HA4 Report

3D Reconstruction

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Q1.1

From the epipolar constraint: $x_1'^T F x_2 = 0$, and x_1, x_2 are at the coordinate origin.

$$0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore f_{33} = 0$

Q1.2

$\because$ second camera's translation is parallel to the x-axis
$\therefore$ translation matrix: $t = [t_x, 0, 0]^T$
$\therefore$

$$t_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$\because$ Rotation matrix:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Essential matrix:

$$E = t_\times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$\therefore$ Epipolar Lines:

$$L_1^T = [x_2 \ y_2 \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [0 \ \ t_x \ \ -t_x y_2] \Rightarrow t_x y - t_x y_2 = 0$$

$$L_2^T = [x_1 \ y_1 \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix} = [0 \ \ -t_x \ \ t_x y_1] \Rightarrow t_x y - t_x y_1 = 0$$

$\therefore$ Line1 and Line2 are both parallel to the x-axis.

Q1.3

We assume the point in real world is $w = [u, v, w]^T$.
Then at time1: $w_1 = R_1 w + t_1$, at time2: $w_2 = R_2 w + t_2$
$\therefore w = R_1^{-1}(w_1 - t_1)$
$w_2 = R_2 R_1^{-1}(w_1 - t_1) + t_2 = R_2 R_1^{-1} w_1 - R_2 R_1^{-1} t_1 + t_2$
$\therefore R_{rel} = R_2 R_1^{-1}$
$t_{rel} = -R_2 R_1^{-1} t_1 + t_2$
$F = (K^{-1})^T E K^{-1} = (K^{-1})^T t_{rel\times} R_{rel} K^{-1}$

Q1.4

We assume point p and p' which are original point and its reflection point in the mirror. Suppose point p's coordinates on image1 and image2 and p''s coordinates on image1 and image2 are x_1, x_2, x_1', x_2' repectively.

$\therefore \tilde{x_1}^T F \tilde{x_1} = 0 \Rightarrow \tilde{x_1}^T F^T \tilde{x_2} = 0$

$\because$ image2 is the reflection image1 in a plane mirror

$\therefore M_2 = TM_1, T^T T = I, T$ is a transition matrix of reflection

$\because \tilde{x_1} = KM_1\tilde{p}, \tilde{x_2} = KTM_1\tilde{p}, \tilde{x_2}^T F \tilde{x_1} + \tilde{x_1}^T F^T \tilde{x_2} = 0$

$\Rightarrow \tilde{p}^T M_1^T (T^T K^T F K + K^T F^T K T) M_1 \tilde{p} = 0$

$\Rightarrow T^T K^T F K + K^T F^T K T = 0$

$\Rightarrow K^T (F + F^T) K = 0$

$\therefore F + F^T = 0 \Rightarrow F = -F^T$

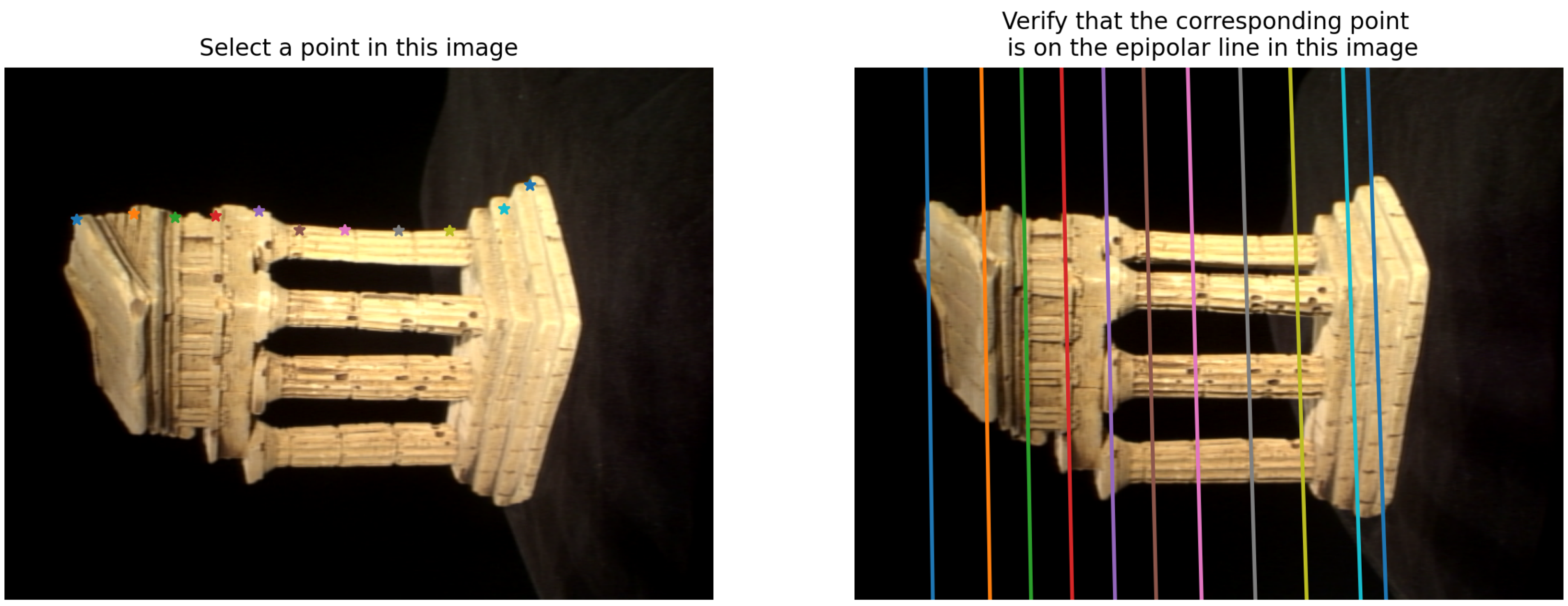
So the fundamental matrix is skew-symmetric.

Q2.1

- Function:

$$F = eightpoint(pts1, pts2, M)$$

```
F=[[ 9.78833288e-10 -1.32135929e-07 1.12585666e-03]
[-5.73843315e-08 2.96800276e-09 -1.17611996e-05]
[-1.08269003e-03 3.04846703e-05 -4.47032655e-03]]
```



Q2.2

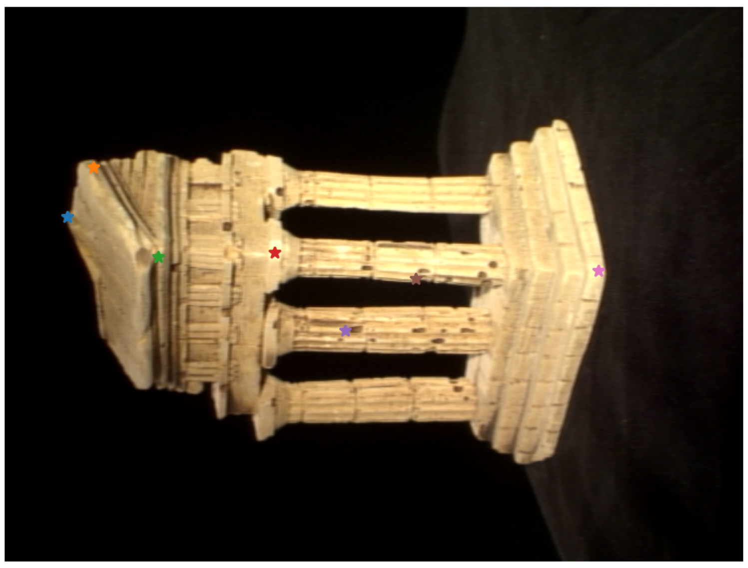
- Fucntion:

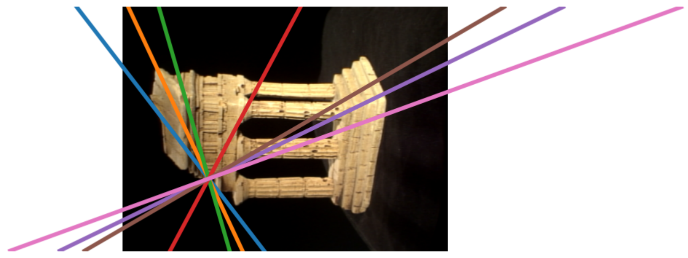
$$Farray = sevenpoint(pts1, pts2, M)$$

```
[[[ 2.63175907e-08 3.04551042e-06 -9.62248443e-04]
[-2.54572685e-06 2.53415440e-07 7.09171846e-04]
[ 7.96232035e-04 -1.02029740e-03 7.29983586e-02]]

[[-1.11942873e-07 1.87735783e-06 -6.08578982e-04]
[-1.88861222e-06 1.34760459e-08 3.11219438e-04]
[ 6.55900434e-04 -3.21241197e-04 -2.30968811e-03]]

[[-1.30664781e-07 1.71917779e-06 -5.60688451e-04]
[-1.79963205e-06 -1.90142486e-08 2.57332535e-04]
[ 6.36898072e-04 -2.26581702e-04 -1.25071825e-02]]]
```





Q3.1

Fucntion:

$$E = essentialMatrix(F8, K1, K2)$$

```
E = [[ 2.26268685e-03 -3.06552495e-01  1.66260633e+00]
 [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
 [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

Q3.2

$\because C_1 = K_1 M_1 = K_1[I|0]$
$\therefore C_1 \tilde{w_i} = \lambda_{i1} \tilde{x_{i1}}$
$\because C_2 = K_2 M_2 = K_2[R|t]$
$\therefore C_2 \tilde{w_i} = \lambda_{i2} \tilde{x_{i2}}$
$C_1 = [C_{ij}^{(1)}], 1 \leq i \leq 3, 1 \leq j \leq 4$
$C_2 = [C_{ij}^{(2)}], 1 \leq i \leq 3, 1 \leq j \leq 4$
$\tilde{w_i} = [x_i, y_i, z_i, 1]^T, \tilde{x_{i1}} = [x_{i1}, y_{i1}, 1]^T, \tilde{x_{i2}} = [x_{i2}, y_{i2}, 1]^T$
$P_i = [x_i, y_i, z_i]^T, A_i P_i = 0 \Rightarrow A\tilde{w_i} = 0$
$\therefore A = [[c_{11}^{(1)} - c_{31}^{(1)} x_{i1}, c_{12}^{(1)} - c_{32}^{(1)} x_{i1}, c_{13}^{(1)} - c_{33}^{(1)} x_{i1}, c_{14}^{(1)} - c_{34}^{(1)} x_{i1}], [c_{21}^{(1)} - c_{31}^{(1)} y_{i1}, c_{22}^{(1)} - c_{32}^{(1)} y_{i1}, c_{23}^{(1)} - c_{33}^{(1)} y_{i1}, c_{24}^{(1)} - c_{34}^{(1)} y_{i1}], [c_{11}^{(2)} - c_{31}^{(2)} x_{i2}, c_{12}^{(2)} - c_{32}^{(2)} x_{i2}, c_{13}^{(2)} - c_{33}^{(2)} x_{i2}, c_{14}^{(2)} - c_{34}^{(2)} x_{i2}], [c_{21}^{(2)} - c_{31}^{(2)} y_{i2}, c_{22}^{(2)} - c_{32}^{(2)} y_{i2}, c_{23}^{(2)} - c_{33}^{(2)} y_{i2}, c_{24}^{(2)} - c_{34}^{(2)} y_{i2}]]$

- Performance check:

$$err = \sum_i ||p_{i1}, \hat{p_{i1}}||^2 + ||p_{2i}, \hat{p_{2i}}||^2$$

- Function:

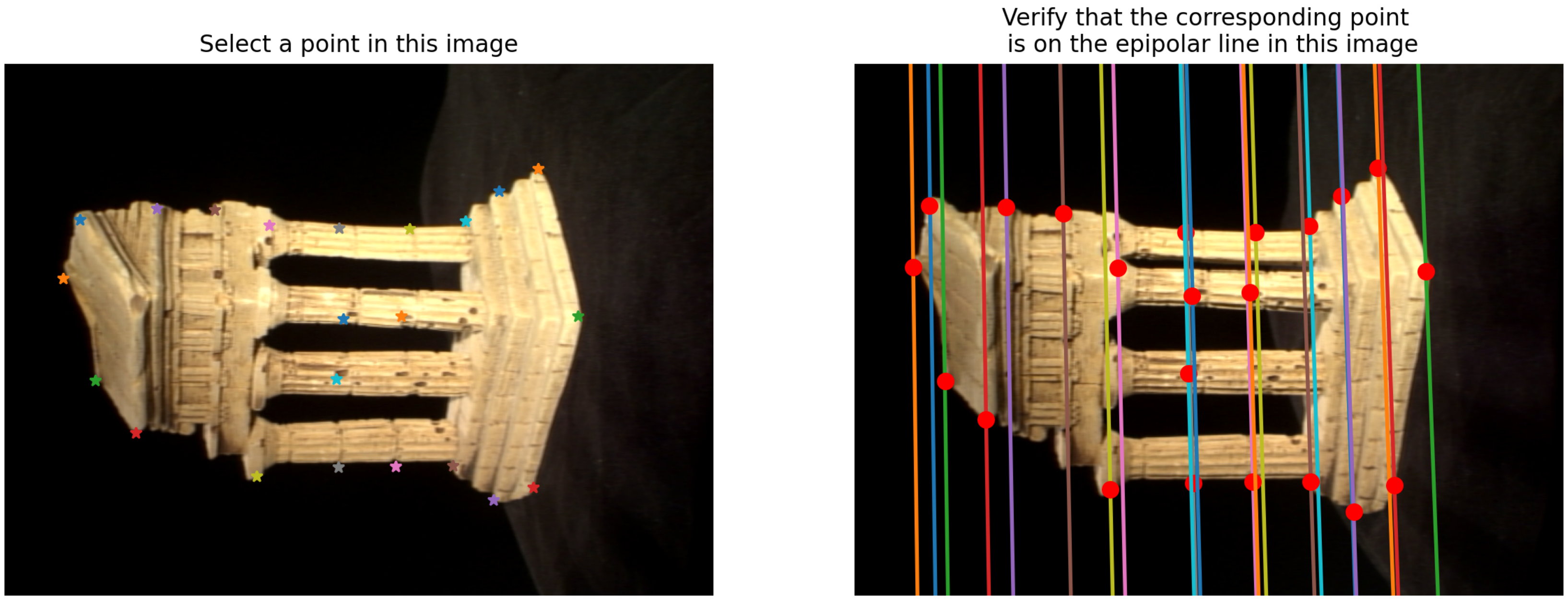
$$[w, err] = triangulate(C1, pts1, C2, pts2)$$

Q4.1

- Function:

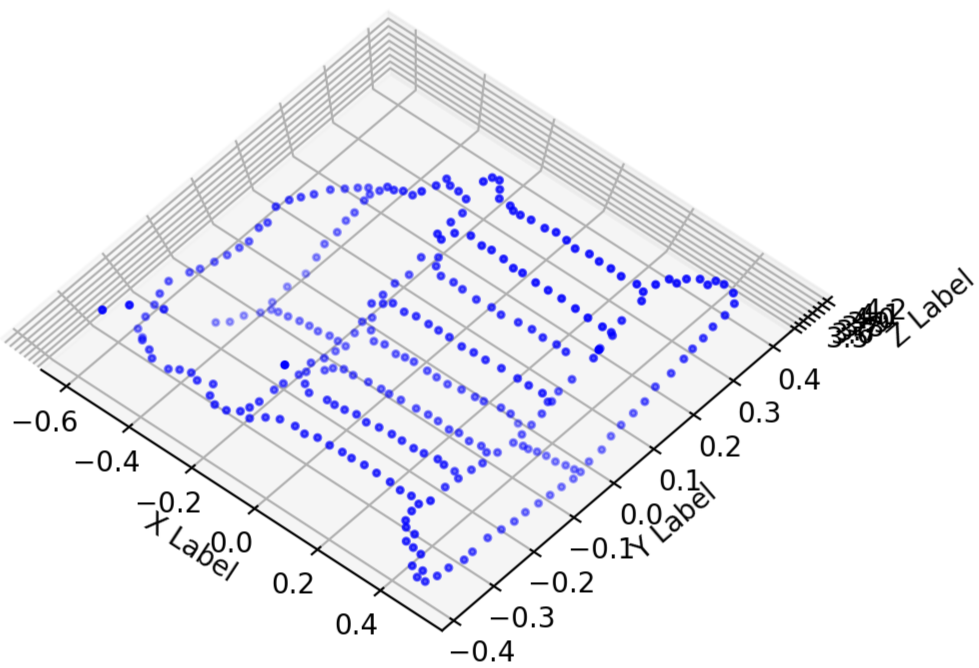
$$[x2, y2] = epipolarCorrespondence(im1, im2, F, x1, y1)$$

- This function is implemented to estimate the coordinates of the pixel on im2 which correspondence to the input on im1

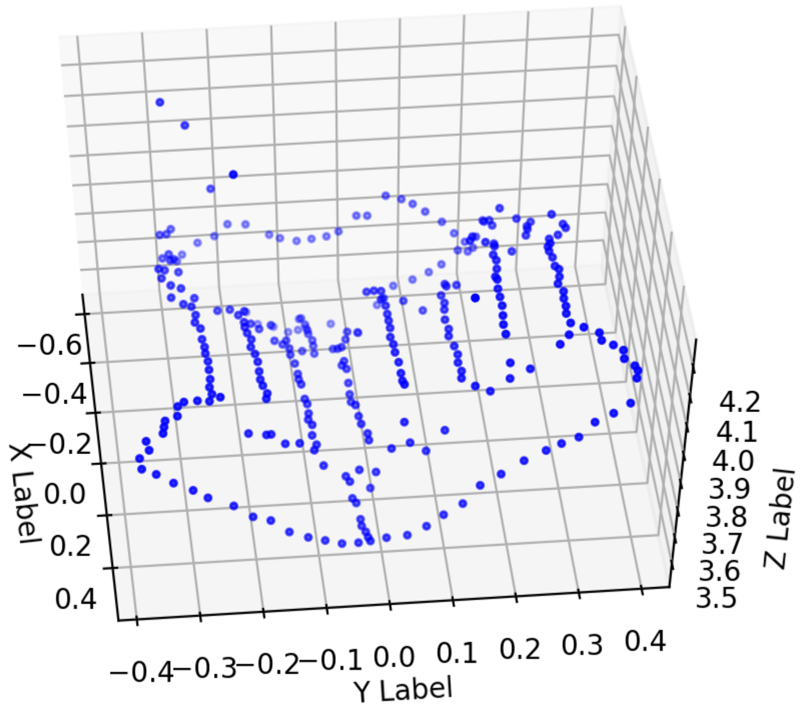


Q4.2

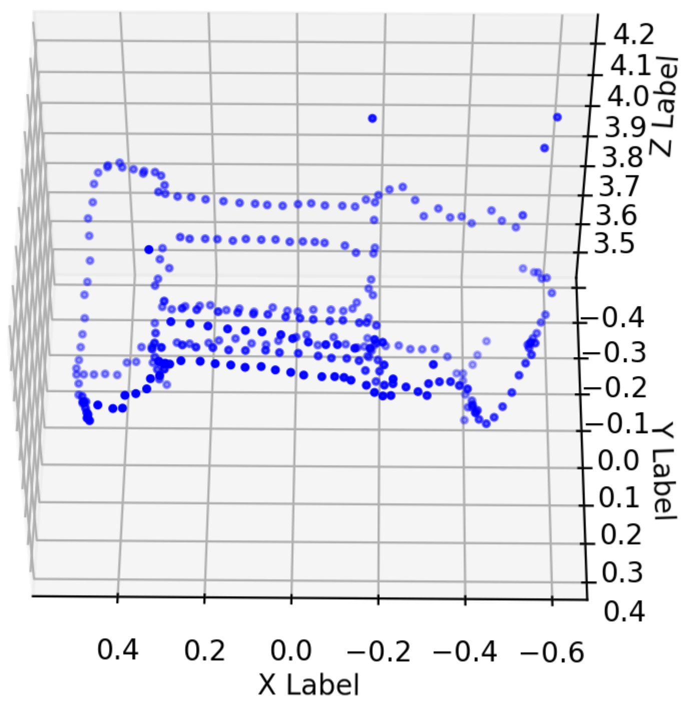
- This part is finished in `visualize.py`, which is used to generate the 3D reconstruction using $F, M1, M2, C1, C2$.
- Fig1:



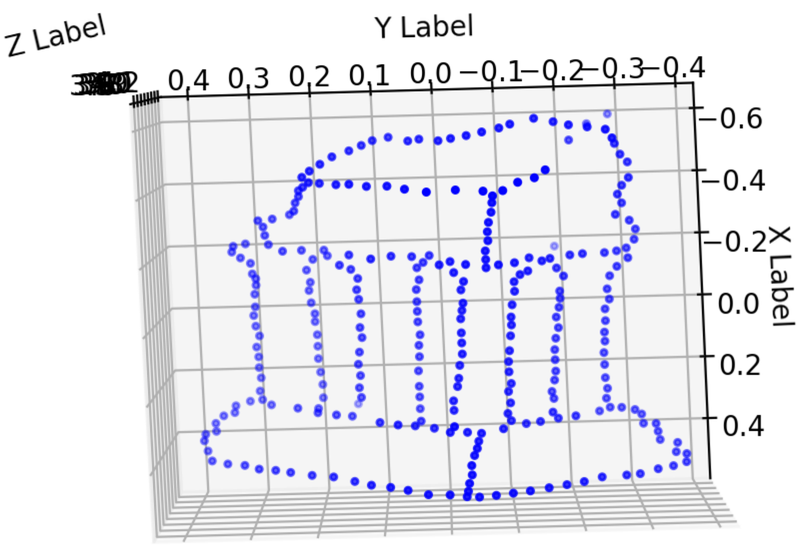
- Fig2:


X Label
Y Label
Z Label

- Fig3:


X Label
Y Label
Z Label

- Fig4:

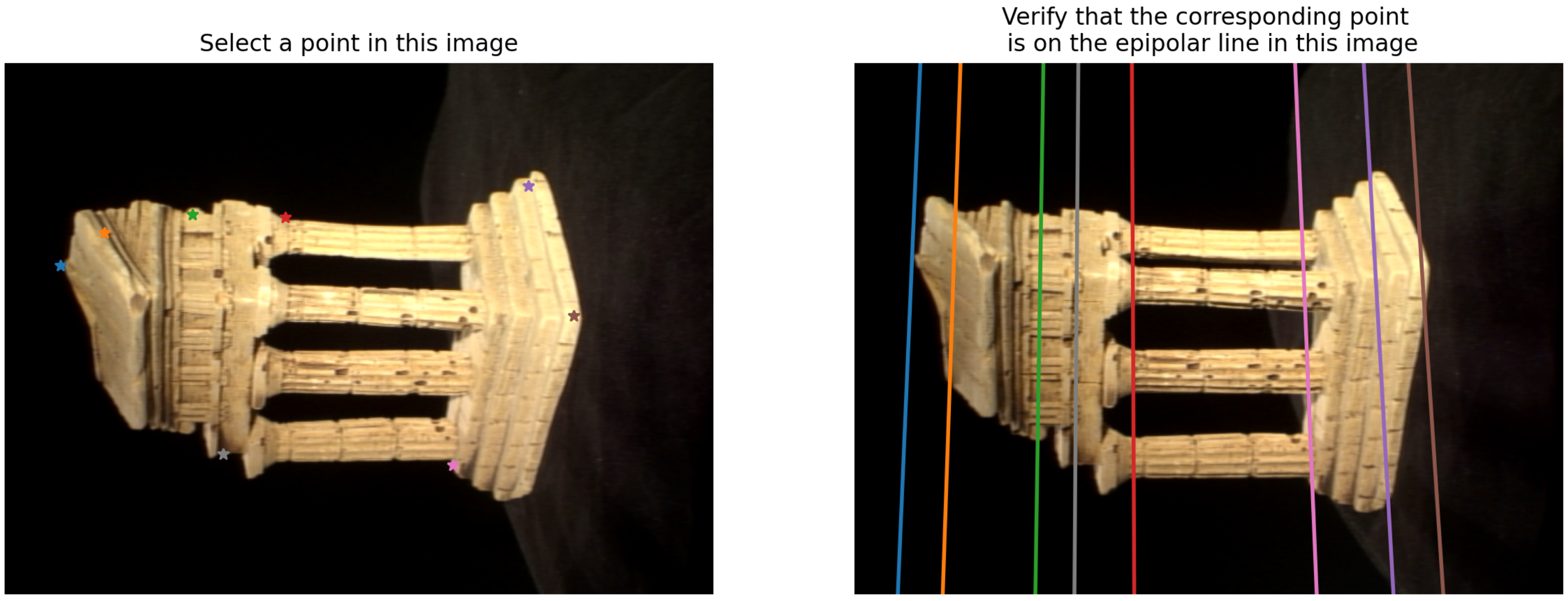

Z Label
Y Label
X Label

Q5.1

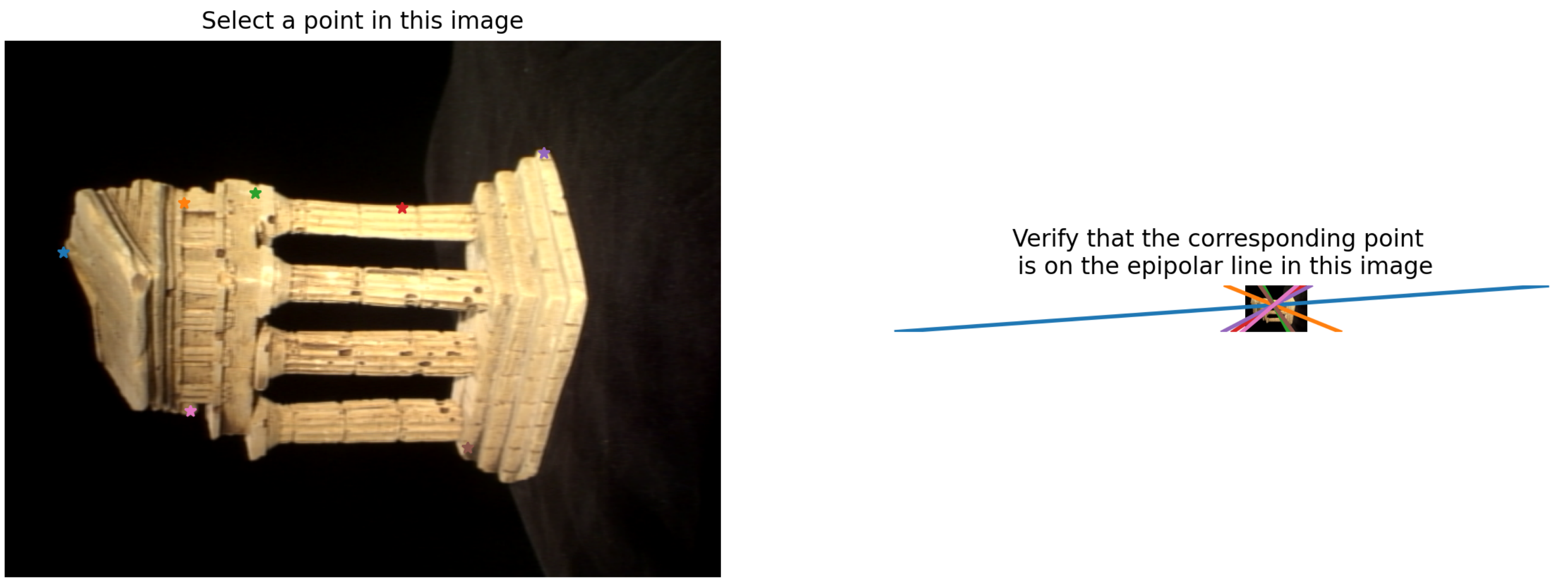
- Function:

$$[F, inliers] = ransacF(pts1, pts2, M)$$

- Using RANSAC:



- Using eightpoint:



Q5.2

- Function:

$$R = rodrigues(r)$$

- Function:

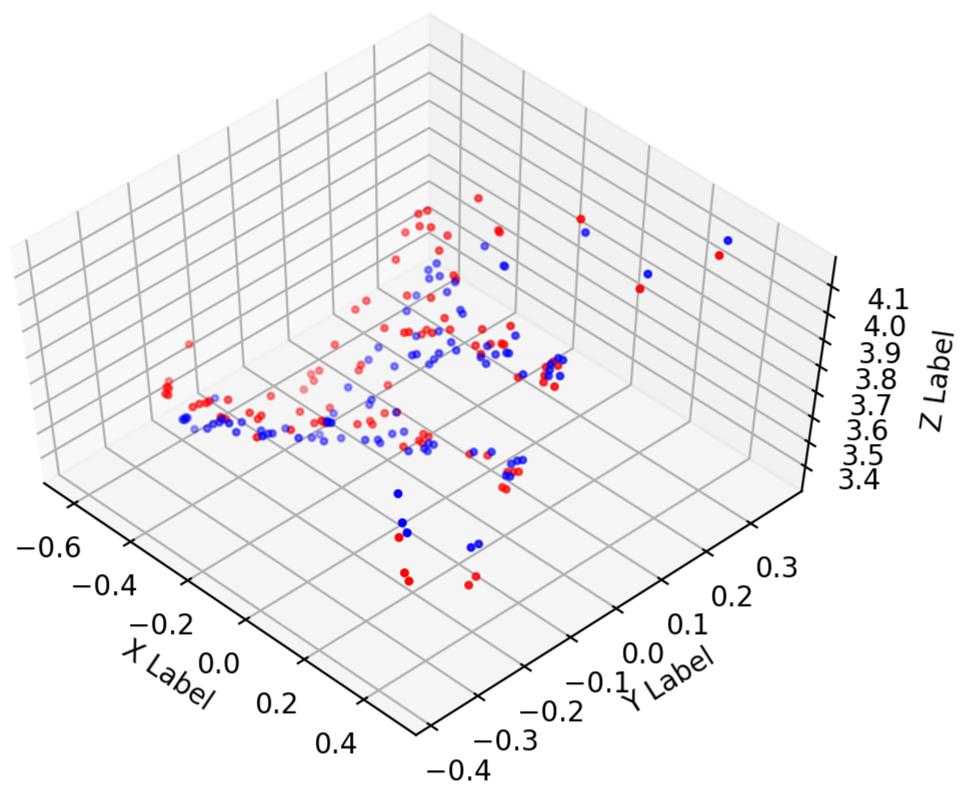
$$r = invRodrigues$$

Q5.3

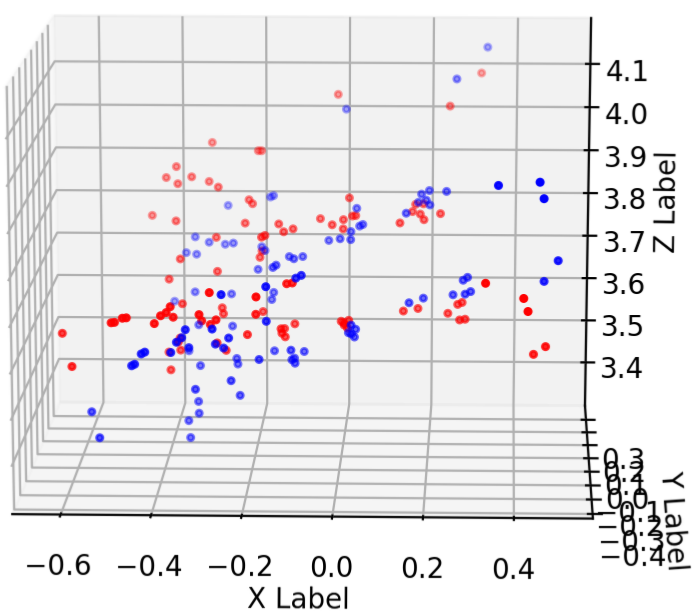
- Function:

$$M2, P = bundleAdjustment(K1, M1, p1, K2, M2, M2_{i}nit, p2, P_{i}nit)$$

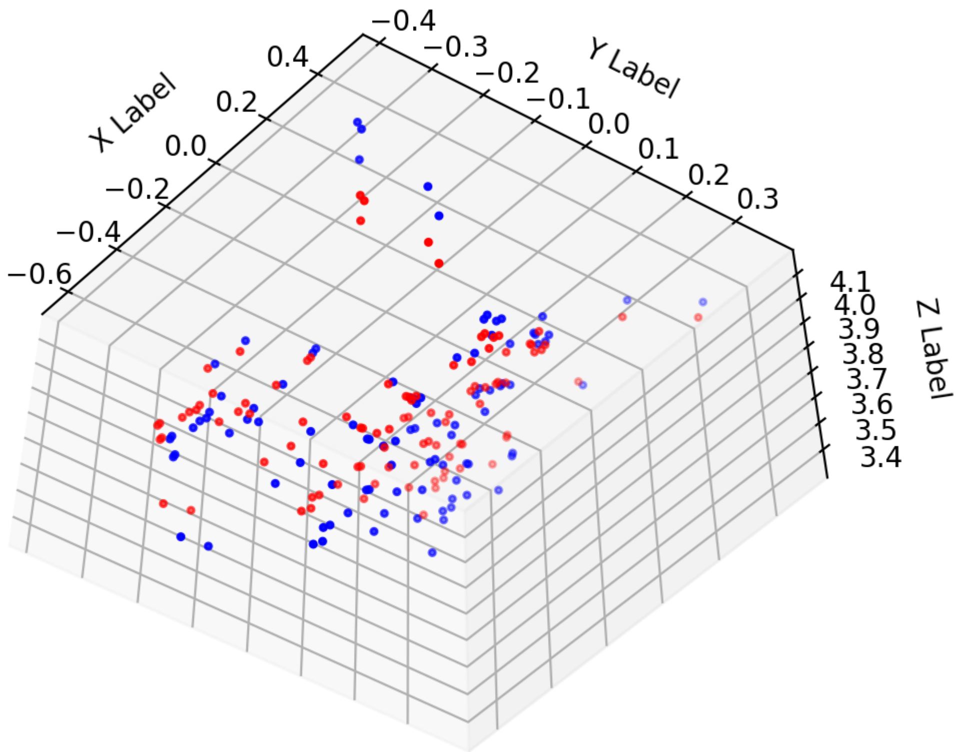
- Optimized matrices:
- Fig1:



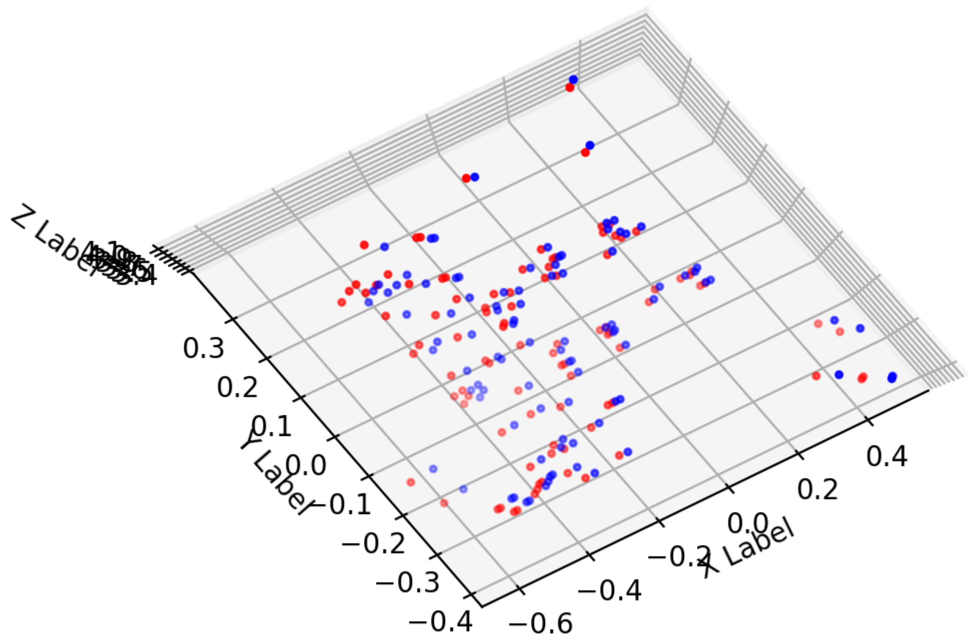
- Fig2:



- Fig3:



- Fig4:



- Reprojection error with initial M_2 and P: 15194.853165587034
- Reprojection error with the optimized matrices: 10.39769714618452