Expansion of the Terminal Identity Equation

Recursive Horizon Theory – Phase 1

Symbol Definitions

- $I(x)$: Recursive memory (identity) field
- $S_{\mu\nu}(x)$: Entropy flux tensor
- $\Phi(x)$: Gravitational potential field
- $g_{\mu\nu}(x)$: Metric tensor
- ∇_μ: Covariant derivative
- α: Coupling constant (surface tension scaling)

Original Terminal Identity Equation

$$\Box I(x) = \alpha \nabla^\mu \left(S_{\mu\nu}(x) \nabla^\nu \Phi(x) \right)$$

Expansion of the Left-Hand Side

The d'Alembertian (wave operator) in curved spacetime:

$$\Box I(x) = g^{\mu\nu} \nabla_\mu \nabla_\nu I(x)$$

In flat or weak-field approximation:

$$\Box I(x) \approx -\partial_t^2 I(x) + \nabla^2 I(x)$$

Expansion of the Right-Hand Side

Apply the product rule for covariant derivatives:

$$\nabla^\mu \left(S_{\mu\nu} \nabla^\nu \Phi(x) \right) = (\nabla^\mu S_{\mu\nu}) \nabla^\nu \Phi(x) + S_{\mu\nu} \nabla^\mu \nabla^\nu \Phi(x)$$

Fully Expanded Terminal Identity Equation

Thus:

$$-\partial_t^2 I(x) + \nabla^2 I(x) = \alpha \left[(\nabla^\mu S_{\mu\nu}) \nabla^\nu \Phi(x) + S_{\mu\nu} \nabla^\mu \nabla^\nu \Phi(x) \right]$$

Physical Interpretation

- Left-hand side ($\Box I(x)$): Propagation and evolution of the identity field.
- First term on the right-hand side: How divergence of entropy fields interacts with gravitational gradients.
- Second term on the right-hand side: How entropy structure directly modifies gravitational curvature.