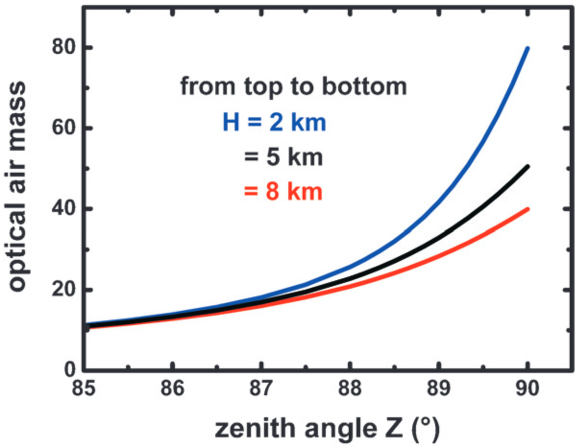


Figure 5. The optical air mass for uniformly distributed tropospheric aerosols, whose height distribution is characterized by a scale height H, computed from equation (12).

colder atmosphere should produce more vivid sunset colours. When $T = 288$ K, $M(90^\circ) = 38.09$, which compares closely to the tabulated value of 38.89 for the standard atmosphere. However, for a typical aerosol scale height, $H_{\rm aer} \approx 2$ km, $M(90^\circ)_{\rm aer} \approx 77$, roughly double that of the molecular atmosphere.

For other values of the zenith angle, the distance, r, from the observer to the top of the atmosphere is shown in figure 4(b) and is found by using the cosine law

$$(R_{\rm E} + H)^2 = r^2 + R_{\rm E}^2 - 2rR_{\rm E}\cos(180 - Z). \tag{11}$$

Solving for r and dividing by H yield the equation for the optical air mass in a homogeneous atmosphere, namely,

$$M(Z) = \frac{r(Z)}{H} = \left[-\frac{R_{\rm E}}{H}\cos(Z) + \sqrt{\left(\frac{R_{\rm E}}{H}\cos(Z)\right)^2 + 2\frac{R_{\rm E}}{H} + 1}\right]. \tag{12}$$

When $Z = 90^\circ$, equation (12) reduces to equation (10).

$M(Z)$ depends only on the zenith angle Z and the ratio $R_{\rm E}/H$. The percentage difference between the results of equation (12) (upper curve in figure 3) and the tabulated values for the standard atmosphere is surprisingly small given the simplicity of the model and its neglect of refraction. The error is about 2% at the horizon ($Z = 90^\circ$), reaches a maximum of 6.3% for a narrow range of zenith angles centred at $Z = 88.6^\circ$ and then decreases rapidly as Z decreases.

The geometric model can also be applied to uniform aerosol layers with different scale heights. Figure 5 shows the optical air mass $M_{\rm Aer}$ as a function of zenith angle for $H =$ 2, 5 and 8 km. Note that the smaller the scale height, the larger the optical air mass near the horizon. This is why thin haze layers, as often occur in Los Angeles, reduce horizontal visibility markedly while leaving the zenith sky blue.

5. Elevated aerosol layers

The geometric model for the optical air mass can easily be applied to elevated aerosol layers with thickness ΔH (figure 6). In that case, the optical air mass is given by

$$M(Z) = \frac{\Delta r(Z)}{\Delta H}. \tag{13}$$

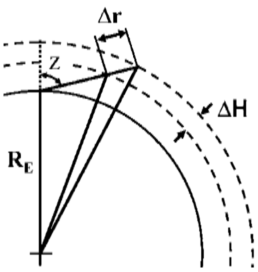
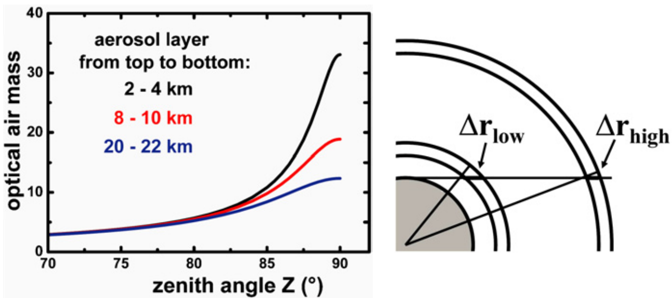


Figure 6. Geometry for calculating the optical air mass of elevated aerosol layers for arbitrary zenith angles Z.



(a) (b)

Figure 7. Calculated optical air mass (equation (13)) for an aerosol layer of 2 km thickness each at various heights (a) and geometries (b).

Here, Δr is the difference in distances r (see equation (12)) from the observer to the top and bottom of the layer divided by its thickness ΔH.

Figure 7(a) shows that when $Z > 85°$, M_{Lay} decreases dramatically as the central height of a 2 km thick layer increases because any straight line starting from the Earth's surface crosses an elevated layer closer to its normal than at the ground (see figure 7(b)), i.e., Δr_{high} will be smaller than Δr_{low}. For a horizontal beam at sea level the M_{Lay} value decreases from 33.1 to 12.3 as the central height of a 2 km thick layer increases from 3 to 21 km.

6. Elevated observers

Astronauts, observers in an airplane or mountaineers look through much less air near the zenith but up to twice as much air at the horizon than observers at sea level. The resulting visual consequences can be striking. Photographs of the setting Sun or Moon taken by astronauts are much redder and more squashed than at sea level. Because the optical air mass is defined as the optical path from sea level, an effective optical air mass M^* that includes the viewer's altitude z_{Obs} can be defined for $Z \leqslant 90°$ as

$$M^*(Z, z) = M(Z) \cdot \frac{p_{\max}(z)}{p_0}, \tag{14}$$

where $p_{\max}(z)$ is the maximum pressure along the viewer's line of sight and p_0 is the sea level pressure.

The geometry for calculating M^* in the homogeneous atmosphere is shown in figure 8(a) for an observer at height z_{Obs} above sea level looking upwards ($Z \leqslant 90°$). The maximum

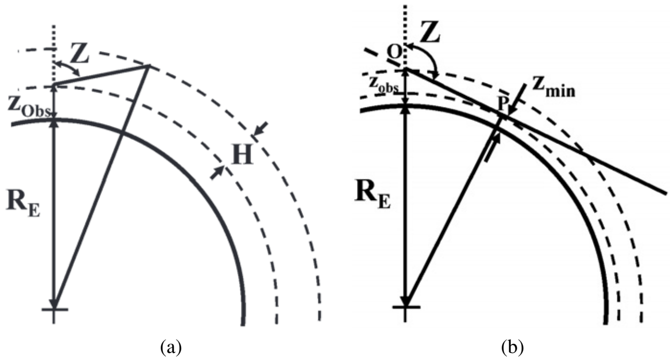


Figure 8. Geometry for calculating the optical air mass of an elevated observer at height, $z = y$ above sea level when zenith angle $Z < 90°$.

pressure along the line of sight is denoted as $p_{\max}(z) = p(z_{\mathrm{Obs}})$, i.e., the pressure at the height of the observer. Pressure in the standard atmosphere can be found, e.g., from tables [7].

However, when looking downwards (figure 8(b), $Z \geqslant 90°$), $p_{\max}(z) > p(z_{\mathrm{Obs}})$. In the latter case, $p_{\max}$ is determined from the minimum height, $z_{\min}$ of the beam:

$$z_{\min} = (R_{\mathrm{E}} + z)\sin(Z) - R_{\mathrm{E}}. \tag{15}$$

The simplest way to calculate the optical air mass for elevated observers and $Z > 90°$ is to split the light path into two parts. The first part extends from observer O to point P of minimum height, i.e. maximum pressure (note the similarity to figure 4(b)). The second part equals the air mass from P for $Z = 90°$. Thus, $M_{\mathrm{O}}(Z > 90°) = M_{\mathrm{OP}} + M_{\mathrm{P}}(Z = 90°)$.

7. Colours of the Sun and Moon

The Sun or Moon exhibit a range of colours that can be simulated by including the impact of the optical air mass in Bouguer's law on the spectrum of sunlight. The Sun or Moon appears slightly yellow when high in the sky but reddens near the horizon because air molecules and most particles scatter short waves more efficiently than long waves and because the optical air mass increases. On rare occasions, when the upper troposphere or stratosphere is laden with particles from large forest fires or droplets from volcanic eruptions, the Sun and Moon turn green or blue because the particles fall in the size range that scatters long waves more efficiently than short waves [11].

Modelling the colours of the Sun or Moon, which can be taught in undergraduate courses, may be done in a first step by discussing the changes of the relevant spectra (see also the Excel spreadsheet, available on the webpage [1]). These spectra may be used in a second step [12] to calculate chromaticity diagrams and quantitative colour coordinates. Here we restrict ourselves to a qualitative analysis of colour by discussing spectral changes.

The spectrum of sunlight (or moonlight) atop the atmosphere can be approximated by a Planck radiator at 5800 K. Its spectral radiance is given by

$$I_0(\lambda) \approx \frac{2hc^2}{\lambda^5}\left(\mathrm{e}^{\frac{hc}{k\lambda T}} - 1\right)^{-1}, \tag{16}$$

where $h = 6.626 \times 10^{-34}$ J s^{-1} is Planck's constant, $k = 1.38 \times 10^{-23}$ J K^{-1} is Boltzmann's constant and $c = 2.998 \times 10^8$ m s^{-1} is the speed of light. Such a spectrum produces white or

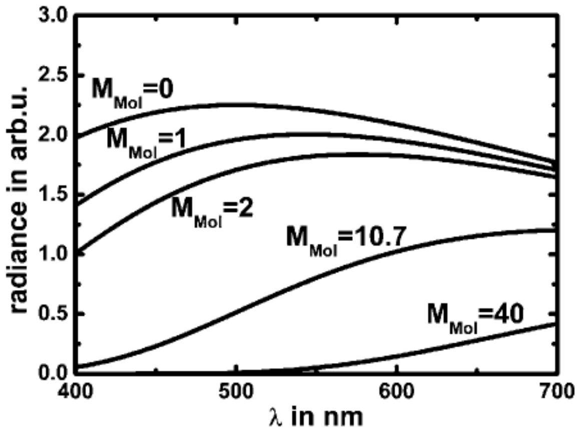


Figure 9. Spectra of sunlight (from equation (17)) approximated by a Planck radiator at 5800 K for various values of optical air mass, M_{Mol}, in a molecular atmosphere ($K_{\mathrm{Mol}} = 8.66 \times 10^{-27}\ \mathrm{m}^{-4}$). $M_{\mathrm{Mol}} = 0$ represents the spectrum at the top of the atmosphere. The other lines represent solar spectra at sea level when the solar zenith angles are $Z = 0$ ($M_{\mathrm{Mol}} = 1$), $Z = 60^\circ$ ($M_{\mathrm{Mol}} = 2$), $Z = 85$ ($M_{\mathrm{Mol}} = 10.7$) and $Z = 90^\circ$ ($M_{\mathrm{Mol}} = 40$).

slightly yellow light. Here, we discuss the changes of this spectrum due to scattering in the atmosphere.

In a pure, dry molecular atmosphere, scattering obeys Rayleigh's law, i.e. the normal optical thickness is given by $\tau_{\mathrm{Mol}} = K_{\mathrm{Mol}}/\lambda^4$ (equation (4)). Under these conditions, roughly 3% of the shortest visible violet light waves ($\lambda \approx 400$ nm) and 30% of the longest visible red light waves ($\lambda \approx 700$ nm) of a vertical sunbeam are scattered on its passage through the atmosphere. This mixture of preponderantly short waves of scattered sunlight is what makes the sky blue, while the greater removal of short waves from the direct beam turns the Sun or Moon slightly yellow even at the zenith.

Aerosol particles, which are much larger than air molecules, scatter light in the more complex manner that is often approximated by Mie scattering by spheres and, furthermore, absorb light [13, 14]. Typical uniformly distributed tropospheric aerosols are of the order of 100 nm and scatter light roughly proportional to λ^{-1}, [15], i.e. $\tau_{\mathrm{Aer}} = K_{\mathrm{Aer}}/\lambda$, while forest fires or volcanic eruptions often produce aerosol layers with micrometre size particles that scatter light roughly proportional to λ^{+1} [11], i.e. $\tau_{\mathrm{Lay}} = K_{\mathrm{Lay}}\lambda$. As a result, the radiance of direct sunlight seen by an observer at sea level and reduced by scattering can be approximated by

$$I(\lambda) \approx I_0(\lambda) \exp\left(-\left(\frac{M_{\mathrm{Mol}}K_{\mathrm{Mol}}}{\lambda^4} + \frac{M_{\mathrm{Aer}}K_{\mathrm{Aer}}}{\lambda} + M_{\mathrm{Lay}}K_{\mathrm{Lay}}\lambda\right)\right), \qquad (17)$$

where K_{Mol}, K_{Aer} and K_{Lay} are the scattering coefficients for the molecular atmosphere, typical tropospheric aerosols and volcanic aerosols or droplets, distributed in layers, respectively.

The spectrum of visible sunlight for a pure, molecular atmosphere as seen at sea level is shown in figure 9 for various values of M_{Mol} or Z, and compared to the spectrum of a Planck radiator at the top of the atmosphere ($M = 0$). At the top of the atmosphere, where sunlight is near white, peak radiance occurs at $\lambda = 500$ nm. By sea level, the peak has shifted to $\lambda = 545$ nm when the Sun is overhead ($M = 1$), and to the longest visible red light once the Sun lies within about 5° of the horizon ($M > 10.7$). In very clean air, light of the setting Sun is still so intense that it saturates our cones and only appears orange when a dark filter is used.

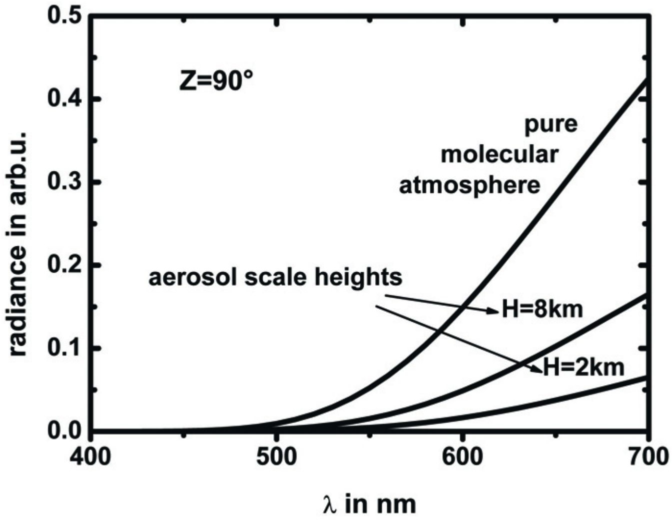


Figure 10. Spectra of sunlight (from equation (17)) at the horizon approximated by a Planck radiator with $T = 5800$ K in a purely molecular atmosphere compared with a molecular atmosphere which also contains uniformly distributed aerosols with $K_{\mathrm{aer}} = 1.63 \times 10^{-8}\ \mathrm{m}^{-1}$ for two different values of aerosol scale height.

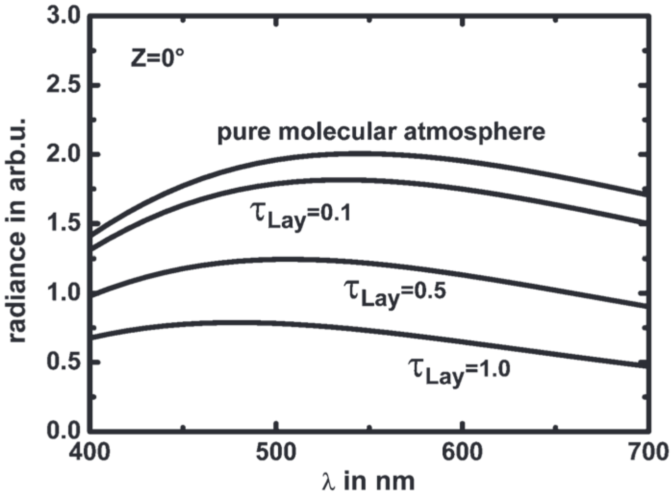


Figure 11. Spectra of sunlight (from equation (17)) at the zenith ($M_{\mathrm{Mol}} = 1$, $M_{\mathrm{Lay}} = 1$) approximated by a Planck radiator at $T = 5800$ K at sea level in a purely molecular atmosphere compared with a molecular atmosphere which also contains layers of volcanic aerosols. The layers were 5 km thick centred at 22.5 km above sea level. Their optical thicknesses $\tau_{\mathrm{Lay}} = 0.1, 0, 5$ and 1 (from top to bottom) correspond to scattering coefficients $K_{\mathrm{Lay}} = 1.82 \times 10^{5}$ m, 9.09×10^{5} m and 1.82×10^{6} m, respectively.

Typical tropospheric aerosols dim and redden sunlight. Figure 10 shows the impact of aerosols and aerosol scale height on the brightness and colour of the Sun at the horizon ($M_{\mathrm{Mol}} = 40$, M_{Aer}, see figure 5) in an atmosphere where the weighted mean aerosol optical thickness integrated over the Planckian solar spectrum is 28.5% of that of the molecular atmosphere ($K_{aer} = 1.63 \times 10^{-8}\,\mathrm{m}^{-1}$). Even this modest aerosol loading dims and reddens the Sun substantially near the horizon. Decreasing the aerosol scale height, H_{aer} (e.g., from 8 to 2 km) of a layer with base at the ground further dims and reddens the horizon Sun.

The Sun or Moon may turn blue or green when elevated aerosol layers produced by forest fires or volcanic eruptions are present. Particles with diameters of about 800 nm act in an anomalous manner in that they scatter long waves more efficiently than short waves. The scattering of predominantly red light when these particles occupy the stratosphere also leads to the vibrant crimson twilights seen after volcanic eruptions. Figure 11 gives an example of the change of Sun spectra, if layered aerosols are present.

8. Conclusions

A geometrical model of the optical air mass, M, for a homogeneous atmosphere was presented. Except for the highly anomalous atmospheric conditions that lead to mirages, the values of M calculated with this model can be used to determine e.g. atmospheric visibility, distance to the horizon and the colours of the Sun and Moon with good accuracy. The model, which could be used in undergraduate courses, shows that the Sun (and Moon) turn orange-red as they get low in the sky, because M becomes so large that scattering by air molecules and typical aerosols removes much of the short waves from the direct solar beam. It also shows that the Sun or Moon may even turn green or blue well above the horizon when there are enough aerosols or droplets produced by large forest fires and by volcanic eruptions because these particles scatter long waves more efficiently than short waves.

Acknowledgments

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