Subformula Linking

Semantics of drag-and-drop (Chaudhuri 2013)

Idea: bring matching subformulas through switch rules

$$
\begin{aligned}
\text{switch} \left\{
\begin{array}{l}
\quad \underline{A} \wedge B \ogreaterthan B \wedge (\underline{A} \vee C) \wedge D \\
\to B \wedge (\underline{A} \wedge B \ogreaterthan (\underline{A} \vee C) \wedge D) \\
\to B \wedge (\underline{A} \wedge B \ogreaterthan \underline{A} \vee C) \wedge D \\
\to B \wedge ((\underline{A} \wedge B \ogreaterthan \underline{A}) \vee C) \wedge D \\
\to B \wedge ((B \Rightarrow (\underline{A} \ogreaterthan \underline{A})) \vee C) \wedge D
\end{array}
\right. \\
\text{identity} \left\{ \to B \wedge ((B \Rightarrow \top) \vee C) \wedge D \right. \\
\text{unit elimination} \left\{
\begin{array}{l}
\to B \wedge (\top \vee C) \wedge D \\
\to B \wedge \top \wedge D \\
\to B \wedge D
\end{array}
\right.
\end{aligned}
$$

Variant of the **Calculus of Structures** (Guglielmi 1999)

$$\exists y.\ \forall x. R(x, y) \ogreaterthan \forall a. \exists b. R(a, b)$$

- Unify linked subformulas

$$\exists y.\ \forall x.\underline{R(x,y)} \ogreaterthan \forall a.\exists b.\underline{R(a,b)} \qquad \begin{array}{c} x \longmapsto a \\[2em] y \longmapsfrom b \end{array}$$

Linking under quantifiers (Donato, Strub, and Werner 2022)

- Unify linked subformulas
- **Check** for $\forall\exists$ dependency cycles

$$x \longmapsto a$$

$$\exists y.\ \forall x.\underline{R(x,y)} \ogreaterthan \forall a.\exists b.\underline{R(a,b)}$$

$$y \longmapsfrom b$$

- Unify linked subformulas
- **Check** for ∀∃ dependency cycles
- **Switch** uninstantiated quantifiers

$$x \longmapsto a$$

$$\exists y.\ \forall x.\underline{R(x,y)} \otimes \forall a.\exists b.\underline{R(a,b)}$$

$$\to \forall y.\left(\forall x.\underline{R(x,y)} \otimes \forall a.\ \exists b.\underline{R(a,b)}\right)$$

$$y \longleftarrow b$$

$$\to \forall y.\forall a.\left(\forall x.\underline{R(x,y)} \otimes \exists b.\ \underline{R(a,b)}\right)$$

✓

- Unify linked subformulas
- **Check** for $\forall\exists$ dependency cycles
- **Switch** uninstantiated quantifiers
- **Instantiate** unified variables

$$x \longmapsto a$$

$$y \longleftarrow\!\!\dashv b$$

$$\exists y.\ \forall x.\underline{R(x,y)} \otimes \forall a.\exists b.\underline{R(a,b)}$$

$$\to \forall y.\left(\forall x.\underline{R(x,y)} \otimes \forall a.\ \exists b.\underline{R(a,b)}\right)$$

$$\to \forall y.\forall a.\left(\forall x.\underline{R(x,y)} \otimes \exists b.\ \underline{R(a,b)}\right)$$

$$\to \forall y.\forall a.\left(\forall x.\ \underline{R(x,y)} \otimes \underline{R(a,y)}\right)$$

$$\to \forall y.\forall a.(\underline{R(a,y)} \otimes \underline{R(a,y)})$$

$$\to^{*} \top$$

✓

- Unify linked subformulas

$$\forall a.\exists b.\underline{R(a,b)} \otimes \exists y.\forall x.\underline{R(x,y)}$$

$$x \longleftarrow\!\!\dashv\ a$$

$$y \longmapsto b$$

- Unify linked subformulas
- **Check** for $\forall\exists$ dependency cycles

$$\forall a.\exists b.\underline{R(a,b)} \ogreaterthan \exists y.\forall x.\underline{R(x,y)}$$

$$\begin{tikzcd}
x \arrow[d, bend right] & a \arrow[l, mapsto] \\
y \arrow[r, mapsto] & b \arrow[u, bend right]
\end{tikzcd}$$

✗

Completeness

Conclusion

- Quelques features manquantes, mais déjà utilisable
- Mathis a simplifié l'install -> n'hésitez pas à installer !
- Bientôt prêt pour évaluation sur des étudiants, voire experts

Bibliography

Chaudhuri, Kaustuv. 2013. "Subformula Linking as an Interaction Method". Edited by Sandrine Blazy, Christine Paulin-Mohring, David Pichardie, David Hutchison, Takeo Kanade, Josef Kittler, Jon M. Kleinberg, et al.. *Interactive Theorem Proving*. Berlin, Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-39634-2_28

Donato, Pablo, Pierre-Yves Strub, and Benjamin Werner. 2022. "A Drag-and-Drop Proof Tactic". In *Proceedings of the 11th ACM SIGPLAN International Conference on Certified Programs and Proofs*, 197–209. CPP 2022. Philadelphia, PA, USA: Association for Computing Machinery. https://doi.org/10.1145/3497775.3503692

Guglielmi, Alessio. 1999. "A Calculus of Order and Interaction". https://www.researchgate.net/publication/2807151_A_Calculus_of_Order_and_Interaction