

data analysis

course 01

overall

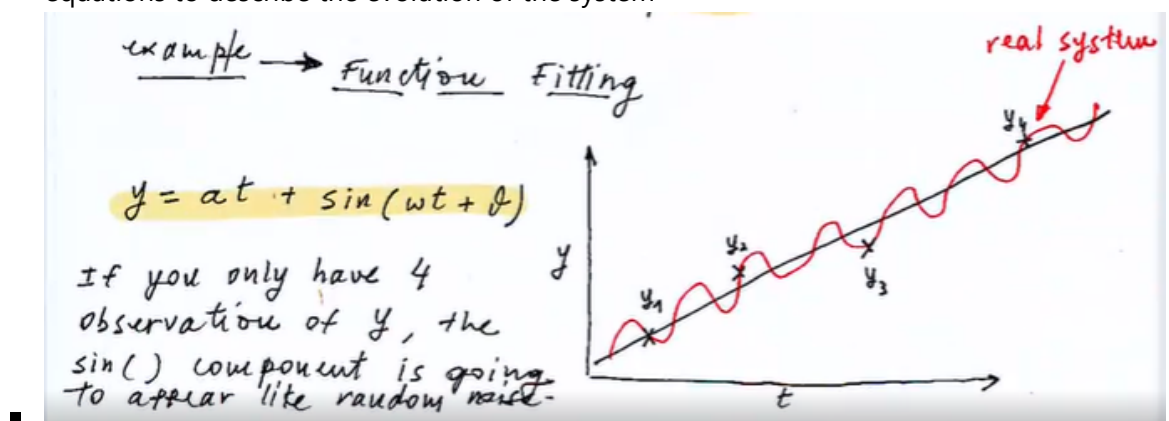
1. part two of the course is hard
 - how to combine the observational data with the theoretical model
2. part three is about forecasting and extrapolation (外推)
 - linear model
 - nonlinear model
3. signal decomposition dimensionality reduction
 - eof
 - pca
 - machine learning
 - ...

topics01 why statistics?

assumption: all environmental variable are controlled by a large **deterministic** system

properties

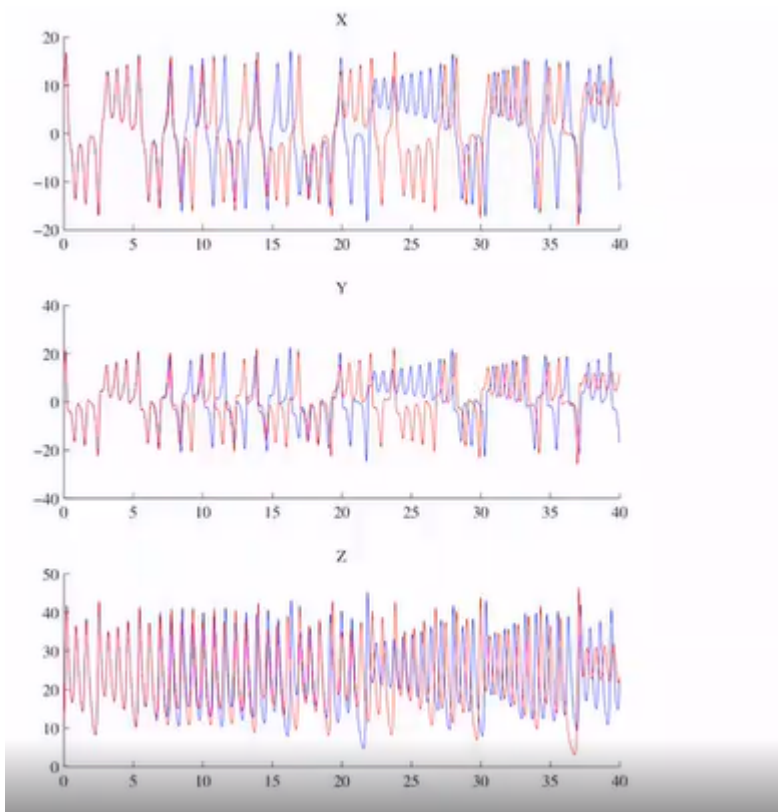
1. system is complex
 - more degrees of freedom than one can observe
 - thus the system will appear to be "non-deterministics" and introduce a "random" component
 - if you have a deterministic system, then you could write down a set of dynamical equations to describe the evolution of the system



in this example, \sin function acts as a "random" component

2. system is nonlinear
 - variable could not be studied in isolation
 - ENSO should couple ocean and atmosphere together
3. dynamics are often unpredictable

- small changes in initial condition $O(\varepsilon)$ would lead to order 1 $O(1)$ changes in the state of the system at future times
- note: **unstable linear system are also unpredictable**



three variable

blue and red curve have great difference in trajectories while they only have $O(\varepsilon)$ difference in initial time, which tells us:

1. small changes in X initial conditions lead to dramatic differences in future state
2. variable Y studied in isolation appears to develop random fluctuations even though at time zero both the red and blue system have exactly the same state, which suggests a nonlinear correlation between X AND Y

chaos

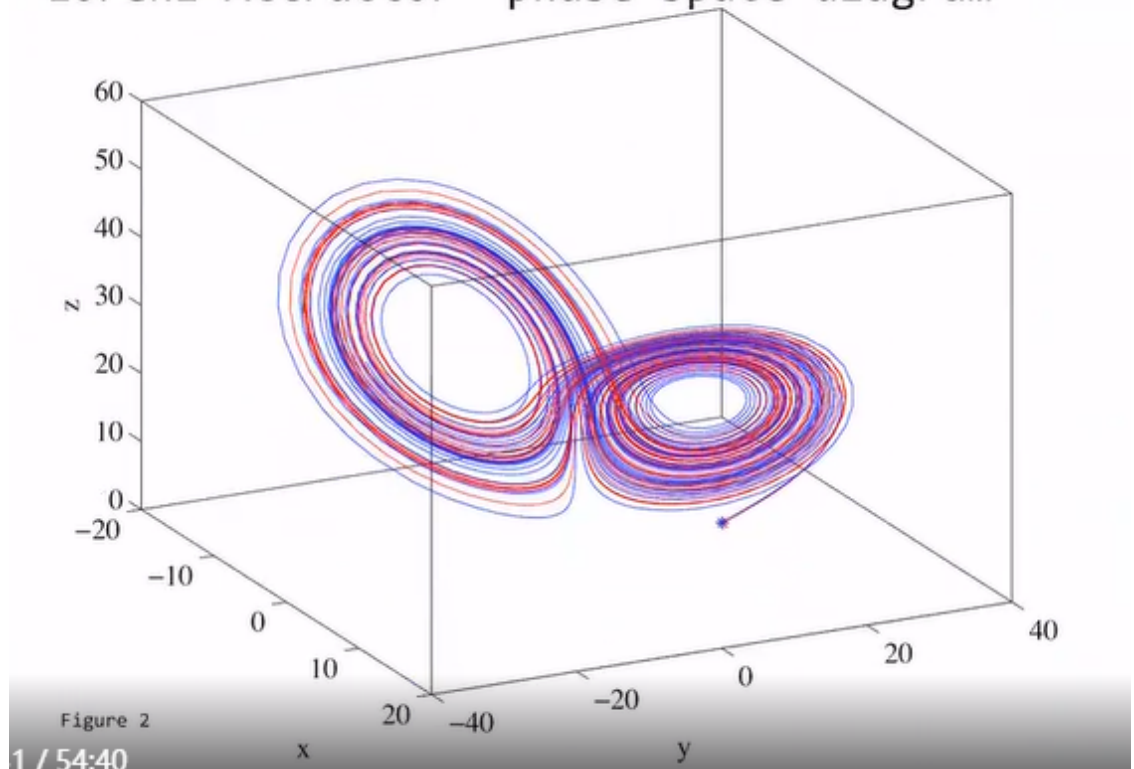
This simple deterministic system is chaotic,
the governing dynamics are

$$\begin{aligned} \frac{dX}{dt} &= -aX + aY \\ \frac{dY}{dt} &= rX - Y - XZ \\ \frac{dZ}{dt} &= -bZ + XY \end{aligned} \quad \left. \vphantom{\begin{aligned} \frac{dX}{dt} &= -aX + aY \\ \frac{dY}{dt} &= rX - Y - XZ \\ \frac{dZ}{dt} &= -bZ + XY \end{aligned}} \right\} \text{Lorenz Attractor}$$

lorenz attractor

- nonlinear deterministic system
 - nonlinear term such as XZ and XY
 - question: how to describe the system
- phase space diagram

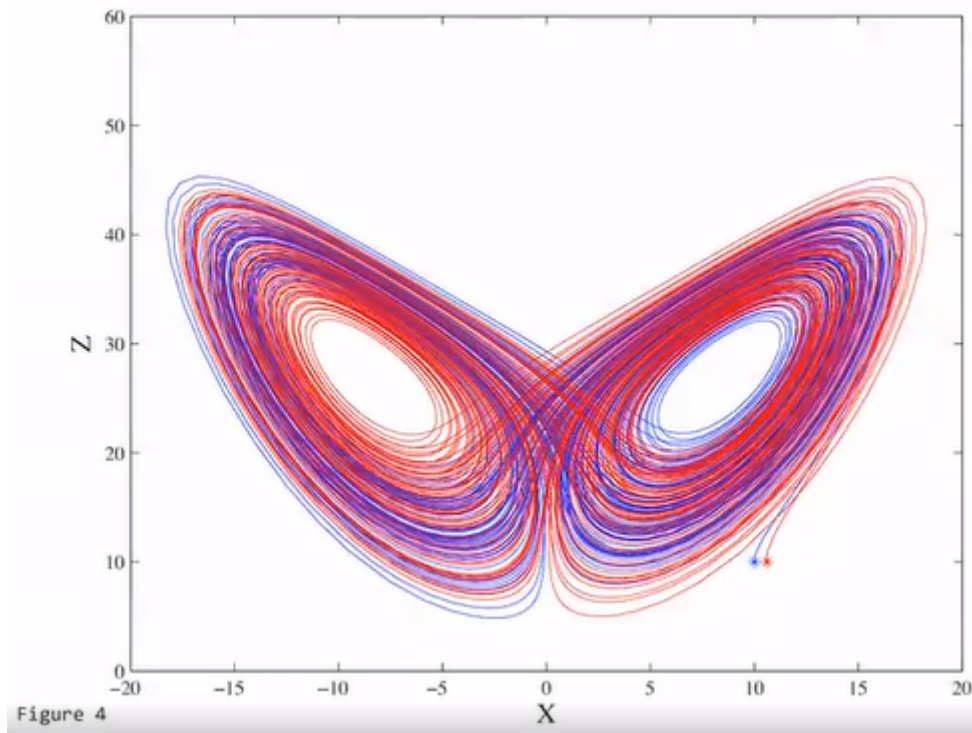
Lorenz Attractor “phase space diagram”



1. this system only have three variable thus it is easy to plot the phase space diagram while in the real case of weather, it may have millions of freedom.
2. the trajectory of the state in phase space collapses around the attractor!

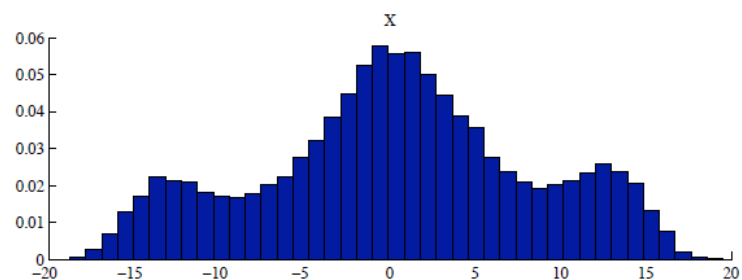
- assume we could not observe Y

A 2D view of the phase space



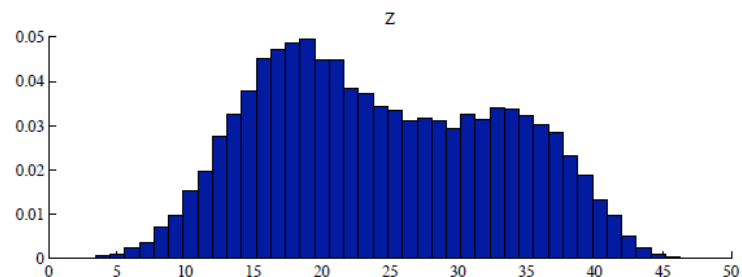
- PDF of the variable

Probability Distribution Function



Area below histograms=1

Figure 5



1. in nature, PDF of most of the environmental data have the shape of Gaussian distribution because of the central limit theory.

2. However, for a nonlinear system, the shape is non-gaussian
3. **Note:** most of the statistics that you would learn is based on the assumption that the distribution of the data is gaussian-shape, so the first thing when you get the data is to