Appendix A

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1 Introduction

Appendix A: Full Mathematical Expansion of Recursive Horizon Theory

A.1: Gravitational Field from Entropy Flow

We define a scalar entropy field $S(x)$ over a 3D spatial hypersurface embedded in a Lorentzian manifold M. Let $\Phi(x)$ be the gravitational potential induced by entropy distribution.

We postulate that entropy gradients generate gravitational potential via:

$$\frac{\delta S}{\delta x^i} = -\partial_i \Phi(x) \quad or \quad \nabla S(x) = -\nabla \Phi(x)$$

Taking divergence of both sides yields:

$$\nabla \cdot \nabla S(x) = -\nabla \cdot \nabla \Phi(x) \Rightarrow \nabla^2 S(x) = -\nabla^2 \Phi(x)$$

We invert this to derive the Poisson equation for gravitational potential:

$$\nabla^2 \Phi(x) = 4\pi G \, \frac{\delta S}{\delta V}$$

This implies: - Gravitational curvature is sourced by local entropy density per unit volume, - Entropic force arises as:

$$\vec{F}(x) = -\nabla \Phi(x)$$

This connects entropy geometry directly to Newtonian gravity through a statistical interpretation of mass-energy localization as entropy compression.

A.2: Surface Gauge Field Equations

We begin by interpreting surface memory deformation as an emergent gauge structure.

Let $S(x)$ be the entropy field defined on a 2D surface embedded in a 4D Lorentzian manifold M. The recursive deformation of this surface produces local curvature patterns, which we associate with emergent gauge fields $A_\mu^a(x)$.

Define the covariant derivative:

$$D_\mu = \partial_\mu + igA_\mu^a(x)T^a$$

where:

- $A_\mu^a(x)$ are the gauge fields emerging from surface tiling symmetry,
- T^a are the generators of the gauge group G,
- g is the coupling constant associated with the recursive symmetry breaking level.

We define the gauge field strength tensor via:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

where f^{abc} are the structure constants of the Lie algebra of G.

The Lagrangian for the gauge field is then given by:

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}$$

This expression emerges not from imposed symmetry, but from the recursive patterning of entropy gradients on the surface.

To show this, we consider local deformation of the entropy gradient:

$$\nabla_\mu S(x) \mapsto \nabla_\mu S(x) + \delta_\epsilon \nabla_\mu S(x)$$

where small transformations δ_ϵ correspond to group-valued mappings preserving local entropy invariance under recursive collapse.

To first order:

$$\delta_\epsilon \nabla_\mu S(x) = i\epsilon^a(x)T^a \nabla_\mu S(x)$$

Therefore, the entropy-covariant field coupling takes the form:

$$\mathcal{L}_{int} = -\nabla^\mu S(x) \cdot D_\mu S(x)$$

This produces minimal coupling between surface entropy flow and gauge curvature fields, tying local memory conservation to topological invariants of the surface tiling group G.

Gauge symmetry thus emerges as a natural recursive deformation symmetry on entropy surfaces — not as a postulated spacetime symmetry, but as a collapse-preserving equivalence under localized horizon information flow.