Appendix A

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1 Introduction

Appendix A: Full Mathematical Expansion of Recursive Horizon Theory

A.1: Gravitational Field from Entropy Flow

We define a scalar entropy field $S(x)$ over a 3D spatial hypersurface embedded in a Lorentzian manifold M. Let $\Phi(x)$ be the gravitational potential induced by entropy distribution.

We postulate that entropy gradients generate gravitational potential via:

$$\frac{\delta S}{\delta x^i} = -\partial_i \Phi(x) \quad or \quad \nabla S(x) = -\nabla \Phi(x)$$

Taking divergence of both sides yields:

$$\nabla \cdot \nabla S(x) = -\nabla \cdot \nabla \Phi(x) \Rightarrow \nabla^2 S(x) = -\nabla^2 \Phi(x)$$

We invert this to derive the Poisson equation for gravitational potential:

$$\nabla^2 \Phi(x) = 4\pi G \, \frac{\delta S}{\delta V}$$

This implies: - Gravitational curvature is sourced by local entropy density per unit volume, - Entropic force arises as:

$$\vec{F}(x) = -\nabla \Phi(x)$$

This connects entropy geometry directly to Newtonian gravity through a statistical interpretation of mass-energy localization as entropy compression.

A.2: Surface Gauge Field Equations

We begin by interpreting surface memory deformation as an emergent gauge structure.

Let $S(x)$ be the entropy field defined on a 2D surface embedded in a 4D Lorentzian manifold M. The recursive deformation of this surface produces local curvature patterns, which we associate with emergent gauge fields $A_\mu^a(x)$.

Define the covariant derivative:

$$D_\mu = \partial_\mu + igA_\mu^a(x)T^a$$

where:

- $A_\mu^a(x)$ are the gauge fields emerging from surface tiling symmetry,
- T^a are the generators of the gauge group G,
- g is the coupling constant associated with the recursive symmetry breaking level.

We define the gauge field strength tensor via:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

where f^{abc} are the structure constants of the Lie algebra of G.

The Lagrangian for the gauge field is then given by:

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}$$

This expression emerges not from imposed symmetry, but from the recursive patterning of entropy gradients on the surface.

To show this, we consider local deformation of the entropy gradient:

$$\nabla_\mu S(x) \mapsto \nabla_\mu S(x) + \delta_\epsilon \nabla_\mu S(x)$$

where small transformations δ_ϵ correspond to group-valued mappings preserving local entropy invariance under recursive collapse.

To first order:

$$\delta_\epsilon \nabla_\mu S(x) = i\epsilon^a(x)T^a \nabla_\mu S(x)$$

Therefore, the entropy-covariant field coupling takes the form:

$$\mathcal{L}_{int} = -\nabla^\mu S(x) \cdot D_\mu S(x)$$

This produces minimal coupling between surface entropy flow and gauge curvature fields, tying local memory conservation to topological invariants of the surface tiling group G.

Gauge symmetry thus emerges as a natural recursive deformation symmetry on entropy surfaces — not as a postulated spacetime symmetry, but as a collapse-preserving equivalence under localized horizon information flow.

A.3: SU(5) Breaking from Recursive Collapse

We postulate that early recursive entropy tiling on surface boundaries admits an effective SU(5) gauge symmetry. This symmetry arises from five-dimensional entropic configurations in a locally isotropic surface tiling space.

Let the gauge field $A_\mu \in su(5)$ be decomposed under the subgroup chain:

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

We define the SU(5) field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_5[A_\mu, A_\nu]$$

We construct the full gauge Lagrangian:

$$\mathcal{L}_{gauge} = -\frac{1}{2}Tr(F_{\mu\nu}F^{\mu\nu})$$

To realize spontaneous symmetry breaking, introduce a Higgs scalar Φ in the adjoint representation (24-dimensional), with vacuum expectation value:

$$\langle\Phi\rangle = v\, diag(2, 2, 2, -3, -3)$$

This breaks SU(5) down to:

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

The potential responsible for this breaking is:

$$V(\Phi) = -\mu^2 Tr(\Phi^2) + \lambda\, Tr(\Phi^2)^2$$

The recursive collapse mechanism selects this vacuum by entropy minimization — the surface tension gradients evolve toward the configuration that minimizes the curvature-induced entropy functional:

$$\mathcal{S}_{rec}[\Phi] = \int \left(\frac{1}{2}Tr(\nabla_\mu \Phi \nabla^\mu \Phi) - V(\Phi)\right) \sqrt{-g}\, d^4x$$

This entropy-action governs the recursive selection of vacuum symmetry. Higher-entropy configurations (e.g., full SU(5)) are unstable under recursive collapse due to their internal curvature tension. The collapse favors lower-energy topologies where field tilings align with minimal entropy curvature, i.e., Standard Model symmetry.

Therefore, symmetry breaking is not imposed but *selected* through recursive entropy relaxation.

Summary

- SU(5) arises from isotropic entropy tiling symmetry.
- Recursive collapse selects lower-entropy field alignment.
- Higgs VEV induces breaking to SM group via curvature-tension minimization.

A.4: Field Quantization from Surface Fluctuations

Let $S(x)$ represent a stable entropy configuration over a 2D surface embedded in spacetime. We model localized surface deviations as small fluctuations $\phi(x)$ around a background $S_0(x)$:

$$S(x) = S_0(x) + \phi(x)$$

We assume $S_0(x)$ satisfies:

$$S_0(x) = 0$$

and fluctuations evolve via:

$$\phi(x) + m^2\phi(x) = 0$$

This is the Klein-Gordon equation, where m is the effective mass generated by curvature tension in the surface memory.

Promote $\phi(x)$ to an operator-valued field and define canonical momentum:

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)} = \dot{\phi}(x)$$

Canonical quantization imposes:

$$[\phi(\vec{x}, t), \pi(\vec{y}, t)] = i\hbar\, \delta^3(\vec{x} - \vec{y})$$

Fourier expansion:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left(a_k e^{-ik\cdot x} + a_k^\dagger e^{ik\cdot x} \right)$$

where $\omega_k = \sqrt{\vec{k}^2 + m^2}$, and $a_k, a_k^\dagger$ are annihilation and creation operators.

These operators satisfy:

$$[a_k, a_{k'}^\dagger] = (2\pi)^3\delta^3(k - k')$$

The Hamiltonian becomes:

$$H = \int d^3k\, \omega_k\, a_k^\dagger a_k + vacuumenergy$$

Thus, every mode of entropy surface fluctuation corresponds to a particle excitation.

This field arises not from imposed quantization, but from the recursive tiling's inability to stabilize exact equilibrium in localized regions — the leftover oscillations manifest as particle states.

Summary

- Fluctuations in entropy surface generate Klein-Gordon fields.
- Canonical quantization yields creation-annihilation structure.
- Quantum fields emerge from recursive instability of local memory.