

Homework 8 - System Analysis with the CTFT

Spring 2023

Exercise 1. Convolution and CTFTs

For each of the following, calculate $X(\omega)$, $Y(\omega)$, $Z(\omega) = X(\omega)Y(\omega)$, and $z(t)$.

(a) $x(t) = e^{-t}u(t)$ and $y(t) = e^{-t}u(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} e^{-t}u(t)e^{-j\omega t}dt = \int_0^{\infty} e^{-t(1+j\omega)}dt = \left. \frac{-e^{-t(1+j\omega)}}{1+j\omega} \right|_0^{\infty} = \frac{1}{1+j\omega}$$

$$Y(\omega) = \frac{1}{1+j\omega}, \quad Z(\omega) = X(\omega)Y(\omega) = \frac{1}{(1+j\omega)^2}$$

$$z(t) = x(t) * y(t) = \int_{-\infty}^t e^{-\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau = \int_0^t e^{-t}d\tau = te^{-t}u(t)$$

[Not Finished]