Theory of Computation

ChanBong

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Lecture 1: Introduction

1.1 Basic Defination

To get started, we will define what is computation and various jargons we'll be using throughout this course

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Definition 1.1 Strings: set of alphabets. Denoted by Σ

Example 1.1: Some examples of strings are:

- $u = aabbca$

- $v = bac$

- $uv = aabbcca$

Definition 1.2 Length of a string: Number of alphbets in the string

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Empty strings are denoted by λ.

- $|\lambda| = 0$.

- $\lambda w = w\lambda = w$.

If $w = uv$ then u is called **prefix** of w and v is called **suffix** of w.

Example 1.2: Consider $w = abcab$. Then $\{\lambda, a, ab, abc, abca, abcab\}$ are all prefixes of w and $\{\lambda, b, bc, bca, bcab\}$ are all suffixes of w.

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Lecture 2: Grammer

Theorem 2.1: $|uv| = |u| + |v|$.

Proof: We will use mathematical induction to prove this.
Base case: $|\lambda v| = |v| = |\lambda| + |v|$.
Inductive step: Assume $|uv| = |u| + |v|$. Then $|u'v| = |u'| + |v| = |u| + |v| + 1$.
Thus, $|uv| = |u'v| = |u| + |v| + 1 = |u| + |v|$.
Thus, $|uv| = |u| + |v|$.
$\square$

2.1 Grammer

Definition 2.1 Grammer: A grammer $\mathbb{G} = \{\mathbb{V}, \mathbb{T}, \mathbb{S}, \mathbb{P}\}$
- $\mathbb{V}$ is finite set of objects called variables.

- $\mathbb{T}$ is finite st of objects called terminals.

- $\mathbb{S} \in \mathbb{V}$: special symbol called start variable

- $\mathbb{P}$ is a finite set of rules

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Lecture 3: Deterministic Finite Acceptors

3.1 Language from grammer

Example 3.1: Consider the grammer $\mathbb{G} = (\{s\}, \{a, b\}, \mathbb{S}, \mathbb{P}); \quad \mathbb{P}_1 \mathbb{S} \to a\mathbb{S}b/\lambda$ then language generated by this grammer is

$$\mathbb{L}(G_1) = \{a^m b^m : m \geq 0\}$$

Example 3.2: Consider the grammer $\mathbb{G} = (\{s, \mathbb{A}\}, \{a, b\}, \mathbb{S}, \mathbb{P}); \quad \mathbb{P}_2 \mathbb{S} \to a\mathbb{S}b/\lambda, \mathbb{A} \to a\mathbb{A}b/\lambda$ then language generated by this grammer is

$$\mathbb{L}(G_2) = \{a^m b^m : m \geq 0\}$$

Definition 3.1 : Two grammers are equivalent iff $L(G_1) = L(G_2)$

3.2 Deterministic Finite Acceptors (DFA)

Definition 3.2 : A DFA is defined by $\mathbb{M} = (\mathbb{Q}, \mathbb{E}, \mathbb{D}, \mathbb{Q}_0, \mathbb{F})$ where
- $\mathbb{Q}$ is a finite set of states

- $\mathbb{E}$ is a finite set of input symbols

- $\mathbb{D} : \mathbb{Q} \times \mathbb{E} \to \mathbb{Q}$ is a transition function

- $\mathbb{Q}_0 \in \mathbb{Q}$ is the start state

- $\mathbb{F} \subseteq \mathbb{Q}$ is the set of final states

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Example 3.3: Consider the DFA $\mathbb{M} = (\{q_0, q_1, q_2\}, \{a, b\}, \mathbb{D}, q_0, \{q_1\})$. $\mathbb{D}(q_0, a) = q_1, \mathbb{D}(q_0, b) = q_0, \mathbb{D}(q_1, a) = q_2, \mathbb{D}(q_1, b) = q_0, \mathbb{D}(q_2, a) = q_2, \mathbb{D}(q_2, b) = q_2$

3.3   Graphical Representation of DFA

Definition 3.3 :   A DFA is represented by a directed graph $G = (V, E)$ where

- $V = \mathbb{Q}$

- $E = \{(q, \mathbb{D}(q, \mathbb{E}), \mathbb{E}) : q \in \mathbb{Q}, \mathbb{E} \in \mathbb{E}\}$

- an open $\rightarrow$ indicates the start state

- $\bigcirc$ indicates the final state

Example 3.4:   Consider the DFA $\mathbb{M} = (\{q_0, q_1, q_2\}, \{a, b\}, \mathbb{D}, q_0, \{q_1\})$. $\mathbb{D}(q_0, a) = q_1, \mathbb{D}(q_0, b) = q_0, \mathbb{D}(q_1, a) = q_2, \mathbb{D}(q_1, b) = q_0, \mathbb{D}(q_2, a) = q_2, \mathbb{D}(q_2, b) = q_2$
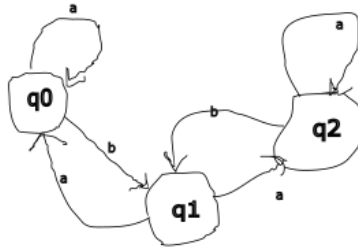

Figure 3.1: DFA

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Lecture 4: DFA Acceptance

4.1 Walk in DFA

Definition 4.1 Trap State: A state q is a trap state iff $\mathbb{D}(q, \mathbb{E}) = q$ for all $\mathbb{E} \in \mathbb{E}$

Example 4.1: $\mathbb{M} = (\{q_0, q_1, q_2\}, \{a, b\}, \mathbb{D}, q_0, \{q_1\})$. $L(M) = \{a^n b : n \geq 0\}$
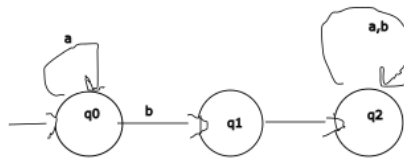

Figure 4.1: DFA

Here q_2 is a trap state.

Theorem 4.2: Let $\mathbb{M}$ be a DFA and let G_m be its graphical representation. Then for every $q_i \in Q$ and $w \in \Sigma^+$ we have $D^+(q_i, w) = q_j$ iff there is a walk from q_i to q_j with label w.

Proof: We will use mathematical induction on the length of w.
Base case: $w = a$. Then $D^+(q_i, a) = q_j$ iff there is a edge from q_i to q_j with label a.
Inductive step: Assume for all $|u| = n < m$, if $D^+(q_i, u) = q_k$ then there is a walk form q_i to q_j with label u.
Now if $w \in \Sigma^+$, with $|w| = n + 1 = m$, say $w = ua$.
Then $D^+(q_i, w) = D^+(D^+(q_i, v), a) = D^+(q_k, a) = q_j \implies$ there is a walk from q_i to q_j with label w.

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Backward direction: Assume for all $|u| = n < m$, if there is a walk from q_i to q_j with label u then $D^+(q_i, u) = q_j$.

$\square$

Remark: If there is a walk from q_i to q_j with label w, then $D^+(q_i, w) = q_j$

4.2 DFA Acceptance

Definition 4.2 : A DFA accepts a string w iff w is accepted by $\mathbb{M}$.

Definition 4.3 Regular Language: A language L is called regular iff there is a DFA $\mathbb{M}$ such that $L(M) = L$.

Example 4.3: Find a DFA for the language $L = \{abw : w \in \{a, b\}\}$.

Figure 4.2: Solution to the above example

Example 4.4: Show that the language $L = \{awa : w \in \{a, b\}^+\}$ is regular.

Figure 4.3: Solution to the above example

$\oint$ How to prove that a language is not regular?

Definition 4.4 Concatanation of two languages: