ELE8059 Coursework Assignment 1

Your coursework should be submitted on Canvas, submitting a single PDF file. If you wish, you can write your answers by hand, take images of your pages, combine these into a document (for example using MS Word or other software) together with matlab codes and figures, and create a PDF from that; please make sure though that your handwriting is readable. Of course, you can also write the answers directly in MS Word (or other tools, such as Latex), using the equation typesetting tools available in such software for mathematical expressions. All figures plotted with Matlab and all Matlab codes should included in the PDF submission (e.g. you can copy and paste the code into your document). You should label figures with numbers, and where appropriate, refer to these in the text.

The assignments are to be done strictly individually; students can give each other advice on the course material of the sessions, and may also help each other understanding the assignment questions. However, a student should strictly adhere to not supplying any information about any of her/his assignment answers to another student. Evidence of collusion (such as the same unlikely mistake in one particular answer made by a number of students) will be looked at very critically Evidence of collusion will be investigated under the University's academic offense procedures, and can lead to a mark of zero for the whole assignment (in some cases, with no option to re-submit the work). It is therefore extermely important that students:

(a) do not collaborate on the coursework,

(b) do not to copy anything from another student,

(c) do not to pass on any information regarding her/his answers to another student (also not after the specified submission dead-line).

The above holds for all parts of the assignment, including Matlab code, written answers, and figure plots.

For written answers, supply some text that explains your reasoning; in other words, do not restrict the submitted to purely mathematical calculations, but supply (brief!) information as to why you are taking each calculational step. Please ensure however that each of your answers directly addresses the question, and is not a compilation of your knowldege about the main topic. For Matlab coding, avoid using Simulink, and avoid also any software shortcut to an end result that does not follow the specific instructions given in the question.

Mark Scheme

The points available for each individual subquestion are listed in the table below.

Tab. 1: *Marking scheme.*

	(a)	(b)	(c)	(d)	(e)	(f)
Q1	6	9	7			
Q2	6	5	7	5	6	6
Q3	7	8	6	5		
Q4	6	6	5			

QUESTION 1

Consider a continuous-time system in which the relationship between input (x) and output (y) is:

$$y(t) + 0.5\frac{\partial y}{\partial t} = 3\frac{\partial x}{\partial t}, \tag{1}$$

where t denotes time.

(a) Specify the transfer function and the zeros and poles of the system.

(b) Specify the output signal as a function of time when the input signal is a pulse of width T, as plotted in Figure 1.

(c) Write a Matlab script that plots the output signal (as obtained under (b)) for $T = 1$ between $t = 0$ and $t = 4$ seconds. Include both the plot and a printout of the Matlab script in your submission.

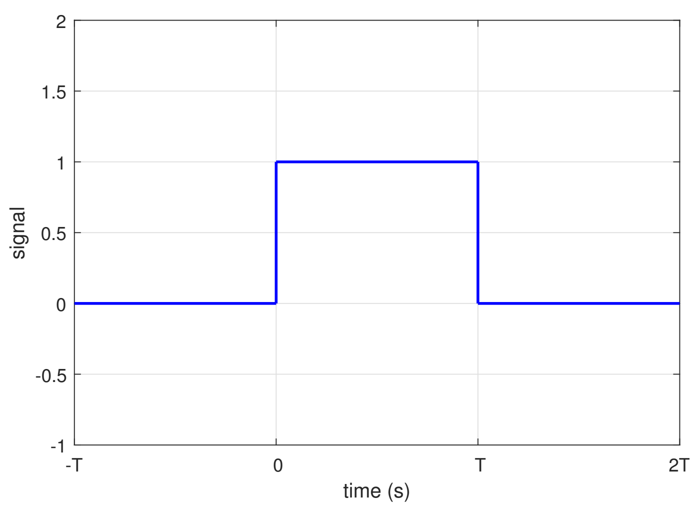

Fig. 1: A pulse of finite width T.

QUESTION 2

The transfer function of a digital system, running at a sample rate $f_s = 10$ kHz is given by

$$H(z) = \frac{a + z^{-1}}{1 + az^{-1}}, \tag{2}$$

where a is a real-valued coefficient.

(a) Derive the difference equation of this filter.

(b) Specify the range of (real) values of a for which the system is stable.

(c) Show (analytically) that, for any a, the power response equals unity at all frequencies i.e. $|H(\omega)|^2 = 1$ for any frequency ω.

(d) What is the effect of the system on a sinusoidal input signal? In words, describe any differences between the input and the output signal.

(e) Write a Matlab script that plots the impulse response $h(n)$ for $a = 0.5$, in the range $0 \leq n \leq 10$, where n indicates time (in samples). Include both the plot and a printout of the Matlab script in your submission.

(f) Sketch a realisation (signal diagram) of the system that uses only one multiplier.

QUESTION 3

A Discrete Fourier Transform (DFT) processor has, as default settings, an input block length of $N = 128$ samples and a sampling frequency of 64 kHz. An analog signal comprising 3 sinusoids of equal amplitude at frequencies of 11.5 kHz, 20.7 kHz and 38 kHz is sampled at the default sample rate. A 128-sample sampled-data sequence is then routed to the input of the DFT processor.

(a) Write a matlab script that performs the DFT on the described block of samples. Plot the resulting (linear-scale) amplitude response. Include the plot in your submission, as well as a printout of the Matlab script.

(b) Explain any non-ideal responses found under (a). Detail how these problems can be eliminated.

(c) Now consider a scenario where the input only contains the 11.5 kHz and 20.7 kHz components, and in which - in contrast to the scenarion in question (b) - no changes can be made to either the signal block length or the sampling period of the processor. Explain what could still be done to at least alleviate any problems associated with the spectral representation of the signal.

(d) For a a block length $N = 128$, by what ratio is the computational complexity (as defined by the number of complex multiplications) of the processor reduced when using an FFT instead of a DFT? In other words, how many times more complex multiplications are required with the DFT than with the FFT in this case?

QUESTION 4

Consider the two following digital sequences:

$$x_1(n) = [2, 3, 0, 1], \tag{3}$$

$$x_2(n) = [1, -1, 0, 2, 1]. \tag{4}$$

(a) Calculate, by hand, the linear convolution of these two signals.

(b) Write a Matlab script that performs the linear convolution of these two signals via frequency-domain multiplication, making use of the FFT and IFFT commands to compute any Discrete Fourier Transforms and/or Inverse Discrete Fourier Transforms. Copy the result in your submission, and supply a printout of the code.

(c) Explain in what scenario(s) a blockwise approach is typically employed in convolution, and why; name two possible methods for block-wise convolution.