# Chapter 1

# Bisection Method

In this chapter, we solve the following equation using the **Bisection Method**:

$$f(x) = x^3 - 10x^2 + 5 = 0 \tag{1.1}$$

## Step 1: Identify the Interval

First, we identify the interval where the root lies. Below is the plot of the function:

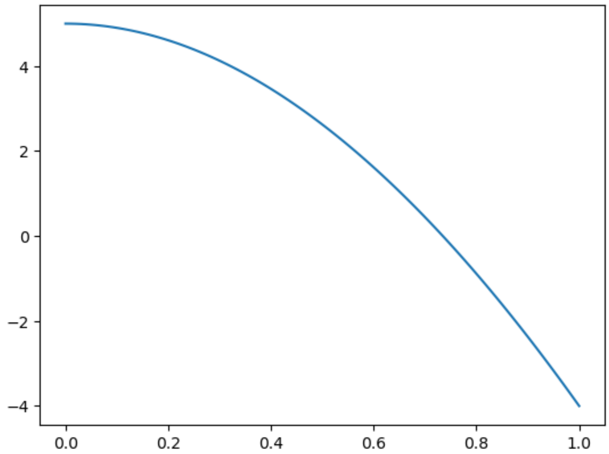


Figure 1.1: Plot of $f(x) = x^3 - 10x^2 + 5$

From the graph in Figure 1.1, the root lies between $x = 0.6$ and $x = 0.8$. Let us take these as the initial guesses.

## Step 2: Manual Calculation

Using the Bisection Method, we iteratively compute the midpoints and narrow the interval until we find the root.

**Initial Calculations:**

- For $x = 0.6$, $f(x) = 0.6^3 - 10(0.6)^2 + 5 = 1.616$
- For $x = 0.8$, $f(x) = 0.8^3 - 10(0.8)^2 + 5 = -0.888$

**Iteration Results:**

| **Midpoint ($x$)** | $f(x)$ | **Interval** |
|---|---|---|
| 0.7 | 0.443 | $(0.7, 0.8)$ |
| 0.75 | -0.203 | $(0.7, 0.75)$ |
| 0.725 | 0.125 | $(0.725, 0.75)$ |
| 0.7344 | 0.000 | — |

Table 1.1: Manual Calculation of the Bisection Method

## Step 3: Approximate Root

From the table above, the root of the equation is approximately:

$$\boxed{x = 0.7344}$$

We've successfully found the root using the Bisection Method.

Let's see the python implementation

```

def f(x):
return x**3 -10*x**2 +5

import matplotlib.pyplot as plt
import numpy as np
x=np.linspace(0,1,100)
y=[]

for i in range(len(x)):
    y.append(f(x[i]))
```

```python

    plt.plot(x,y)


tolerance=0.0001
x3=[]
x2=0.8
x1=0.6
x3_=(x2+x1)/2
x3.append(x3_)
i=0



while abs(f(x3_))>0 :
    x3_=(x2+x1)/2
    x3.append(x3_)
    if f(x3_)*f(x1)>0 :
        x1=x3_

    elif f(x3_)*f(x1)<0 :
        x2=x3_
    i+=1

print(x3_)
```

**Answer got from python code : 0.7346035077893033**