ASSIGNMENT 1: GATE PHYSICS IN: INSTRUMENTATION ENGINEERING

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Some useful physical constants, symbols and formulae

Speed of light in free space, c	3.0×10^{8} m s^{-1}
Atomic mass unit, amu	1.66×10^{-27} kg
Avogadro's number, N_A	6.02×10^{23} mole^{-1}
Bohr magneton, μ_B	9.27×10^{-24} A m^{2}
Boltzmann constant, k_B	1.38×10^{-23} J K^{-1}
Electron charge, e	1.60×10^{-19} C
Planck's constant, h	6.63×10^{-34} J s
Rest mass of electron, m_e	9.11×10^{-31} kg
Reduced Planck's constant, $\hbar$	1.05×10^{-34} J s
Permeability of free space, μ_0	1.26×10^{-6} N A^{-2}
Permittivity of free space, ε_0	8.85×10^{-12} C^{2} N^{-1} m^{-2}

$$\nabla() = \hat{r}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\right) + \hat{\theta}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta) + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

1) The eigenvalues of a matrix are i, $-2i$ and $3i$. The matrix is

(GATE IN 2007)

a) unitary.
b) anti-unitary.
c) Hermitian.
d) anti-Hermitian.

2) A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. This orbit is

(GATE IN 2007)

a) a larger circle.
b) a smaller circle.
c) an ellipse.
d) a parabola.

3) A power amplifier gives 150 W output for an input of 1.5 W. The gain, in dB, is

(GATE IN 2007)

a) 10
b) 20
c) 54
d) 100

4) Four point charges are placed in a plane at the following positions: $+Q$ at $(1,0)$, $-Q$ at $(-1,0)$, $+Q$ at $(0,1)$, and $-Q$ at $(0,-1)$. At large distances the electrostatic potential due to this charge distribution will be dominated by the

(GATE IN 2007)

a) monopole moment.
b) dipole moment.
c) quadrupole moment.
d) octopole moment.

5) A charged capacitor (C) is connected in series with an inductor (L). When the displacement current reduces to zero, the energy of the LC circuit is

(GATE IN 2007)

a) stored entirely in its magnetic field.
b) stored entirely in its electric field.
c) distributed equally among its electric and magnetic fields.
d) radiated out of the circuit.

6) Match the following

(GATE IN 2007)

P. Franck-Hertz experiment	1. electronic excitation of molecules
Q. Hartree-Fock method	2. wave function of atoms
R. Stern-Gerlach experiment	3. spin angular momentum of atoms
S. Franck-Condon principle	4. energy levels in atoms

(A) P-4, Q-2, R-3, S-1
(B) P-1, Q-4, R-3, S-2
(C) P-3, Q-2, R-4, S-1
(D) P-4, Q-1, R-3, S-2

7) The wavefunction of a particle, moving in a one-dimensional time-independent potential $V(x)$, is given by

$$\psi(x) = e^{-ax^2+b}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

This means that the potential $V(x)$ is of the form

(GATE IN 2007)

a) $V(x) \propto x$
b) $V(x) \propto x^2$
c) $V(x) = 0$
d) $V(x) \propto e^{-ax}$

8) The D_1 and D_2 lines of Na ($3^2P_{1/2} \to 3^2S_{1/2}$, $3^2P_{3/2} \to 3^2S_{1/2}$) will split on the application of a weak magnetic field into

(GATE IN 2007)

a) 4 and 6 lines respectively.
b) 3 lines each.
c) 6 and 4 lines respectively.
d) 6 lines each.

9) In a He-Ne laser, the laser transition takes place in

(GATE IN 2007)

a) He only.

b) Ne only.

c) Ne first, then in He.

d) He first, then in Ne.

10) The partition function of a single gas molecule is Z_a. The partition function of N such non-interacting gas molecules is then given by

(GATE IN 2007)

a) $\frac{(Z_a)^N}{N!}$

b) $(Z_a)^N$

c) $N(Z_a)$

d) $\frac{(Z_a)^N}{N}$

11) A solid superconductor is placed in an external magnetic field and then cooled below its critical temperature. The superconductor

(GATE IN 2007)

a) retains its magnetic flux because the surface current supports it.

b) expels out its magnetic flux because it behaves like a paramagnetic material.

c) expels out its magnetic flux because it behaves like an anti-ferromagnetic material.

d) expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field.

12) A particle with energy E is in a time-independent double well potential as shown in the figure.

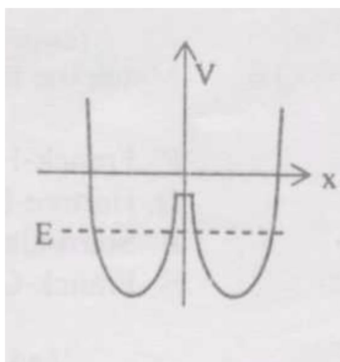


Fig. 12

Which of the following statements about the particle is **NOT** correct?

(GATE IN 2007)

a) The particle will always be in a bound state.

b) The probability of finding the particle in one well will be time-dependent.

c) The particle will be confined to any one of the wells.

d) The particle can tunnel from one well to the other, and back.

13) It is necessary to apply quantum statistics to a system of particles if

(GATE IN 2007)

a) There is substantial overlap between the wavefunctions of the particles.

b) The mean free path of the particles is comparable to the inter-particle separation.

c) The particles have identical mass and charge.

d) The particles are interacting.

14) When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is

(GATE IN 2007)

a) Repelled towards the lower field because it is diamagnetic.

b) Attracted towards the higher field because it is diamagnetic.

c) Repelled towards the lower field because it is paramagnetic.

d) Attracted towards the higher field because it is paramagnetic.

15) Fission fragments are generally radioactive as

(GATE IN 2007)

a) They have excess of neutrons.

b) They have excess of protons.

c) They are products of radioactive nuclides.

d) Their total kinetic energy is of the order of 200 MeV.

16) In a typical *npn* transistor the doping concentrations in emitter, base and collector regions are C_E, C_B and C_C respectively. These satisfy the relation,

(GATE IN 2007)

a) $C_E > C_C > C_B$

b) $C_E > C_B > C_C$

c) $C_C > C_B > C_E$

d) $C_E = C_C > C_B$

17) The allowed states for He $(2p^2)$ configuration are

(GATE IN 2007)

a) ${}^1S_0$, ${}^3S_1$, ${}^1P_1$, ${}^3P_{0,1,2}$, ${}^1D_2$, ${}^3D_{1,2,3}$

b) ${}^1S_0$, ${}^3P_{0,1,2}$, ${}^1D_2$

c) ${}^1P_1$, ${}^3P_{0,1,2}$

d) ${}^1S_0$, ${}^1P_1$

18) The energy levels of a particle of mass m in a potential of the form

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2, & x > 0 \end{cases}$$

are given, in terms of quantum number $n = 0, 1, 2, \ldots$, by

(GATE IN 2007)

a) $\left(n + \frac{1}{2}\right)\hbar\omega$

b) $\left(2n + \frac{1}{2}\right)\hbar\omega$

c) $\left(2n + \frac{3}{2}\right)\hbar\omega$

d) $\left(n + \frac{3}{2}\right)\hbar\omega$

19) The electromagnetic field due to a point charge must be described by lienard-wechert potentials when

(GATE IN 2007)

a) The point charge is highly accelerated.

b) the electric and magnetic fields are not perpendicular.

c) the point charge is moving with velocity close to that of light.

d) the calculation is done for the radiation zone.

20) The strangeness quantum number is conserved in

(GATE IN 2007)

a) strong, weak and electromagnetic interactions

b) weak and electromagnetic interactions only.

c) strong and weak interaction only.

d) strong and electromagnetic interactions only.

21) the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ are

(GATE IN 2007)

a) 6, 1 and $\begin{pmatrix} 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

b) 2, 5 and $\begin{pmatrix} 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

c) 6, 1 and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

d) 2, 5 and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

22) A vector field is defined everywhere as

$$\mathbf{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$$

. The net flux of **F** associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, Z axes is

(GATE IN 20007)

a) $2L^3$

b) $4L^3$

c) $8L^3$

d) $10L^3$

23) If $\mathbf{r} = x\hat{i} + y\hat{j}$, then

(GATE IN 2007)

24) Consider a vector $\mathbf{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\mathbf{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is

(GATE IN 2007)

a) $(1 - \sqrt{3})\hat{i}' + 3\hat{j}' + (1 + \sqrt{3})\hat{k}'$

b) $(1 + \sqrt{3})\hat{i}' + 3\hat{j}' + (1 - \sqrt{3})\hat{k}'$

c) $(1 - \sqrt{3})\hat{i}' + (3 + \sqrt{3})\hat{j}' + 2\hat{k}'$

d) $(1 - \sqrt{3})\hat{i}' + (3 - \sqrt{3})\hat{j}' + 2\hat{k}'$

25) The contour integral $\oint \frac{dz}{z^2+a^2}$ is to be evaluated on a circle of radius $2a$ centered at the origin. It will have contributions only from the points

(GATE IN 2007)

a) $\frac{1+i}{\sqrt{2}}a$ and $\frac{1-i}{\sqrt{2}}a$

b) ia and $-ia$

c) $ia, -ia, \frac{-1+i}{\sqrt{2}}a$ and $\frac{-1-i}{\sqrt{2}}a$

d) $\frac{1+i}{\sqrt{2}}a$, $\frac{-1+i}{\sqrt{2}}a$, $\frac{-1-i}{\sqrt{2}}a$ and $\frac{1-i}{\sqrt{2}}a$

26) Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is

(GATE IN 2007)

a) $\cos 2x + \frac{1}{2}\sin 2x$
b) $\cos x + \frac{1}{2}\sin x$
c) $\cosh x + \frac{1}{2}\sinh x$
d) $\cosh 2x + \frac{1}{2}\sinh 2x$

27) The points, where the series solution of the Legendre differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0$ will diverge, are located at

(GATE IN 2007)

a) 0 and 1
b) 0 and -1
c) -1 and 1
d) $\frac{3}{2}$ and $\frac{5}{2}$

28) Solution of the differential equation $x\frac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$ at $x = 1$, is

(GATE IN 2007)

a) $y = 5x^4 - 4$
b) $y = x^4 + \frac{4x}{5}$
c) $y = \frac{4x^4}{5} + \frac{1}{5x}$
d) $y = x^4 + \frac{4}{5x}$

29) Match the following:

TABLE 29: Matching list for Question 29

P. rest mass	1. timelike vector
Q. charge	2. Lorentz invariant
R. four-momentum	3. tensor of rank 2
S. electromagnetic field	4. conserved and Lorentz invariant

(GATE IN 2007)

a) P-2, Q-4, R-3, S-1
b) P-4, Q-2, R-1, S-3
c) P-2, Q-4, R-1, S-3
d) P-4, Q-2, R-3, S-1

30) The moment of inertia of a uniform sphere of radius r about an axis passing through its centre is given by $\frac{2}{5}\left(\frac{4\pi r^3}{3}\rho\right)$. A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radius $R/2$ hollowed out of it, as shown in the figure. The moment of inertia of the resulting body about the Y-axis is

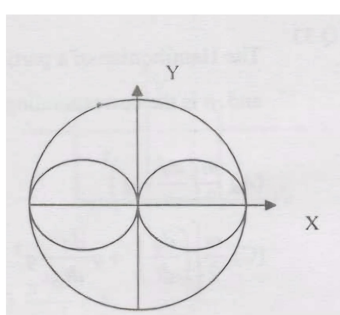


Fig. 30

Which of the following statements about the particle is **NOT** correct?

(GATE IN 2007)
(GATE IN 20007)

a) $\frac{\pi\rho R^5}{4}$ b) $\frac{5\pi\rho R^5}{12}$ c) $\frac{7\pi\rho R^5}{12}$. d) $\frac{3\pi\rho R^5}{4}$

31) The Lagrangian of a particle of mass m is

$$L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{\nu}{2}\left(x^2 + y^2\right) + W' \sin(\omega t),$$

where ν, W' and ω are constants. The conserved quantities are:

(GATE IN 20007)

a) energy and z-component of linear momentum only.

b) energy and z-component of angular momentum only.

c) 2-components of both linear and angular momenta only.

d) energy and z-components of both linear and angular momenta.

32) Three particles of mass m each situated at $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is:

(GATE IN 20007)

a) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 + \frac{k}{2}(x_1 - x_2 - l)^2$

b) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_9 - l)^2 + \frac{k}{2}(x_1 - x_2 - l)^2$

c) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 + l)^2 - \frac{k}{2}(x_1 - x_2 + l)^2$

d) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 - \frac{k}{2}(x_1 - x_2 - l)^2$

33) The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$ where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is:

(GATE IN 20007)

a) $\frac{m}{2}(\dot{q} + q)^2$

b) $\frac{m}{2}(\dot{q} - q)^2$

c) $\frac{m}{2}\left[\dot{q}^2 + q\dot{q} - q^2\right]$

d) $\frac{m}{2}\left[\dot{q}^2 - q\dot{q} + q^2\right]$

34) A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permeability μ_r. The magnitude of magnetic induction $\mathbf{B}$ inside the toroid, at a radial distance r from the axis, is given by:

(GATE IN 20007)

a) $\mu_r\mu_0\frac{NI}{r}$

b) $\mu\,\mu_0\frac{NI}{r}$

c) $\frac{\mu_r\mu_0 NI}{2\pi r}$

d) $2\pi\,\mu_r\mu_0\frac{NI}{r}$

35) Can the following scalar and vector potentials describe an electromagnetic field?

$$\phi(x, y, z) = 3xyz - 4r \tag{1}$$

$$\mathbf{A}(\mathbf{x}, t) = (2x - \omega t)\,\hat{i} + (y - 2z)\,\hat{j} + (z - 2xe^{i\omega t})\,\hat{k} \tag{2}$$

(GATE IN 20007)

a) Yes, in the Coulomb gauge.

b) Yes, in the Lorentz gauge.

c) Yes, provided $\omega = 0$.

d) No.

36) An electromagnetic wave $\tilde{E}(z,t) = E_0 \cos(\alpha v t - kz)\,\hat{z}$ is traveling in free space and crosses a disc of radius 2 m placed perpendicular to the z-axis. If $E_z = 60$ V m^{-1}, the average power, in Watt, crossing the disc along the z-direction is:
(GATE IN 20007)

a) 30
b) 60
c) 120
d) 270

37) For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2 x^2$. the first excited energy eigenstate is $\psi(x) = x\,e^{-\alpha\omega x^2}$ The value of α is:
(GATE IN 20007)

a) $\frac{m\omega}{4\hbar}$
b) $\frac{m\omega}{3\hbar}$
c) $\frac{m\omega}{2\hbar}$
d) $\frac{2m\omega}{3\hbar}$

38) If $[x, p] = i\hbar$, the value of $[x^3, p]$ is:
(GATE IN 20007)

a) $2i\hbar x^2$
b) $-\frac{2}{\hbar}x^2$
c) $3i\hbar x^2$
d) $-3i\hbar x^2$

39) There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies

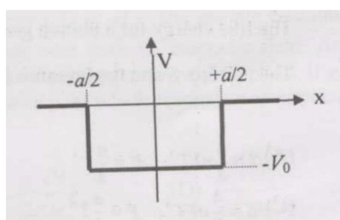


Fig. 39

(GATE IN 20007)

a) $\frac{m}{2}\left(\frac{dq}{dt} + q\right)^2$
b) $\frac{m}{2}\left(\frac{dq}{dt} - q\right)^2$
c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 + q\frac{dq}{dt} - q^2\right]$
d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 - q\frac{dq}{dt} + q^2\right]$

40) A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permeability μ_r. The magnitude of magnetic induction **B** inside the toroid, at a radial distance r from the axis, is given by:
(GATE IN 20007)

a) $\mu_r\mu_0\frac{NI}{r}$
b) $\mu\,\mu_0\frac{NI}{r}$
c) $\frac{\mu_r\mu_0 NI}{2\pi r}$
d) $2\pi\,\mu_r\mu_0\frac{NI}{r}$

41) An electromagnetic wave $\tilde{E}(z,t) = E_0 \cos(\alpha v t - kz)\,\hat{z}$ is traveling in free space and crosses a disc of radius 2 m placed perpendicular to the z-axis. If $E_z = 60$ V m^{-1}, the average power, in Watt, crossing the disc along the z-direction is:
(GATE IN 20007)

a) 30 b) 60 c) 120 d) 270

42) For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2 x^2$, the first excited energy eigenstate is $\psi(x) = x\,e^{-\alpha\omega x^2}$. The value of α is:

(GATE IN 20007)

a) $\frac{m\omega}{4\hbar}$ b) $\frac{m\omega}{3\hbar}$ c) $\frac{m\omega}{2\hbar}$ d) $\frac{2m\omega}{3\hbar}$

43) If $[x, p] = i\hbar$, the value of $[x^3, p]$ is:

(GATE IN 20007)

a) $2i\hbar x^2$ b) $-\frac{2}{\hbar}x^2$ c) $3i\hbar x^2$ d) $-3i\hbar x^2$

44) The free energy for a photon gas is given by $F = -\frac{\alpha}{3}\nu T^4$, where α is a constant. The entropy S and the pressure P of the photon gas are

(GATE IN 2007)

a) $S = \frac{4}{3}aVT, \quad P = \frac{a}{3}T^4$

b) $S = \frac{1}{3}aVT^4, \quad P = \frac{4a}{3}T^3$

c) $S = \frac{4}{3}aVT^3, \quad P = \frac{a}{3}T^3$

d) $S = \frac{1}{3}aVT^3, \quad P = \frac{4a}{3}T^4$

45) A system has energy levels $E_0,\ 2E_0,\ 3E_0, \ldots$ where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is

(GATE IN 2007)

a) 2 b) 3 c) 4 d) 5

46) In accordance with the selection rules for electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states

(GATE IN 2007)

a) $2^1S_0,\ 2^1P_1,\ 3^1D_2$

b) $3^1P_1,\ 3^1D_2,\ 3^1S_0$

c) $3^3P_2,\ 3^3D_3,\ 3^3P_0$

d) $2^3S_1,\ 3^3D_2,\ 3^3D_1$

47) If an atom is in the $^3D_3$ state, the angle between its orbital and spin angular momentum vectors ($\mathbf{L}and\mathbf{S}$)is

(GATE IN 2007)

a) $\cos\left(\frac{1}{\sqrt{3}}\right)$ b) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\cos^{-1}\left(\frac{1}{2}\right)$ d) $\cos\left(\frac{\sqrt{3}}{2}\right)$

48) The hyperfine structure of Na ($3^2P_{3/2}$) with nuclear spin $I = \frac{3}{2}$ has

(GATE IN 2007)

a) 1 state b) 2 states c) 3 states d) 4 states

49) The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J + 1)$ where B is the rotational constant and J is a rotational quantum number.
In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass changes to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line corresponding to a transition $J = 4 \to J = 5$ is

(GATE IN 2007)

a) $0.475B$ b) $0.50B$ c) $0.95B$ d) $1.0B$

50) The number of fundamental vibrational modes of CO_2 molecule is

(GATE IN 2007)

a) Four: 2 Raman active and 2 infrared active

b) Four: 1 Raman active and 3 infrared active

c) Three: 1 Raman active and 2 infrared active

d) Three: 2 Raman active and 1 infrared active

51) A piece of paraffin is placed in a uniform magnetic field H_0. The sample contains hydrogen nuclei of mass m_p which interact only with the external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorption. If g is the g-factor of the hydrogen nucleus, the frequency at which resonance absorption takes place is given by

(GATE IN 2007)

a) $\frac{3g_1 eH_0}{2\pi m_p}$ b) $\frac{3g_j eH_0}{4\pi m_p}$ c) $\frac{g_z eH_0}{2\pi m_p}$ d) $\frac{S_r eH_q}{4\pi m_p}$

52) The solid phase of an element follows van der Waals bonding with inter-atomic potential $V(r) = -\frac{P}{r^5} + \frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(GATE IN 2007)

a) $\left(\frac{2P}{Q}\right)^{-6}$ b) $\left(\frac{P}{Q}\right)^{-6}$ c) $\left(\frac{P}{2Q}\right)^{-6}$ d) $\left(\frac{P}{Q}\right)^{-6}$

53) Consider the atomic packing factor (APF) of the following crystal structures:
P. Simple Cubic
Q. Body Centred Cubic
R. Face Centred Cubic
S. Diamond
T. Hexagonal Close Packed
Which two of the above structures have equal APF?

(GATE IN 2007)

a) P and Q b) S and T c) R and S d) R and T

54) In a powder diffraction pattern recorded from a face-centred cubic sample using x-rays, the first peak appears at 30°. The second peak will appear at

(GATE IN 2007)

a) 32.8° b) 33.7° c) 34.8° d) 35.3°

55) Variation of electrical resistivity ρ with temperature T of three solids is sketched ($ondifferentscales$) in the figure, as curves P, Q and R.

Which one of the following statements describes the variations most appropriately?

(GATE IN 2007)

a) P is a superconductor, and R for a semiconductor.

b) Q is a superconductor, and P for a conductor.

c) Q is a superconductor, and R for a conductor.

d) R is a superconductor, and P for a conductor.

56) An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the doping concentration varies as $N_D(x) = N_0 \exp(-x/L)$. Assume that the mobility of the majority carriers remains constant. The resistance R of the sample is given by:

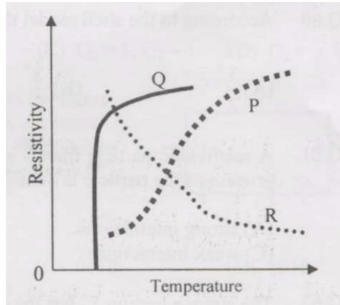


Fig. 55

(GATE IN 2007)

a) $R = \frac{L}{A\mu N_0}\left[\exp(1) - 1\right]$

b) $R = \frac{L}{\mu\pi r N_0}\left[\exp(1) - 1\right]$

c) $R = \frac{L}{A\epsilon N_0}\left[\exp(-1) - 1\right]$

d) $R = \frac{L}{A\mu N_g}$

57) A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of 1.3×10^6 A/m. Assume that the total number of atoms per unit volume is 8×10^{28} m^{-3}. The magnetic moment of an iron atom, in terms of the Bohr magneton, is:

(GATE IN 2007)

a) 1.7

b) 2.3

c) 2.9

d) 3.8

58) Half-life of a radio-isotope is 4×10^4 years. If there are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^5 years ago were:

(GATE IN 2007)

a) 1.28×10^5

b) 2.56×10^5

c) 5.12×10^5

d) 1.024×10^6

59) In the deuterium + tritium (D + T) fusion, more energy is released compared to deuterium + deuterium (D + D) fusion because:

(GATE IN 2007)

a) Tritium is radioactive

b) More nucleons participate in fusion

c) The Coulomb barrier is lower for the D+T system than D+D system

d) The reaction product He is more tightly bound

60) According to the shell model, the ground state spin of the ^{17}O nucleus is:

(GATE IN 2007)

a) $3/2^+$

b) $5/2^+$

c) $3/2^-$

d) $5/2^-$

61) A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by:

(GATE IN 2007)

a) Strong interactions
b) Electromagnetic interactions
c) Weak interactions
d) Gravitational interactions

62) The strange baryon Ξ^- has the quark structure:

(GATE IN 2007)

a) uds
b) uud
c) uus
d) $u\bar{s}$

63) A neutron scatters elastically from a heavy nucleus. The initial and final states off the neutron have the

(GATE IN 2007)

a) same energy.
b) same energy and linear momentum.
c) same energy and angular momentum.
d) same linear and angular momentum.

64) The circuit shown is based on ideal operational amplifiers. It acts as a

(GATE IN 2007)

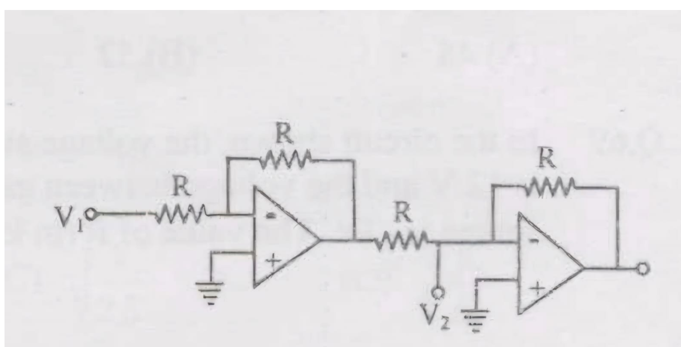


Fig. 64

a) subtractor
b) buffer amplifier
c) adder
d) divider

65) Identify the function F generated by the logic network shown

(GATE IN 2007)

a) F = $(X + Y)$Z
b) F = Z + Y +YX
c) ZY$(Y + X)$
d) XYZ

66) In the circuit shown, the ports Q_1 and Q_2 are in the state $Q_1 = 1$, $Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes

(GATE IN 2007)

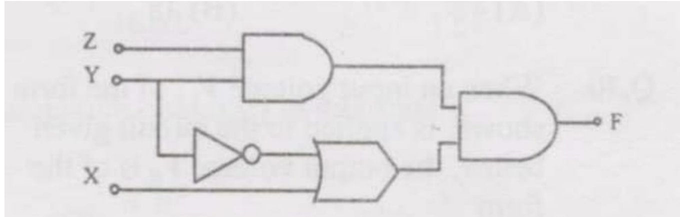


Fig. 65

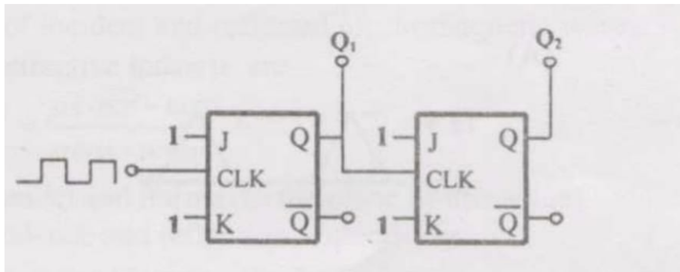


Fig. 66

a) $Q_2 = 1, Q_1 = 0$ b) $Q_2 = 0, Q_1 = 1$ c) $Q_2 = 1, Q_1 = 1$ d) $Q_2 = 0, Q_1 = 0$

67) The registers Q_D, Q_C, Q_B and Q_A shown in the figure are initially in the state 1010 respectively. An input sequence SI = 0101 is applied. After two clock pulses, the state of the shift registers is

(GATE IN 2007)

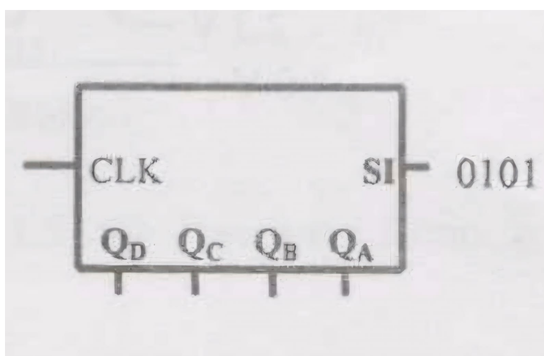


Fig. 67

a) 1001 b) 0100 c) 0110 d) 1010

68) For the circuit shown, the potential difference in volts across R_L is

(GATE IN 2007)

a) 48 b) 52 c) 56 d) 65

69) In the circuit shown, the voltage at test point P is 12 V and the voltage between gate and source is -2V. The value of R in kΩis

(GATE IN 2007)

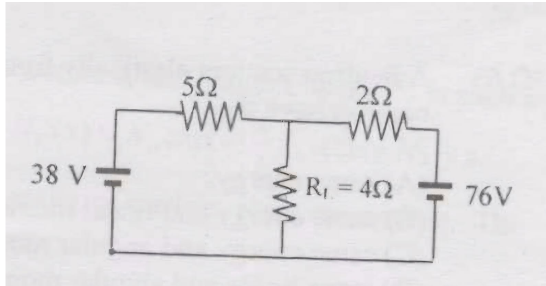


Fig. 68

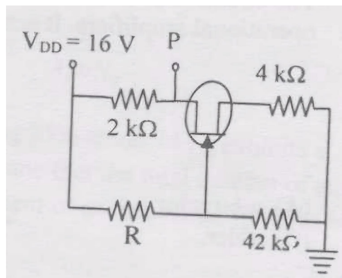


Fig. 69

a) 42 b) 48 c) 56 d) 70

70) when an input voltage V_i, of the form shown, is applied to the circuit given below, the output voltage is of the form (GATE IN 2007)

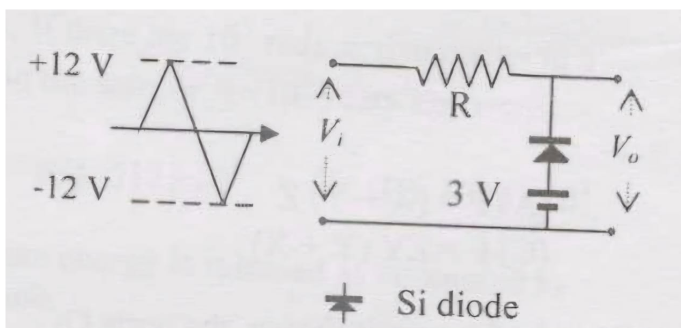
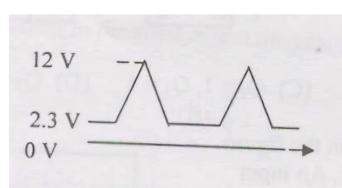
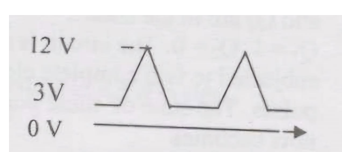


Fig. 70

a) 

b) 

c) 

d) 

Common Data Questions

Common Data for Questions 71,72,73:

A particle of mass m is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = 2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_A \cos\left(\frac{\pi x}{4L}\right)$, where ψ_A is a constant.

71) The normalization factor of this wavefunction is:

(GATE IN 2007)

a) $\sqrt{2/L}$

b) $\sqrt{1/(4L)}$

c) 24

d) $\sqrt{1/L}$

72) The energy eigenvalue corresponding to this state is:

(GATE IN 2007)

a) $\frac{\hbar^2\pi^2}{2mL^2}$

b) $\frac{\hbar^2\pi^2}{4mL^2}$

c) $\frac{\hbar^2\pi^2}{16mL^2}$

d) $\frac{\hbar^2\pi^3}{32mL^2}$

73) The expectation value of p^2 (momentum operator) in this state is:

(GATE IN 2007)

a) 0

b) $\frac{h^2\pi^2}{32L^2}$

c) $\frac{h^2\pi^3}{16L^2}$

d) $\frac{h^2\pi^3}{8L^2}$

Common Data for Questions 74,75:

The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ are:

$$E_{\parallel}^{\text{ref}}/E_{\parallel}^{\text{inc}} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i}, \quad E_{\perp}^{\text{ref}}/E_{\perp}^{\text{inc}} = \frac{\cos i - \mu\cos r}{\cos i + \mu\cos r}$$

where i and r are the angles of incidence and refraction respectively.

74) The condition for the reflected ray to be completely polarized is:

(GATE IN 2007)

a) $\mu\cos i = \cos r$

b) $\cos i = \mu\cos r$

c) $\mu\cos i = -\cos r$

d) $\cos i = -\mu\cos r$

75) For normal incidence at an air-glass interface with $\mu = 1.5$, the fraction of energy reflected is:

(GATE IN 2007)

a) 0.40

b) 0.20

c) 0.16

d) 0.04

Linked Answer Questions: Q.76 to Q.81 carry two marks each.

Statement for Linked Answer Questions 76 & 77:
In the laboratory frame, a particle P of rest mass m_0 is moving in the positive x direction with speed $5c/19$. It approaches an identical particle Q, moving in the negative x direction with speed $2c/5$.

76) The speed of the particle P in the rest frame of particle Q is:

(GATE IN 2007)

a) $\frac{7c}{95}$ b) $\frac{13c}{85}$ c) $\frac{3c}{5}$ d) $\frac{63c}{95}$

77) The energy of the particle P in the rest frame of particle Q is:

(GATE IN 2007)

a) $\frac{1}{2}m_0\omega^2$ b) $\frac{5}{4}m_0c^2$ c) $\frac{19}{13}m_0c^2$ d) $\frac{11}{9}m_0c^2$

Statement for Linked Answer Questions 78 & 79:
The atomic density of a solid is 5.85×10^{28} m^{-3}. Its electrical resistivity is 1.6×10^{-4} $\Omega \cdot$m. Assume electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

78) The drift mobility (m^2 V^{-1}s^{-1}) of the conduction electrons is:

(GATE IN 2007)

a) 6.67×10^{-3} b) 6.67×10^{-6} c) 7.63×10^{-1} d) 7.63×10^{-4}

79) The relaxation time (mean free time), in seconds, of the conduction electrons is:

(GATE IN 2007)

a) 3.98×10^{-15} b) 3.79×10^{-14} c) 2.84×10^{-12} d) 2.64×10^{-11}

Statement for Linked Answer Questions 80 & 81:
A sphere of radius R carries a polarization $\overline{P} = k\mathbf{r}$, where k is a constant and $\mathbf{r}$ is measured from the centre of the sphere.

80) The bound surface and volume charge densities are given, respectively, by:

(GATE IN 2007)

a) $-k|\mathbf{r}|$ and $3k$ b) $k|\mathbf{r}|$ and $-3k$ c) k and $-4kR$ d) $-k|\mathbf{r}|$ and $4kR$

81) The electric field $\mathbf{E}$ at a point outside the sphere is given by:

(GATE IN 2007)

a) $\hat{E} = 0$ b) $\overline{E} = \frac{kR(R^2 - r^2)}{\epsilon_0 r^3}\hat{r}$ c) $\mathbf{E} = \frac{kR(R^3 - r^2)}{\kappa_a r^3}\hat{r}$ d) $\mathbf{E} = \frac{3k(r - R)}{4\pi\epsilon_0 r^4}\hat{r}$

Statement for Linked Answer Questions 82 & 83:
An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/(k_B\beta)$.

82) The partition function of a single oscillator with energy levels $(m + 1/2)\hbar\omega$ is:
(GATE IN 2007)

a) $Z = e^{-\beta\hbar\omega/2}\frac{1}{1-e^{-\beta\hbar\omega}}$
b) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$
c) $Z = e^{-\beta\hbar\omega/2}\frac{1}{1+e^{-\beta\hbar\omega}}$
d) $Z = \frac{1}{1+\mu^{-\beta km}}$

83) The average number of energy quanta of the oscillators is given by:
(GATE IN 2007)

a) $\langle n \rangle = \frac{1}{e^{2\lambda+\nu}-1}$
b) $\langle H \rangle = \frac{e^{-j\lambda x}}{e^{j\lambda x}-1}$
c) $\langle n \rangle = \frac{1}{e^{x\lambda x}+1}$
d) $\langle n \rangle = \frac{e^{-\beta\beta w}}{e^{\beta\beta w\sigma}+1}$

Statement for Linked Answer Questions 84 & 85:
A $16\,\mu$A beam of alpha particles, having cross-sectional area $10^{-4}\,\text{m}^2$, is incident on a rhodium target of thickness $1\,\mu$m. This produces neutrons through the reaction:

$$\alpha + {}^{100}\text{Rh} \rightarrow {}^{100}\text{Pd} + 3n$$

84) The number of alpha particles hitting the target per second is:
(GATE IN 2007)

a) 0.5×10^{14}
b) 1×10^{14}
c) 2×10^{10}
d) 4×10^{11}

85) The neutrons are observed at the rate of $1.306 \times 10^4\ \text{s}^{-1}$. If the density of rhodium is approximated as $10\,\text{kg/m}^3$, the cross-section (in barns) is:
(GATE IN 2007)

a) 0.1
b) 0.2
c) 0.4
d) 0.8

END OF THE QUESTION PAPER

Space for Rough Work: