Deep Inference for Graphical Theorem Proving

Pablo Donato

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Institut Polytechnique de Paris, LIX, PARTOUT

PhD defense, Palaiseau

Supervised by Benjamin Werner

Introduction

Logic

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- Hidden assumptions ⇒ lack of **certainty**

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↳ Formal essence of logical reasoning

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- **Proof theory:** design & study of *rule systems* capturing maths

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- Problem: maths is *hard* ⇒ need for a **human** in the loop

↳ Interactive Theorem Provers (ITPs)

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💻 : “✅ OK here is the result!”

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1st contribution: build proofs by direct manipulation of *formulas*

↳ No need to *memorize* the rules

↳ No risks of *errors*

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2nd contribution: replace logical symbols by geometrical diagrams

Symbolic Manipulations

Proof-by-Action

Actema

Paradigm

Subformula Linking

Semantics of drag-and-drop

Linking under quantifiers

Rewriting equalities

Completeness

Integration with Coq

Architecture

Protocol

Compiling actions

Conclusion

- Quelques features manquantes, mais déjà utilisable
- Mathis a simplifié l'install -> n'hésitez pas à installer !
- Bientôt prêt pour évaluation sur des étudiants, voire experts

Iconic Manipulations

Symmetric Bubble Calculus

The chemical metaphor

Adding bubbles

Reducing non-determinism

Asymmetric Bubble Calculi

Coloring bubbles

Red bubbles

Blue bubbles

Classical vs. Intuitionistic

A change of viewpoint

Existential Graphs

The three systems of EGs

The three icons of Alpha

Illative transformations

Example: modus ponens

Flowers

The scroll

The n-ary scroll

Blooming

Corollaries

Gardens

Flower Calculus

Iteration and Deiteration

Insertion and Deletion

Instantiation and Abstraction

Case reasoning

Scrolling

QED

Metatheory: Nature vs. Culture

Natural rules

Cultural rules

Hypothetical provability

Cult-elimination

Flower Prover

Demo

Paradigm

Towards Curry-Howard

Thank you!