Integrating Graphical Proofs in Coq

A description of the `coq-actema` system

Pablo Donato Benjamin Werner Kaustuv Chaudhuri

Partout team — LIX

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Boston

Context

Goal: Make proof assistants *easier* to use

- **Intuitive** and **discoverable** for newcomers
- **Productive** and **beautiful** for experts

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Disclaimer: WIP, still at an experimental stage...

Graphical Proofs

coq-actema

“A demo is worth a thousand words…”

Paradigm

- Fully graphical: no textual proof language
- Both spatial and temporal:

$$\text{proof} \quad = \quad \text{gesture sequence}$$

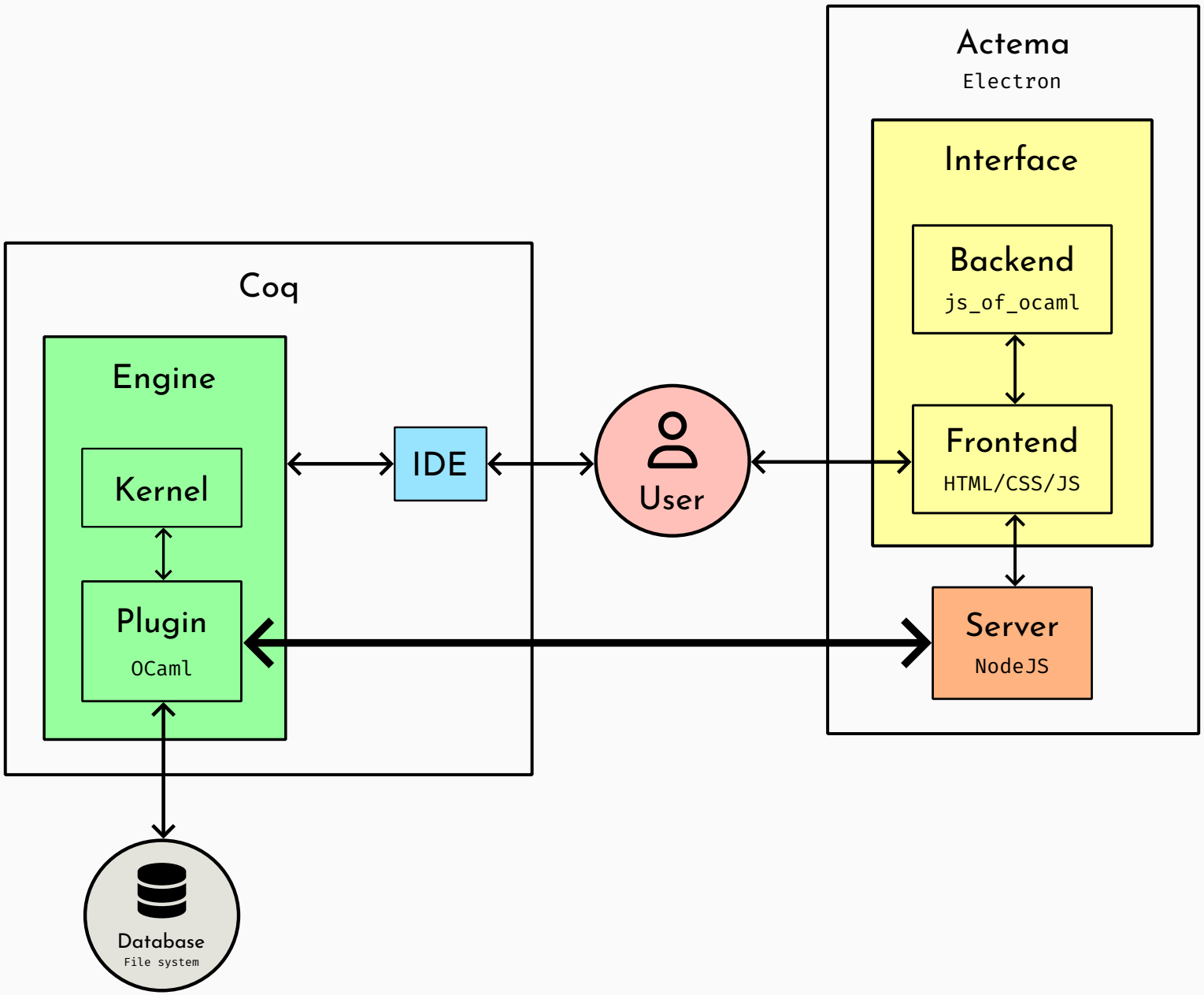
- **Different modes** of reasoning with a **single "syntax"**:

$$\begin{aligned} \text{Click} &\iff \text{introduction/elimination} \\ \text{Drag-and-Drop} &\iff \text{backward/forward} \end{aligned}$$

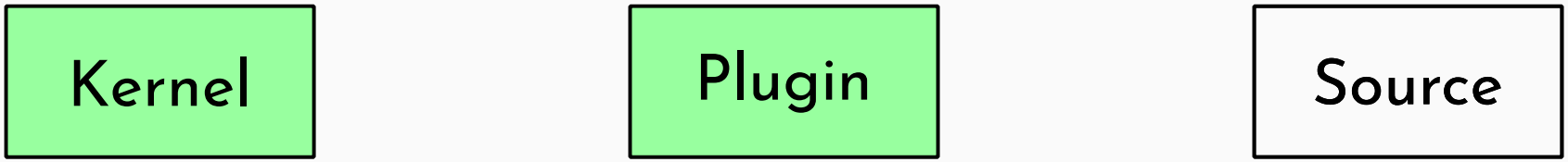
*Sound and **complete** for iFOL!*

Integration with Coq

Architecture



Protocol



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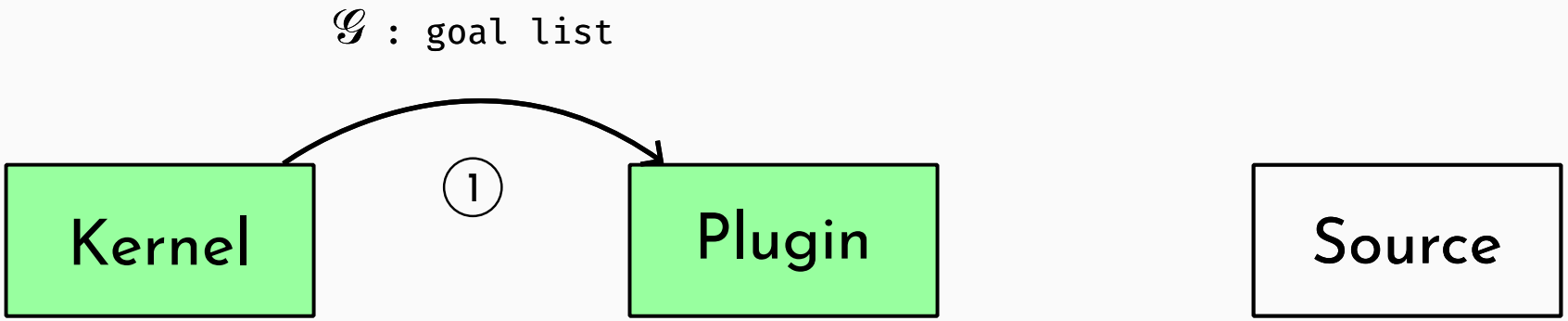

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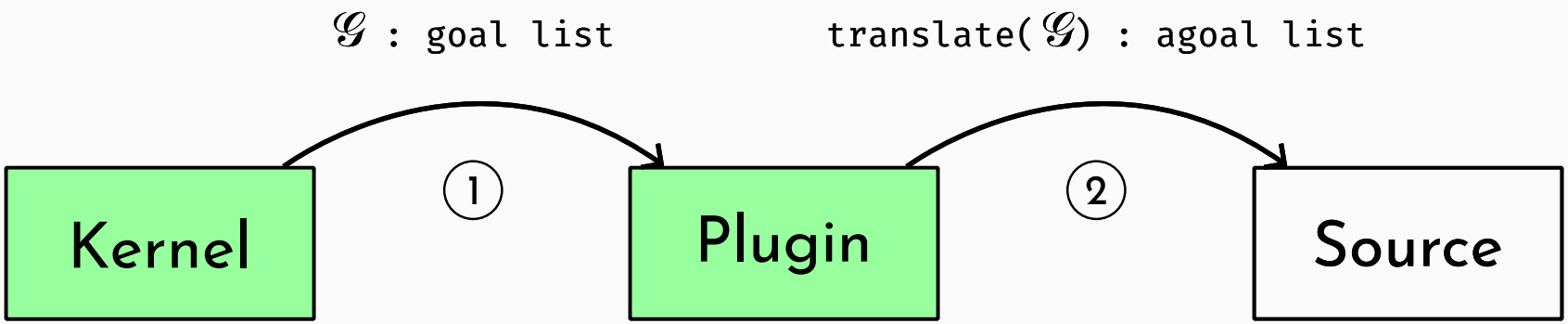
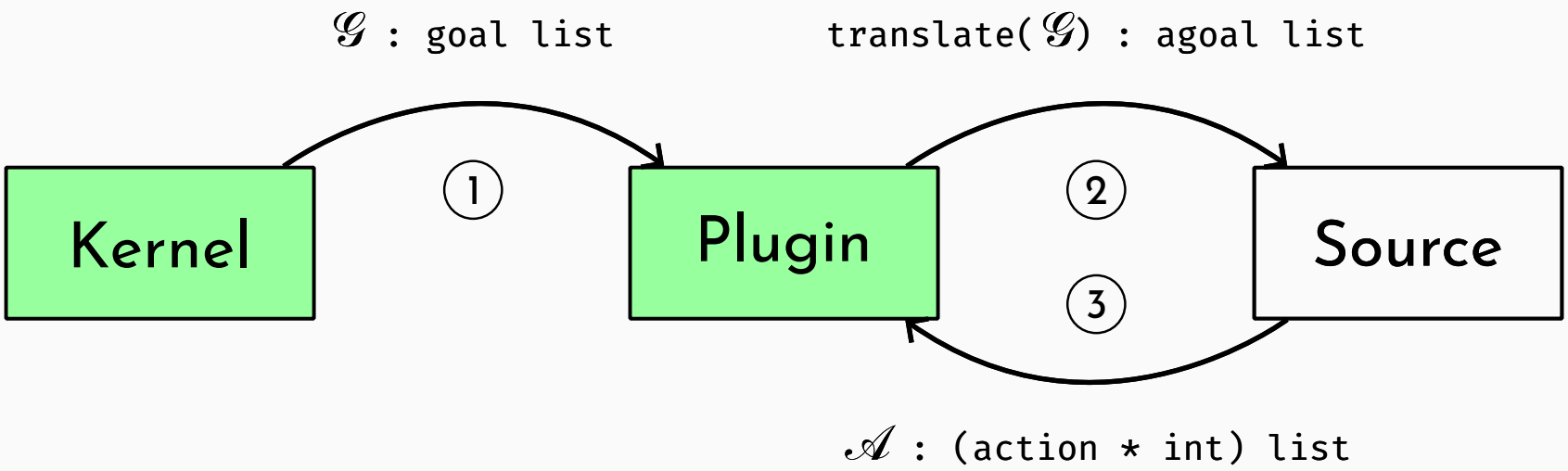

$\mathscr{G}$: goal list
translate($\mathscr{G}$) : agoal list
Kernel
1
Plugin
2
Source

Protocol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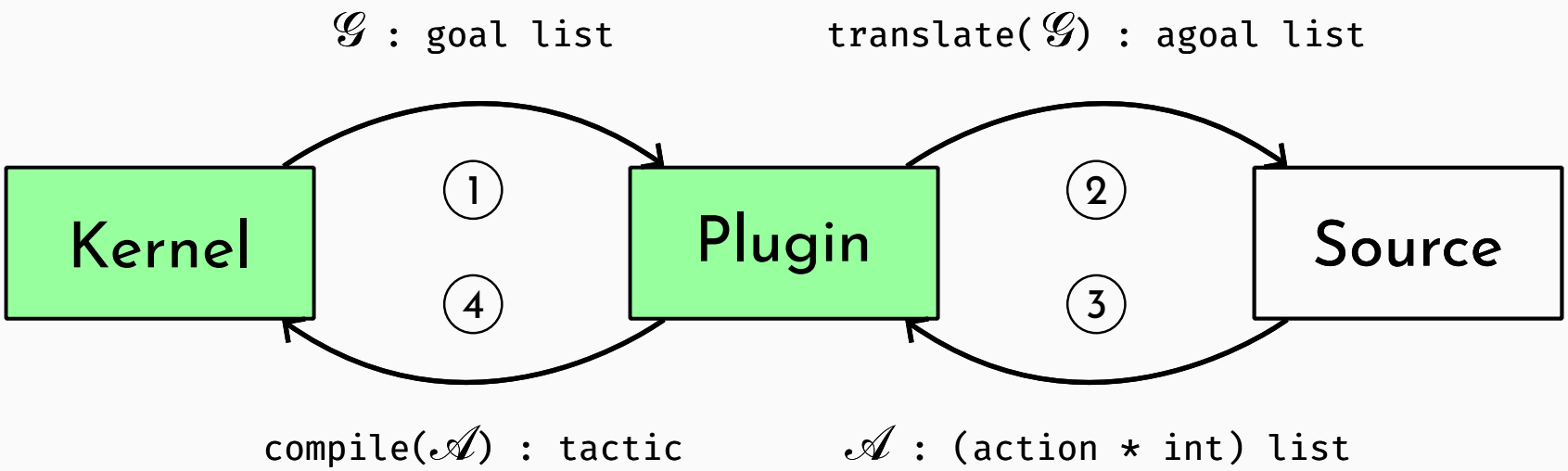
$\mathscr{G}$: goal list
translate($\mathscr{G}$) : agoal list
Kernel
1
4
Plugin
2
3
Source
compile($\mathscr{A}$) : tactic
$\mathscr{A}$: (action * int) list

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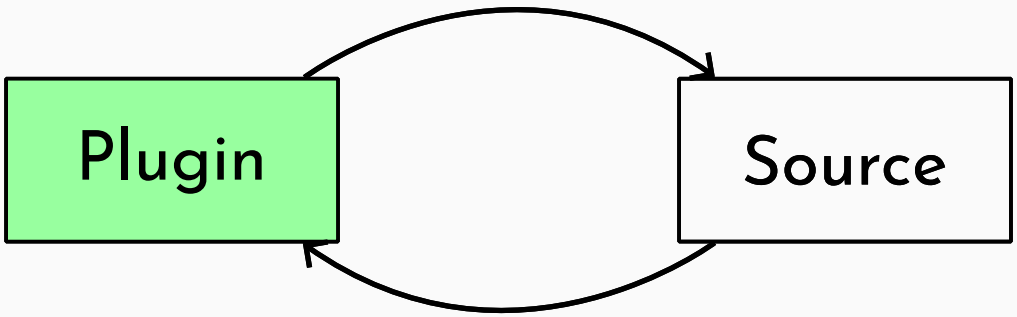
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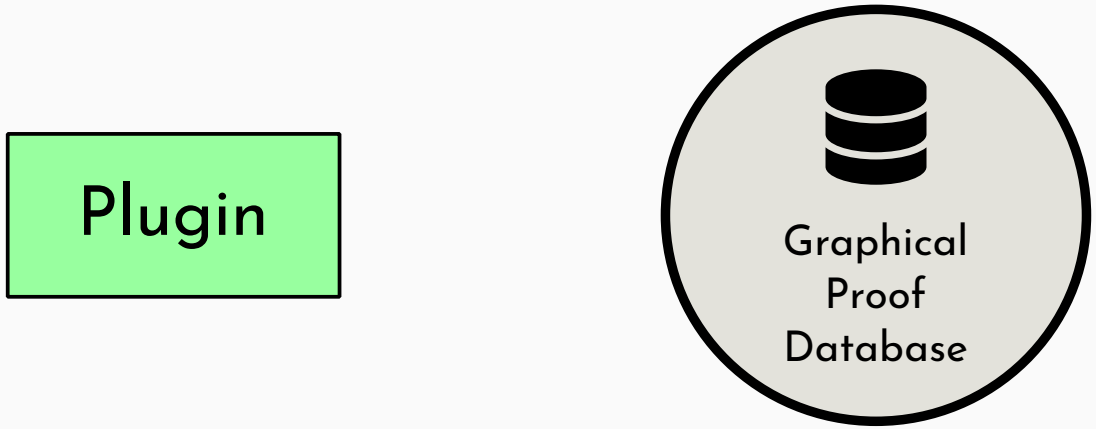
Protocol (non-interactive)



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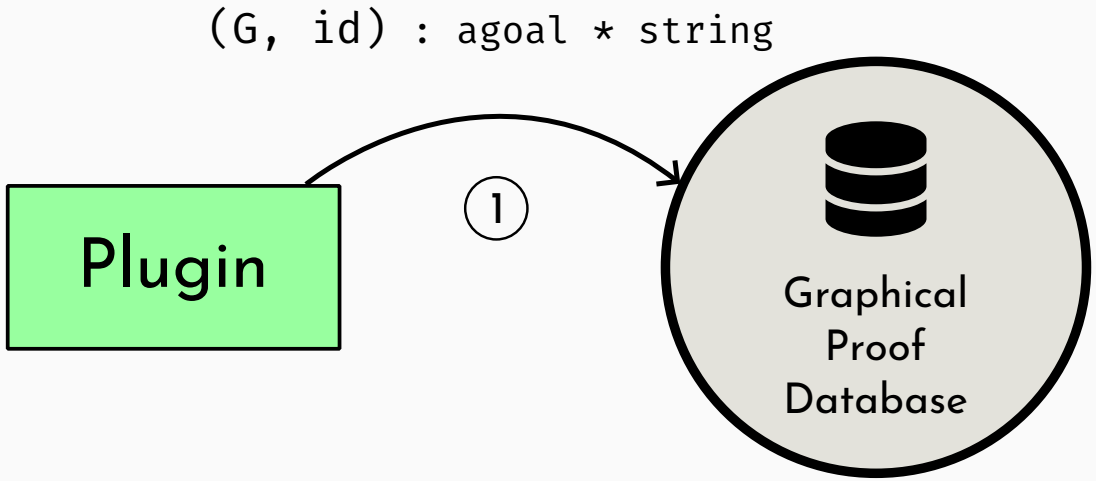

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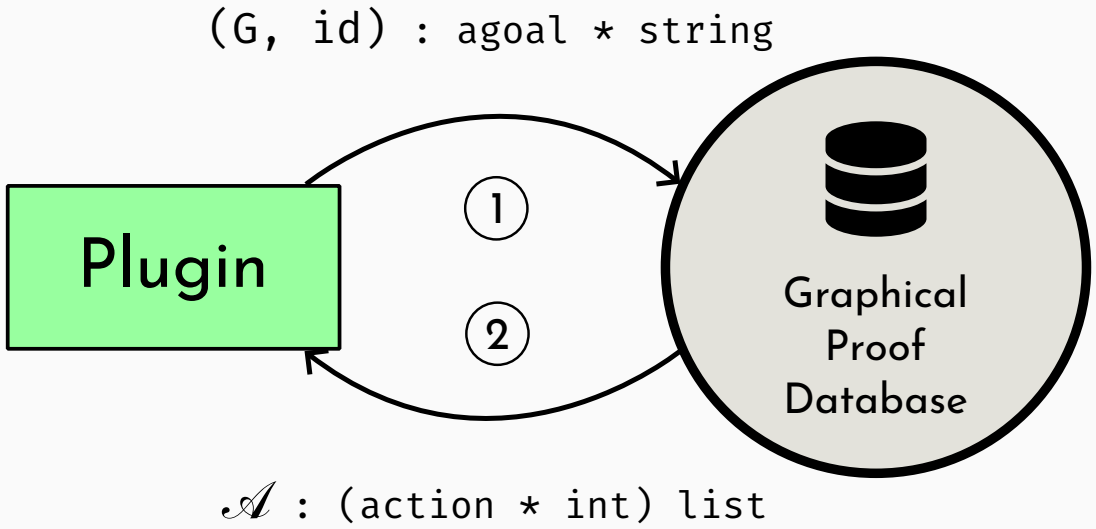

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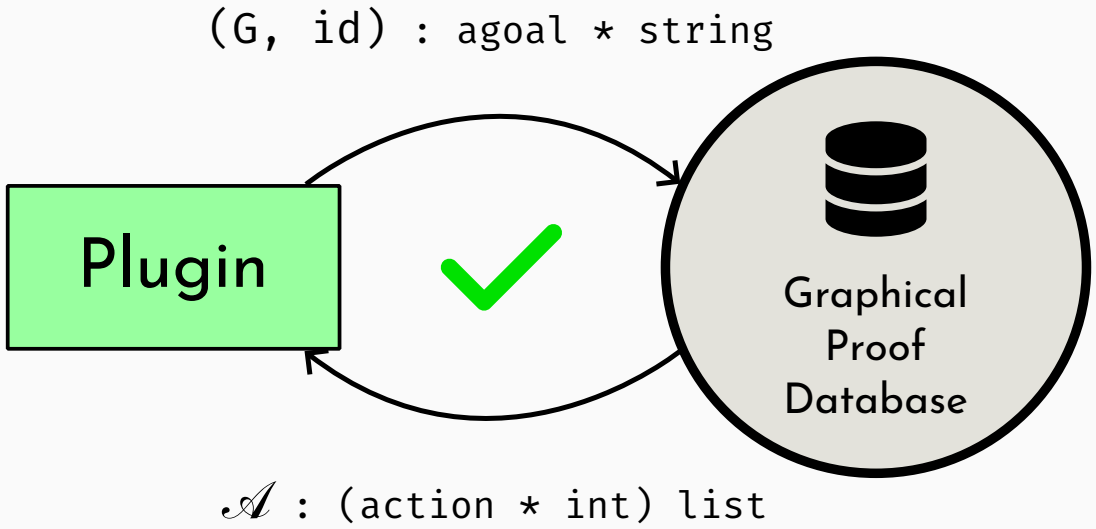

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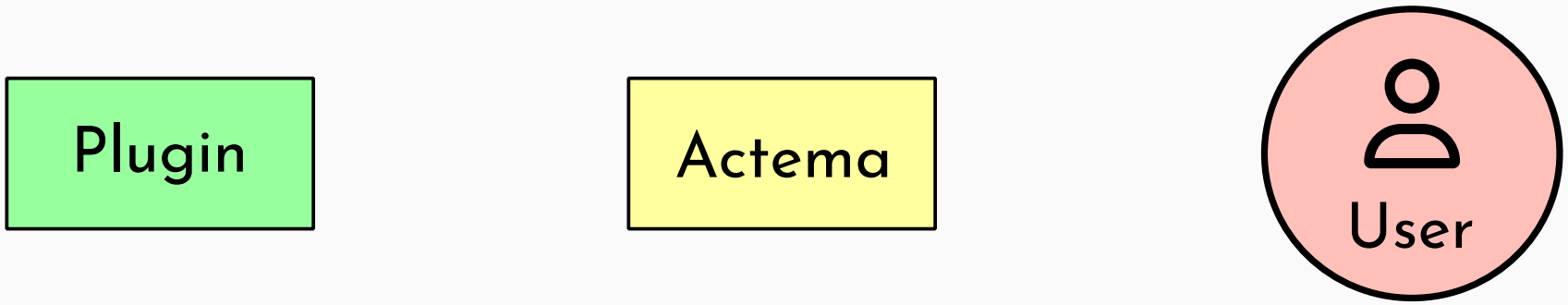

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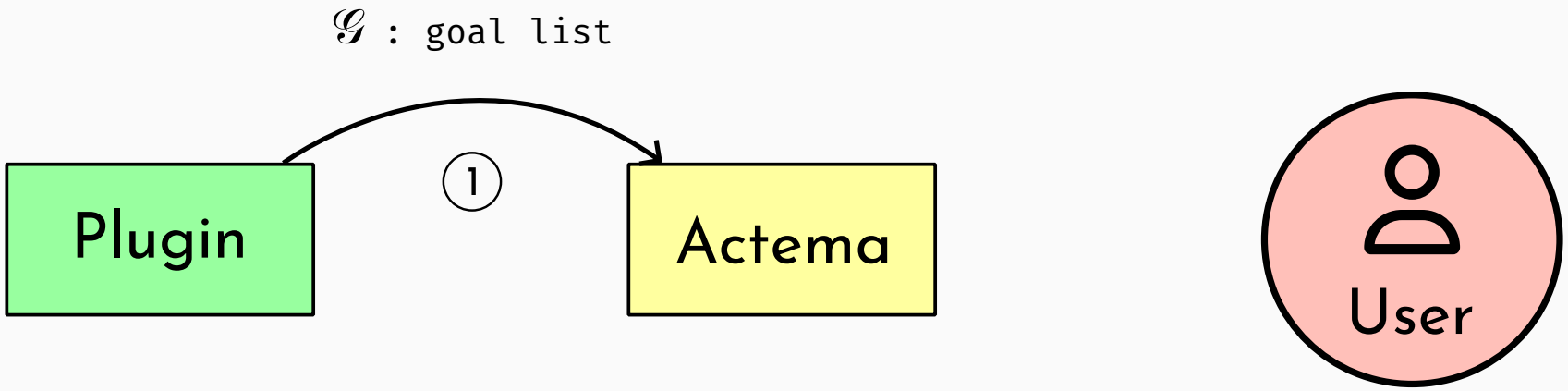

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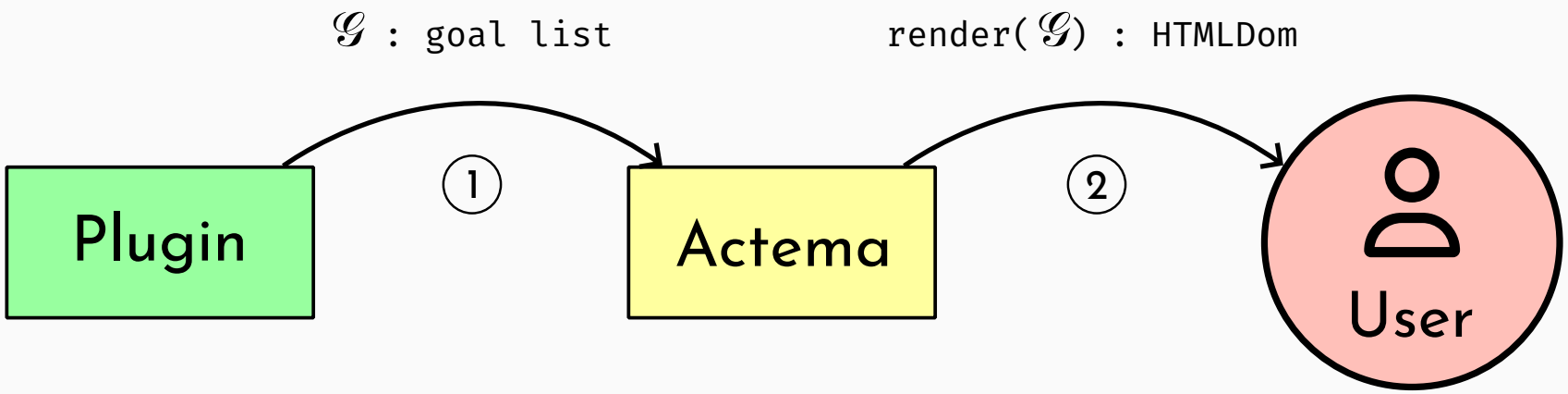

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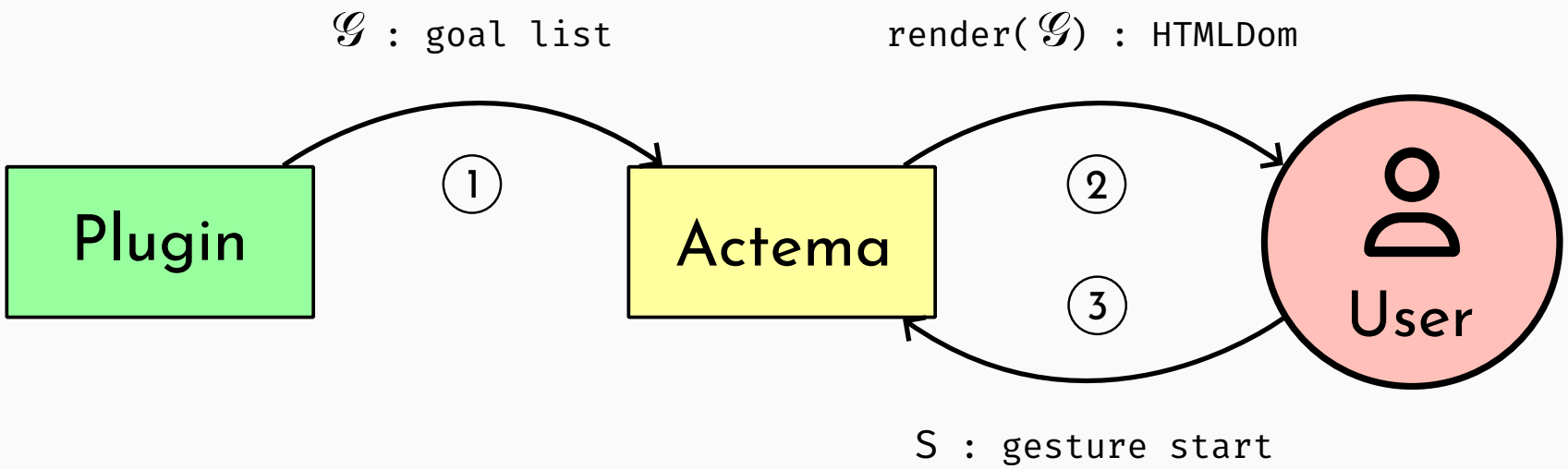

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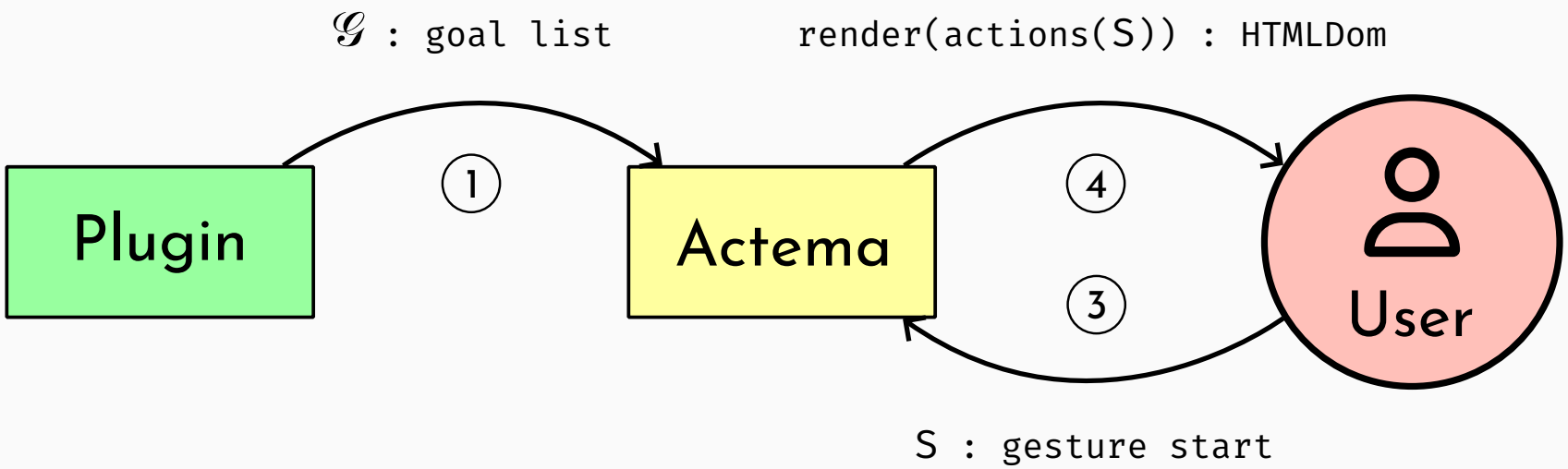

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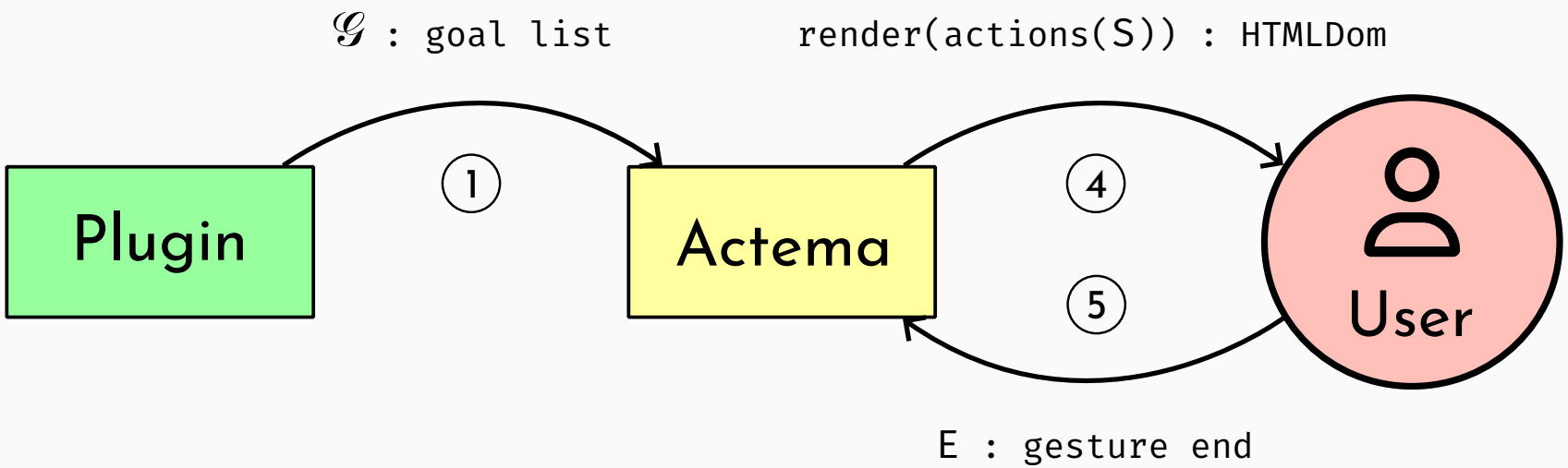

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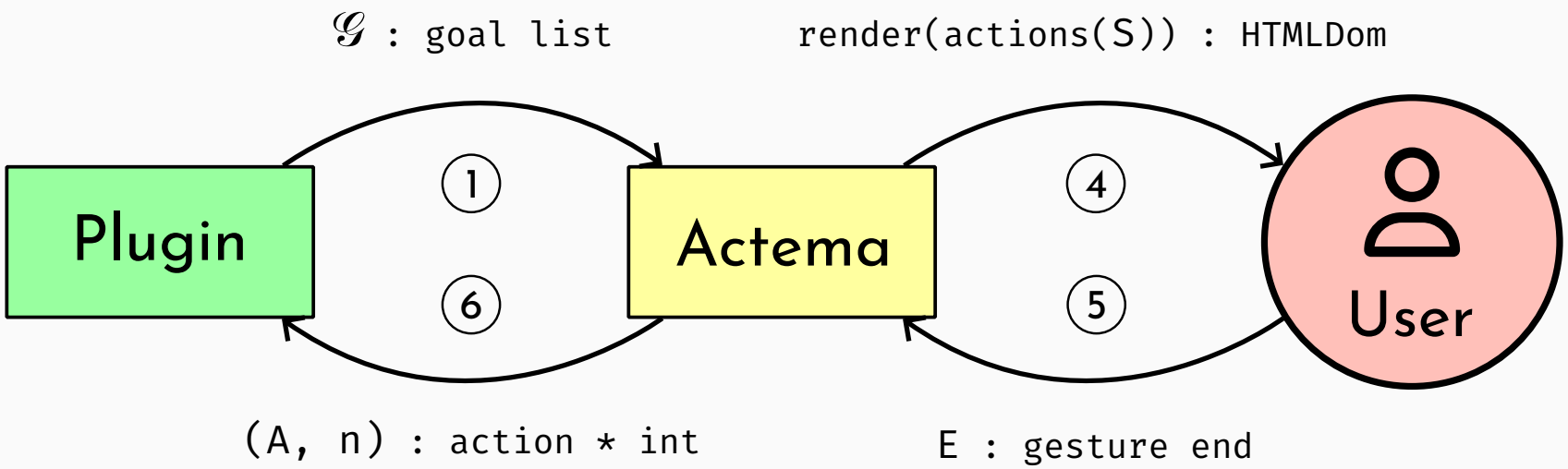

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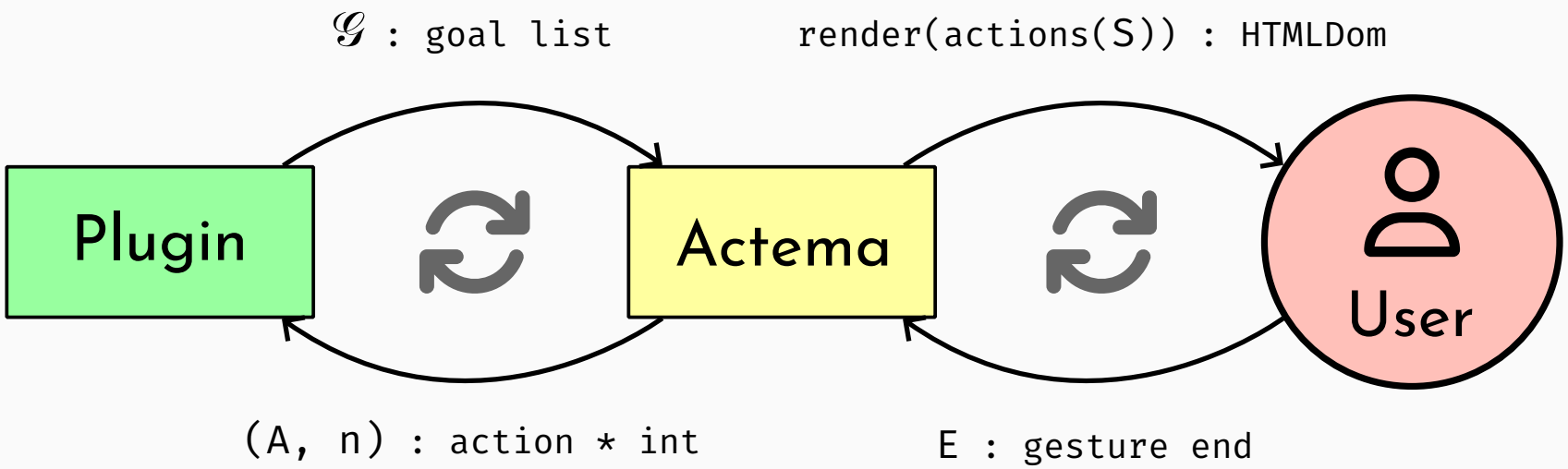

Compiling actions (B. Werner, K. Chaudhuri)

- **Click** actions: standard Coq tactics
- **Drag-and-Drop** actions: $\sim$ 3000 lines of Coq/Ltac
  - **Deep embedding** of goal $\Gamma \vdash C$ in FOL
  - Subterm selection as **paths**, i.e. `list nat`
  - **Computational reflection** for *deep inference* semantics [Donato et al. (2022)]
    - Backward: new conclusion C'
    - Forward: new hypothesis A
  - Final tactic = apply **soundness** theorem
    - Backward: $\Gamma \Rightarrow C' \Rightarrow C$
    - Forward: $\Gamma \Rightarrow A$

Conclusion

FAQ

What are the most useful usecases of Actema?

- Proof **exploration**
- **Educational** setting

FAQ

What were the infrastructure challenges/solutions?

- Interaction protocol that can handle **arbitrary goals and tactics** (still a WIP, because of FOL and notations)
- Generic protocol **independent of the specifics of Coq** (simpler with FOL)
- **Portable API with reusable boilerplate** for serialization on both sides (atdgen)
- **Linking external libraries** in Coq plugin, for serialization/HTTP (currently falls out of dune capabilities, need coq_makefile)

Related works (non-exhaustive)

- **Proof-by-Pointing** [Bertot et al. (1994)]
- **Subformula linking** [Chaudhuri (2013), Chaudhuri (2021)]
- **ProofWidgets** [Ayers et al. (2021)]
  - Framework for user-defined graphical notations
  - PA serves the GUI, instead of requesting from it
  - Relies on Lean's metaprogramming capabilities

Future works

For more complex theories:

- Support arbitrary **Coq notations** (and more?)
- Selection-based **lemma search**
- Extend to **HOL**

For proof evolution:

- Translate graphical proof into *readable* and *reusable* **tactic invokations** (avoid paths)
- Replay/Edit graphical proof through **animations**

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Thank you!

References

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