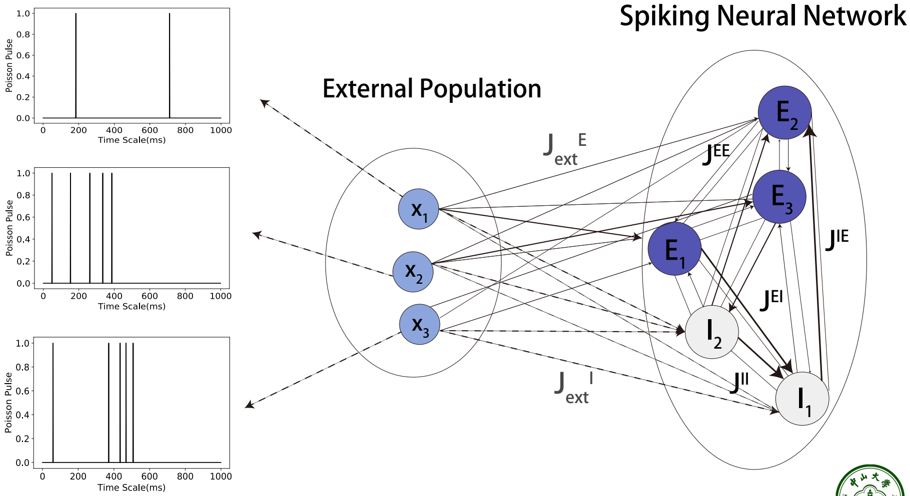
Content

1. structure of models

2. Three usual models for EI balance.


Spiking Neural Network
External Population
Poisson Pulse
Time Scale(ms)
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
X_1
X_2
X_3
E_1
E_2
E_3
I_1
I_2
J_{ext}^E
J_{ext}^I
J^{EE}
J^{IE}
J^{EI}
J^{II}

Content

Three usual models for EI balance.

- Model1: Amit, Brunel, 1997

$$\tau_m \dot{V}^{\alpha}(t) = -V^{\alpha}(t) + V_{leak} + I^{\alpha}(t), \tag{1a}$$

$$\tau_\alpha \dot{I}_i^{\alpha}(t) = -I_i^{\alpha}(t) + \tau_m \sum_{\beta} \sum_{j}^{C} J_{ij}^{\alpha\beta} \sum_{k} \delta\left(t_{ij}^{k} - t\right). \tag{1b}$$

- Updating functions based on one-order Euler method

$$(V^{\alpha}(t) - V^{\alpha}(t-1)) = \frac{dt}{\tau_m}(-V^{\alpha}(t) + V_{leak} + I^{\alpha}(t)), \tag{2a}$$

$$(I_i^{\alpha}(t) - I_i^{\alpha}(t-1)) = -\frac{dt}{\tau_\alpha} I_i^{\alpha}(t) + \frac{\tau_m}{\tau_\alpha} \int_{t-dt}^{t} \sum_{\beta} \sum_{j}^{C} J_{ij}^{\alpha\beta} \sum_{k} \delta\left(t_{ij}^{k} - t\right) \tag{2b}$$

- The definition of currents.

$$I_E^{\alpha}(t) = \int_t^{t-dt} \left(\sum_{j=1}^{N_E} J_{ij}^{\alpha E} \sum_k \delta\left(t - t_{jk}^E\right) + \sum_{j=1}^{N_E} J_{ij}^{out-\alpha} \sum_k \delta\left(t - t_{jk}^{out}\right) \right) c \tag{3a}$$

$$I_I^{\alpha}(t) = \int_t^{t-dt} - \sum_{j=1}^{N_I} J_{ij}^{\alpha I} \sum_k \delta\left(t - t_{jk}^I\right) dt, \tag{3b}$$

- See more details in the python code.

Three usual models for EI balance.

- Model2: Amit, Brunel, 1997 and Brunel 2020. If the time constant of the currents is much shorter than that of the potentials,

$$\tau \frac{dV_i^{\alpha}(t)}{dt} = -V_i^{\alpha}(t) + V_{leak} + \tau \left(\sum_{\beta} \sum_{j=1}^{C} J_{ij}^{\alpha\beta} \sum_k \delta\left(t - t_{jk}^{\beta}\right) \right), \tag{4a}$$

$$I_E^{\alpha}(t) = \int_{t-dt}^{t} \left(\sum_{j=1}^{N_E} J_{ij}^{\alpha E} \sum_k \delta\left(t - t_{jk}^{E}\right) + \sum_{j=1}^{N_E} J_{ij}^{out-\alpha} \sum_k \delta\left(t - t_{jk}^{out}\right) \right) dt \tag{4b}$$

$$I_I^{\alpha}(t) = \int_{t-dt}^{t} - \sum_{j=1}^{N_I} |J_{ij}|^{\alpha I} \sum_k \delta\left(t - t_{jk}^{I}\right) dt, \tag{4c}$$

Three usual models for EI balance.

- Model3: EIM models (Dynamics of the exponential integrate-and-fire model

$$C_m \frac{dV_j^a}{dt} = -g_L \left(V_j^a - E_L \right) + D_T e^{\left(V_j^a - V_T \right)/D_T} + T_j^a(t), \tag{5a}$$

$$T_j^a(t) = E_j^a(t) + I_j^a(t) + X_j^a(t), \tag{5b}$$

$$T_j^a(t) = \sum_b \sum_k J_{jk}^{ab} \sum_n \alpha_b \left(t - t_{k,n}^b \right), \tag{5c}$$

where $\alpha_b(t) = e^{-t/\tau_b}/\tau_b H(t)$.

Three usual models for EI balance.

- Updating functions

$$V_j^a(t) - V_j^a(t-1) = \frac{dt}{Cm}(-g_L\left(V_j^a - E_L\right) + D_T e^{\left(V_j^a - V_T\right)/D_T}) \quad (6a)$$

$$+ \int_{t-dt}^{t} T_j^a(t), \quad (6b)$$

$$T_j^a(t) = E_j^a(t) + I_j^a(t) + X_j^a(t), \quad (6c)$$

$$\int_{t-dt}^{t} T_j^a(t) = \sum_b \sum_k J_{jk}^{ab} \sum_n (e^{-t+dt/\tau_b} - e^{-t/\tau_b})H(t) \quad (6d)$$

where $\alpha_b(t) = e^{-t/\tau_b}/\tau_b H(t)$.

Three usual models for EI balance.

- Other models: Somplinsiky 1996 and 2013
- 2013

$$I^A_{syn,i}(t) = \sum_{B=0,E,I} \sum_{j=1}^{N^B} M^{AB}_{ij} \bar{J}^{AB} E^B_j(t) \tag{7}$$

where $M^{AB}_{ij} \in \{0,1\}$ is the random connectivity matrix and $\bar{J}^{AB}$ are the strengths of non-zero synapses. The postsynaptic currents $E^B_j(t)$ are linear sums of the contributions from individual presynaptic spikes given by a difference of exponentials,

$$\varepsilon(t) = \frac{1}{\tau_1 - \tau_2}\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right), \quad t > 0 \tag{8}$$

Three usual models for EI balance.

- 1996: The updated state of the updating neuron at time t is

$$\sigma_k^i(t) = \Theta\left(u_k^i(t)\right) \tag{9}$$

where $\Theta(x)$ is the Heaviside function, $\Theta(x) = 0$ for $x \leq 0$ and $\Theta(x) = 1$ for $x > 0$. The total synaptic input, u_k^i to the neuron, relative to the threshold, θ_k, at time t is

$$u_k^i(t) = \sum_{l=1}^{2} \sum_{j=1}^{N_l} J_{kl}^{ij} \sigma_l^j(t) + u_k^0 - \theta_k \tag{10}$$