A Small GreenREFORM-like CGE model

Rasmus K. Skjødt Berg*

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The note outlines a smaller version of the computable general equilibrium (CGE) model GreenREFORM. The model is a multisector, deterministic, dynamic general equilibrium model with the same roughly 60 production sectors as in GreenREFORM; the specialized modules of GreenREFORM are not included here, though. The CGE model is generated using the `GamsPython` class; a general introduction to this tool can be found in the online documentation.[1]

At the current stage, we use the following data explicitly in the model:

- The Danish national accounts data in *values*. Only for *2018*.
- The Danish national accounts data on investments, durables (capital), and depreciation rates in *values*. Only for *2018*.
- The general nested production structure of GreenREFORM is included. This further allows us to borrow estimated elasticities of substitution.
- Very little data on regulation is currently used in the model:
  - For the household module we include a few large blocks including average labor income tax, lump-sum transfers (e.g. from unemployment).
  - For other sectors, we use data from the national accounts on VAT and other production taxes/subsidies to identify simple, flat, and permanent rates.
- No price data is currently used: All prices on non-durables are set to 1 in the baseline year.
- No data from the energy balance is currently used.

The economy consists of domestic production sectors, foreign sectors, households, investment sectors, inventory sectors, and a government sector. The following outlines the implementation of each of them in turn.

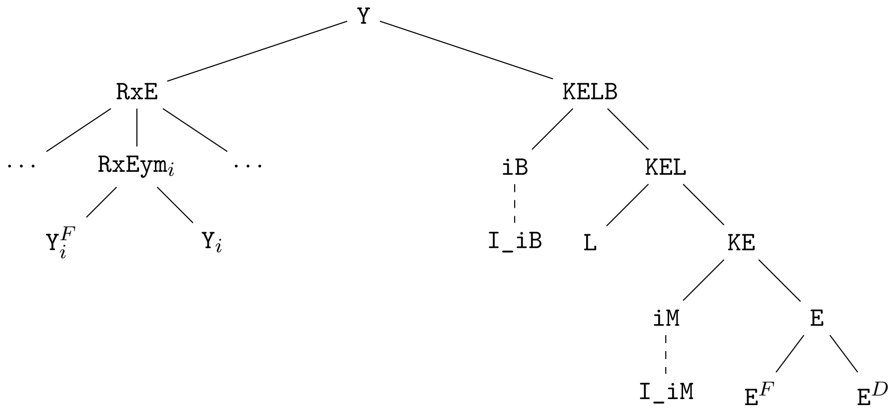
1 Domestic production sectors

The general nesting structure for each domestic sector `s` is defined as in figure 1.1.

*University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: rasmus.kehlet.berg@econ.ku.dk.

[1]The current version is GPM_v071

Figure 1.1: Production nest



Non-energy materials nest:

The figure illustrates that the output from a sector (`Y`) is split into a non-energy materials nest (`RxE`) and other (`KELB`). Materials can come from any other sector (here indexed i). All domestic sectors are inherently tied to a similar foreign sector: If a sector relies on both, they are combined in the final nest here as $\texttt{RxEym}_i$ with $\texttt{Y}_i^F$ denoting the foreign type i and $\texttt{Y}_i$ the domestic type.

Durables, labor, and energy nesting:

The rest of the nesting tree sequentially splits out buildings (`iB`), labor (`L`), and machines (`iM`) and energy (`E`). The two durable inputs (`iM`,`iB`) are accrued through purchasing corresponding investment goods (`I_iM`,`I_iB`) from the investment sectors – and is subject to installation costs.

All solid lines indicate a standard CES-like nesting. The dashed lines from `iB`, `iM` indicates that the production sectors accumulate these in a non-CES manner (elaboration below). At the current stage, the energy input `E` is simply the output from one of the domestic/foreign production sectors.[2]

To do: (1) Split energy into clean, dirty etc. or (2) into energy goods.

1.1 The optimization problem

Let $n \in \texttt{in}_s$ denote the subset of inputs in sector s and $n \in \texttt{dur}_s$ the set of durables (`iB`,`iM`). Note that the durables are not included in the set of inputs, only the corresponding investment goods are. In other words: $\texttt{I\_iB} \in \texttt{in}_s$, but $\texttt{iB} \notin \texttt{in}_s$. The firm s maximizes its discounted future profits from t_o:

$$\begin{aligned} \max \quad \sum_{t=t_o}^{T} R_{t,t_o} \Bigg\{ & p_{(t,s,Y)} \cdot q_{(t,s,Y)} - \sum_{n \in \texttt{in}_s} \left(p_{(t,s,n)} \cdot q_{(t,s,n)} \right) \\ & - \sum_{n \in \texttt{dur}_s} p_{(t,s,n)} \left[q^I_{(t,s,n)} + q^K_{(t,s,n)} \Psi_{(s,n)} \left(\frac{q^I_{(t,s,n)}}{q^K_{(t,s,n)}} \right) \right] \Bigg\}, \end{aligned} \tag{1a}$$

[2]The `E` sectors currently cover the (i) electricity production, (ii) transmission, distribution, and trade in electricity, and (iii) supply of heat.

where R_{t,t_0} is the accumulated discount factor between t and t_0, $\Psi_{(s,n)}$ is a measure of installation costs and superscripts I, K indicate the relevant flow, stock variables the specific type of durable. The optimization is subject to a set of technology functions (nested CES) and the law of motion for durables. For every *knot* in the nesting tree, $n \in \texttt{knots}_s$, a CES function produces $q_{(t,s,n)}$ by combining all relevant *branches* denoted by $n' \in \texttt{map}_{s,n}$:

$$q_{(t,s,n)} = \left(\sum_{n' \in \texttt{map}_{s,n}} \left(\mu_{(s,n,n')}\right)^{\frac{1}{\sigma_{(s,n)}}} \left(q_{(t,s,n')}\right)^{\frac{\sigma_{(s,n)}-1}{\sigma_{(s,n)}}} \right)^{\frac{\sigma_{(s,n)}}{\sigma_{(s,n)}-1}}, \qquad \forall n \in \texttt{knots}_s \tag{1b}$$

For example, the element $n = \texttt{KE}$ is a knot in the nesting tree in figure 1.1 with branches $n' \in \{\texttt{iM}, \texttt{E}\}$. For the durables part, we add the law of motion for all types of durables $n \in \texttt{dur}_s$:

$$q^K_{(t+1,s,n)} = q^K_{(t,s,n)}(1 - \delta_{(s,n)}) + q^I_{(t,s,n)}, \qquad \forall t_0 < t \leq T \tag{1c}$$

$$q^K_{(T+1,s,n)} = q^K_{(T,s,n)}, \qquad q^K_{(t_0,s,n)} > 0 \text{ given,} \tag{1d}$$

where (1d) is a simple transversality condition. We assume quadratic installation costs Ψ:

$$\Psi_{(s,n)}(x) = \frac{\phi_{(s,n)}}{2} \left(x - \delta_{(s,n)}\right)^2. \tag{1e}$$

Solving this yields the following system of equations for the nesting tree for all t:

$$\text{For } n \in \texttt{knots}_s \qquad p_{(t,s,n)} \cdot q_{(t,s,n)} = \sum_{n'} q_{(t,s,n')} \cdot p_{(t,s,n')} \tag{2a}$$

$$\text{For } (n', n) \in \texttt{map}_s \qquad q_{(t,s,n)} = \mu_{(s,n',n)} \left(\frac{p_{(t,s,n')}}{p_{(t,s,n)}}\right)^{\sigma_{(s,n')}} q_{(t,s,n')}. \tag{2b}$$

We note that this solution introduces new auxiliary variables, in particular the price indices for all intermediate goods in the nesting tree. This includes prices on intermediate goods like KE, but also a price on the durables (iB,iM). The prices on durables are similarly only auxiliary variables used to characterize the solution. In addition to the law of motions and the TVC condition, we can combine relevant first order conditions for the durables and investment goods into the following requirement for the shadow value of the durables $p^K_{(t,s,n)}$ for $t_0 < t < T$:

$$\begin{aligned} p^K_{(t,s,n)} =& R_t p^I_{(t-1,s,n)} \left[1 + \phi_{(s,n)} \left(\frac{q^I_{(t-1,s,n)}}{q^K_{(t-1,s,n)}} - \delta_{s,n)}\right)\right] \\ &+ p^I_{(t,s,n)} \left[\frac{\phi_{(s,n)}}{2}\left(\delta^2_{(s,n)} - \left(\frac{q^I_{(t,s,n)}}{q^K_{(t,s,n)}}\right)^2\right) - (1 - \delta_{(s,n)}) \left(1 + \phi_{(s,n)} \left[\frac{q^I_{(t,s,n)}}{q^K_{(t,s,n)}} - \delta_{(s,n)}\right]\right)\right]. \end{aligned} \tag{2c}$$

In the final period, we approximate this shadow value by the value in steady state, that is:

$$p^K_{(T,s,n)} = R_T p^I_{(T-1,s,n)} \left(1 + \phi_{(s,n)} \frac{q^I_{(T-1,s,n)}}{q^K_{(T-1,s,n)}}\right) - (1 - \delta_{(s,n)}) p^I_{(T,s,n)}. \tag{2d}$$

Causally, the system of equations in (2) takes as given: (i) Technical parameters, (ii) the prices of inputs as given ($p_{(t,s,n)}$, $n \in \texttt{in}_s$), and (iii) the scale of output ($q_{(t,s,Y)}$). It then determines: (i) Intermediate quantities and prices (including durables), (ii) the demand for all inputs ($\texttt{q}_{(t,s,n)}$, $n \in \texttt{in}_s$), and (iii) the cost index on output ($\texttt{p}_{(t,s,Y)}$).

1.2 Regulation, profits, and prices

The optimization problem above generally relies on sector-specific prices input prices, but generally does not include any description of competition (and thus profits) nor do we include regulation (e.g. through taxes). To account for these perspectives, we include a block of equations that map from *effective* sector prices to *equilibrium* ones. These are the equations we need to adjust, if we want to introduce new regulation or alter the rates of existing taxes.

In the baseline version of the model, we include the following three equations:

Profits and equilibrium prices:

Equilibrium prices (p_t^n) as a function of the cost index ($p_{(t,s,Y)}$), a sector specific markup (m_s), regulation in the form of a unit tax on output and a lump-sum transfer ($\tau^S_{(t,s)}, \tau^L_{(t,s)}$), and installation costs ($\Psi_{(t,s)}$):

$$p_t^n = (1 + m_s)\left(p_{(t,s,Y)} + \tau^S_{(t,s)} + \frac{\tau^L_{(t,s)} + \Psi_{(t,s)}}{q_{(t,s,Y)}}\right). \tag{3a}$$

In this way, the firm s enters the market for the good n with a markup over their average costs, taking installation costs and taxes into account. In the baseline version of the model, this equation is used to identify the equilibrium price p_t^n on the output from the sector.

Effective input prices:

A mapping from equilibrium prices (p_t^n) to effective input prices in sector s ($p_{(t,s,n)}$) that takes regulation of inputs into account. For instance, assuming that the input is associated with $\nu_{(t,s,n)}$ emissions and a uniform CO_2 tax of $\tau_t^{CO_2}$ implies the mapping:[3]

$$p_{(t,s,n)} = p_t^n + \nu_{(t,s,n)} \cdot \tau_t^{CO_2}. \tag{3b}$$

In the baseline version of the model, this equation is used to identify the effective input prices $p_{(t,s,n)}$ given regulation.

Tax revenue:

Finally, we include a variable that sums up the net transfers to the public sector $T_{(t,s)}$.[4] With the

[3]Note that taxes levied on inputs are automatically included in the cost index $p_{(t,s,Y)}$ as this ultimately relies on effective input prices $p_{(t,s,n)}$. If, however, there are *nonlinear* taxation of inputs, this no longer holds. In this case, the marginal tax rate should be used to model effective input prices, $p_{(t,s,n)}$, and the difference between marginal and average rates should be added to the price index in (3a).

[4]`TotalTax` in the code.

regulation outlined in this example, the total transfer is defined as:

$$T_{(t,s)} = \tau^L_{(t,s)} + q_{(t,s,Y)} \cdot \tau^S_{(t,s)} + \sum_{n \in \texttt{in}_s} q_{(t,s,n)} \cdot \nu_{(t,s,n)} \cdot \tau_t^{CO_2}. \tag{3c}$$

In the baseline version of the model, this equations is used to identify $T_{(t,s)}$.

1.3 Calibration

In calibrating the model, the relevant input-output data in the baseline year t_o is targeted by adjusting relevant technical parameters. Each of the targets in data identify a specific technical parameter:

i. Data on levels of demand ($q_{(t_o,s,n)}$) is used to identify relevant share parameters in the production function ($\mu_{(s,n',n)}$).

ii. Data on the equilibrium price on the output from the sector ($p^n_{t_o}$) is used to identify the markup from the sector (m_s).

iii. Data on the total net transfer from regulation ($T_{(t_o,s)}$) is used to identify the level of one of the tax rates. We adjust the unit tax on outputs ($\tau^S_{(t,s)}$) permanently to target this.

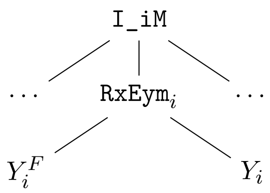
iv. Data on the initial level of investments ($q^I_{(t_o,s,n)}$) is used to identify the share parameter on the corresponding durable ($\mu^K_{(s,n',n)}$).

Other technical parameters like elasticities (σ) and scale of installation costs (ϕ) have to be determined outside of the model.[5]

2 The Investment module

The investment sectors essentially work as simplified versions of the production sectors outlined in section 1 with two main differences: First, the output from these sectors are investment goods (`I_iB`, `I_iM`) that are solely bought by domestic production firms. Second, they use a simplified nesting structure similar to the non-energy materials nest in the production sectors (i.e. no durables). Third, there are no profits and thus no markup in this sector.

Figure 2.1: Investment production nest



[5] The share parameters on intermediate goods are determined by value shares from the IO data in the baseline year.

3 The inventory module

The module is a small add-on that *adhoc* accounts for inventory "investments". We only include this for completeness – i.e. to cover all categories of the national accounts.

4 The Household Module

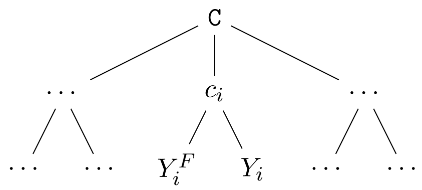
Households are modelled as Ramsey agents with endogenous labor supply. The instantaneous utility is separable in a consumption aggregate `C` and labor `L`. Consumption preferences are constant relative risk aversion (CRRA, θ) and (dis)utility from labor supply is iso-elastic with a constant Frisch elasticity of supply ξ. The discounted utility of the households are then given by:

$$U = \sum_{t=t_0}^{T} \beta_s^{t-t_0} \left[\frac{\left(q_{(t,s,C)}\right)^{1-\theta_s}}{1-\theta_s} - \frac{\xi_s}{1+\xi_s} \left(\frac{q_{(t,s,L)}}{\gamma_s} \right)^{\frac{1+\xi_s}{\xi_s}} \right], \tag{4a}$$

where β is the yearly discount factor, $q_{(t,s,C)}$ and $q_{(t,s,L)}$ is quantity of consumption/labor supply, and γ is a scale parameter. Note that, symbolically, households are also added as a sector with index s. In the current model, we only have a single household sector s, but we can easily extend the setup to include heterogeneity in this regard.

The consumption aggregate `C` is a CES aggregate of the final goods of the economy. The nesting structure is similar to the one applied in the investment module (figure 4.1): For all roughly 60 types of goods, the lower nest reflects competition between foreign/domestic goods. The upper nest reflects how various types of goods are combined into the aggregate `C`. The implication is that

Figure 4.1: Household intratemporal nest



$q_{(t,s,C)}$ is modeled as a nesting tree – as in the production module: For every *knot* in the nesting tree, $n \in \texttt{knots}_s$, a CES function produces $q_{(t,s,n)}$ by combining all relevant *branches* denoted by $n' \in \texttt{map}_{s,n}$:

$$q_{(t,s,n)} = \left(\sum_{n' \in \texttt{map}_{s,n}} \left(\mu_{(s,n,n')}\right)^{\frac{1}{\sigma_{(s,n)}}} \left(q_{(t,s,n')}\right)^{\frac{\sigma_{(s,n)}-1}{\sigma_{(s,n)}}} \right)^{\frac{\sigma_{(s,n)}}{\sigma_{(s,n)}-1}}, \qquad \forall n \in \texttt{knots}_s \tag{4b}$$

In this example, the knots include the aggregate `C` and the intermediate goods c_i for each type of good.

The household has savings $v_{t,s}$ that earns an interest i_t in each period. The household thus faces the law of motion and corresponding transversality condition:

$$v_{(t+1,s)} = (1+i_t) \cdot v_{(t,s)} + p_{(t,s,L)} \cdot q_{(t,s,L)} - \sum_{n \in \mathtt{in}_s} p_{(t,s,n)} \cdot q_{(t,s,n)} - \tau^L_{(t,s)}, \qquad \forall t_0 < t \leq T \tag{4c}$$

$$v_{(T+1,s)} = v_{(T,s)}, \qquad v_{(t_0,s)} \text{ given,} \tag{4d}$$

where $\tau^L_{(t,s)}$ is a lump sum tax on households.

4.1 The optimization problem

The household maximizes (4a) subject to (4b)-(4d). We can split the solution into an intra- and an intertemporal part. The intratemporal part specifies the components of the consumption aggregate – and how labor is supplied given $\mathtt{C}$. First, for every t we have:

$$\text{For } n \in \mathtt{knots}_s \qquad p_{(t,s,n)} \cdot q_{(t,s,n)} = \sum_{n'} q_{(t,s,n')} \cdot p_{(t,s,n')} \tag{5a}$$

$$\text{For } (n',n) \in \mathtt{map}_s \qquad q_{(t,s,n)} = \mu_{(s,n',n)} \left(\frac{p_{(t,s,n')}}{p_{(t,s,n)}} \right)^{\sigma_{(s,n')}} q_{(t,s,n')}. \tag{5b}$$

This is a copy of the CES system from the production module. Second, given $q_{(t,s,C)}$, optimal labor supply follows:

$$q_{(t,s,L)} = \gamma_s \left(\frac{p_{(t,s,L)}}{p_{(t,s,C)} \left(q_{(t,s,C)}\right)^{\theta_s}} \right)^{\xi_s}. \tag{5c}$$

Intertemporal optimality is represented by the Euler equation, the budget, and the transversality condition:

$$q_{(t,s,C)} = q_{(t-1,s,C)} \left(\beta_s \cdot (1+i_t) \cdot \frac{p_{(t-1,s,C)}}{p_{(t,s,C)}} \right)^{\frac{1}{\theta_s}} \tag{5d}$$

$$\text{Equations (4c), (4d).} \tag{5e}$$

Causally, the system of equations in (5) takes as given: (i) Preference parameters, (ii) effective input prices ($p_{(t,s,n)}$), and (iii) the wage rate net of taxes ($p_{(t,s,L)}$). It then determines: (i) Intermediate quantities and prices, (ii) consumption, (iii) supply of labor, and (iv) savings.

4.2 Regulation and prices

To account for regulation and interactions with the government sector, we add the following simple system of equations:

$$p_{(t,s,L)} = p^L_t - \tau^S_{(t,s,L)} \tag{6a}$$

$$p_{(t,s,n)} = p^n_t + \tau^D_{(t,s,n)}, \qquad \forall n \in \mathtt{in}_s \tag{6b}$$

$$T_{(t,s)} = \tau^L_{(t,s)} + \tau^L_{(t,s,L)} \cdot q_{(t,s,L)} + \sum_{n \in \texttt{in}_s} \tau^D_{(t,s,n)} \cdot q_{(t,s,n)}. \tag{6c}$$

Here $\tau^S_{(t,s,L)}$ is the income tax, $\tau^D_{(t,s,n)}$ is consumption taxes, and $T_{(t,s)}$ is the total net transfer to the government from the household sector. This type of regulation can easily be altered.

4.3 Calibration

In calibrating the model, the relevant input-output data in the baseline year t_0 is targeted by adjusting relevant preference parameters. Each of the targets in data identify a specific technical parameter:

i. Data on levels of demand $(q_{(t_0,s,n)})$ is used to identify relevant share parameters in the nested utility function $(\mu_{(s,n',n)})$.

ii. Data on the equilibrium supply of labor $(q_{(t,s,L)})$ is used to identify the scale parameter γ_s.

iii. The total transfer $T_{(t_0,s)}$ is targeted by adjusting the lump-sum tax rate.

5 The Trade Module

The trade module determines the foreign demand for domestically produced goods. At the moment, we model financial markets in a very simple manner – capital is perfectly mobile with a fixed interest rate. The implication is that we do not need to specify the demand for domestic assets; the asset market is simply represented by the interest rate.

The implementation of the trade module currently follows a simple Armington setup. As outlined in previous sections, each type of good can be either produced domestically or in a foreign corresponding sector. This is identified by a mapping $(n, n') \in \texttt{map}_s$ where n indicates the domestic type of good, n' the foreign counterpart, and s in this instance indicates the foreign sector. Using this, exports are defined from:

$$\text{For all } (n, n') \in \texttt{map}_s: \qquad q_{(t,s,n)} = \gamma_{(s,n)} \left(\frac{p^{n'}_t}{p_{(t,s,n)}} \right)^{\sigma_{(s,n)}}, \tag{7a}$$

where $\sigma_{(s,n)}$ is the Armington elasticity. Similar to previous modules, we may add regulation to the exports as well, e.g. by imposing a unit tax on exports:

$$p_{(t,s,n)} = p^n_t + \tau^D_{(t,s,n)}, \tag{7b}$$

where we recall that p^n_t is the equilibrium price and $p_{(t,s,n)}$ the effect price paid by the foreign sector s.

The total tax revenue from the foreign sector is then defined by

$$T_{(t,s)} = \sum_{n \in \texttt{in}_s} \tau^D_{(t,s,n)} \cdot q_{(t,s,n)}. \tag{7c}$$

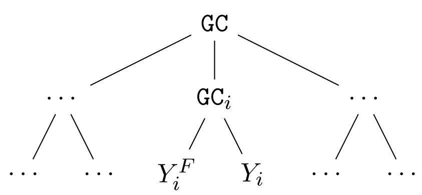
The causality of this module is straightforward: Given equilibrium prices on foreign and domestic goods, (7b) identifies the effective input price, (7a) then determines the exports, and (7c) sums up the total tax transfers. When calibrating the model to a baseline year, the scale parameter $\gamma_{(s,n)}$ is used to ensure the observed levels of exports in the baseline year ($q_{(t_0,s,n)}$) and the tax rates $\tau^D_{(t,s,n)}$ are moved uniformly to ensure the model replicates the observed level of taxes $T_{(t_0,s)}$.

6 The Government Module

The government sector imposes regulation and collects tax revenue from all other sectors in the economy. The government sector also has net wealth of $v_{(t,s)}$ available that pays an interest i_t. The revenue from regulation and net wealth is ultimately used to finance government consumption.

To measure the *service level* provided by government consumption, we use the aggregate government consumption good `GC`: This is produced using the same general preference structure as private households, cf. figure 6.1.

Figure 6.1: Government Consumption Nest



In this way, the equations defining optimal government consumption thus resemble those of the household solution:

$$\text{For } n \in \texttt{knots}_s \qquad p_{(t,s,n)} \cdot q_{(t,s,n)} = \sum_{n'} q_{(t,s,n')} \cdot p_{(t,s,n')} \tag{8a}$$

$$\text{For } (n', n) \in \texttt{map}_s \qquad q_{(t,s,n)} = \mu_{(s,n',n)} \left(\frac{p_{(t,s,n')}}{p_{(t,s,n)}} \right)^{\sigma_{(s,n')}} q_{(t,s,n')}. \tag{8b}$$

The government sector – also similar to the household's problem – face a law of motion for their net wealth $v_{(t,s)}$. However, the stream of revenue looks quite different here:

$$v_{(t+1,s)} = (1 + i_t) \cdot v_{(t,s)} + \sum_{s'} T_{(t,s')} - \sum_{n \in \texttt{in}_s} p_{(t,s,n)} \cdot q_{(t,s,n)}. \tag{8c}$$

This states that the revenue comes from other sectors' taxes ($T_{(t,s')}$) and interests on its net wealth; this is used to finance its consumption ($q_{(t,s,n)}$).

Even though it may seem odd at first, we also include the potential for the government to tax itself. Thus, as we did earlier, we also define the total transfer from the government to itself:

$$T_{(t,s)} = \sum_{n \in \texttt{in}_s} \tau^D_{(t,s,n)} \cdot q_{(t,s,n)}. \tag{8d}$$

Finally, we need to close the government in two ways: (1) What determines the *level* of government consumption ($q_{(t,s,GC)}$) and (2) how do the government finance it? In the baseline version of the model, we assume that the level of government consumption is kept fixed over time and policy scenarios, i.e. we fix the level $q_{(t,s,GC)}$ and we assume that the government always run a balanced budget by adjusting the labor income tax rate $\tau^{s}_{(t,hh,L)}$. In other words, we include the condition:

$$v_{(t+1,s)} = v_{(t,s)}. \tag{8e}$$

7 The General Equilibrium Module

All of the modules outlined above can be run as partial equilibrium models. In general equilibrium, all equations from the various modules are combined along with the general equilibrium conditions:

$$\forall n \in \texttt{dEqui}: \qquad \sum_{s \in \texttt{dS}_n} q_{(t,s,n)} = \sum_{s \in \texttt{dD}_n} q_{(t,s,n)}, \tag{9}$$

where `dEqui` indicates the markets for which we establish an equilibrium, $\texttt{dS}_n$ indicates all sectors that supply good n, and $\texttt{dD}_n$ all sectors that demand it.

In the small open economy we consider here, the markets in `dEqui` include: (i) all domestic goods, (ii) investment goods, and (iii) labor labor.

Appendices