Deep Inference for Graphical Theorem Proving

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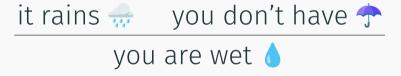
PhD defense, Palaiseau

Supervised by Benjamin Werner

Introduction

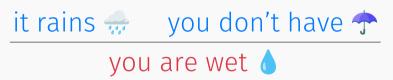
Study of sound reasoning

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- Example of everyday life deduction:



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premisses
conclusion



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you are wet •
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Hidden assumptions ⇒ lack of certainty

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Socrates is human All humans are mortal Socrates is mortal

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→ Formal essence of logical reasoning

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Generic patterns of deduction as rules

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- Generic patterns of deduction as rules
- Formalist school (Hilbert 20th century):

Maths as a huge game

Goal: to prove theorems by following rules

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Proof theory: design & study of rule systems capturing maths

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- · Computers very good at manipulating symbols and following rules
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- Problem: maths is $hard \Rightarrow$ need for a **human** in the loop
 - → Interactive Theorem Provers (ITPs)

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: "Please apply this rule"

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: " OK here is the result!"

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: "X ERROR: dkfsljfjdklsfjdkfjsldjfkdlsfj"

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1st contribution: build proofs by direct manipulation of formulas

- → No need to *memorize* the rules
- → No risks of errors

Symbolic vs. Iconic

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 - ▶ learn ⇒ purely conventional meaning
 - ► manipulate ⇒ need for very precise gestures

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2nd contribution: replace logical symbols by geometrical diagrams

Symbolic Manipulations

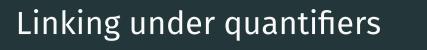
Proof-by-Action



Paradigm

Subformula Linking









Integration with Coq





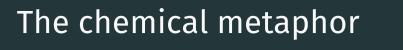


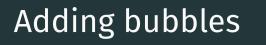
Conclusion

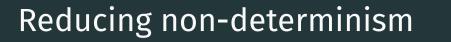
- · Quelques features manquantes, mais déjà utilisable
- Mathis a simplifié l'install -> n'hésitez pas à installer!
- · Bientôt prêt pour évaluation sur des étudiants, voire experts

Iconic Manipulations

Symmetric Bubble Calculus



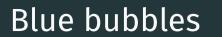


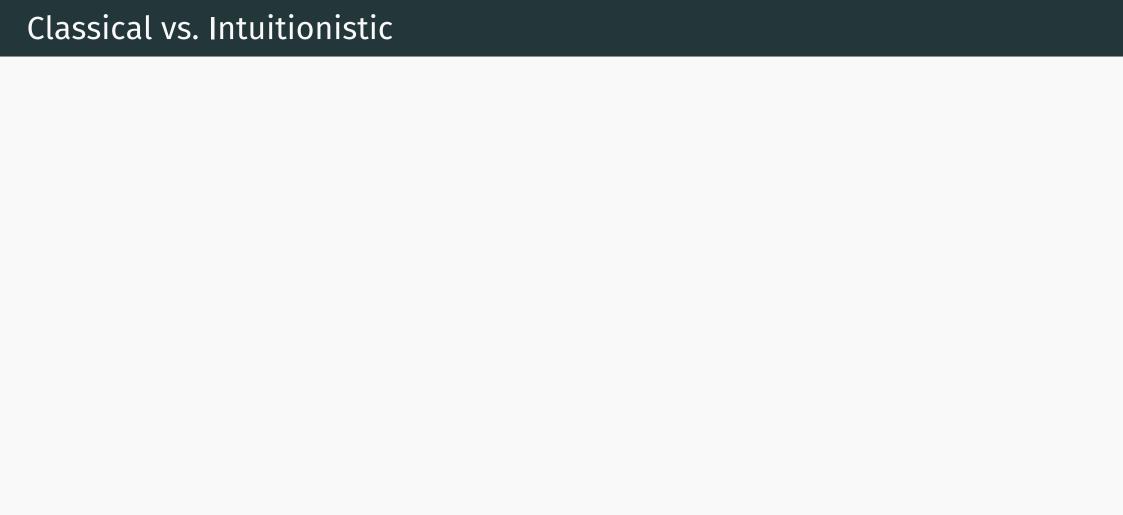


Asymmetric Bubble Calculi



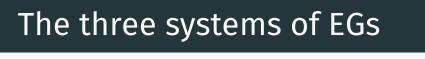
Red bubbles





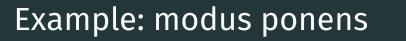


Existential Graphs



The three icons of Alpha





Flowers

The scroll

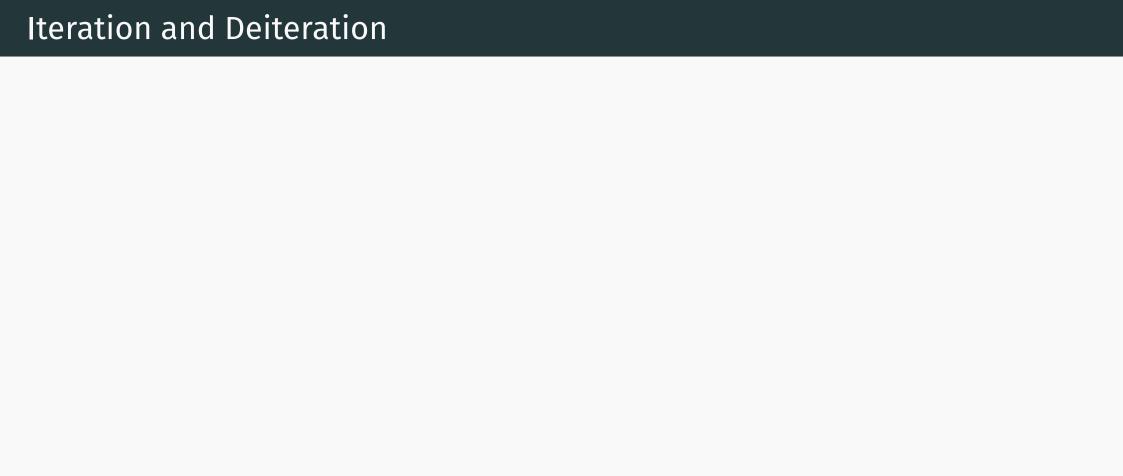
The n-ary scroll

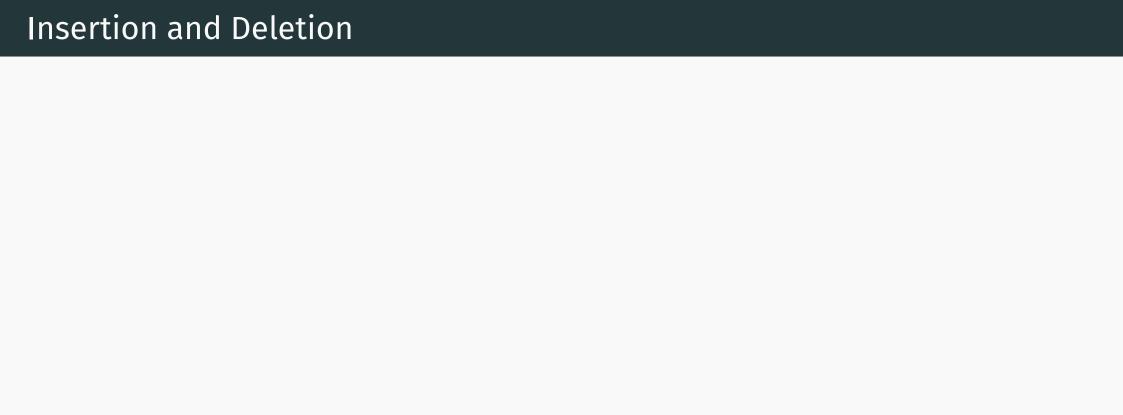
Blooming

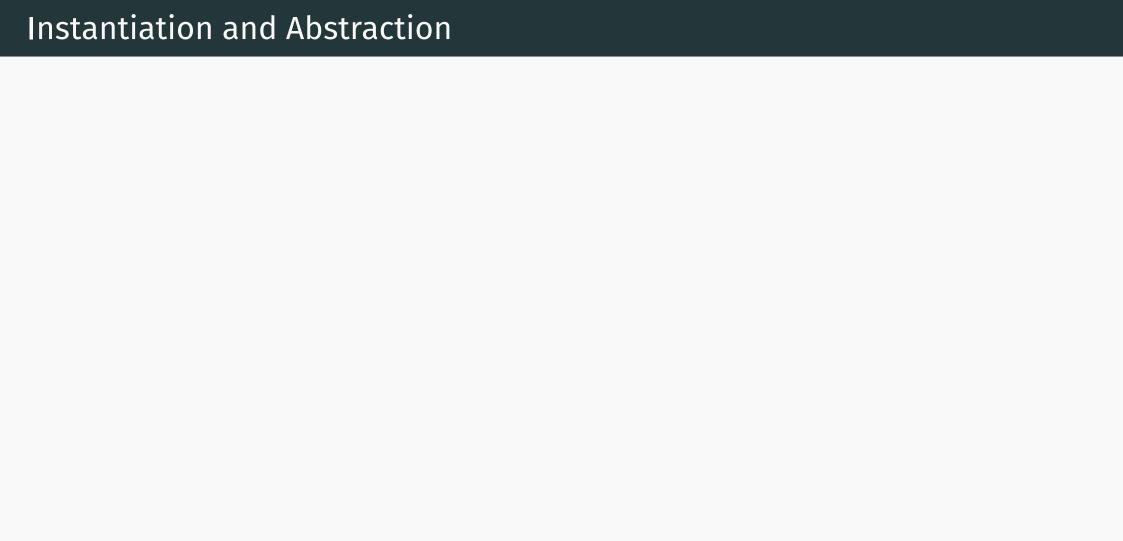




Flower Calculus





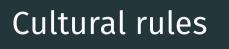






Metatheory: Nature vs. Culture

Natural rules







Flower Prover



Paradigm



Thank you!