Combinatorics

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Lecture 8: Introduction

1.1 Examples on Generating Functions

- $(1-x)^{-r} = \sum_{n=0}^{\infty} \binom{r+n-1}{r-1} x^n$
- $G_a(x) = \sum_{n=0}^{\infty} a_n x^m$ is generating function for $\{a_n\}$

Example 1.1: Find the generatign function for $a_n = \binom{n}{2} = \frac{n(n-1)}{2}$
$a_n = \frac{1}{2}[b_n - c_n]; b_n = n^2, c_n = n$

Theorem 1.2 Addition and Multiplication of GF:

Given sequence $\{a_n\}$, $\{b_n\}$ and their generating functions $G_a(x) = \sum_{n=0}^{\infty} a_n x^m$ and $G_a(x) = \sum_{n=0}^{\infty} b_n x^m$

- Addition
  $G_a + G_b = \sum_{n=0}^{\infty} (a_n + b_n) x^m$
- Multiplication
  $G_a * G_b = \sum_{n=0}^{\infty} (c_n) x^m$, where $c_n = \sum_{k=0}^{n} a_k b_{n-k}$

Example 1.3:

- Do the examples in class.

Definition 1.1 Triangular Numbers: $T_n = 1+2+\ldots+n = \binom{n+1}{2}; n \geq 1$