A short note on the Diamond and Dybvig (1983) model

Disclaimer: The note is a very brief and probably incomplete presentation of the DD-model. It is meant to give a simple overview before proceeding to more advanced texts in this framework. Please use as such and beware of typos, mistakes etc..

The setup

The general model framework is as follows:

- Three periods, $t = 0, 1, 2$. One consumption good.
- Continuum of consumers with a unit endowment of the consumption good.
- Consumers are identical at $t = 0$. With probability π they are impatient and probability $1 - \pi$ they are patient. Formally this entails a utility function of the form:

$$U = \begin{cases} u(C_1), & \text{if impatient, with probability } \pi, \\ u(C_2), & \text{if patient with probability } 1-\pi \end{cases}$$
$$EU = \pi u(C_1) + (1-\pi)u(C_2),$$

  where C_i denotes consumption at $t = i$.

- To meet their end of consuming either all they have when $t = 1$ (if impatient) or at time $t = 2$ (if patient), consumers can save their unit endowment using two technologies:

  Storage technology/short term: One-period storage yielding return of 1.

  Illiquid investment/long term: Two-period investment yielding return $R > 1$. Can be liquidated early $t = 1$ at return $L < 1$.

To sum up: The main 'non-trivial' ingredients in our model is the **uncertainty** of consumers coupled with **incomplete financial markets** (so far at least). Another fine buzzword here is that we have **liquidity risk** as we might end up in a situation where we have to prematurely (and inefficiently) liquidate the long term investment.

Inefficiency in autarky

In the autarky case (no-trade/financial intermediaries/bonds) consumers choose investment in illiquid technology I to maximize expected utility. This is subject to budgets:

$$\begin{aligned} &\text{w. pr. } \pi && C_1 = 1 - I + LI \leq 1 \quad \text{and} \quad C_2 = 0 \\ &\text{w. pr. } 1-\pi && C_2 = 1 - I + RI \leq R \quad \text{and} \quad C_1 = 0. \end{aligned}$$

This states that (1) if the consumers turns out to be impatient he prematurely liquidates the entire long term investment I at $t = 1$ (he gets no utility from C_2). And (2) if the consumers turns out to be patient, the savings in short term technology $(1-I)$ is saved in the short term technology again from $t = 1$ to $t = 2$. Thus to determine I we solve

$$\max_I \ \pi u\big(1 - I + LI\big) + (1-\pi)u\big(1 - I + RI\big).$$

Finally, note that combining consumption levels:

$$\pi C_1 + (1-\pi)\frac{C_2}{R} < 1.$$

The inefficiency in autarky arises from the uncertainty. The efficient allocation is found by allowing a planner to invest I without the uncertainty, thus having to prematurely liquidate long run investments:[1]

$$\max_I \ \pi u\left(\frac{1-I}{\pi}\right) + (1-\pi)u\left(\frac{RI}{1-\pi}\right).$$

Note that combining consumption levels

$$\pi C_1 + (1-\pi)\frac{C_2}{R} = 1,$$

i.e. that total consumption is increased compared to autarky. Secondly, note that first order conditions imply

$$u'\big(C_1^*\big) = Ru'\big(C_2^*\big), \qquad \Rightarrow \qquad C_2^* > C_1^*.$$

Once again to sum up: The uncertainty and incomplete markets make autarky inefficient. The liquidity risk cannot be mitigated by any market-based insurance scheme (in autarky!). Let us now look at ways to remedy this inefficiency.

Bonds and banks; restoring efficiency?

Bonds market:

Introducing a bonds market we allow consumers to trade a bond for a price p at time $t = 1$ when uncertainty of type is revealed. Budgets are then given by

$$C_1 = 1 - I + pRI$$
$$C_2 = \frac{1-I}{p} + RI.$$

Patient consumers can use short run investment $1 - I$ to purchase bonds receiving $(1-I)/p$ when $t = 2$. Impatient consumers receive p for each unit of income they can repay when old RI. In this way investments are not prematurely liquidated. In equilibrium $p = 1/R$ yielding the market allocation $C_1^M = 1$ and $C_2^M = R$. Note as in the first best allocation we now have

$$\pi C_1^M + (1-\pi)\frac{C_2^M}{R} = 1,$$

but generally we do not have $u'(C_1^M) = Ru'(C_2^M)$. Thus the bonds market improves on autarky, but does generally not ensure optimality. The reason why this is not optimal, is that we are still not able to mitigate the liquidity risk as financial markets are incomplete.[2]

Banks/financial intermediary:

[1] A utilitarian planner is assumed here.

[2] There are a lot of nice literature on what complete financial markets can do for welfare. A somewhat vague explanation of what a 'complete' market is; the contract you enter into has to be able to be *state contingent*. In other words a complete financial market would offer investment contracts at time 0 where the return may be contingent on what 'type' the consumer is when $t = 1$. This can effectively eliminate risk. The bank contract offered below actually does this implicitly.

Optimality can be achieved through an intermediary (making markets complete). To mitigate risk we could have the state contingent contract: Deposit your entire endowment at $t = 0$. When $t = 1$ you can withdraw C_1^* (efficient allocation) or wait and withdraw C_2^* when $t = 2$. This contract is clearly optimal as it achieves the first best by construction. Furthermore, if patient households withdraw when $t = 2$ then the bank can finance the contract by investing πC_1^* in short term technology and $1 - \pi C_1^*$ in illiquid investments (same as the social planner would have done).

(inefficient) Bank runs

Clearly, the contract (C_1^*, C_2^*) is optimal and constitutes an equilibrium when all patient households do not withdraw early. However, there might still exist a bank run equilibrium based on the consumers' beliefs on what others do.

If a patient household expect all other patient households to withdraw early (run), then the bank will pay out C_1^* when $t = 1$. To finance this they have to liquidate long term investments prematurely. If they liquidate all of it, the bank can at most at time $t = 1$ pay out:

$$\pi C_1^* + (1 - \pi C_1^*)L < 1.$$

Thus in cases when $C_1^* > 1$ (the Diamond-Dybvig assumption) the bank cannot stay solvent in the case of a bank run. Thus it can be rational for a patient consumer to 'run the bank' trying to withdraw early, if he suspects all other patient households are doing the same.

Institutional arrangements

So how can we then avoid bank runs (a non-exhaustive list):

Narrow banking:

- *Strong version:* Liquid bank reserves must cover 100% of deposits.
- *Weak version:* Bank reserves (including long-run illiquid) must cover 100% of deposits.
- *Modern version:* Require banks to securitize all their investments.

Suspension of convertibility:

- Bank announces that it will not liquidate long term investments early, thus at most pay out πC_1^* at $t = 1$.
- If announcement is credible $\Rightarrow$ rational never to expect bank runs.
- Time-inconsistency in *optimal convertibility*: It may be ex-post inefficient to stop payments at πC_1^*. The idea is: If banks are rational, consumers know that if a bank run occurs, then it may be optimal for the bank to keep paying withdrawals beyond the announced limit πC_1^*. If this is the case, then there is no reason to believe the announcement of stopping payments at πC_1^*. For more see Ennis and Keister (2009).

Deposit insurance:

- Deposit insurance offers guarantee on deposits, financing through premiums paid by banks at time $t = 0$ (or taxes in public system).
- This eliminates risk of bank runs and yields insurance premium of zero in equilibrium.