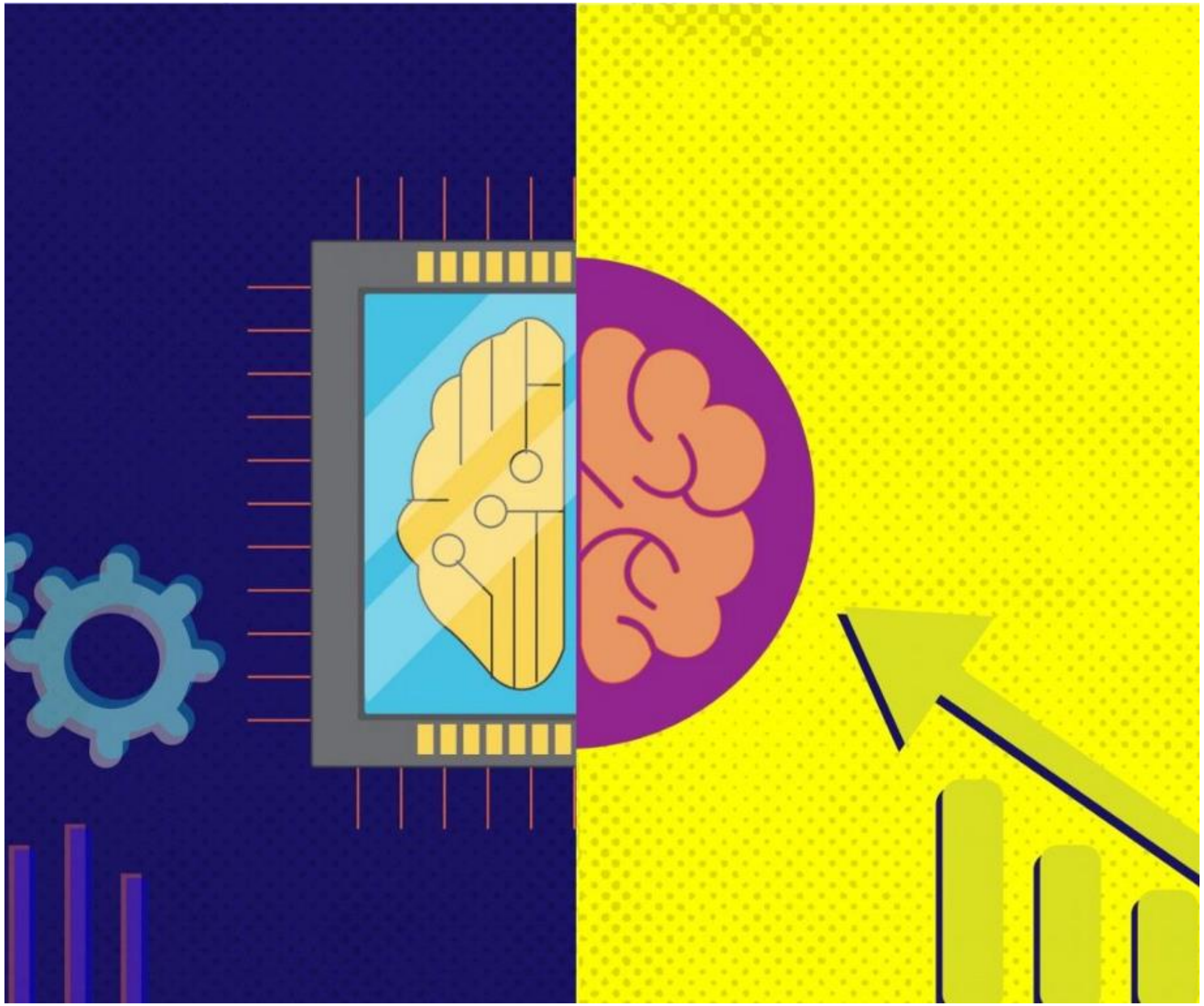
Discrete Probability Distributions

Probability & Statistics (IT2110)



Content

- Discrete Probability Distributions
  - Bernoulli Distribution
  - Binomial Distribution
  - Poisson Distribution

Bernoulli Distribution

Bernoulli Distribution

- Identified by Jacob Bernoulli (Switzerland Mathematician)
- Simplest Distribution
- Focuses on a random experiment with exactly **two outcomes**
- Bernoulli Variable can have only 2 outcomes

Eg: Tossing a coin

Rolling a die, and six is the success

Bernoulli Distribution

- Probability Distribution Function of X can be defined as follows.

$$P(X) = \begin{Bmatrix} p & X = 1 \\ 1 - p & X = 0 \end{Bmatrix}$$

- That is,

$$P(X) = \begin{Bmatrix} p^x (1 - p)^{1-x} & X = 1,0 \\ 0 & Otherwise \end{Bmatrix}$$

- Properties of Bernoulli Distribution

$$E(X) = p$$
$$V(X) = p(1 - p)$$

Bernoulli Distribution

Binomial Distribution

Binomial Distribution

- **Assume there are n independent trials**
- There are **2 outcomes in each trial, success or failure**
- Probability of Success, **p at each trial is equal**
- Let X be equal to number of successes in n independent trials
- Then

$$X \sim Binomial(n, p)$$

Eg: Number of break down buses in a sample of 100 lot, Number of deaths for a sample of 200 patients

Binomial Distribution

$$X \sim Binomial(n, p)$$

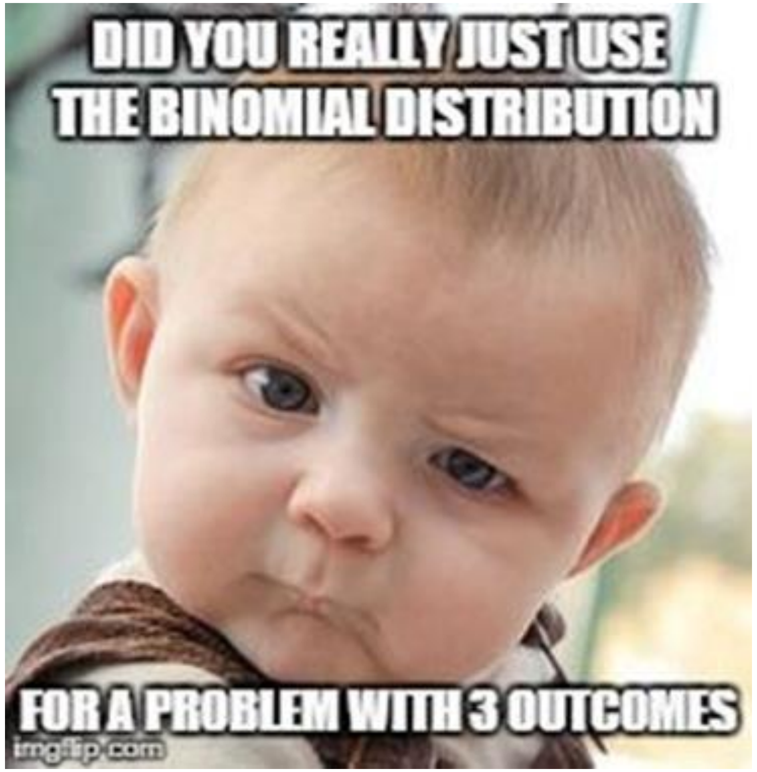
Probability Distribution Function of X

$$P(X = x) = n_{C_x} p^x (1-p)^{n-x}$$

Properties of Binomial Distribution

$$E(X) = np$$

$$V(X) = np(1-p)$$



Binomial Distribution

Example

It is known that screws produced by a certain machine will be defective with probability 0.01 independently of each other. If we randomly pick 10 screws produced by this machine, what is the probability that

a) exactly six screws will be defective?

b) at most 3 screws will be defective?

c) at least 2 screws will be defective?

d) What is the expected number of defectives?

e) What is the variance of defectives?

Poisson Distribution

Poisson Distribution

- Is used to model the number of events occurring within a given time interval/ Region/ Area

Eg: Number of hits to a particular website during 1-2 pm

Number of calls to a particular phone during 8-9 am

Number of telephone calls that arrive on Mondays on your mobile

"My husband always loves your Poisson distribution – it's something to do with him being a mathematician."

Poisson Distribution

$$X \sim Poisson(\lambda)$$

- Poisson distribution has one parameter
- λ is the average number of events in the given time interval
- It is known as the "shape parameter" of the distribution
- Probability Distribution Function of X is,

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

- Properties of Poisson Distribution are

$$E(X) = \lambda$$
$$V(X) = \lambda$$

Example

A small life insurance company has determined that on average it receives 6 death claims per day. Find the probability that the company receives at least seven death claims on a randomly selected day.

Binomial Approximation to Poisson

- The Binomial and Poisson distributions are both discrete probability distributions.
- In some circumstances the distributions are very similar.
- **Condition**
  - n is Large (n > 50)
  - P is small (p < 0.1)

  Then $\mathrm{X} \sim Bin(n, p)$ can be approximated with $Poisson(np)$

Example

Suppose 8% of the tires manufactured at a particular plant are defective. Find the probability of obtaining exactly one defective tire from a sample of 200.

Thank You

Questions?