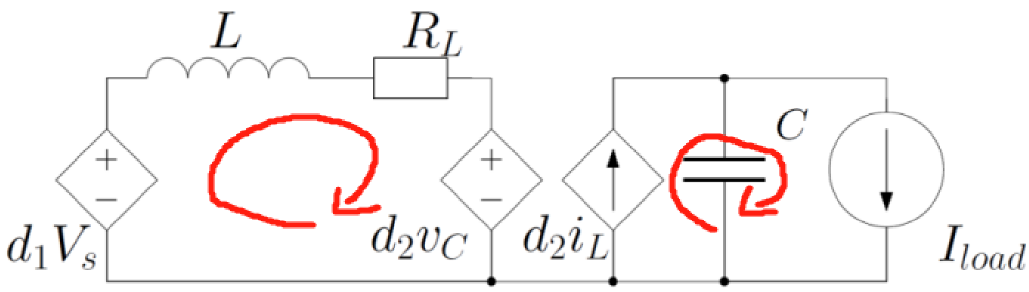
1.



In the left circle

$$d_1V_s = L\frac{di}{dt} + i_LR_L + d_2v_c$$

$$-L\frac{di}{dt} = -d_1V_s + i_LR_L + d_2v_c$$

$$\frac{di}{dt} = \frac{d_1V_s}{L} - \frac{i_LR_L}{L} - \frac{d_2v_C}{L}$$

In the right point

$$d_2i_L = C\frac{dv}{dt} + I_{load}$$

$$-C\frac{dv}{dt} = -d_2i_L + I_{load}$$

$$\frac{dv}{dt} = \frac{d_2i_L}{C} - \frac{I_{load}}{C}$$

So

$$\frac{di_L}{dt} = \frac{d_1V_s}{L} - \frac{i_LR_L}{L} - \frac{d_2v_C}{L}$$

$$\frac{dv_C}{dt} = \frac{d_2i_L}{C} - \frac{I_{load}}{C}$$

And the state space representation

$$\frac{d}{dt}\begin{bmatrix} v_c \\ i_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R_L}{L} \end{bmatrix}\begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{i_L}{C} \\ \frac{V_s}{L} & -\frac{v_C}{L} \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} -\frac{I_{load}}{C} \\ 0 \end{bmatrix}$$

The overall system is a switching system of the form

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R_L}{L} \end{bmatrix}x + \begin{bmatrix} 0 & \frac{i_L}{C} \\ \frac{V_s}{L} & -\frac{v_C}{L} \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} -\frac{I_{load}}{C} \\ 0 \end{bmatrix}$$

So

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{0.2}{0.0002} \end{bmatrix} \begin{bmatrix} 20 \\ 0.4 \end{bmatrix} + \begin{bmatrix} 0 & \frac{0.4}{0.000022} \\ \frac{15}{0.0002} & -\frac{20}{0.0002} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} -\frac{0.2}{0.000022} \\ 0 \end{bmatrix}$$

Thus

$$0 = 0 + \frac{0.4}{0.000022} d_2 - \frac{0.2}{0.000022}$$

$$0 = -400 + \frac{15}{0.0002} d_1 - \frac{20}{0.0002} d_2$$

So

$d_2 = 0.5$

$d_1 = 0.672$

with the state space representation being of the form

$\dot{x} = f(x) + g(x)u + a$

And we have

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R_L}{L} \end{bmatrix} x + \begin{bmatrix} 0 & \frac{i_L}{C} \\ \frac{V_s}{L} & -\frac{v_C}{L} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} -\frac{I_{load}}{C} \\ 0 \end{bmatrix}$$

Which is

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{0.2}{0.0002} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{x_2}{0.000022} \\ \frac{15}{0.0002} & -\frac{x_1}{0.0002} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -\frac{0.2}{0.000022} \\ 0 \end{bmatrix}$$

And

$$x_{eq} = \begin{bmatrix} 20 \\ 0.4 \end{bmatrix} \quad u_{eq} = \begin{bmatrix} 0.672 \\ 0.5 \end{bmatrix} \quad g(x) = \begin{bmatrix} 0 & \frac{x_2}{0.000022} \\ \frac{15}{0.0002} & -\frac{x_1}{0.0002} \end{bmatrix}$$

$$\dot{x}_1 = f_1(x,u) = \frac{x_2}{0.000022} u_2 - \frac{0.2}{0.000022}$$

$$\dot{x}_2 = f_2(x,u) = -\frac{0.2}{0.0002} x_2 + \frac{15}{0.0002} u_1 - \frac{x_1}{0.0002} u_2$$

Besides this

$\dot{z} = Az + Bv \qquad w = Cz$

So

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{u_2}{0.000022} \\ -\frac{u_2}{0.0002} & -1000 \end{bmatrix} = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix}$$

$$B=\begin{bmatrix}\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2}\\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2}\end{bmatrix}=\begin{bmatrix}0 & \frac{0.4}{0.000022}\\ \frac{15}{0.0002} & -\frac{20}{0.0002}\end{bmatrix}=\begin{bmatrix}0 & 18181.82\\ 75000 & -100000\end{bmatrix}$$

$C=[1,0]$

$D=0$

Thus

$$\dot{z}=\begin{bmatrix}0 & 22727.27\\ -2500 & -1000\end{bmatrix}z+\begin{bmatrix}0 & 18181.82\\ 75000 & -100000\end{bmatrix}v$$

$$w=[1,0]z$$

2.

Here is the code:

nonlinear_vs_linearized.m | nonlinear_example.m | +

```
r = 1; % initializationsr
L = 1;
perturbation=1*rand(1,2);
X = [20 0.4]+perturbation;
U = [0 0];
Z = [];
Time = [0];
u=[0.672;0.5];
tinterval=5;

i=1:length(u)
    x0 = X(end,:);
   [t,x]=ode45(@(t,x) nonlinear_example(t,x,u(i)),[0 0.01],x0);
    X=[X;x];
    Time=[Time;t];

Y=X(:,1);

%linearized - system
A=[0 22727.27;-2500 -1000];
B=[0 18181.82;75000 -100000];
C=[1 0];
D=[0];

sys=ss(A,B,C,D)
```

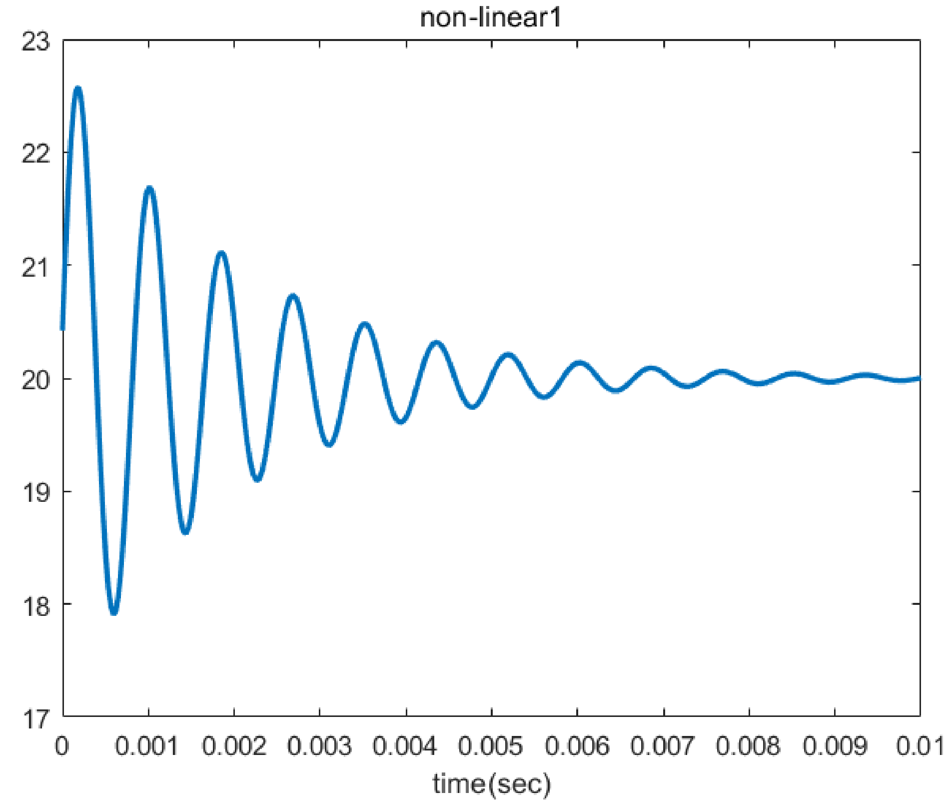
nonlinear_vs_linearized.m | nonlinear_example.m

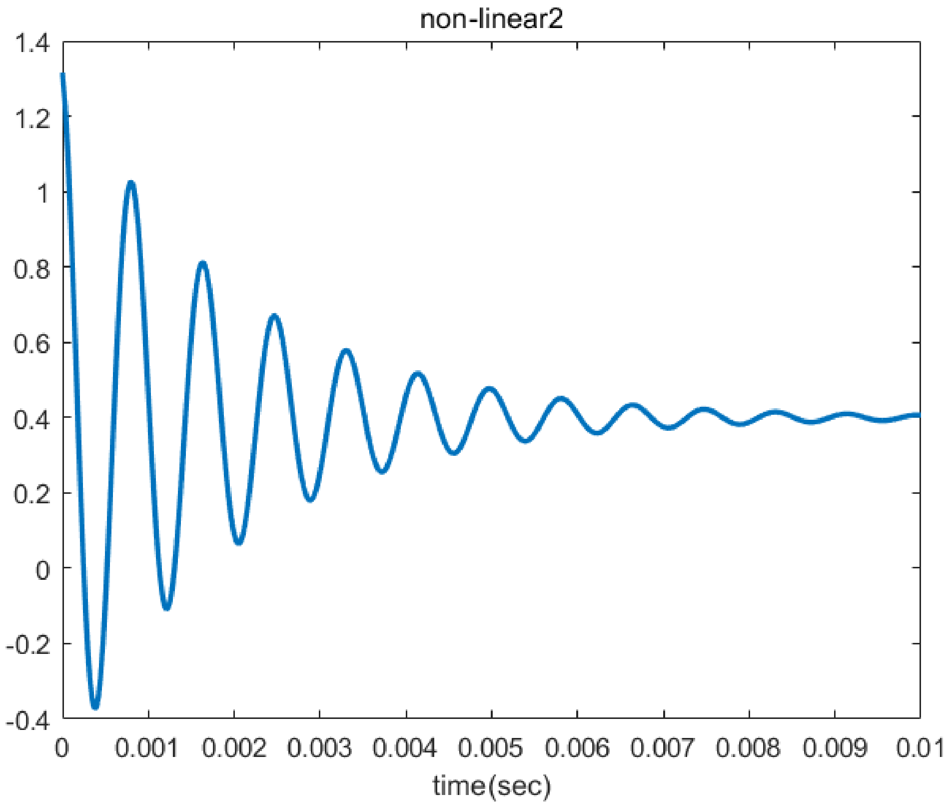
```
t = 0:0.00001:0.01;
perturbation=1*rand(1,2);
X0 = [0 0]+perturbation;
U = zeros(length(t),2);
[ylin,tlin,xlin]=lsim(sys,U,t,X0');

y_eq=20
 ylin=ylin+y_eq

figure(1)
plot(Time,X(:,1),'LineWidth',2)
xlabel('time(sec)');
title('non-linear1')
figure(2)
plot(Time,X(:,2),'LineWidth',2)
xlabel('time(sec)');
title('non-linear2')

 figure(3)
plot(tlin,xlin(:,1)+1,'LineWidth',2)
title('linear system')

figure(4)
plot(tlin,xlin(:,2),'LineWidth',2)
title('linear system')
```

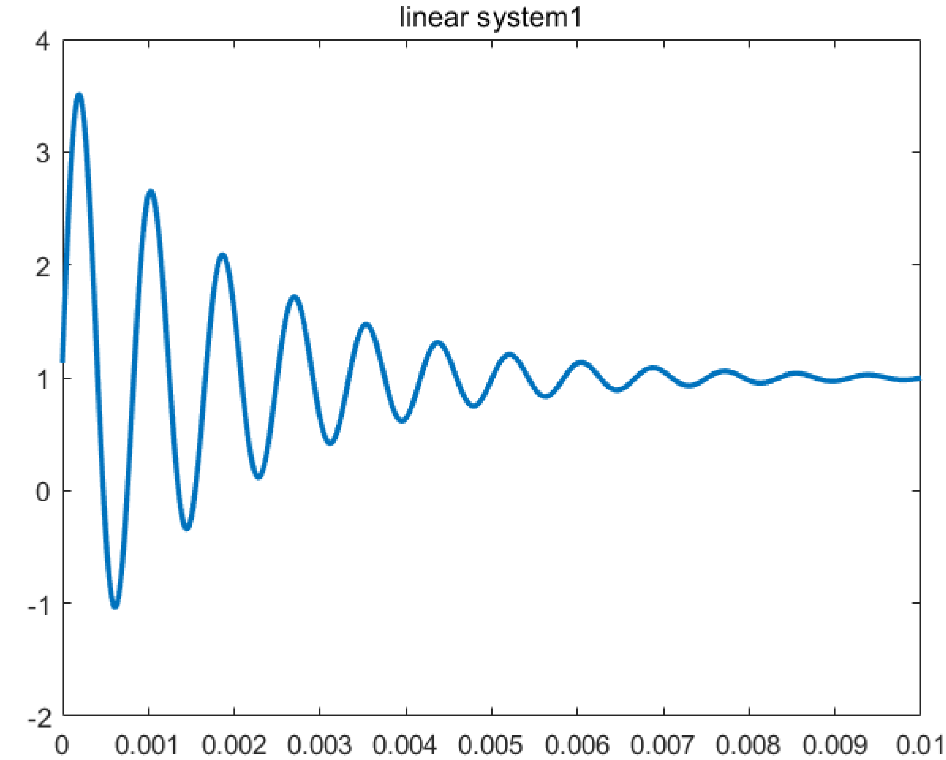
nonlinear_vs_linearized.m | nonlinear_example.m

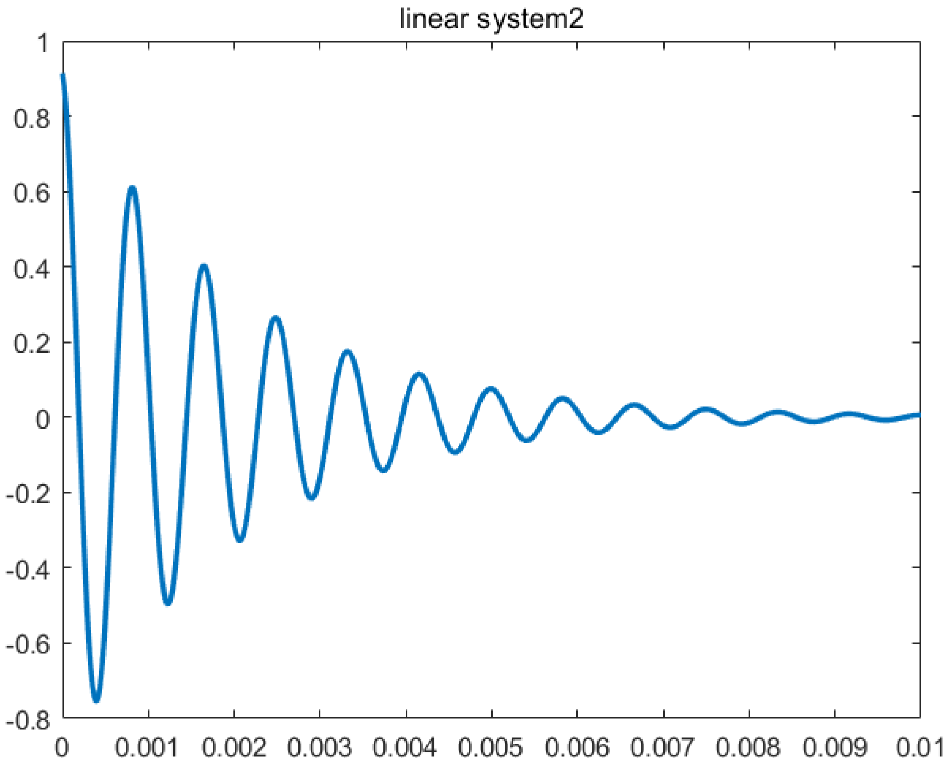
```
function dxdt = nonlinear_example(~,x,u)

dxdt=zeros(2,1);
dxdt(1) = 45454.5455* (u(2)*x(2)-0.2);
dxdt(2) = 5000*(15*u(1)-0.2*x(2)-x(1)*u(2));

end
```

And the figures

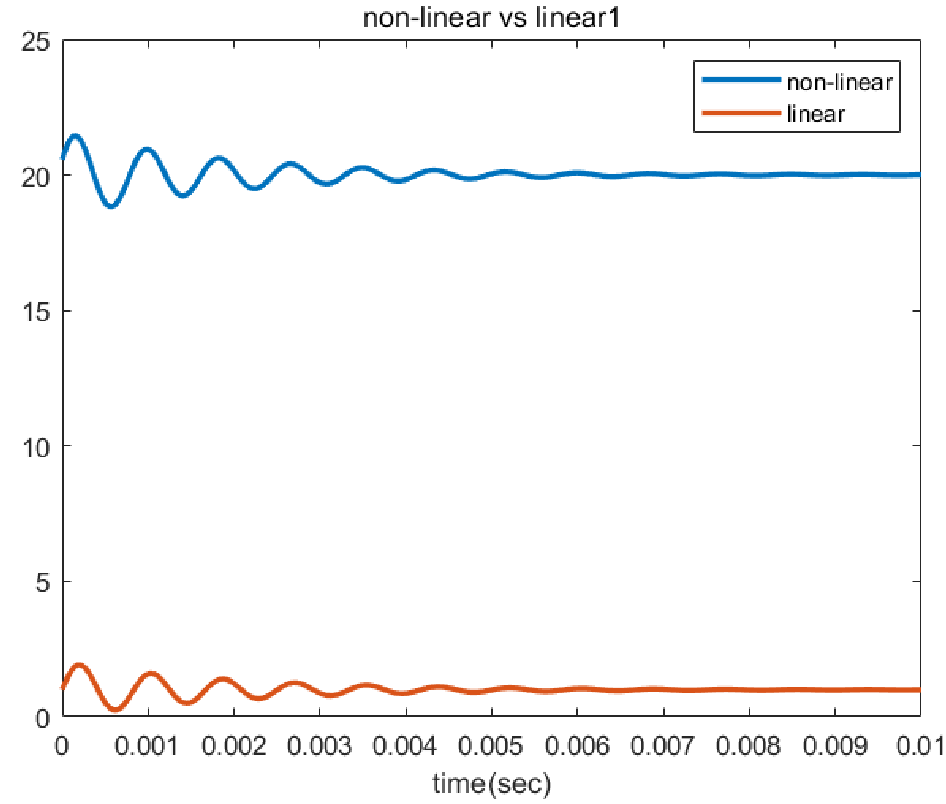

non-linear1
23
22
21
20
19
18
17
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
time(sec)

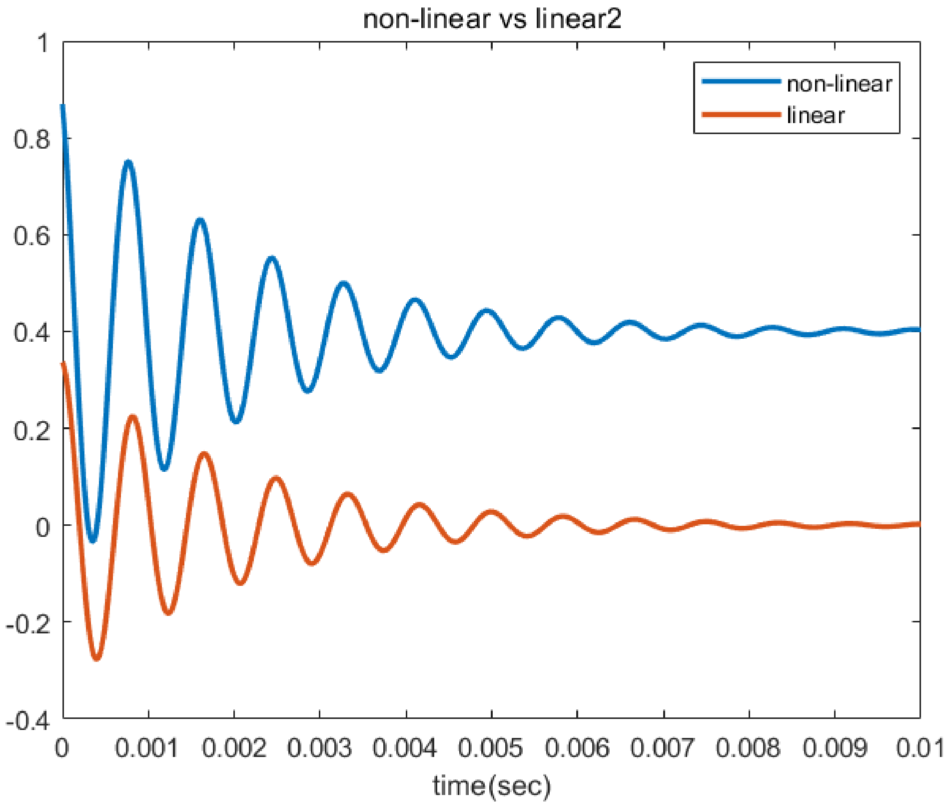

non-linear2
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
time(sec)





We can put them together





3.

We have
$x(t+1) = A_d x(t) + B_d u(t)$

$y(t) = C_d x(t) + D_d u(t)$

Where

$$A_d = e^{AT} = e^{\begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} 0.00001} = e^{\begin{bmatrix} 0 & 0.2272727 \\ -0.025 & -0.01 \end{bmatrix}} = \begin{bmatrix} 0.9972 & 0.2259 \\ -0.0249 & 0.9872 \end{bmatrix}$$

$$B_d = \int_0^T e^{A(T-\tau)} Bu(\tau) d\tau = \int_0^T e^{A(T-\tau)} Bu(\tau) d\tau = \int_0^{0.00001} e^{\begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix}(0.00001-\tau)} \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix} u(\tau) d$$

$$= \begin{bmatrix} 0.0849 & 0.0684 \\ 0.7456 & -0.9963 \end{bmatrix}$$

$C_d = [1,0]$

$D_d = 0$

Thus

$$x(t+1) = \begin{bmatrix} 0.9972 & 0.2259 \\ -0.0249 & 0.9872 \end{bmatrix} x(t) + \begin{bmatrix} 0.0849 & 0.0684 \\ 0.7456 & -0.9963 \end{bmatrix} u(t)$$

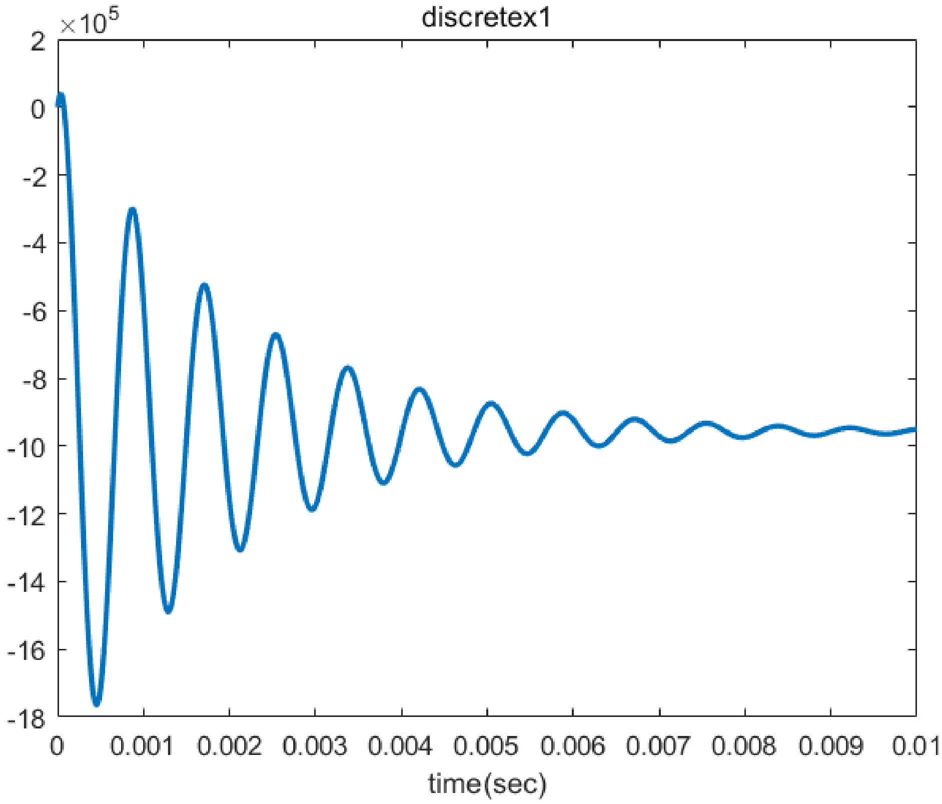
$y(t) = [1,0]x(t)$

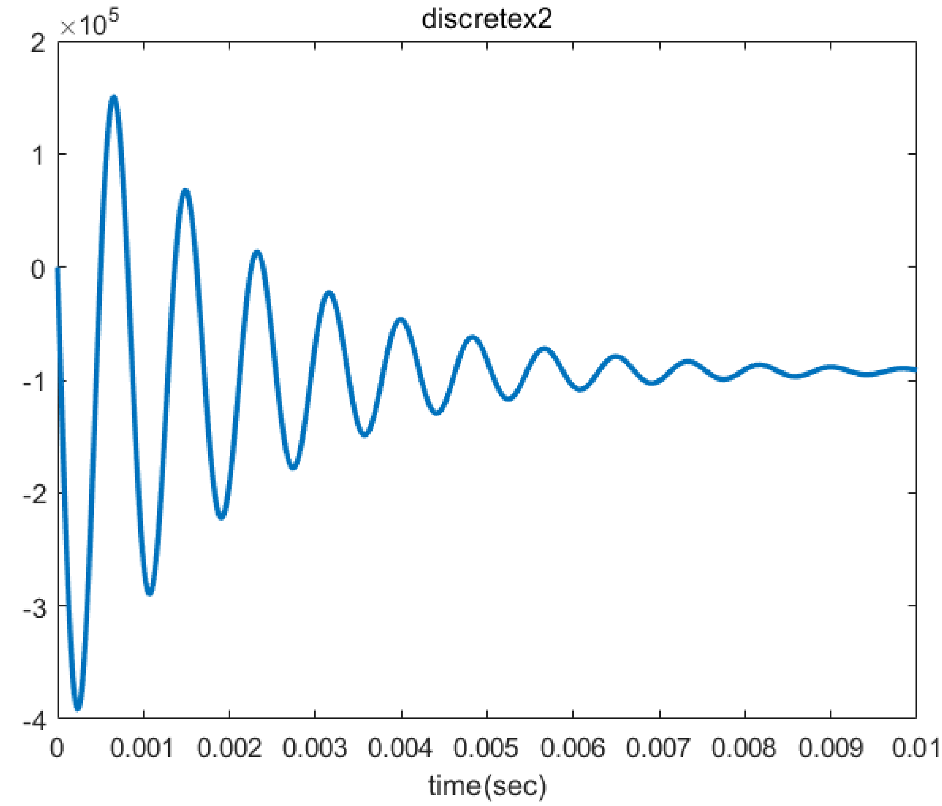
And the code

Eulerforward.m | ELE8066Q3_Discrete.m | nc

```
ts = 0.00001;
Ad = [0.9972 0.2259;-0.0249 0.9872]
Bd = [0 18181.82;75000 -100000];
Cd = [1 0];
Dd = 0;
type = 'zoh';
t = 0:0.00001:0.01;
u = ones(length(t),2);
x0 = [0 0];

sys = ss(Ad,Bd,Cd,Dd,ts);
[y_d,t_d,x_d] = lsim(sys,u,t,x0,type)

figure(1)
plot(t_d,x_d(:,1),'LineWidth',2)
xlabel('time(sec)');
title('discrete_x1')
figure(2)
plot(t_d,x_d(:,2),'LineWidth',2)
xlabel('time(sec)');
title('discrete_x2')
```

The zoh discretisation scheme





And in the Discretization-forward Euler difference we have

f(x(t),u(t)) = Ax(t)+Bu(t)

x(t+1) = x(t)+Tf(x(t),u(t))

So

x(t+1) = x(t)+T(Ax(t)+Bu(t))

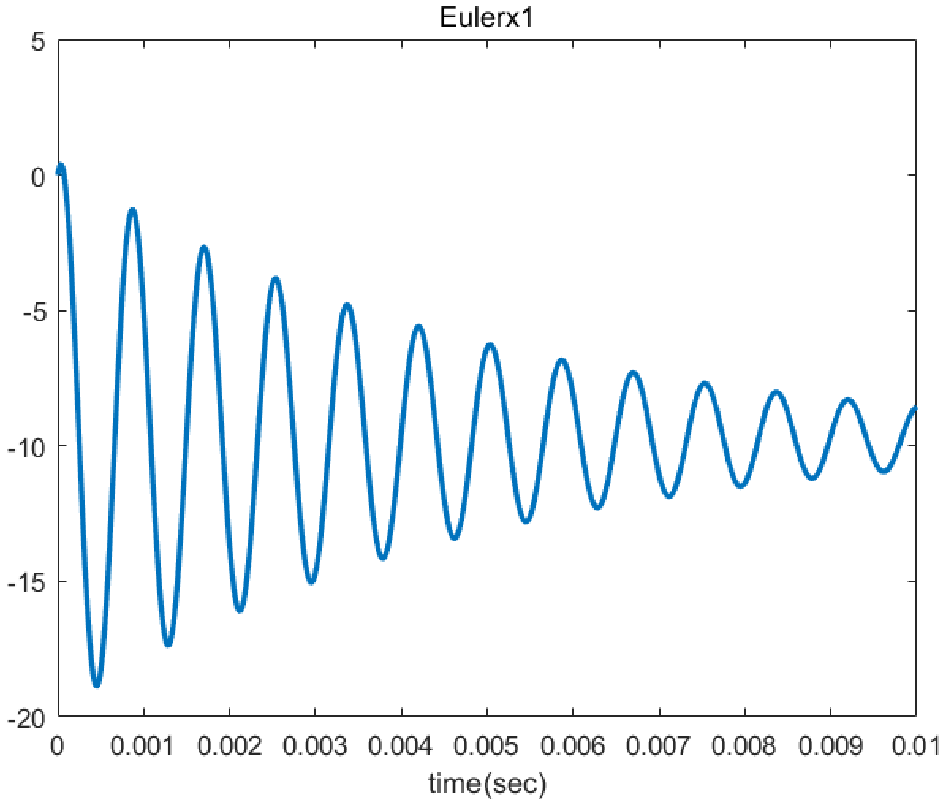
Which is

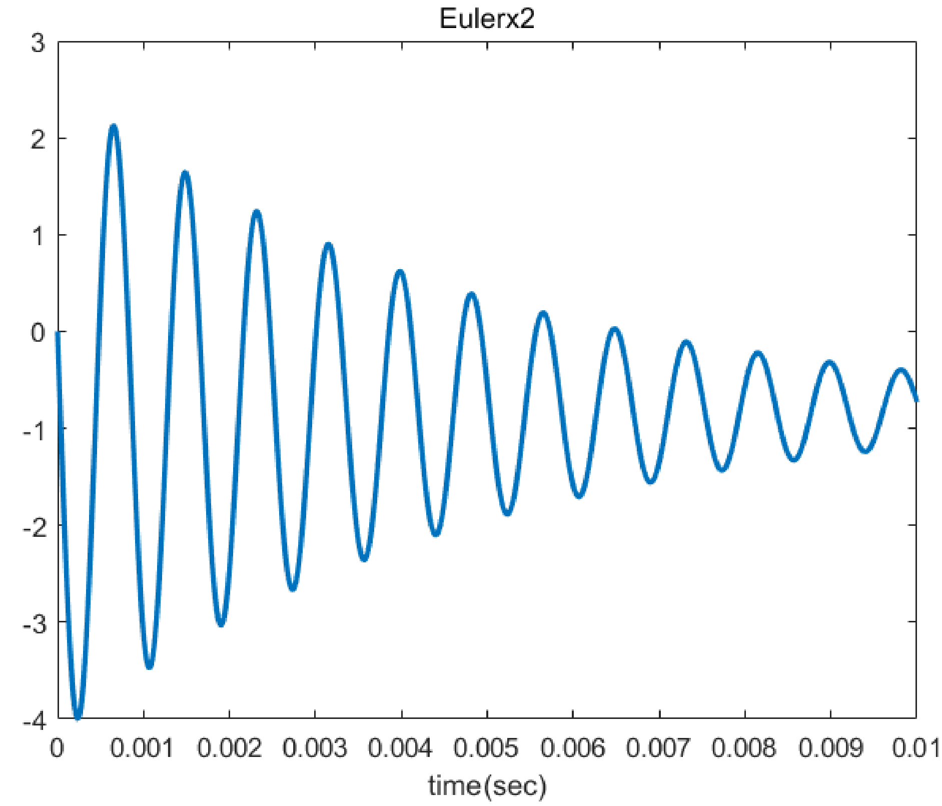
x(t+1) = (1+TA)x(t)+TBu(t)

The code and figures below

Eulerforward.m | ELE8066Q3_Discrete.m | nor

```
ts = 0.00001;
A = [0 22727.27;-2500 -1000]
B = [0 18181.82;75000 -100000];
C = [1 0];
D = 0;
t = 0:0.00001:0.01;
u = ones(length(t),2);
x0 = [0 0];

B = ts * B;
A = eye(2) + ts * A;

sys = ss(A,B,C,D,ts);
[y_e,t_e,x_e] = lsim(sys,u,t,x0,'zoh')

figure(1)
plot(t_e,x_e(:,1),'LineWidth',2)
xlabel('time(sec)');
title('Eulerx1')
figure(2)
plot(t_e,x_e(:,2),'LineWidth',2)
xlabel('time(sec)');
title('Eulerx2')
```





4.

Since we have

$$\dot{z} = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} z + \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix} v$$

And

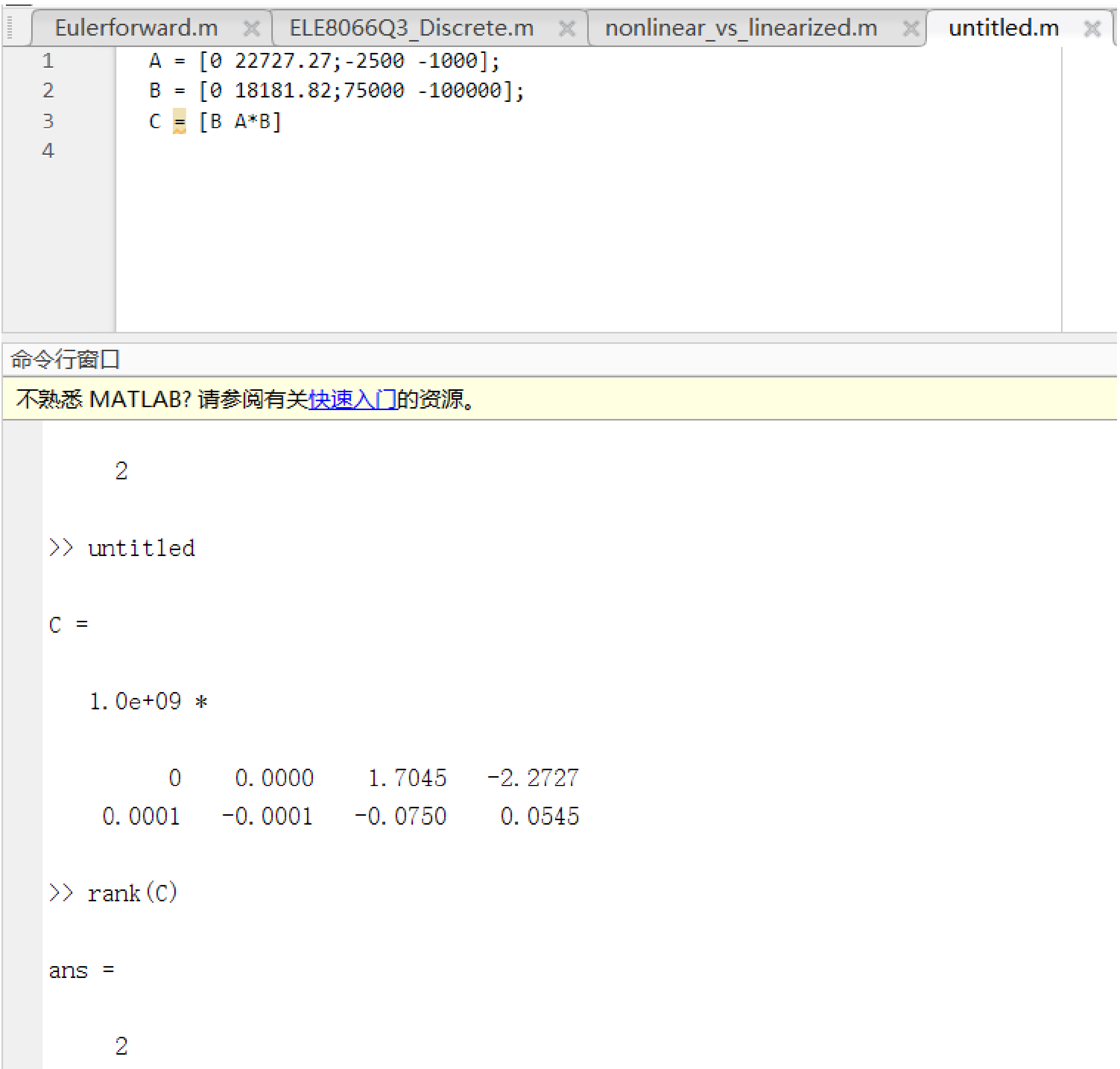
$$A = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix}$$

So

$C = [B \;\; AB]$

In the matlab it is easy for us to know this system is controllable since the rank = 2.



And we have

$$\dot{x} = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} x + \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix} u$$

$y = [1,0]x$

Since $\omega_d = \omega_n\sqrt{1-\zeta^2}$

So

$\omega_n = 2294.16$

And

$s^2 + 4129.488s + 5263170.1 = 0$

Thus

$p = 2064.7 \pm 1000i$

$$K = \begin{bmatrix} 0.1047 & 1.7542 \\ 0.1136 & 1.3050 \end{bmatrix}$$

untitled.m | Eulerforward.m | ELE8066Q

```
syms s
A = [0 22727.27;-2500 -1000];
B = [0 18181.82;75000 -100000];
wd = 1000;
zeta = 0.9;

wn = wd/sqrt(1-zeta*zeta);
s = [1 2*zeta*wn wn*wn];
p = roots(s)

K = place(A,B,p)
[K,prec,message] = place(A,B,p)
```

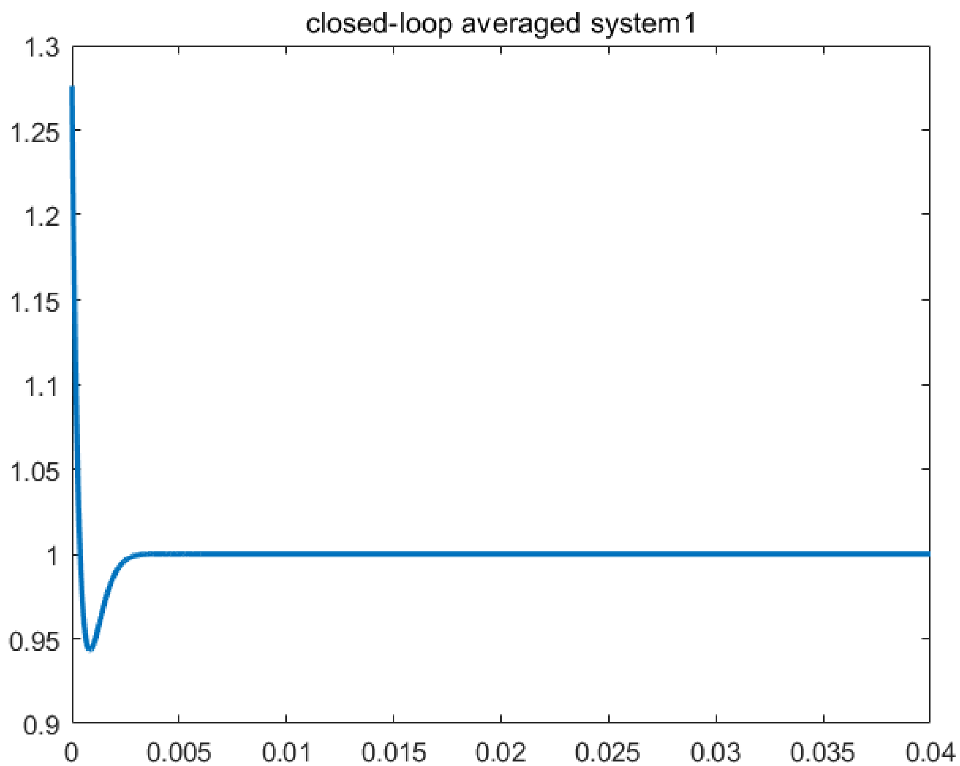
```
p =

   1.0e+03 *

  -2.0647 + 1.0000i
  -2.0647 - 1.0000i
```

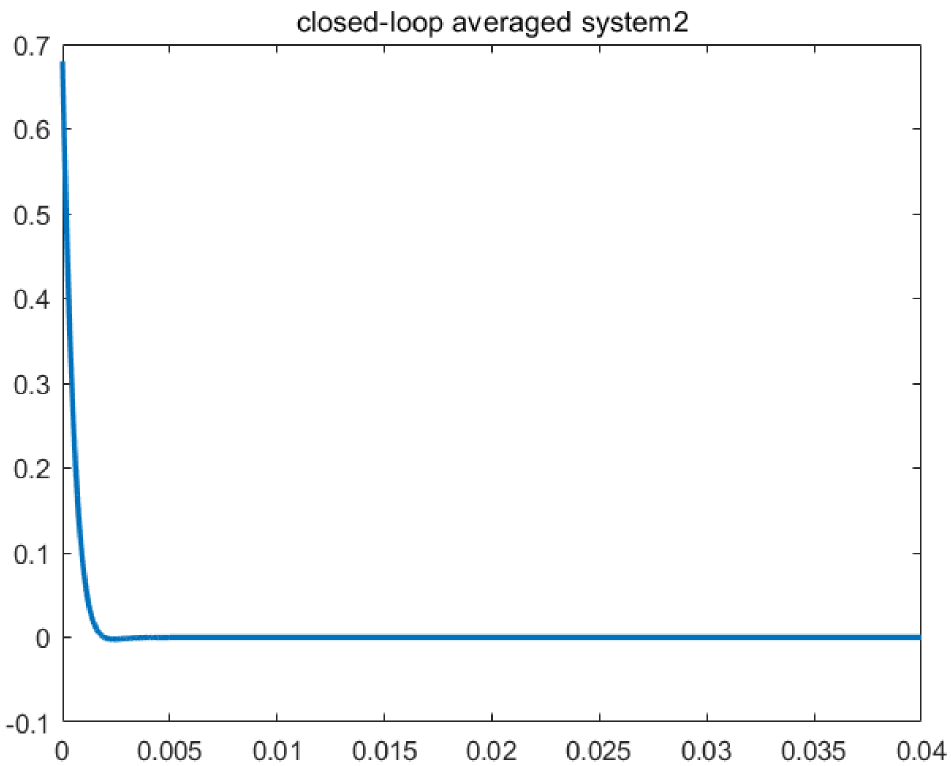
```
K =

    0.1047    1.7542
    0.1136    1.3050
```

Since $u = -\begin{bmatrix} 0.1047 & 1.7542 \\ 0.1136 & 1.3050 \end{bmatrix} x$

In the $\dot{x} = (A - BK)x$,we get the closed-loop averaged system closeloop.m | nonlinear_vs_linearized.m | Eulerfo

```
A=[0 22727.27;-2500 -1000];
B=[0 18181.82;75000 -100000];
C=[1 0];
D=[0];
K = [0.1047 1.7542;0.1136 1.3050];

perturbation=1*rand(1,2);
X = [20 0.4]+perturbation;
U = [0 0];
Z = [];
Time = [0];
tinterval=5;

A = A-B*K

sys=ss(A,B,C,D)

t = 0:0.00001:0.04;
perturbation=1*rand(1,2);
X0 = [0 0]+perturbation;
U = zeros(length(t),2);
[yca,tca,xca]=lsim(sys,U,t,X0');

y_eq=20
 yca=yca+y_eq

 figure(1)
plot(tca,xca(:,1)+1,'LineWidth',2)
title('closed-loop averaged system1')

figure(2)
plot(tca,xca(:,2),'LineWidth',2)
title('closed-loop averaged system2')
```

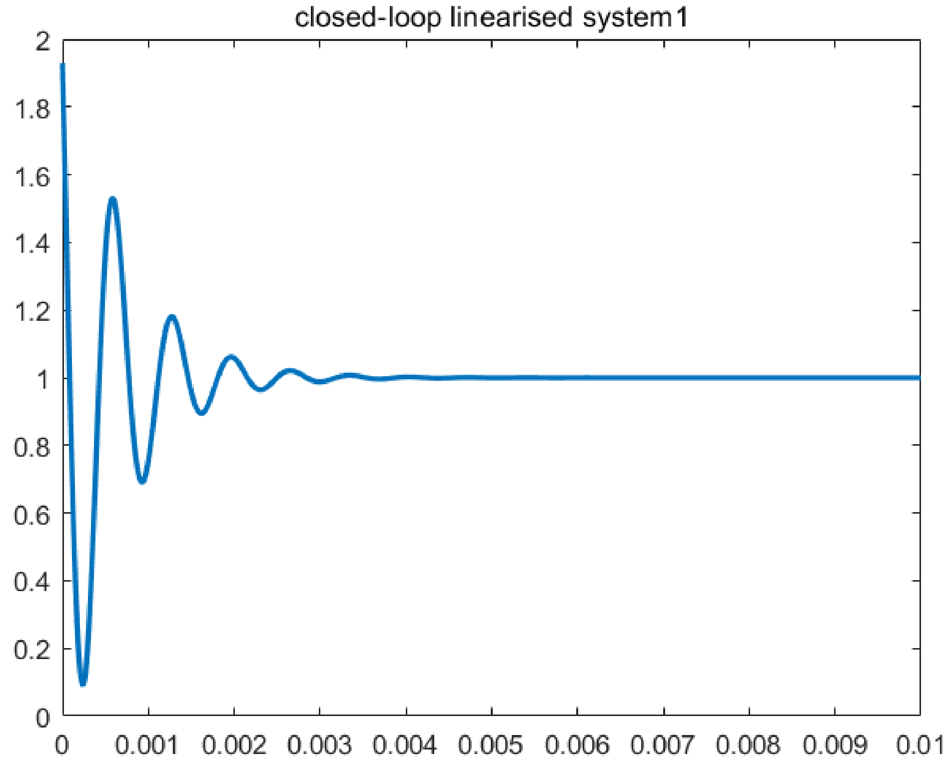


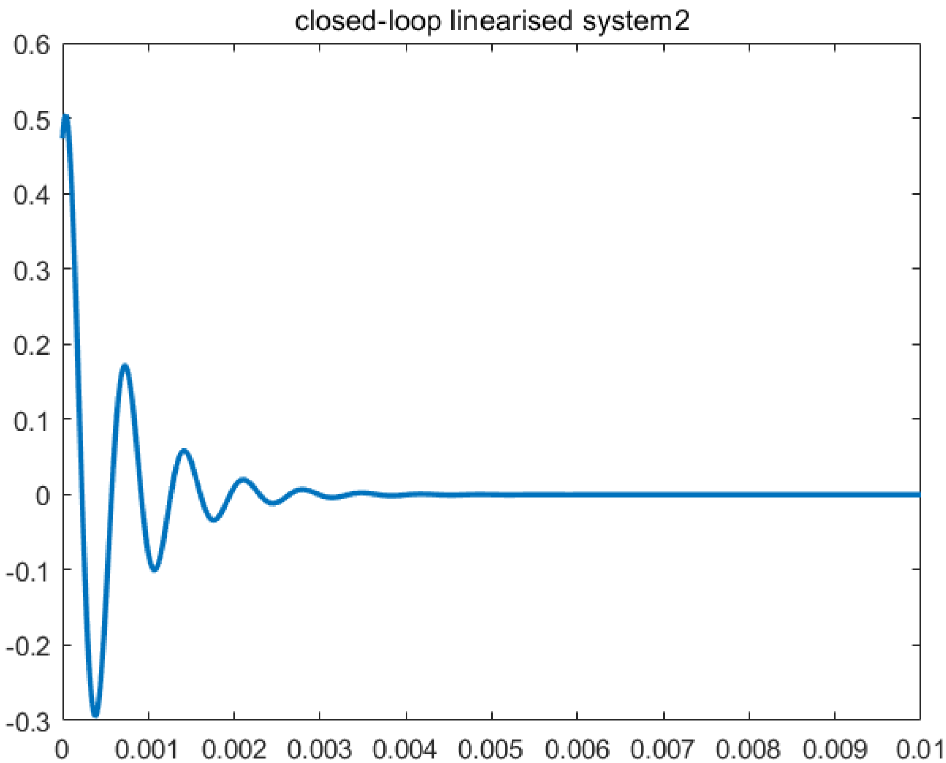


And replace A and B into Ad and Bd respectively.

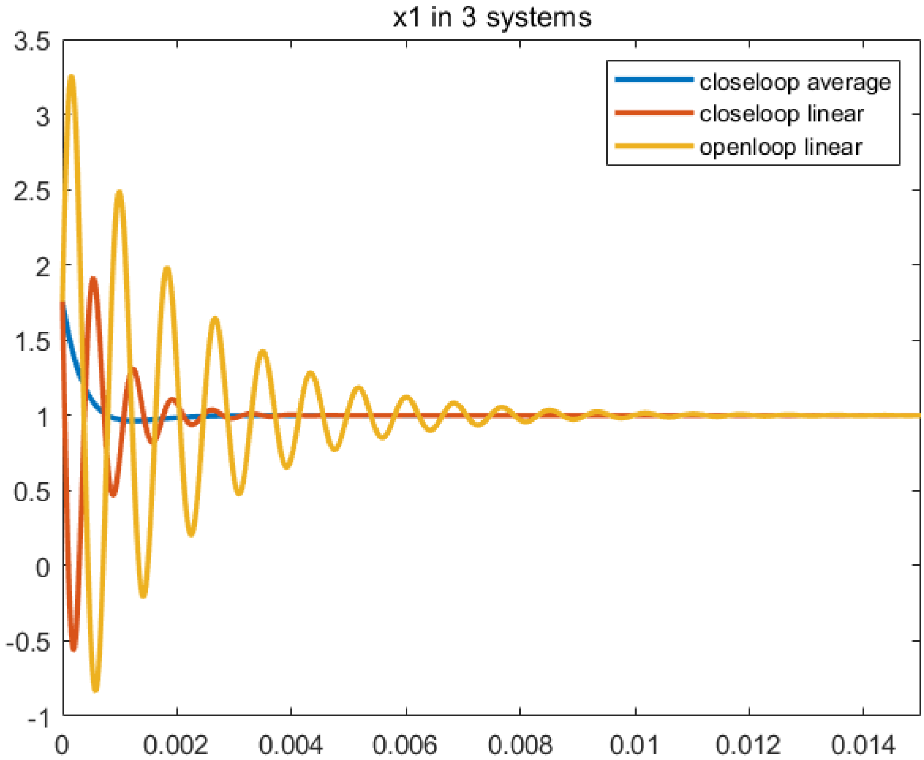
closeloop_average.m | closelooplinear.m

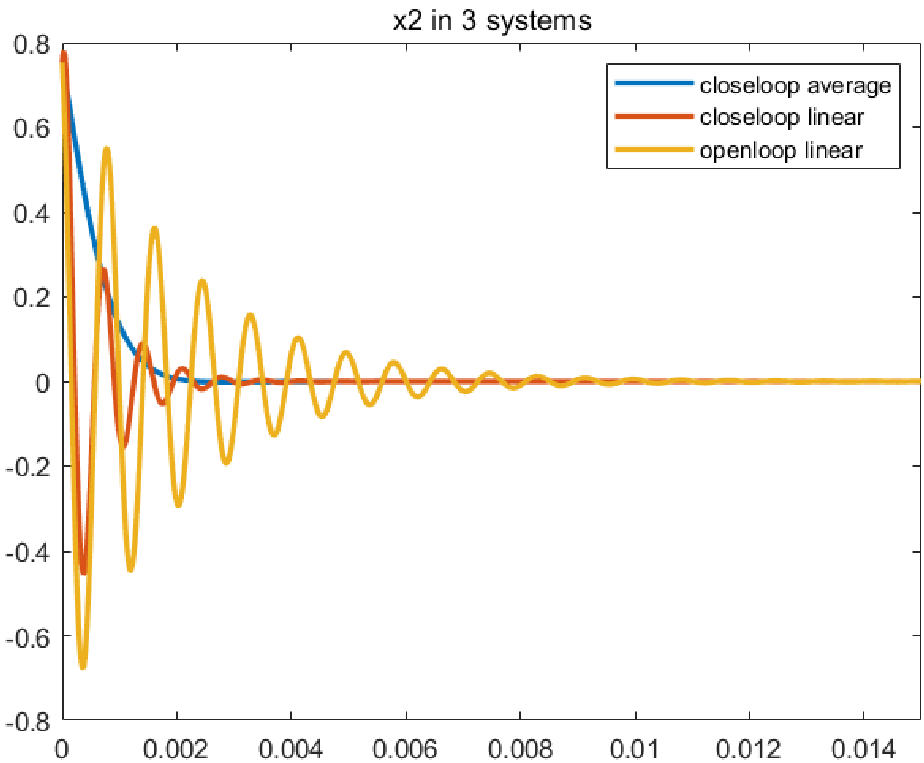
```
Ad=[0.9972 0.2259;-0.0249 0.9872];
Bd=[0 18181.82;75000 -100000];
C=[1 0];
D=[0];
K = [0.1047 1.7542;0.1136 1.3050];

perturbation=1*rand(1,2);
X = [20 0.4]+perturbation;
U = [0 0];
Z = [];
Time = [0];
tinterval=5;

Ad = Ad-Bd*K

sys=ss(Ad,Bd,C,D)

t = 0:0.00001:0.01;
perturbation=1*rand(1,2);
X0 = [0 0]+perturbation;
U = zeros(length(t),2);
[ycl,tcl,xcl]=lsim(sys,U,t,X0');

y_eq=20
 ycl=ycl+y_eq

 figure(1)
plot(tcl,xcl(:,1)+1,'LineWidth',2)
title('closed-loop linearised system1')

figure(2)
plot(tcl,xcl(:,2),'LineWidth',2)
title('closed-loop linearised system2')
```





We can put the three kinds of systems together


x1 in 3 systems
closeloop average
closeloop linear
openloop linear
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014


x2 in 3 systems
closeloop average
closeloop linear
openloop linear
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014

5.

We have

$$\dot{x} = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix} u(t)$$

$$y = [1,0]x$$

So

$$O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 22727.27 \end{bmatrix}$$

Thus we can easily tell that the system is observable

```
>> O

O =

   1.0e+04 *

    0.0001         0
         0    2.2727

>> rank(O)

ans =

     2
```

We would like the close-loop observer to place the eigenvalues of the error system to -50.

$$P_d(\lambda) = (\lambda + 50)(\lambda + 50) = \lambda^2 + 100\lambda + 2500$$

$$\begin{aligned} |\lambda I - (A - LC)| &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [1,0] \\ &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} + \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \lambda + l_1 & -22727.27 \\ 2500 + l_2 & \lambda + 1000 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} det|\lambda I - (A - LC)| &= (\lambda + l_1)(\lambda + 1000) + 22727.27(2500 + l_2) \\ &= \lambda^2 + 1000\lambda + l_1\lambda + 1000l_1 + 56818175 + 22727.27l_2 \\ &= \lambda^2 + (1000 + l_1)\lambda + 1000l_1 + 56818175 + 22727.27l_2 \end{aligned}$$

So

$$\begin{cases} 1000 + l_1 = 100 \\ 56818175 + 22727.27l_2 = 2500 \end{cases} \Rightarrow \begin{cases} l_1 = -900 \\ l_2 = -2499.89 \end{cases}$$

And

$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu$$

So

$$A - LC = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} - \begin{bmatrix} -900 & 0 \\ -2499.89 & 0 \end{bmatrix} = \begin{bmatrix} 900 & 22727.27 \\ -0.11 & -1000 \end{bmatrix}$$

Thus

$$\dot{\hat{x}} = \begin{bmatrix} 900 & 22727.27 \\ -0.11 & -1000 \end{bmatrix}\hat{x} + \begin{bmatrix} -900 \\ -2499.89 \end{bmatrix}y + \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix}u$$

6.

We have

$u = -K\hat{x}$

$\dot{x} = Ax - BK\hat{x}$

Which is

$$u = -\begin{bmatrix} 0.1047 & 1.7542 \\ 0.1136 & 1.3050 \end{bmatrix}\hat{x}$$

Besides this

$e = x - \hat{x} \Rightarrow \hat{x} = x - e$

Thus

$\dot{x} = Ax - BKx + BKe$

$\dot{e} = (A - LC)e$

So

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix}$$

We can get the eigenvalue

$$A - BK = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} - \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix}\begin{bmatrix} 0.1047 & 1.7542 \\ 0.1136 & 1.3050 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix} - \begin{bmatrix} 2065.45 & 23727.27 \\ -105747.5 & -1173435 \end{bmatrix}$$

$$= \begin{bmatrix} -2065.45 & -1000 \\ -103247.5 & 1172435 \end{bmatrix}$$

$$BK = \begin{bmatrix} 2065.45 & 23727.27 \\ -105747.5 & -1173435 \end{bmatrix}$$

$$A - LC = \begin{bmatrix} 900 & 22727.27 \\ -0.11 & -1000 \end{bmatrix}$$

As a result

$$\begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} = \begin{bmatrix} -2065.45 & -1000 & 2065.45 & 23727.27 \\ -103247.5 & 1172435 & -105747.5 & -1173435 \\ 0 & 0 & 900 & 22727.27 \\ 0 & 0 & -0.11 & -1000 \end{bmatrix}$$

The eigenvalue of matrix $\begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}$

eigenvalue.m | closeloop_average.m | closelooplinear.m | nonlinear_vs_linearized.m

```
A = [-2065.45 -1000 2065.45 23727.27;-103247.5 1172435 -105747.5 -1173435;
    0 0 900 22727.27;0 0 -0.11 -1000];
[V,D] = eig(A)
```

命令行窗口

不熟悉 MATLAB? 请参阅有关快速入门的资源。

```
V =

   -0.9962    0.0009    0.5389   -0.0660
   -0.0876   -1.0000    0.1227   -0.0123
         0         0    0.8334   -0.9943
         0         0   -0.0000    0.0831


D =

   1.0e+06 *

   -0.0022         0         0         0
         0    1.1725         0         0
         0         0    0.0009         0
         0         0         0   -0.0010
```

So we get -2200,1172500,900,-1000 as the eigenvalue

When eigenvalue is negative that the system is stable.

And

$$\dot{\hat{x}} = A\hat{x} + Bu + L(Cx - C\hat{x})$$

Which is

$$\dot{\hat{x}} = A\hat{x} + Bu + LCe$$

$$\dot{\hat{x}} = A(x - e) + Bu + LCe$$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 22727.27 \\ -2500 & -1000 \end{bmatrix}(x - e) + \begin{bmatrix} 0 & 18181.82 \\ 75000 & -100000 \end{bmatrix}u + \begin{bmatrix} -900 & 0 \\ -2499.89 & 0 \end{bmatrix}e$$